Mortgage Options: A Primer

October 25, 2004

Fei Zhou
Interest Rates Modeling
212-526-8886
feiz@lehman.com

Prasanth Subramanian
Mortgage Strategies
212-526-8311
psubrama@lehman.com

Mortgage options are options on TBA passthroughs. The primary difference between mortgage options and other bond options is that the underlying asset is short options. The valuation of mortgage options needs to take into account the interplay between these two sources of optionality. Our analysis follows closely that presented in Prendergast [2003].

VALUATION OF MORTGAGE OPTIONS

We establish a valuation framework for pricing mortgage options efficiently. This is achieved by separating out the prepayment option from the mortgage option. We do this by fitting the duration profile of a mortgage to a functional form. Numerical methods can then be used to price the option.

RISK EXPOSURES OF MORTGAGE OPTIONS

Mortgage options have unique risk exposures. Given that mortgage calls are long options on an underlying that is short options, the risk characteristics of mortgage calls are far richer than mortgage puts. Specifically, it is rather interesting to see that the duration of a call extends initially as the call steps into the money and then shortens into a further rally. Mortgage calls can therefore be negatively convex deep in the money. They are also usually short Vega for similar reasons.

APPLICATIONS OF MORTGAGE OPTIONS

Mortgage options have multiple applications in the context of mortgage portfolios and as hedging instruments. Mortgage portfolios can use these options in familiar covered calls or asset + put combinations to enhance income, or to preserve the value of the asset respectively. These options are also ideal hedges for the convexity risks arising from MSR portfolios and origination pipelines. The last application of mortgage options is in conditional mortgage trades, which can be structured similar to conditional spread and curve trades in the rates market. Before considering a conditional trade, it is always essential to figure out what is priced into the market currently. Our valuation model helps estimate the spread directionality in mortgages that is priced in, and serves as a guide towards analyzing conditional trades.

CONCLUSIONS

Mortgage options are unique instruments given that they are written on negatively convex securities. The combined effect of negative convexity of the underlying and the positive convexity of the option leads to fairly interesting risk characteristics of mortgage options. Mortgage options are useful both for mortgage portfolios and as hedging instruments. They contain rich information on mortgage spread directionality priced in the market, and provide guidance for relative value trades.

TABLE OF CONTENTS

INTRODUCTION

As the name suggests, mortgage options are options on TBA passthroughs. A mortgage call gives the holder the right to buy a passthrough at an agreed upon price on the option expiry date. Similarly, a mortgage put gives the holder the right to sell the passthrough at a pre-specified price on the expiry date. The exercise and quotation conventions of mortgage options are similar to other bond options. The primary difference, however, is the complication in the valuation of mortgage options, because the underlying mortgage is in itself short options. The interplay between the prepayment option and the overlaid vanilla option brings with it all the pricing complexities and unique characteristics of mortgage options. Our analysis follows closely that presented in Prendergast [2003].

1 Trading Conventions

Mortgage option expiry is closely linked to TBA settlement cycles; with the ATM strike the forward TBA price today.

In Figure 1, we show a typical pricing grid of mortgage options. The first option is a call on 30yr FN 5% TBAs. A mortgage option may be written on any coupon in the 30yr or 15yr stack, but liquidity is usually limited to the closest to par value coupons. The FN 5 call is priced for expiry in December. Mortgage options expire 7 days prior to the PSA settlement day of the TBA. In this case, the call would expire on the 6th of December (PSA settlement for Dec 2004, is the 13th). Liquidity is usually limited to the following three TBA settlement cycles, in line with the liquidity of the dollar roll market. The ATM strike on mortgage options is the quoted TBA price for the settlement month. Strikes on mortgage options are quoted in increments of half a point away from this ATM price. Assuming the quoted December TBA price on FN 5s was 99-02, a call ½ a point out would be struck at 99-18. The price of mortgage options is typically quoted in 32nds (It may also be quoted as an implied price volatility). The other options in the grid help illustrate differences in maturity, type, and underlying coupon. We discuss the pricing across coupons in a later section but point out here that the prices of ATM options on premium mortgages are lower than those on discounts. For instance, ATM options on FN 6s for October are only 7/32nds compared to 14/32nds on FN 5s of October.

2 Who uses Mortgage Options?

Mortgage originators, banks supplementing income and levered accounts structuring conditional trades are typical users of mortgage options.

The biggest demand for mortgage options is from originators who use these instruments to hedge their pipeline pull through rates (loan conversion rates). Also, mortgage portfolios use mortgage puts as protection from a downtrade. The biggest source of supply of mortgage options is from bank portfolios selling covered calls for yield enhancement. Levered accounts can use these options to structure conditional trades on mortgages. We will discuss the applications of these options in length in a later section.

Figure 1. **Trading Conventions of Mortgage Options**

Option Type	Underlying TBA	Option Expiration Month	TBA Settlement Date	Option Expiry Date	ATM Strike	Option Prc	ATM + 0.5	Option Prc
Call	FN 5s	Dec-04	12/13/2004	12/06/2004	99-02	0-14	99-18	0-05
Put	FN 5s	Dec-04	12/13/2004	12/06/2004	99-02	0-14	99-18	0-21
Call	FN 5s	Jan-05	01/13/2005	01/06/2005	98-24+	0-26	99-08+	0-17
Put	FN 5s	Jan-05	01/13/2005	01/06/2005	98-24+	0-26	99-08+	1-03
Call	FN 6s	Dec-04	12/13/2004	12/06/2004	103-12	0-07	103-28	0-05
Put	FN 6s	Dec-04	12/13/2004	12/06/2004	103-12	0-07	103-28	0-21

Prices shown are hypothetical for the purpose of illustration only and are not indicative levels of any security

VALUATION OF MORTGAGE OPTIONS

Despite the growing popularity of mortgage options, their complexity is sometimes underappreciated. Unlike an option on a plain bond, an option on a mortgage is also one on the prepayment option. In pricing these securities one needs to explicitly account for the negative convexity of the underlying.

1 Price Volatility: The Missing Input

The price volatility of mortgages is not easy to estimate because the durations of the underlying change with rates.

For interest bearing bonds, one can price a call or put on the bond knowing the volatility in forward prices, the forward price, the strike price and the time to maturity. These are inputs into the industry standard Black option pricing formula. The only input that is unknown is the volatility of forward prices. In the case of bond options, it is straightforward to translate this yield volatility into price volatility. The price volatility in this case is a simple function of the duration of the bond. For a mortgage options, the conversion from its mortgage yield volatility to price volatility is not as straightforward because the underlying duration varies considerably with yields.

2 Understanding the Prepayment Option

Prepayments follow a skewed "S" profile which matches the duration profile of mortgages.

The prepayment option that the mortgage is short causes durations to shorten in rallies, and extend in sell offs. Such a variation is closely linked to the observed prepayment behavior of homeowners. Prepayments gradually increase as rates drop, until the rate incentive crosses the elbow of the prepayment function, at which point the fixed costs of refinancing are overcome. At the elbow, prepayments start increasing at an increasing pace. As mortgages get deep in the money, the pace of prepayments slows down, and becomes a decreasing function of rates, till prepayments level off. The duration of a mortgage mirrors this profile. As rates rally, the duration shortens at an increasing pace due to accelerated prepayments. Duration tapers off at low levels of rate (Figure 2). This duration profile leads to the negative convexity in mortgage prices (Figure 3).

3 Valutaion Framework

Mortgage options can be valued similar to Mortgages. In our model we decouple the prepayment and the mortgage option.

We could use the same approach to value mortgage options as is used for mortgages. Similar to the Monte-Carlo simulation procedures used for mortgages, the cash flows of mortgage options can be simulated under multiple rate scenarios to price the option. However, given that the mortgage option is almost always short dated and the underlying
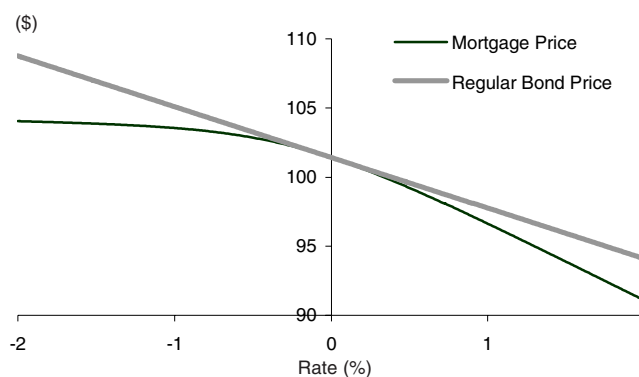
Figure 2.   **Duration Profile of a Mortgage**
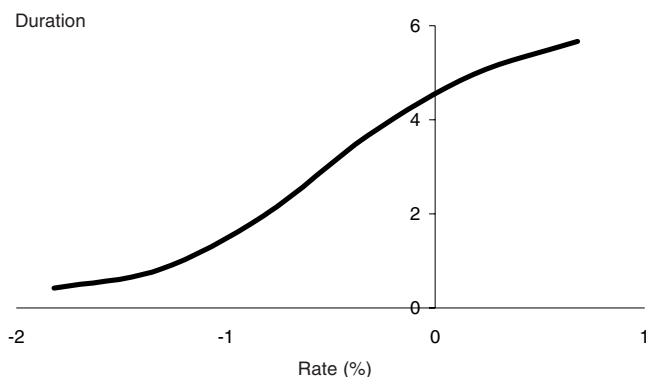
Shows the change in duration of a mortgage with a base case duration of 4yrs based on the LB Prepayment Model

Figure 3.   **Price Profile of a Mortgage**

Price profile shown at constant OAS based on LB Prepayment model

prepayment option is long dated, we split up the prepayment option from the mortgage option in our model. A more detailed description of the methodology can be found in Appendix A. In Figure 4 we show a flowchart outlining the modelling procedure, proposed recently by Prendergast [2003]. We start off with the duration profile of the underlying, and finish at the option price in a process we describe next.

*Fitting the duration profile of a mortgage to a functional form achieves this decoupling.*

a. **Input, Duration Profile:** The first input to our valuation model is the duration profile of the underlying mortgage with rates. We use the Lehman Brothers mortgage prepayment model to obtain the duration profile. This duration profile entirely captures the valuation of the prepayment option.

b. **Fitting Durations to a Function:** We fit this duration profile to an analytical function linking rates to durations[1]. Once durations are expressed in the form of a function, the prepayment option is captured.

The duration function leads to a function for prices with rates…

c. **Obtaining the Price Function:** Knowing the duration of an asset across rates, we can derive the function of its price across rates[2]. For instance, if durations were constant across rates, prices would move linearly with rates. Given the price function of a mortgage across rates, the valuation of the options on the mortgage is similar to the valuation of the options on other rate sensitive assets.

d. **Input, the Mortgage Rate Process:** We have established a way to convert changes in rates into changes in prices. The next input needed is a process for mortgage rates. For instance, a simple rate process could be a normal process, with a bp volatility. The mortgage rate volatility can be estimated from the implied volatilities of swaptions. For example, changes in current coupon rates today have a 30% sensitivity to the 2yr point of the curve, and 70% to the 10yr point. Knowing the

---

[1] Please see the appendix or Prendergast [2003] for a more detailed explanation.
[2] Since duration is the rate of change of price with rates, integration of the duration with respect to rates gives us an expression for prices as a function of rates.

Figure 4. **Steps in the Valuation of Mortgage Options**
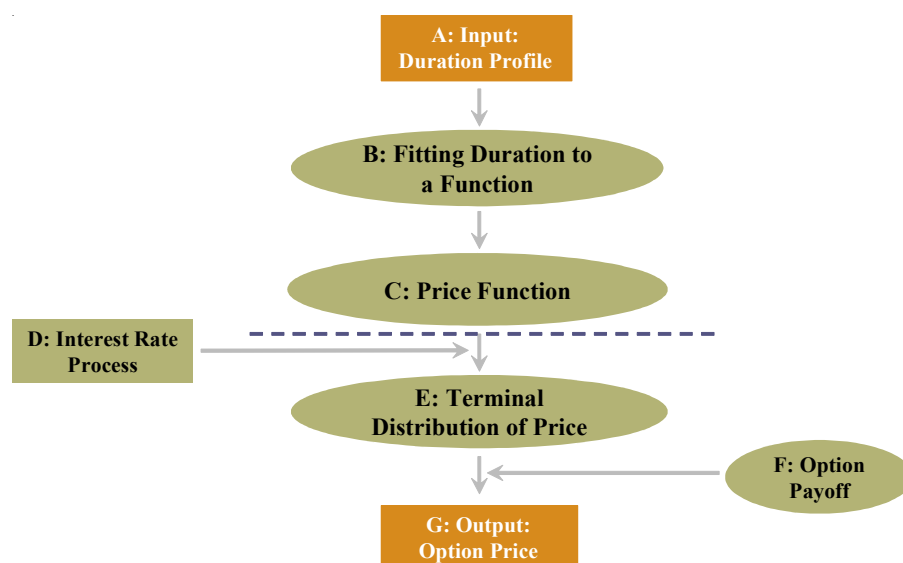
…the price function combined with a rate process gives the terminal distribution of prices.

… The distribution of prices weighted by the option payoff gives the option price!

implied rate volatility of 1mx2yr and 1mx10yr swaptions, we can estimate the volatility of mortgage rates.

e. **Terminal Distribution of Prices:** Knowing prices as a function of rates, along with the terminal distribution of yields, we can obtain the terminal distribution of prices.

f. **Option Payoff:** We also need the option payoff rule for the option to be valued. For instance the payoff rule for a European call option is Max(0,S-K), where S is the price at expiry and K is the strike of the option.

g. **Distribution Weighted Payoff** = **Option Price:** The weighted option payoff gives the fair option price. For example, for a lognormal process, this weighted average over payoff can be carried out analytically, leading to the popular Black formula. For an underlying with a complex process, such as that of a mortgage, numerical methods can be used to arrive at an option price.

4 Back to Price Volatility

Across strikes, price volatility is negatively skewed; lower strikes have a higher volatility than higher strikes...

Our valuation model for mortgage options provides us with the fair price for a mortgage option. The corresponding price volatility[3] can then be implied directly from the option price. For a bond whose durations do not change much with rates, such as a Treasury note, price volatilities are fairly constant across strikes. Since mortgage durations vary considerably across rates, the price volatility of a mortgage option varies noticeably across strikes. In fact, there is a significant negative skew, i.e. higher volatility for low strikes and low volatility for high strikes. In Figure 5, we compare the price volatility skew of a mortgage option with that of a Treasury note option.

… ATM options on lower coupon mortgages are worth more than higher coupons

The price volatility also varies widely with coupons. As shown in Figure 6, the ATM price volatility of options on a 4.5% coupon is 7.7% while that on a 7.5% coupon is 2.0%. The reason for this difference is that a lower coupon mortgage has a higher duration and a more dispersed distribution of prices than a mortgage with a lower duration. ATM options on higher coupon mortgages will therefore be worth less than those on lower coupon mortgages. For a treasury option, the coupon on the underlying has moderate impact on the pricing volatility of an ATM option (Figure 6).

---

[3] With the Black formula, the option price can be translated into a price volatility.

Figure 5. **Pricing Volatility across Strikes**

Black Volatility (%)
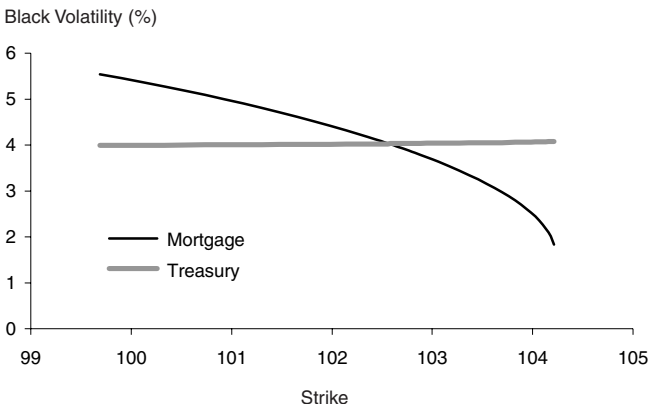
Shows the one year price volatility of a mortgage and a Treasury note (that has the same duration at a price of 101) across strikes. For instance the mortgage option at a strike of 100 will be priced closed to 6% price volatility while at a strike of 104 it will be priced closed to 2%

Figure 6. **Pricing Volatilities across Coupons**

Coupon	ATM Mortgage Vol	ATM Treasury Vol
4.5	7.69	5.93
5.5	5.74	5.74
6.5	3.25	5.58
7.5	2.03	5.44

Shows the price volatility of ATM options on various mortgage coupons. The ATM pricing vols display a distinctive skew across coupons.

RISK EXPOSURES OF MORTGAGE OPTIONS

Understanding the risk exposures of mortgage options helps to better use these options in mortgage portfolios, and hedging instruments. The reason the risks on mortgage options are different from plain vanilla options, is the negative convexity of the underlying. In this section, we compare the risk exposures of mortgage options, to options on a constant duration bond[4] (CDB). We are interested in understanding the risk exposures of mortgage options[5]. We start off looking at their price profiles.

1 Price Profiles of Mortgage Options

1.1 Mortgage Calls are worth less in the money than CDB Calls

Mortgage calls hit the price compression of the underlying and are worth less than plain vanilla calls in the money.

In Figure 7, we show the price profile of a mortgage call option struck ATM on FN 5.5s, and another call struck ATM on a constant duration bond. The price profile is shown for an instantaneous change in forward rates. At the money, the mortgage and CDB call will be priced close to each other (around 20/32nds in this case). As rates rally, both the mortgage call and the CDB call get deeper in the money, and are worth more than in the base case. In the case of the CDB, the duration of the underlying does not change with rates, while the mortgage shortens in a rally. Due to this shortening and corresponding price compression, a mortgage does not increase as much in price as a CDB for the same change in rates. The prices of mortgages calls therefore appreciate slower than CDB calls for the same rate move.

1.2 Mortgage Puts are worth more in the money than CDB Puts

The opposite holds true for mortgage puts which are always worth more than plain vanilla puts in the money.

The exact opposite holds true for mortgage puts, which are worth more than CDB puts when in the money (Figure 8). All the reasons why mortgage calls are worth less than CDB calls as rates rally, work in the opposite direction for puts. As rates sell off, durations extend on the underlying mortgage. The price depreciation on a mortgage is greater than that on a CDB because of this extension, and this works in favour of the put.

_____

[4] In all the discussions that follow we assume a normal distribution in mortgage rates with a volatility of 110bp/yr. Unless otherwise specified, we compare one month ATM calls and puts on FN 5.5s with corresponding CDB options; by corresponding we mean that the CDB has the same duration as FN 5.5s in the base case. The constant duration bond (CDB) concept is similiar to the zero convexity bond (ZCB) in Prendergast [2003].
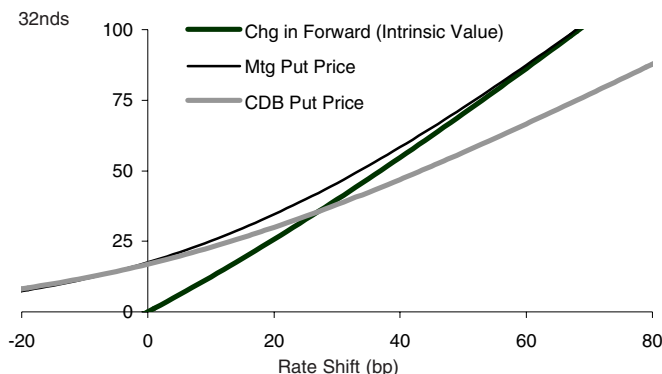[5] Some of the duration and convexity profiles shown here have been analyzed in Prendergast [2003]

Figure 7.   **Price Profiles of Calls**
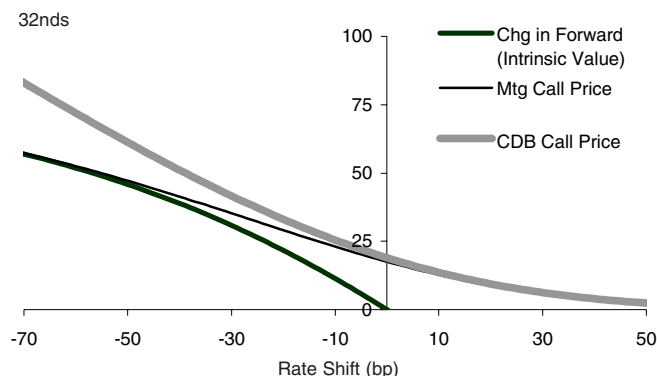
Shows the price of ATM calls on FN 5.5s and equivalent CDB calls as rates shift around based on LB mortgage options valuation model

Figure 8.   **Price Profiles of Puts**

1.3   Coupon Effects

The differences, in the money between mortgage & plain vanilla calls, are a function of the call risk in the mortgage.

ATM the price of mortgage calls and calls on a bond with no convexity (CDB) are very close to each other. The differences in prices get larger as the calls get deeper into the money. Across coupons, this price differential is more pronounced for coupons with more call risk. For example, the difference between the prices of calls struck ATM on FN 4.5s and equivalent CDB calls is 9/32nds, when 50bp ITM (Figure 9). For a coupon with limited call risk such as FN 6.5s the difference is only 3/32nds for the same moneyness.

For mortgage puts the differences are a function of the amount of extension risk.

The reverse holds true for puts. The more the extension risk in the underlying mortgage, the higher the difference in the two prices (mortgage and CDB put), for the same moneyness. For example the difference between the prices of puts struck ATM on FN 4.5s and equivalent CDB puts is 3/32nds, in a 50bp sell off. For a coupon with large extension risk such as FN 6.5s the difference almost triples to 9/32nds.

2 Delta & Duration of Mortgage Options

Mortgage put and call delta is steadily increases as it moves deeper into the money.

The delta of a derivative instrument is the price sensitivity of the derivative with respect to the price of the underlying. For assets whose price depends on the level of rates, a more useful risk metric for an option is its duration. The duration measures the sensitivity of the option price to a change in rates.

2.1   Delta of Mortgage Options

The delta of an option is a useful metric to hedge the option with the underlying asset. For instance, if the delta of an option is 0.4, 100 face of the option is equivalent to 40 face of the underlying. The delta of a mortgage call (Figure 10) is always positive and

Figure 9.   **Greater the Call Risk, Less Calls are Worth in the Money**

Coupon	4.5	5.0	5.5	6.0	6.5
ATM Price of Mtg Call/Put	19	17	13	7	3
ATM Price of CDB Call/Put	19	17	13	7	3
Price of Mortgage Call in a 50bp Rally	**84**	71	46	22	**10**
Price of CDB Call in a 50bp Rally	**93**	81	60	32	**13**
Price of Mtg Put in a 50bp Sell Off	89	**84**	71	**46**	22
Price of CDB Put in a 50bp Sell Off	86	**75**	55	**28**	9

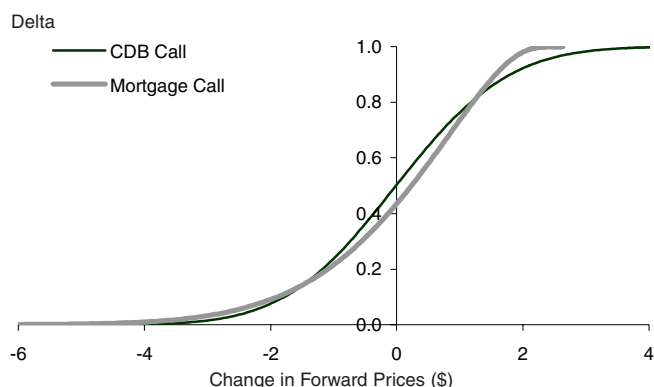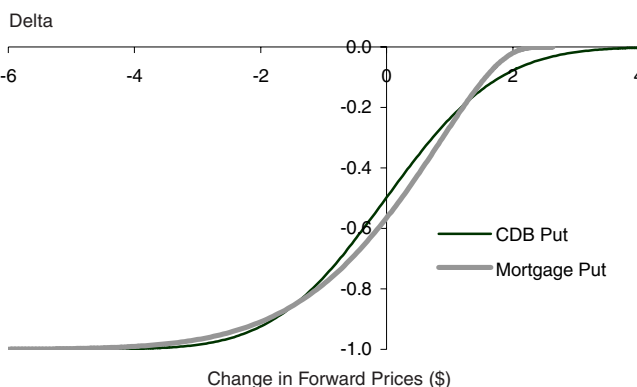Figure 10.   **Delta of a Call Option**

Figure 11.   **Delta of a Put Option**

Delta measures the rate of change of price of the option as the price of the underlying changes

increases as the price of the underlying moves up. The delta of a put (Figure 11) is always negative, and the magnitude increases as the underlying price goes down. Though the delta profiles of mortgage options look similar to plain vanilla options, they are not on top of each other. Take the case of just the ATM deltas of these options. Most practitioners are accustomed to understanding the delta of an option, as a crude proxy for the probability of exercise of the option. At the money therefore, the delta is "intuitively" understood to be very close to 0.5, because the probability that the option will be in the money at expiry is the same as that it will be out of the money.

This is not the case for mortgage options. The price distribution of a mortgage is skewed towards lower prices, (see the Appendix), and therefore, the probability that an ATM call option will be in the money at expiry is less than 0.5. ATM calls on mortgage options have a delta less than 0.5, and correspondingly puts have a delta less than -0.5 (see Figures 10 & 11).
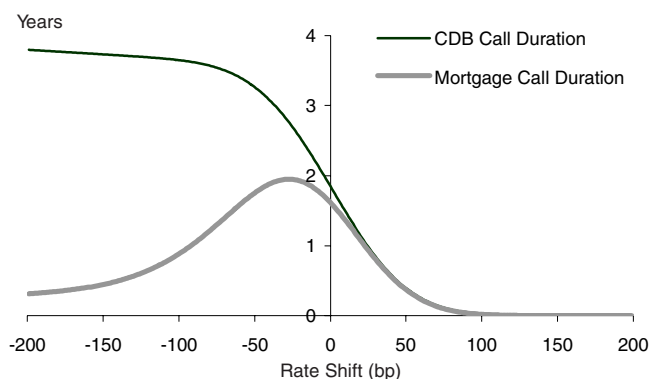
2.2 Duration of Mortgage Options

Durations of mortgage calls follow a bell shaped profile with durations shortening into a rally sufficiently deep in the money.

The duration[6] measures the change in price of a derivative instrument, for a parallel shift in rates. Duration is an important metric for a mortgage option. It determines the hedge ratio for the option against other interest rate instruments. The duration of a mortgage option is closely tied to the duration of the underlying. In fact the duration is simply the delta times the duration of the underlying. For a plain vanilla call option, the duration steadily increases as it gets deeper into the money until it stabilizes to the duration of the underlying[7]. When mortgage calls are out of the money, they too follow this profile with durations extending as the call moves deeper into the money. Deep into the money however, the fact that the underlying mortgage is shortening starts to play a big role, and mortgage calls shorten into any further rally. For instance, in Figure 12 the duration of mortgage calls start shortening into a rally when they are 30bp in the money. For a mortgage put, on the other hand, the fact that the underlying is extending as the put gets deeper in the money works in the same direction, with the net effect that the duration of puts steadily increase as they get deeper into the money (Figure 13).

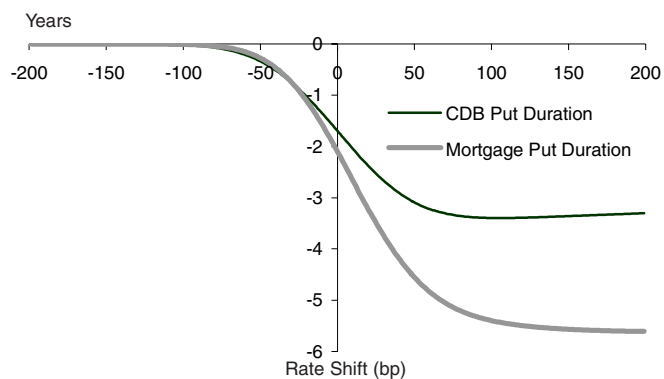[6] We refer to the rate delta of the option as the duration.
[7] This is inline with the steady increase in the delta of the options.

Figure 12. **Bell Shaped Duration of a Call Option**

Duration measures the rate of change of price of the option as the underlying rate (mortgage rate) changes

Figure 13. **Duration of a Put Option**

Mortgage put duration steadily increases

---

3 Convexity of Mortgage Options

Mortgage calls can be negatively convex

We define the convexity of mortgage options as the rate of change of duration with rates. Given the duration profile of mortgage calls, depending on the relative strike, mortgage calls can be negatively convex. The convexity of a regular option is always positive, with the convexity peaking at the money. Mortgage calls, on the other hand, have both a peak and a trough of convexity. They have a peak of positive convexity and a trough of negative convexity (Figure 14). The fact that mortgage calls can be negatively convex is not very surprising. Taking an extreme example, suppose an investor was holding a call on FN 5.5s struck 3 points in the money. The changes to the price of the call will closely match changes in the underlying. The option borrows all the risk characteristics of the underlying, including its negative convexity. A mortgage put option, on the other hand, benefits from the negative convexity of the underlying. The convexity of a put is always positive and much greater in magnitude than a plain vanilla put option (Figure 15).

A mortgage's puts duration steadily increases as it moves deeper into the money.

4 Vega of Mortgage Options

*Mortgage calls are typically short vega while puts are always positive vega.*

The volatility risk of a mortgage option is rather complicated, because of the underlying prepayment option. The mortgage is short volatility, while the overlaid mortgage option is long volatility. The Vega is determined by the combined sensitivity to volatility of these two instruments. For a mortgage call, the short dated option has a mildly positive Vega, while the underlying is short Vega, which is normally larger in magnitude. The Vega exposure of a mortgage call is usually net negative, i.e. a long position in a mortgage call can actually be hurt if volatility increases. Meanwhile, for a mortgage put which is short the underlying, both these optionalities enhance each other, and the combined Vega exposure is still positive. In Figures 16 and 17, we show Vega exposures of a mortgage call, and put across rates. In order to incorporate the contribution to the Vega from the mortgage and the explicit option, we assume that the entire vol surface moves in parallel. The Vega profile of a call option is inverted and skewed compared to the Vega of a CDB. The Vega profile of a put bears no resemblance to that of a CDB and is an "S" shaped function. Just as a check on our model, the sum of the Vega of the ATM put and call is 6/32nds which is the Vega of FN 5.5s on our prepayment model (at a price of 102).

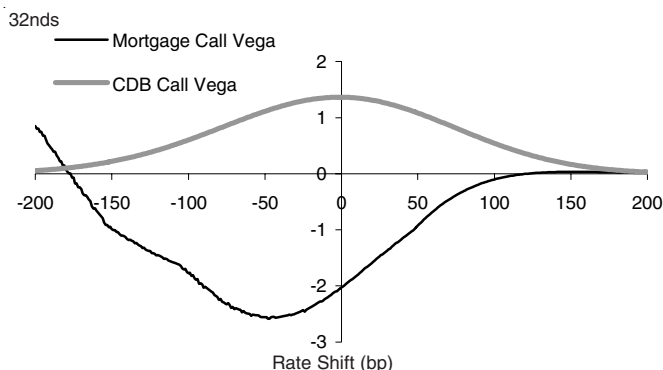Figure 14.   **Negative Convexity in Mortgage Calls**
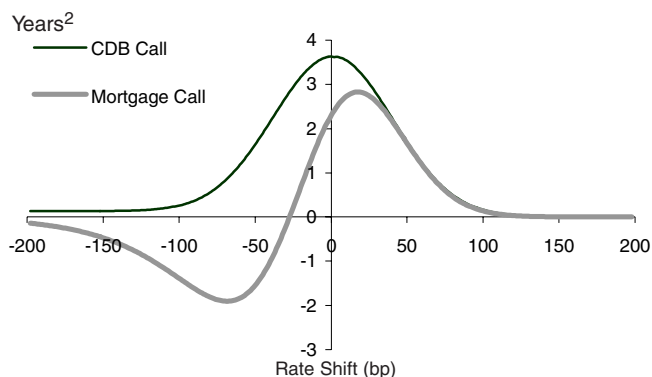
Convexity is defined as the rate of change of duration with rates. Mortgage calls can be negatively convex depending on the strike.
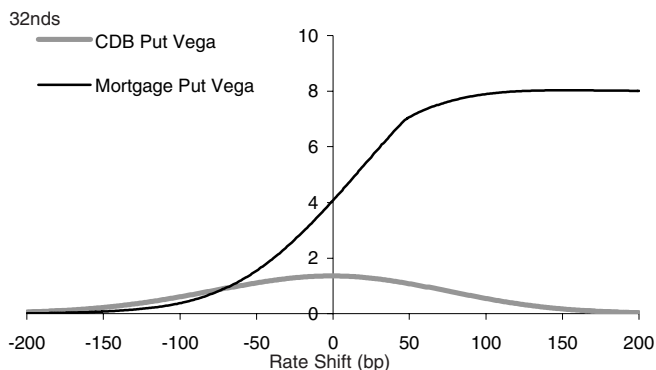
Figure 15.   **Large Convexity of Mortgage Puts**
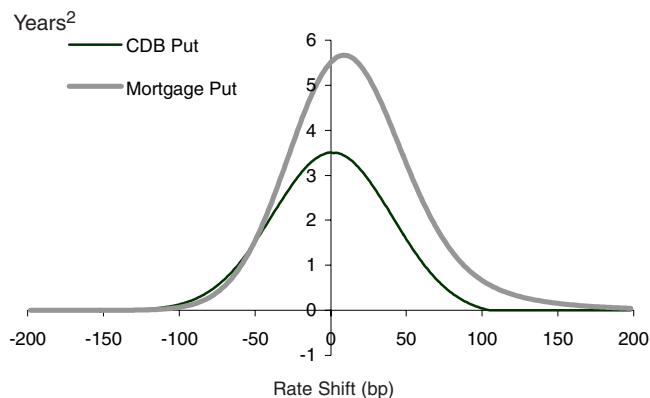
Mortgage puts are always positive convex

Figure 16. **Negative Vega in Mortgage Calls**

Vega measures the change in price of the option (in 32nds) for a 1% parallel
increase in all implied volatilities

Figure 17. **Vega Profiles of Mortgage Puts**

5 Why Buy Mortgage Calls

The carry earned on the mortgage which compensates it for its convexity also compensates mortgage calls for their properties.

At the money, one can buy either a mortgage call, or put at the same price[8]. This however, leads to some interesting issues. As we discussed earlier, the price appreciation of a mortgage call is far lower than a call on a CDB, while a put appreciates much faster. Given that rates are as likely to go up as they are to go down, it appears that the holder of the put has a much better reward profile for the same up front premium.

The fallacy in this argument is that, one can make the same arguments in a mortgage vs. Treasury comparison as well. From just a repricing perspective the reward from a mortgage is lower because of its negative convexity. The compensation for this incremental convexity is the higher coupon and carry on the mortgage. For instance, of the dollar roll of 12/32nds on a mortgage, around 4/32nds is just the premium for being short convexity. If rates were to move along the forwards, one would collect this premium.

This higher coupon works in favor of mortgage calls as well. For instance, assuming the ATM strike on FN 5.5s is 102 today, representing a drop of 12/32nds. If rates were to move along the forwards, the price will not be 102 at expiry but will be 102-04. An ATM call today will therefore be 4/32nds in the money if rates move along the forwards. This is the compensation for all the negatives of mortgage calls. Similarly an ATM put today will find itself 4/32nds OTM if rates move along the forwards, penalizing the put for its better payoff profile.

---

[8] This is just because of the no-arbitrage conditions of the put call parity

TYPICAL STRATEGIES USING MORTGAGE OPTIONS

We look at some typical applications of mortgage options. Mortgage portfolios can use these options in familiar covered calls, or asset + put combinations, to either enhance income, or to preserve the value of the asset. These options can also be used to hedge convexity risks arising from both MSR portfolios, and origination pipelines. Lastly, these options can be used to structure conditional mortgage trades, in ways similar to conditional spread or curve trades in the rates markets.

1 Income Enhancement Strategies

A mortgage holder can sell calls to supplement yield. Bank portfolios are the biggest source of mortgage optionality through this strategy.

In an income enhancement strategy, typically - the holder of a mortgage that has seen some price appreciation sells calls against it to pick up the option premium. The investor gives up any further upside having sold the call. This strategy is just like selling calls against a long position in any other asset such as equity stocks. Covered call strategies are the biggest source of mortgage optionality in the market. The typical users of this strategy are bank portfolios looking to supplement their income. Looking at a hypothetical example, assuming an investor held a pool of 5.5% mortgages which are priced at $102 today, the one month forward price on the pool is $101.5. The investor decides to sell ATM calls on the pool which mature in a month and gets paid 10/32nds for them. Over the next month the investor picks up all the carry in the mortgage and the 10/32nds in premium, but gives up any price upside. We show the one month income generated from a naked position in a mortgage, and with a covered call in Figure 18.
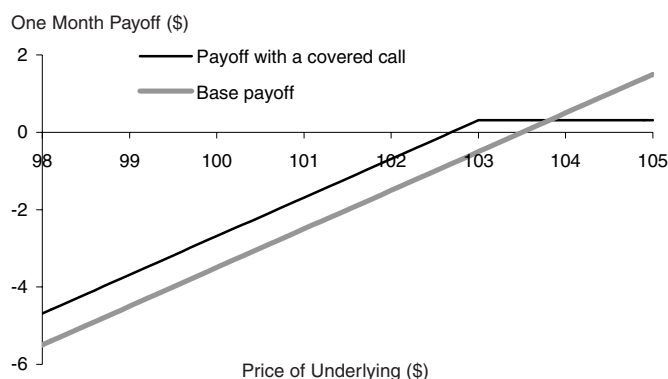
2 Wealth Preservation Strategies

A mortgage holder can buy puts to preserve the value of the asset.

A wealth preservation strategy is in many ways just the opposite of a covered call strategy. Here, he holder of a mortgage buys puts against it, to protect the value of the asset in a downtrade, while giving up some yield in the process, by way of the option premium. The typical users of this strategy are again bank mortgage portfolios. In Figure 19, we show the one month payoff from a naked position in a mortgage and a mortgage + put combination. In the case of the mortgage combined with the put, while the base case income is lower, the portfolio is protected from a downtrade.

Figure 18. **Covered Call Strategies**

One Month Payoff ($)
— Payoff with a covered call
— Base payoff
Price of Underlying ($)

Figure 19. **Asset + Put Strategies**

One Month Payoff ($)
— Payoff with a Put
— Base payoff
Price of Underlying ($)

Covered calls and asset + put strategies shown with options struck at a price of 103-16 today, which is the quoted TBA price on the mortgage one month forward. The figures show the one-month income from both strategies
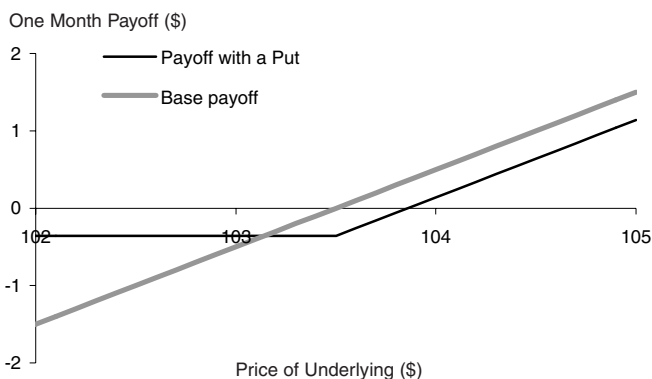
Servicers can use mortgage options to hedge their convexity especially when the MSR is sitting on cuspy sections of the prepayment curve.

3 Hedging Servicing Portfolios

Servicing portfolios are typically duration hedged with a mix of mortgages, and other rate instruments such as swaps. A duration hedged MSR portfolio has substantial negative convexity, from both the MSR and any collateral hedges. For instance, as we show in Figure 20, 5% IOs hedged with 5.5% collateral can lose close to 2 points, due to convexity losses representing around 10% of the value of the IO. Servicers typically use swaptions to hedge this convexity. Mortgage options are an ideal convexity hedge for MSRs, as they do not involve the basis risk that using swaptions exposes them too. If mortgage spreads were very volatile over a period, this basis risk can make the swaptions based convexity hedges ineffective. In Figure 20, we show the convexity profile of a duration hedged combination of 5% IOs with 5.5% TBAs. We also show the profile when hedged with ATM straddles on FN 5.5s.

Mortgage options have disadvantages, including their illiquidity, and the short dated nature of these options. However, these options are ideal convexity hedges, when servicing portfolios are sitting at cuspy points on the prepayment curve. At these points the portfolio is extremely sensitive to mortgage rate volatility. Even if the entire portfolio is not sitting at a cuspy rate, these options could be used to hedge the convexity of just the cuspy portions of the portfolio.

4 Hedging Pipeline Risk

Originators are short the lock-in option to homeowners which is substantially convex.

Originators are the biggest users of mortgage options, using these to hedge their pipeline pull through risks. Pipeline risk is the risk associated with taking applications from prospective mortgage borrowers, who may opt to decline a quoted mortgage rate within a certain grace period. The number of borrowers, who opt to accept a mortgage rate, is variable on what rates do in the interim period. This is the lock in option that originators are short which results in significant negative convexity. For instance, in the base case an originator might expect that in a pipeline of 100 loans, 70 would close. These loans can be sold forward in the TBA market. If rates were to rally 50bp in the interim, only 50 loans would close, and some of the mortgages sold forward would need to be bought back. In a 50bp sell off on the other hand, 90 loans would close and, some more mortgages need to be sold forward. This situation, where the originator needs to be constantly buying high and selling low to duration rebalance the pipeline, results in its negative convexity.

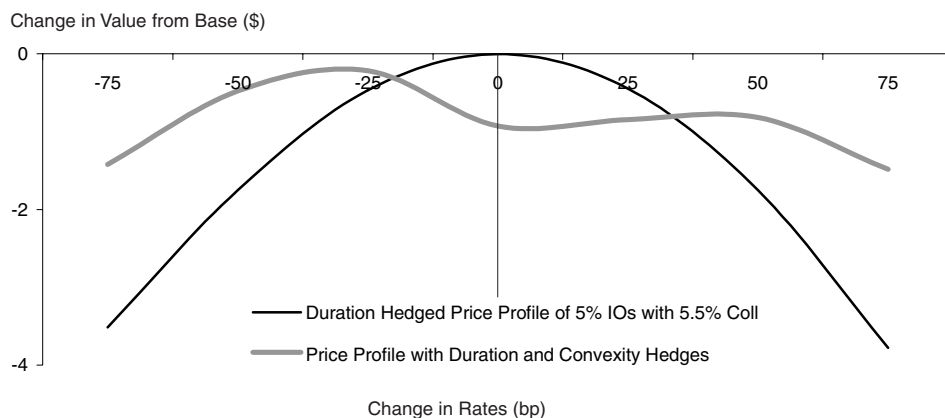Figure 20. **Mortgage Options as Convexity Hedges for MSRs**

Change in Value from Base ($)

Duration Hedged Price Profile of 5% IOs with 5.5% Coll

Price Profile with Duration and Convexity Hedges

Change in Rates (bp)

Figure 21.   **Variation of Pull-through % with Rates**

bp
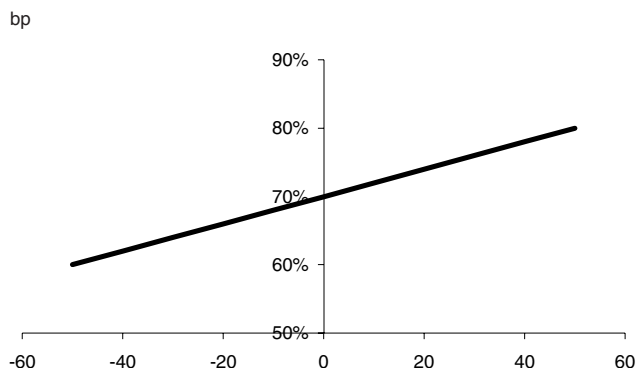
Figure 22.   **Negative Convexity from Pull-through Rates**

bp

Shows convexity losses on a duration hedged pipleine in bp
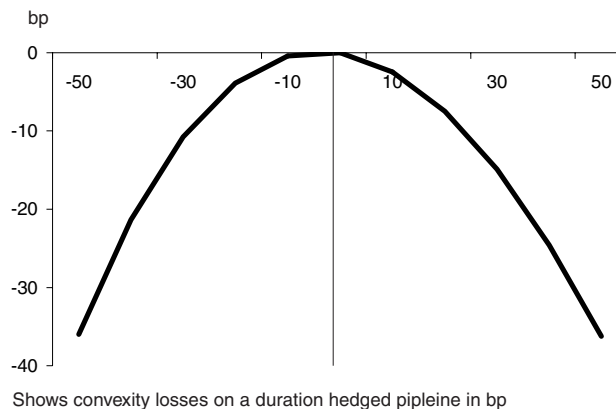
In Figure 21, we show a typical variation in pull through rates, with rates varying from 60% to 80% in a 100bp range. In Figure 22, we show the negative convexity brought into the pipeline, because of this pull through variation. For instance, the originator faces 30bp in convexity losses in a 50bp move in rates, in this scenario.

4.1 How to Hedge this Risk: Calls or Puts?

Originators can use mortgage calls or puts to hedge their risks.

An originator can use any security that has positive convexity, to hedge this pull through risk. The benefit of using mortgage options is that, the pipeline is not exposed to any basis risk between mortgage and other interest rates.

Should an originator be using calls, or puts to hedge the pipeline? Let us look at two different hedging strategies. Consider a case, where pullthrough rates go from 60% in a down 50bp scenario to 80% when rates are up 50bp. The originator can sell 60% of the loans forward and but 20% worth of ATM puts or they could sell 80% of the loans forward and buy 20% ATM calls. Both these scenarios are identical and will produce exactly the same returns over the life of the pipeline because of the put-call parity. It, therefore, does not matter whether the originator uses calls or puts to hedge the pipeline risk. A more significant issue is around what strikes the originator chooses to hedge the convexity.

4.2 Tradeoff between Liquidity and Efficiency

Because of the nature of the pull through risk, strikes on mortgage options to hedge the risk need to be OTM.

In a simplistic scenario, we define the objective of the convexity hedge on a pipeline to be a reduction in returns variability in a +-50bp range in rates. As we see in Figure 22, the convexity risks in the pipeline kick in only post the first 10-15bp move in rates. Therefore, the strikes that would be the most effective would typically have to be out of the money strikes. For instance, if pullthroughs were to vary from 50% to 80% in a 100bp range, the originator can minimize variability in returns to 4bp using just ATM options. On the other hand, using out of the money strikes ($0.5 out strangles in this case), the variability can be brought close to zero (Figure 23). If the pull through profile is steeper, the strikes would need to shift more out of the money. For instance, if pullthroughs were to vary from 40 to 90% in a 100bp range, $1 OTM strangles can reduce variability in returns to 4bp. Using only ATM options the variability would have been 11bp. Given the better hedge effectiveness of OTM strikes, depending on the risk preference of the originator, they may choose to sacrifice some of the liquidity in ATM options and move into OTM strikes.

Figure 23.   **Strikes for Origination Risk Hedging**

			Only ATM Options Allowed	All Options Allowed	
	Scenario		Variation in Returns	Variation in Returns	Most Sensitive Option
Base Case					
Down 50 60%	Base 70%	Up 50 80%	4bp	~ 0bp	$0.5 OTM Strangle
Flat Profile					
Down 50 65%	Base 70%	Up 50 75%	2bp	~ 0bp	$0.5 OTM Strangle
Very Steep Curve					
Down 50 40%	Base 70%	Up 50 90%	11bp	4bp	$1 OTM Strangle

5 Conditional Mortgage Trades

Mortgage options can be used in conditional mortgage trades either as mortgage basis trades or intra-mortgage trades.

Mortgage options can be used to express relative value views in mortgages. This is a unique source of alpha generation for mortgage portfolios. Similar to their counterparts in the rates market, who use rate based options to structure, conditional spread and curve trades; mortgage portfolios can use mortgage options, in conditional mortgage trades. Conditional trades can be in the form of mortgage basis trades (e.g. mortgage spreads will widen in a rally) or intra-mortgage trades (e.g. premiums will widen vs. discounts in a rally). A typical conditional trade for instance, could be one where the view is that mortgage spreads will widen into a rally. As rates rally the objective is to get shorter mortgages vs. swaps. If rates sell off or stay unchanged, the position should expire worthless. Selling mortgage calls and buying swaptions can get this kind of a profile. As rates rally we get shorter more mortgages vs. swaps, and if spreads on mortgages is widening at the same time, the trade will make money.

5.1   Framework for Conditional Mortgage Trades

Our valuation model can be used to estimate the amount of spread directionality being priced in by the options market.

The first step before putting on any conditional trade is to estimate what is being priced into the market currently. Our valuation model can be used to determine what is being priced into mortgage option valuations, for simple conditional spread trades (buying/ selling mortgage options vs. swaptions or treasury options). The first step is to figure out a fair value for mortgage options, based on the valuation model.

- *Implied Volatility:* As we described in our modeling section, and the Appendix, given the duration profile of a mortgage, and a rate process for mortgage rates, we can use the valuation model to estimate the fair value of mortgage options. The first input needed is an implied volatility for mortgage rates. Assuming mortgage rates are normally distributed, the implied volatility of liquid swaptions can help come up with an implied volatility for mortgage rates[9].
- *Fair Prices:* The implied volatility of mortgage rates can give the price of any mortgage option, using our model. For instance, at a volatility of 104bp/yr, the model estimates a fair price of 14/32nds for one month ATM calls and puts on FN 5.5s.

---

[9] For instance if 1m/2yr swaptions are trading at an implied volatility of 103bp/yr and 1m/10yr swaptions are trading at 105bp/yr assuming these two rates are uncorrelated and based on the fact that current coupon mortgage rates are around 30% sensitive to the 2yr point on the curve and 70% to the 10yr point on the curve, the implied volatility of mortgage rates over a month should be 0.3*103 + 0.7*105 = 104bp/yr.

- *Market Implied Spreads:* We assume that any difference between market prices and model prices is due to spread directionality priced into the options market. For instance if the market price for ATM calls on FN 5.5s is 8/32nds and the model price is 14/32nds, we attribute this to the market pricing in a spread widening in a rally.
- *The Trade:* If the market is pricing in a spread widening in a rally, and our view is different, there is a trade. For example, given the fair price of 14/32nds on ATM calls on FN 5.5s and a market price of 8/32nds we can estimate that the market is pricing in a 10bp widening in spreads for every 25bp of rally[10]. In case we believe spreads will not widen, there is a trade (buy mortgage calls sell receivers).

5.2 Risks to Mortgage Conditional Trades

There are a few unique risks to mortgage conditional trades that investors need to be aware of. In a mortgage conditional spread trade, there are strategies that have limited upside and some strategies, that have potentially unlimited upside.

Two conditional spread strategies have unlimited upside while two strategies have limited upside and probably some downside.

Conditional Trades with Potentially "Unlimited" Upside: Consider the two strategies betting on mortgage spread widening in a rally, or spread tightening in a sell-off. In both these cases there is "unlimited upside"[11] however, the positions are negative carry. Taking an example, if we are betting on mortgage spread widening in a rally we would be short mortgage calls vs. treasury calls. As we explained in the risks section, the price profiles of mortgage options are such that as mortgage calls get deeper into the money, they are always worth less than calls on securities that are not negatively convex. Therefore if rates do rally, this price compression of the mortgage will result in "unlimited" upside. The only issue, however, is that (see Section 3.6) a position where one is short mortgage calls vs. treasury calls is inherently negative carry. If rates move along the forwards, the mortgage call will be in the money, and the treasury call will expire worthless. Therefore, there is a minimum rate move required before the price compression of the mortgage wipes out this carry advantage on the call. In this set of strategies therefore, there is a minimum rate move required (even if spread do widen) and potentially unlimited upside beyond.

Conditional Trades with Limited Upside: If one were betting on mortgage spread widening in a selloff, or spread tightening in a rally, in both these strategies, there is limited upside and possibly downside as well. For instance, if we are betting on mortgage spread tightening in a rally we would be long mortgage calls vs. Treasury calls. Since mortgage calls hit the price compression of the underlying mortgage, beyond a certain move in rates even if spreads were to tighten, the position would actually result in losses. There is, therefore, a maximum rate move that is possible in the case of these strategies.

5.3 An Example Conditional Trade

Recommend selling calls on FN 5.5s vs. 2yr and 10yr receivers as a bet on spread widening in a rally.

Having gone over the risks to conditional mortgage trades, we end this section with a current example. In Figure 24, we show current prices (as of Oct 6th) of ATM calls on FN 5.5s in Nov and Dec, and show our model projected fair prices. For instance, calls in Nov are priced at a mid-market price of 16/32nds, vs. a model projected fair price of 15/32nds. Since the options are priced close to fair value, the options market is essentially pricing in no spread directionality in FN 5.5s. Given the cuspy nature of the coupon, any rally from these levels is likely to cause spreads to widen on the coupon.

---

[10] The exact amount of spread widening priced in can be estimated using the Lehman prepayment model
[11] Unlimited upside for all practical purposes.

Figure 24. **Market and Fair Prices on FN 5.5 Calls**

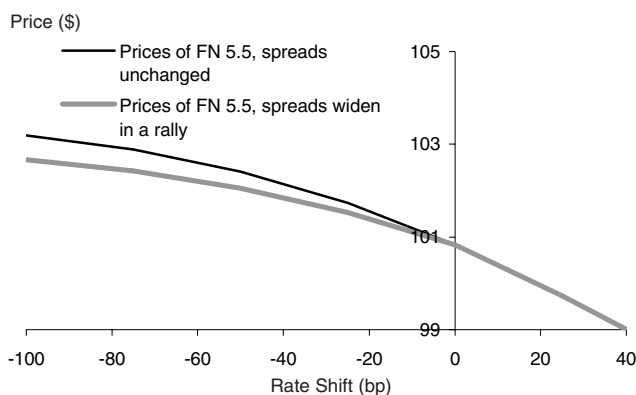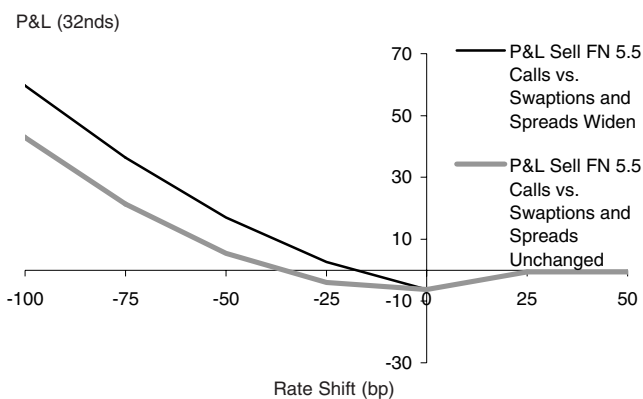| ATM Option Pricing	Fair Prices	Actual Mid-Prices	Difference
FN 5.5s Nov 2004	15/32nds	16/32nds	1/32nd
FN 5.5s Dec 2004	22/32nds	22/32nds	0/32nd

Figure 25. **Market Prices vs. Our View**

Figure 26. **P/L If Spreads Widen in a Rally**

We are recommending selling calls on FN 5.5s and buying 2m and 10yr receivers to hedge the duration exposure. If rates sell off, the position expires worthless. In Figure 25 we show the price profile of FN 5.5s assuming spreads on the coupon remain unchanged with rates. We also show the price profile, assuming spreads widen by 5bp for every 25bp in rally. The first price profile is what is being priced into the market, and the second is our view. We show the P/L profile at expiry from the trade in Figure 26. For instance, if spreads were to widen at a rate of 5bp for every 25bp of rally in a 50bp move the trade makes 20/32nds.

CONCLUSIONS

Mortgage options are unique instruments given that they are one of the few options written on negatively convex securities. Their pricing is rather involved, as we need to value the embedded prepayment option along with the explicit option on the mortgage. The negative convexity of the underlying leads to some unique risk characteristics of mortgage options. For example, contrary to intuition, the holder of a mortgage call could actually be holding a negatively convex security. These options are ideal convexity hedges for the convexity needs of servicing portfolios and mortgage originators. Their biggest benefit, however, is to serve as a guide for the amount of mortgage spread directionality priced into the market. This provides a basis for putting on mortgage trades either in an outright form or through the mortgage options market.

APPENDIX: MODELING MORTGAGE OPTIONALITY

Methodology

For the purpose of pricing an option on a mortgage, it is desirable to separate the modeling of the prepayment option from that of the option on the mortgage. We choose to model the prepayment option through a parameterized function for the duration of the mortgage. We then determine the parameters by calibrating the duration function to the duration profile of a mortgage generated by an existing prepayment model. This calibrated duration function contains sufficient information on the underlying mortgage and it serves as an interface between the embedded prepayment option and the explicit option on the mortgage. The subsequent modeling of the latter option uses only the duration function as the effective prepayment input and is carried out independently of the actual prepayment analysis.

Mortgage Duration Profile

The duration of a bond D is traditionally defined as the negative percentage price sensitivity with respect to underlying rate, that is

$$D(y) = -\frac{1}{P(y)}\frac{dP(y)}{dy} \qquad (1)$$

where $P(y)$ is the price of the bond as a function of underlying rate y. As already discussed in the text, mortgage durations vary considerably with rate moves. A rally shortens mortgage duration while sell off extends it. The duration of a mortgage varies across rates monotonically, rising from a minimal level for low rates to approach a stable level for high rates.

There is a lot of freedom in choosing the actual parameterization of this behavior. For simplicity as well as analytic flexibility[1], we illustrate with the choice

$$\frac{dP}{dy} = -\frac{b}{1 + e^{-c(y-a)}} \qquad (2)$$

where the underlying rate y is defined as the difference between the current coupon rate and the coupon of the underlying mortgage. The parameters a, b and c are constants for a particular coupon and are determined by calibrating the shape of D to the one produced by a separate prepayment model. In Figure 1, we calibrated the Lehman Brothers prepayment model based durations to the analytical function introduced above and show the fitted OADs and OACs vs. the corresponding quantities generated by the prepayment model.

---

[1] See Prendergast [2003] for a more refined choice of the duration function and the corresponding price integration.

Figure 1.   **Comparing Durations of Mortgages with Fitted Durations**

Coupon	Price	MARKET*		FITTED FUNCTION	
		OAD	**OAC**	**OAD**	**OAC**
4.5	96.58	5.67	-0.70	5.61	-0.55
5.0	99.21	4.96	-1.50	4.96	-1.60
5.5	101.43	3.67	-3.10	3.66	-3.16
6.0	102.86	1.96	-3.00	1.97	-3.00
6.5	103.52	0.81	-1.40	0.86	-1.40

* Durations of various coupons based on Lehman Brothers prepayment model

Resultant Convexity Profile

Given the duration of a mortgage, its sensitivity with respect to rates, or equivalently, the convexity, can be deduced accordingly. The availability of an analytic expression for the duration allows us to obtain an explicit profile function of the mortgage convexity. After taking the derivative of (2), we have

$$C(y) = \frac{1}{P} \frac{bc}{\left[2\cosh(c(y-a)/2)\right]^2} \qquad (3)$$

This expression shows that the convexity of the mortgage tapers off for very low and high rates, and peaks at certain rate. In fact, it can be shown that the convexity derived from the analytic duration function peaks exactly at y = a. Since a mortgage convexity peaks when prepayment activity is most intensive, this gives the parameter a the interpretation of being the cost to overcome in prepayment decisions.

Price Function

The price sensitivity function defined in (2) is analytically soluble. We obtain the following explicit functional form for the price of the mortgage by simply integrating (2) over rates.

$$P(y) = P_H - \frac{b}{c}\ln\left[1 + e^{c(y-a)}\right] \qquad (4)$$

where P_H is a constant. It can be shown that for very low rate, the price approaches to P_H. Therefore, P_H measures the level of the mortgage price compression at low rates. This level may be deduced by matching the market price quote with the model price (4) for the underlying mortgage.

Distribution of Price

For bond options, including ones on mortgages, it is the process of interest rate, not the bond price itself, that is normally specified. However, the price-rate relationship of a mortgage derived from the duration profile allows one to derive the corresponding price process of the mortgage, which then can be used to price options written on the mortgage.

Knowing the functional form (4) of the price of a mortgage across rates, we can determine the distribution of prices provided that the distribution of rates is known. Given a rate distribution function $g(y)$, the price distribution function $G(P)$ can be computed as

$$G(P) = \frac{g(y)}{PD(y)} \qquad (5)$$

The utilization of the price-rate function (4) on the right-hand side is implicit, with the rate y viewed as a function of price P through the inverse function of (4).

For instance, if mortgage rates were assumed to be normally distributed (5), the distribution g(y) simply becomes a Gaussian function and the pricing distribution function can then be obtained directly. Take the example of a CDB, G(P) simply becomes a lognormal distribution (Figure 2).

Volatility of Mortgage Rates

In order to derive the distribution of mortgage price via (5), we need to specify the volatility of underlying mortgage rate y. A straightforward approach is to use the quoted rate volatilities in swaption market. Mortgage rates are closely tied to swap rates and the volatility in mortgage rates can be derived from the volatility of swaptions. For example, changes in current coupon rates recently, have 30% sensitivity to the 2yr swap rate and 70% to the 10yr swap rate. Knowing the implied rate volatility of 1M2Y and 1M10Y swaptions along with their correlation, we can estimate the volatility for mortgage rates over a one month period.

Figure 2. **Terminal Distribution of Rates**
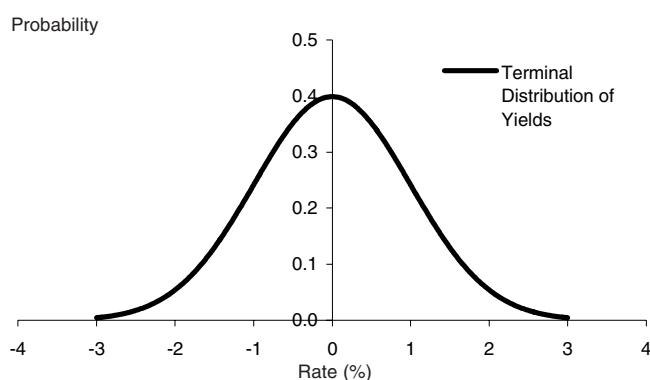
Shows the distribution of rates along the forward assuming a standard deviation of 100bp.

Figure 3. **Terminal Distribution of Prices**
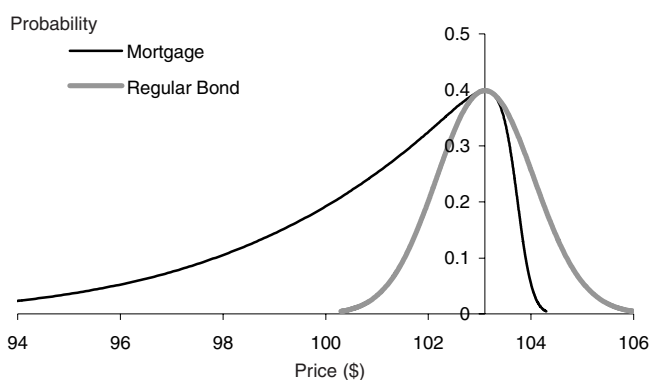
Shows the corresponding distribution of prices on both the mortgage and a constant duration bond based. Mortgage prices are skewed to the negative side.

Mortgage Option Pricing

Once the price distribution function for a mortgage is known, the price of an option on the mortgage can be computed directly as an integration of the distribution function over the payoff at the option expiry. For a call option on a mortgage, it is

$$C = \int_{K}^{\infty} (P - K)G(P)dP \qquad (6)$$

This integral can be carried out, either through established numerical procedures or via approximate analytic solutions.

References

Prendergast, J. "The Complexities of Mortgage Options". The Journal of Fixed Income, March 2003, pp. 7-24.

The views expressed in this report accurately reflect the personal views of Fei Zhou and Prasanth Subramanian the primary analyst(s) responsible for this report, about the subject securities or issuers referred to herein, and no part of such analyst(s)' compensation was, is or will be directly or indirectly related to the specific recommendations or views expressed herein.

Any reports referenced herein published after 14 April 2003 have been certified in accordance with Regulation AC. To obtain copies of these reports and their certifications, please contact Larry Pindyck (lpindyck@lehman.com; 212-526-6268) or Valerie Monchi (vmonchi@lehman.com; 44-(0)207-102-8035).

Lehman Brothers Inc. and any affiliate may have a position in the instruments or the companies discussed in this report. The firm's interests may conflict with the interests of an investor in those instruments.

The research analysts responsible for preparing this report receive compensation based upon various factors, including, among other things, the quality of their work, firm revenues, including trading, competitive factors and client feedback.

Lehman Brothers Inc. managed or co-managed a public offering of Fannie Mae and Ginnie Mae securities in the past year. Lehman Brothers usually makes a market in the securities mentioned in this report. These companies are current investment banking clients of Lehman Brothers or companies for which Lehman Brothers would like to perform investment banking services.

Publications-L. Pindyck, B. Davenport, W. Lee, D. Kramer, R. Madison, A. Acevedo, T. Wan, M. Graham, V. Monchi, K. Banham, G. Garnham, Z. Talbot