Calculation of the Focal Length of an Offset Satellite Dish Antenna

by John A R Legon, B.Sc.

Given an offset satellite dish or antenna without LNB bracket or documentation, it is useful to be able to determine the focal point in order to establish where the feed or LNB should be located. It is generally believed that there is no simple solution to this problem, because the position of the origin or vertex of the parabolic curve isn't known. In fact, however, thanks to some analytical geometry and a little-known property of the parabola, an exact solution is possible using just three dimensions - the height and width of the dish, and the maximum depth.

Conventional offset dishes appear roughly circular but are in fact slightly higher than they are wide, and the outer rims fall on a flat plane. The height and the width are easily measured and the maximum depth can be found with reference to a straight edge laid across the dish from top to bottom. For "shaped" offset dishes, the necessary measurements may be obtained by laying the dish on a level surface and filling it with water.

As shown below, given these three dimensions of height, width and maximum depth, the focal length of the dish is given by the formula:

Focal length = (width ^ 3) / (16 * depth * height)

(Legon's equation for the focal length of an offset dish antenna)

The derivation of this formula depends on the fact that an offset dish antenna represents a plane section through a paraboloid of revolution. Such a section has the following significant properties:

> *1. Every plane section of a paraboloid of revolution, oblique to the axis of the surface, is an ellipse.*
>
> *2. The orthogonal projection of that ellipse onto a plane at right angles to the axis of the surface is a circle.*

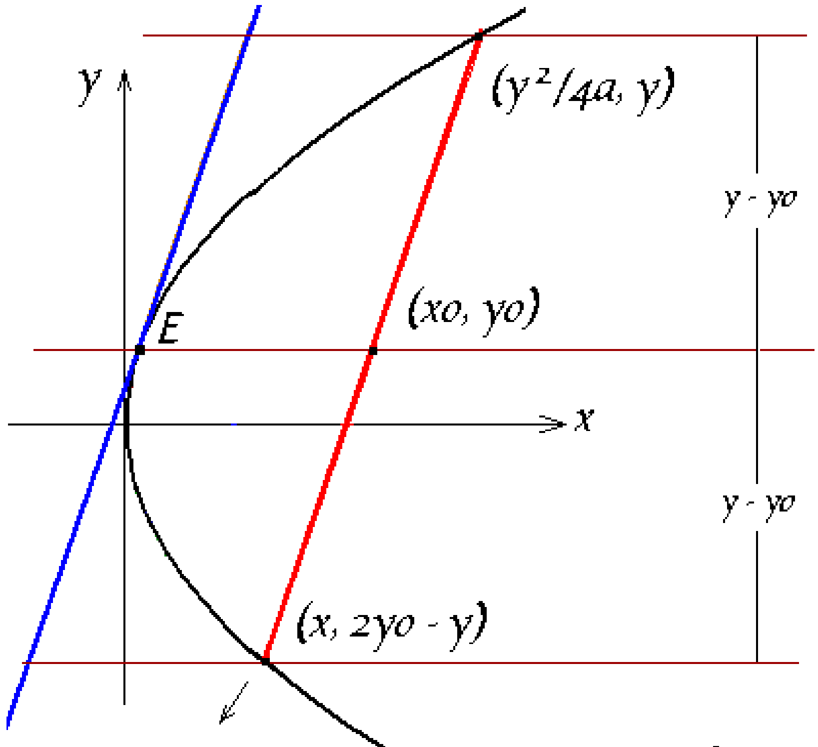
It follows from these properties that the offset angle of the dish, which is the angle between the plane of the section and the plane orthogonal to the axis of revolution, can be calculated from the width and height of the elliptical rim:

cosine (offset angle) = width / height

The properties of a parabola which make it possible to calculate the focal length of an offset dish when the point of origin of the curve isn't known are these:

> *1. A line drawn parallel to the axis of a parabola through the midpoint of any chord, intersects the parabola at a point where the tangent to the parabola is parallel to the chord.*
>
> *2. At this point, the perpendicular distance to the chord is at a maximum.*

This relationship between the slope of the chord and the gradient of the parabola is illustrated in the diagram below. The equation of the parabola is $x = y^2 / 4a$, where 'a' is the focal length.



$$x = \frac{4yo^2 + y^2 - 4yo.y}{4a}$$

For parabola $x = \dfrac{y^2}{4a}$

gradient of tangent

at point E $= \dfrac{yo}{2a}$

gradient of chord =

$$\frac{\dfrac{y^2}{4a} - \dfrac{4yo^2 + y^2 - 4yo.y}{4a}}{2y - 2yo}$$

$$= \frac{yo.y - yo^2}{2a(y - yo)} = \frac{yo}{2a}$$

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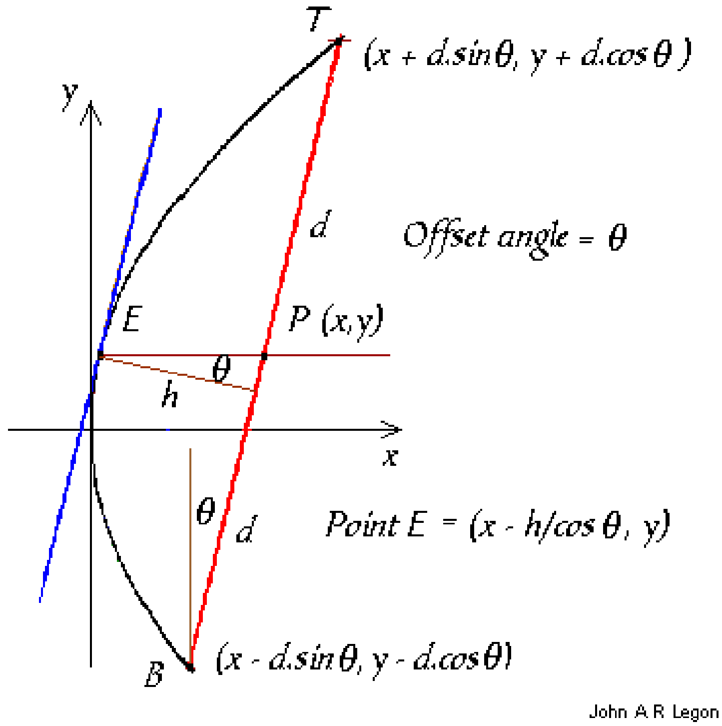
In the following diagram, the point P has been given the coordinates (x,y), and the end points of the chord BT, with length 2d and midpoint P, are derived from the semi-length of the chord and the offset angle theta.



Now calling the maximum depth of the dish curvature h, measured perpendicular to the chord, the depth parallel to the axis of the parabola will be h / cos(theta). The point of maximum depth, E, thus has the coordinates (x - h / cos(theta), y).

At E, the point of maximum depth, the tangent is parallel to the chord, and the gradient is equal to the offset angle Θ
Hence for the gradient (dx/dy) *we have*:

$$\frac{y}{2.a} = \tan\Theta \qquad (1)$$

Also at point E,

$$x - \frac{h}{\cos\Theta} = \frac{y^2}{4.a} \qquad (2)$$

At point B,

$$x - d.\sin\Theta = \frac{(y - d.\cos\Theta)^2}{4.a} \qquad (3)$$

Subtracting (2) *from* (3),

$$\frac{h}{\cos\Theta} - d.\sin\Theta = \frac{(y - d.\cos\Theta)^2}{4.a} - \frac{y^2}{4.a}$$

$$\frac{4.a.h}{\cos\Theta} - 4.a.d.\sin\Theta = y^2 + d^2.\cos^2\Theta - 2.y.d.\cos\Theta - y^2$$

$$4.a.h - 2.a.d.\sin 2\Theta = d^2.\cos^3\Theta - 2.y.d.\cos^2\Theta \qquad (4)$$

But from the gradient at point E,

$$y = 2.a.\tan\Theta = \frac{2.a.\sin\Theta}{\cos\Theta}$$

Hence substituting for y in (4)

$$4.a.h - 2.a.d.\sin 2\Theta = d^2.\cos^3\Theta - 2.a.d.\sin 2\Theta$$

And

$$\textit{Focal Length} \quad a = \frac{d^2.\cos^3\Theta}{4.h}$$

But

$$\cos\Theta = \frac{W}{D}$$

where W = width and D = 2.*d = height of dish antenna*

Hence

$$\textit{Focal Length} = \frac{W^3}{16.h.D}$$

Legon's equation for the focal length of an offset parabolic antenna

To take a practical example, the appendix to http://www.qsl.net/n1bwt/chap5.pdf describes the calculation of the focal length of an offset dish by measuring three points along the curve of the dish, and using the coordinates to solve three quadratic equations with three unknowns - the focal length and the x and y coordinates of the point of origin - a very tedious calculation. The text refers to a dish with a height of 500 mm, a width of 460 mm, and a maximum depth of 43 mm at a point 228 mm up the chord from the bottom edge. This gives the coordinates of (0, 0), (49.8, 226.6) and (196, 460), which are used to write and simultaneously solve three equations of the form

$$4a.(X + Xo) = (Y + Yo)^2$$

where Xo and Yo refer to the unknown position of the origin. Solving these equations gives a focal length of 282.89 mm.

My analysis reduces the problem to just one equation: width^3 / (16 x depth x height) , so for this example we have:

$$\text{focal length} = 460^3 / (16 \times 43 \times 500) = 282.95 \text{ mm}$$

The result is thus in almost perfect agreement with that obtained by the solution of three simultaneous equations - the slight difference being due to the fact that the measurement given for the position of the point of maximum depth isn't strictly accurate. But as we have shown, it is isn't necessary to know this dimension.

Tto find the position of the origin or vertex of the paraboloid, the midpoint of our chord is at a distance from the axis of the parabola of y = 2.a.tan(theta) or 241 mm. But this point is also at a distance from the lower rim of the dish of d.cos(theta) = d.w / 2d = w / 2 =230 mm. This places the axis (241 - 230) = 11 mm outside the lower rim of the dish, as the solution of the quadratic equations for Yo also shows.

The Position of the LNB

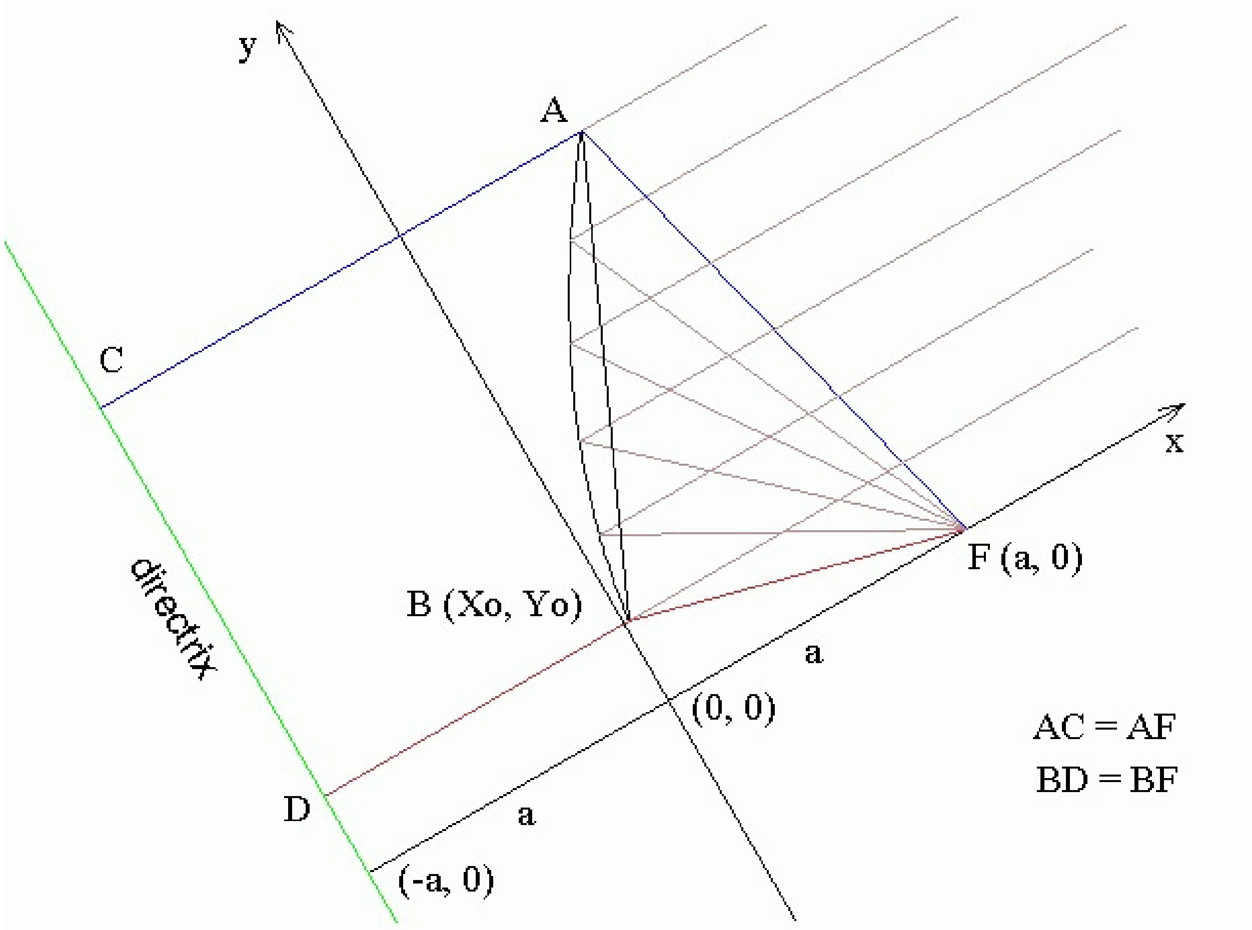
It now remains to determine the correct position of the LNB or feed. This depends on the coordinates (Xo, Yo) of the bottom edge of the dish in relation to the vertex of the parabola at the point (0, 0). From the above discussion we have:

$$Yo = 2.a.\tan(\theta) - w / 2$$

and from the equation of the parabola:

$$Xo = Yo^2 / 4.a$$

The following calculation makes use of the fact that every point on a parabola is the same distance from the focal point as it is from a line known as the directrix, which in the present case is a line drawn parallel to the y axis through the point (-a, 0), where 'a' is the focal length of the parabola.



It follows that the distance BF from the lower rim of the dish to the focal point at the LNB is obtained simply by adding the focal length '*a*' to the x-coordinate:

$$BF = BD = Xo + a$$

The point A on the upper rim of the dish is connected to the point B by the chord AB at an angle to the y axis equal to the offset angle theta. Thus the distance AF to the focal point will be:

$$AF = AC = Xo + a + AB.\sin (theta)$$

where AB is the height of the dish D or 2.d in our previous working. In practice Xo is often close to zero and may be neglected.

Now applying these equations to our worked example, we find that Xo is 0.1 mm, and hence BF is 283 mm and AF is 479 mm. The above-cited text suggests a "top string length" of 466-476 mm, and reaches the conclusion that the origin of the parabola is located on the lower rim of the dish. Since Yo is only one centimetre, this was arguably the designer's intention.

Comments to John Legon
