

CHAPTER

1

INTRODUCTION TO SOIL MECHANICS AND FOUNDATIONS

1.0 INTRODUCTION



Concept Learning

Access Chapter 1, Section 1.0 on the CD for a short introduction to soil mechanics and foundations, and to view a simulation of Coulomb's theory. Take the end of chapter quiz to test your understanding.

Soil is the oldest and most complex engineering material. Our ancestors used soils as construction material to build burial sites, flood protection, and shelters. Western civilization credits the Romans for recognizing the importance of soils in the stability of structures. Roman engineers, especially Vitruvius, who served during the reign of Emperor Augustus in the first century B.C., paid great attention to soil types (sand, gravel, etc.) and to the design and construction of solid foundations. There was no theoretical basis for design; experience from trial and error was relied upon.

Coulomb (1773) is credited as the first person to use mechanics to solve soil problems. He was a member of the French Royal Engineers, who were interested in protecting old fortresses that fell easily from cannon fire. To protect the fortresses from artillery attack, sloping masses of soil were placed in front of them (Fig. 1.1). The enemy had to tunnel below the soil mass and the fortress to attack. Of course, the enemy then became an easy target. The mass of soil applies a lateral force to the fortress that could cause it to topple over or could cause it to slide away from the soil mass. Coulomb attempted to determine the lateral force so that he could evaluate the stability of the fortress. He postulated that a wedge of soil ABC (Fig. 1.1) would fail along a slip plane BC and this wedge would push the wall out or overtopple it as it moves down the slip plane.

Movement of the wedge along the slip plane would occur only if the soil resistance along the wedge were overcome. Coulomb assumed that the soil resistance is provided by friction between the particles and the problem became one of a wedge sliding on a rough (frictional) plane, which you may have analyzed in your physics or mechanics course. Coulomb has tacitly defined a failure criterion for soils. Today, Coulomb's failure criterion and method of analysis still prevail.

From the early twentieth century, the rapid growth of cities, industry, and commerce required myriad building systems: for example, skyscrapers, large public buildings, dams for electric power generation and reservoirs for water supply and irrigation, tunnels, roads and railroads, port and harbor facilities, bridges, airports and runways, mining activities, hospitals, sanitation systems, drainage systems, and towers for communication systems. These building systems require stable and economic foundations and new questions about soils were asked. For example, what is the state of stress in a soil mass, how can one design safe and economic foundations, how much would a building settle, and what is the stability of structures founded on or within soil? We continue to ask these questions and to try to find answers as new issues have confronted us. Some of these new issues include removing toxic compounds from soil and

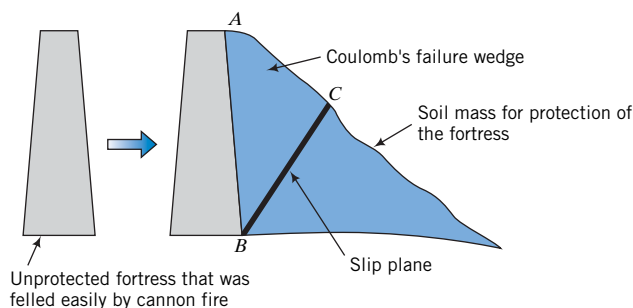


FIGURE 1.1 Unprotected and protected fortress.

water, designing foundations and earth structures to mitigate damage from earthquakes and other natural hazards, and designing systems to protect the environment.

To answer these questions, we need the help of some rational method and, consequently, soil mechanics was born. Karl Terzaghi (1883–1963) is the undisputed father of soil mechanics. The publication of his book *Erdbaumechnik* in 1925 laid the foundation for soil mechanics and brought recognition to the importance of soils in engineering activities. Soil mechanics, also called geotechnique or geotechnics or geomechanics, is the application of engineering mechanics to the solution of problems dealing with soils as a foundation and a construction material. Engineering mechanics is used to understand and interpret the properties, behavior, and performance of soils.

Soil mechanics is a subset of geotechnical engineering, which involves the application of soil mechanics, geology, and hydraulics to the analysis and design of geotechnical systems such as dams, embankments, tunnels, canals and waterways, foundations for bridges, roads, buildings, and solid waste disposal systems. Every application of soil mechanics involves uncertainty because of the variability of soils—their stratification, composition, and engineering properties. Thus, engineering mechanics can provide only partial solutions to soil problems. Experience and approximate calculations are essential for the successful application of soil mechanics to practical problems. Many of the calculations in this textbook are approximations.

Stability and economy are two tenets of engineering design. In geotechnical engineering, the uncertainties of the performance of soils, the uncertainties of the applied loads, and the vagaries of natural forces nudge us to compromise between sophisticated analyses and simple analyses or approximate methods. Stability should never be compromised for economy. An unstable structure compromised to save a few dollars can result in death and destruction.

1.1 MARVELS OF CIVIL ENGINEERING—THE HIDDEN TRUTH

The work that geotechnical engineers do is often invisible once construction is completed. For example, four marvelous structures—the Sears Tower (Fig. 1.2), the Empire State Building (Fig. 1.3), the Taj Mahal (Fig. 1.4), and the Hoover Dam (Fig. 1.5)—grace us with their engineering and architectural beauty. However, if the foundations, which are invisible, on which these structures stand were not satisfactorily designed then these structures would not exist. A satisfactory foundation design requires the proper application of soil mechanics principles, accumulated experience, and good judgment.

The stability and life of any structure—a building, an airport, a road, dams, levees, natural slopes, power plants—depend on the stability, strength, and deformation of soils. If the soil fails, structures founded on or within it will fail or be impaired, regardless of how well these structures



FIGURE 1.2 Sears Tower. (© Bill Bachmann/Photo Researchers.)



FIGURE 1.3 Empire State Building. (© Rafael Macia/Photo Researchers.)



FIGURE 1.4 Taj Mahal. (© Will & Deni McIntyre/Photo Researchers.)

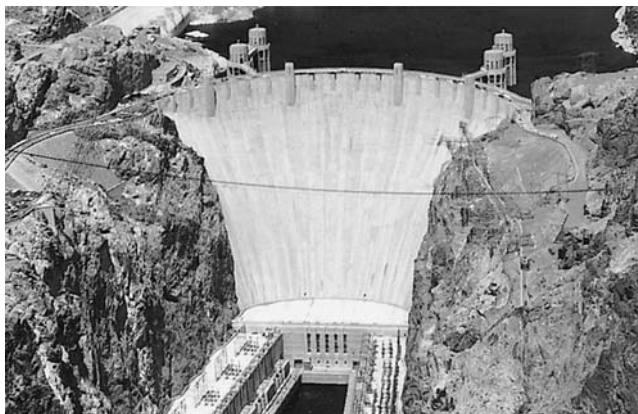


FIGURE 1.5 Hoover Dam. (Courtesy Bureau of Reclamation, U.S. Department of the Interior. Photo by E.E. Hertzog.)

are designed. Thus, successful civil engineering projects are heavily dependent on geotechnical engineering.

1.2 GEOTECHNICAL LESSONS FROM FAILURES

All structures that are founded on earth rely on our ability to design safe and economic foundations. Because of the natural vagaries of soils, failures do occur. Some failures have been catastrophic and caused severe damage to lives and properties; others have been insidious. Failures occur because of inadequate site and soil investigations; unforeseen soil and water conditions; natural hazards; poor engineering analysis, design, construction, and quality control; postconstruction activities; and usage outside the design conditions. When failures are investigated thoroughly, we obtain lessons and information that will guide us to prevent similar types of failure in the future. Some types of failure caused by natural hazards (earthquakes, hurricanes, etc.) are difficult to prevent and our efforts must be directed toward solutions that mitigate damages to lives and properties.

One of the earliest failures that was investigated and contributed to our knowledge of soil behavior is the failure of the Transcona Grain Elevator in 1913 (Fig. 1.6). Within 24 hours after loading the grain elevator at a rate of about 1 m of grain height per day, the bin house began to tilt and settle. Fortunately, the structural damage was minimal and the bin house was later restored. No borings were done to identify the soils and to obtain information on their strength. Rather, an open pit about 4 m deep was made for the foundations and a plate was loaded to determine the bearing strength of the soil.

The information gathered from the Transcona Grain Elevator failure and the subsequent detailed soil investigation was used (Peck and Bryant, 1953; Skempton, 1951) to verify the theoretical soil bearing strength. Peck and Bryant (1953) found that the applied pressure from loads imposed by the bin house and the grains was nearly equal to the calculated maximum pressure that the soil can withstand, thereby verifying the theory for calculating the bearing strength of soils. We also learn from this failure the importance of soil investigations, soils tests, and the effects of rate of loading.

The Transcona Grain Elevator was designed at a time when soil mechanics was not born. One eyewitness (White, 1953) wrote: “Soil Mechanics as a special science had hardly begun at that time. If as much had been known then as is now about the shear strength and behaviour of soils, adequate borings would have been taken and tests made and these troubles would have been avoided. We owe more to the development of this science than is generally recognized.”



FIGURE 1.6 Failure of the Transcona Grain Elevator. (Photo courtesy of Parrish and Heimbecker Limited.)

We have come a long way in understanding soil behavior since its fatherhood by Terzaghi in 1925. We continue to learn more daily though research on and experience from failures and your contribution to understanding soil behavior is needed. Join me on a journey of learning the fundamentals of soil mechanics and its applications to practical problems so that we can avoid failures or, at least, reduce the probability of their occurrence.

CHAPTER 2

GEOLOGICAL CHARACTERISTICS OF SOILS AND SOILS INVESTIGATION

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2.0 INTRODUCTION

The purpose of this chapter is to introduce you to geology, soils, and soils investigation. You will learn basic geology, descriptions of soils, and how to characterize a site for a construction project. One of the primary tasks of a geotechnical engineer is to understand the character of the soil at a site. Soils, derived from the weathering of rocks, are very complex materials and vary widely. There is no certainty that a soil will have the same properties within a few centimeters of its current location. Usually soils investigations are conducted only on a fraction of a proposed site because it would be prohibitively expensive to conduct an extensive investigation of a whole site. We then have to make estimates and judgments based on information from a limited set of observations and field and laboratory test data that will have profound effects on the performance and costs of structures constructed at a site.

When you complete this chapter, you should be able to:

- Appreciate the importance of geology in geotechnical engineering.
- Describe the formation and physical characteristics of soils.
- Plan a soils investigation.

Sample Practical Situation A bridge is required as part of a highway project. You are required to design the bridge foundation and abutment. To initiate a design of the foundation and the abutment, you have to know the geology of the site including the soil types, their spatial variations, groundwater conditions, and potential for damages from natural hazards such as earthquakes.

2.1 DEFINITIONS OF KEY TERMS

Dip is the downward separation of a bedding plane.

Faults are ground fractures.

Minerals are chemical elements that constitute rocks.

Rocks are the aggregation of minerals into a hard mass.

Soils are materials that are derived from the weathering of rocks.

Strike is the horizontal surface separation of a layer or bedding plane.

2.2 QUESTIONS TO GUIDE YOUR READING

1. Why is geology important in geotechnical engineering?
2. What is engineering soil?
3. What are the main minerals in soils?
4. What are the physical parameters that characterize soils?
5. How is soil described and classified?
6. What are the differences between coarse-grained and fine-grained soils?
7. What is a soils investigation?
8. How do you plan a soils investigation?

2.3 BASIC GEOLOGY



Concept Learning and Self-Assessment

Access www.grow.arizona.edu and search using the keyword geology to learn more about geological engineering. Access Chapter 2, Section 2.3 and take Quiz 2.3 to test your understanding.

2.3.1 Importance of Geology

Geological forces and processes often result in inhomogeneous and discontinuous formations that significantly influence the stability and costs of civil engineering works. The amount of investigation needed to characterize a site economically, the type and methods of construction, and natural geological hazards such as earthquakes, volcanic activity, and groundwater conditions are important geological factors that must be considered in the practice of geotechnical engineering. Many failures of structures, causing loss of lives and properties, have resulted from unrealized geological conditions. Consider the geology at a potential construction site in a county as shown in Fig. 2.1. To map these geological features requires applications of geophysical methods and a series of closely spaced boreholes. The precise size of each geological feature is difficult to ascertain. In building a skyscraper,

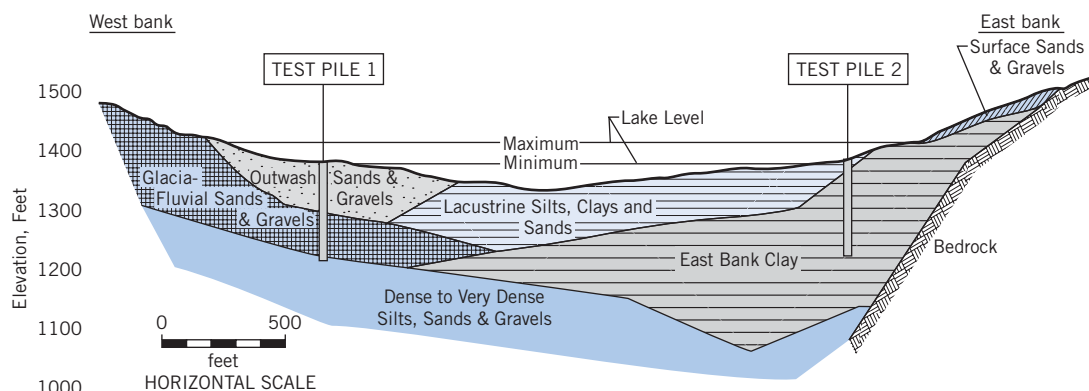


FIGURE 2.1 Soil profile at a potential construction site. (Source: McCammon and Golder, 1970.)

for example, you must have a knowledge of the geological features under and within the vicinity of the building to design a safe and economical foundation.

Most of the theories we will be using to predict and understand the response of a soil are based on the assumption of a homogeneous soil mass—one that has the same properties at all points. But, soils are rarely homogeneous as illustrated in Fig. 2.1. Thus, we will be treating soils as ideal or hypothetical materials and use statistical average properties. However, in many cases, statistical average values could mislead because a soil layer at a particular location may control the stability of a geotechnical system (e.g., a foundation).

2.3.2 Earth's Profile

Our planet Earth has an average radius of 6373 km and a mean mass density of 5.527 g/cm^3 compared with a mean mass density of soil particles of 2.7 g/cm^3 and water of 1 g/cm^3 . Studies from elastic waves generated by earthquakes showed that the earth has a core of heavy metals, mostly iron, of mass density 8 g/cm^3 surrounded by a mantle. The mantle consists of two parts, upper mantle and lower mantle. The upper mantle is solid rock while the lower mantle is molten rock. Above the upper mantle is the crust, which may be as much as 50 km thick in the continental areas (Fig. 2.2) and as thin as 7 km in oceanic areas.

2.3.3 Plate Tectonics

The crust and part of the upper mantle, about 100 km thick, make up the lithosphere. Below the lithosphere is the asthenosphere, which is about 150 km thick. The lithosphere is fragmented into about 20 large plates—large blocks of rocks—that slide against and move toward, away from, and under each other above hot molten materials in the asthenosphere. The theory governing the movements of the plates is called plate tectonics. Plate tectonics is based on *uniformitarianism*, which states that earth's forces and processes are continuous rather than catastrophic and the present is similar to the past.

The plates move relatively to each other slowly but occasionally jerk sending the energy contained in the straining rock in all directions. The energy is transmitted as shock waves. When these waves reach the surface, the ground shaking that occurs is referred to as an earthquake. The adjustment of the plates after an earthquake causes another set of shock waves that is referred to as an aftershock. The point at which the earthquake originates is called the focus and the point directly above it on the earth's surface is called the epicenter.

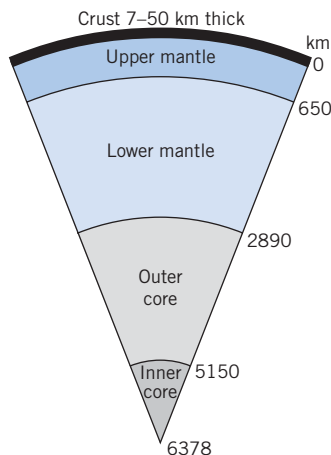


FIGURE 2.2 A sector of the earth.

The shock waves that originate from the focus separate, as they move to the earth's surface, into body waves and surface waves because these waves travel at different velocities. Body waves comprise compression, or primary, P waves, and distortional, or shear, S waves. P waves are the first to arrive at the surface followed by the S waves. Surface waves comprise Love (LQ) waves, and Raleigh (LR) waves. These surface waves have large amplitudes and long periods. The amount of seismic energy released is defined by the magnitude (M) of the earthquake. On the Richter scale, M is a logarithmic scale that ranges from 0 to 9. An earthquake of $M = 2$ is barely felt while an earthquake of $M = 7$ could cause extensive damage.

At the edges of the plates, three phenomena are of particular importance:

1. A fault zone that occurs when the plates slide past each other
2. A subduction zone that occurs when the plates move toward each other causing one plate to move beneath the other
3. A spreading zone that occurs when the plates move away from each other

2.3.4 Composition of Earth's Crust

The materials that compose the earth's crust are sediments and rock. Sediments are solid fragments of inorganic or organic material resulting from the weathering of rocks and are transported and deposited by wind, water, or ice. Rocks are classified into three groups—igneous, sedimentary, and metamorphic—based on the earth's process that forms them.

Igneous rocks are formed from magma (molten rock materials) emitted from volcanoes that have cooled and solidified. Sedimentary rocks are formed from sediments, animals and plant materials that are deposited in water or on land on the earth's surface and then subjected to pressures and heat. The heat and pressures that are involved in forming sedimentary rocks are low in comparison to igneous rocks. Metamorphic rocks are formed deep within the earth's crust from the transformation of igneous, sedimentary, and even existing metamorphic rocks into denser rocks. Their appearance and texture are variable. For engineering purposes, foliation (layering caused by parallel alignment of minerals), weak minerals, and cleavage planes are particularly important because they are planes of weakness. No melting takes place so the original chemical composition of the original rock remains unchanged. The rock texture generally becomes coarser-grained.

Sedimentary rocks are of particular importance to geotechnical engineers because they cover about 75% of the earth's surface area with an average thickness of 0.8 km. The sediments that composed sedimentary rocks may be bonded (cemented) together by minerals, chemicals, and electrical attraction or may be loose. Clastic sedimentary rocks are small pieces of rocks cemented together by minerals such as carbonates (calcite, CaCO_3) or sulphates (gypsum, $\text{CaSO}_4 [+2\text{H}_2\text{O}]$). Examples of clastic sedimentary rocks are sandstones formed from sand cemented by minerals and found on beaches and sand dunes; shales formed from clay and mud and found in lakes and swamps; and conglomerates formed from sand and gravels at the bottom of streams. Chemical sedimentary rocks are minerals such as halide (rock salt), calcite, and gypsum that have been formed from elements dissolved in water (e.g., the material found in Death Valley, California). Organic sedimentary rocks are formed from organic materials such as plants, bones, and shells. Coal is an organic sedimentary rock formed deep in the earth from the compaction of plants.

2.3.5 Discontinuities

Rock masses are seldom homogeneous and continuous. Rather, they consist of discontinuities that control the strength and displacements of the rock mass, and the stability of any structure founded on them. Discontinuities in sedimentary rocks are called bedding planes. These bedding planes are planes that separate different bodies of sedimentary deposits. In metamorphic rocks they are called foliation

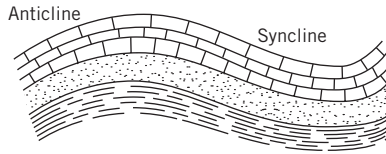


FIGURE 2.3 Simple Folding.

planes. In igneous rocks they are called joints. However, the term joint is used generically to describe most discontinuities in rock masses. The terms strike and dip are used to describe the geometry of a bedding plane. Strike is the horizontal surface separation of a layer or bedding plane. Dip is the downward separation of a bedding plane.

Rock masses may be distorted by folding. There are a variety of folds. Two simple folds (Fig. 2.3) are anticlines—rock mass folded upward (convex)—and synclines—rock mass folded upward (concave). Folding results in unequal distribution of stresses within the rock mass and can cause major problems in civil engineering construction through uneven release of stresses.

The movements of the plates cause ground fractures called faults. The three predominant faults are normal, thrust, and strike/slip. Tension causes normal fault (Fig. 2.4a). An example of a normal fault is the Teton Mountain in Wyoming. Compression causes thrust or reverse fault (Fig. 2.4b). Shear causes strike/slip fault (Fig. 2.4c). An example of a strike/slip fault is the San Andreas Fault in California. Faults are rarely simple. They normally consist of a combination different types of faulting.

2.3.6 Geologic Cycle and Geological Time

The formation of rocks and sediments is a continuous process known as the geologic cycle. Sediments are transformed by heat and pressure into rocks and then the rocks are eroded into sediments. The cycle has neither a starting point nor an ending point. There are three main geological principles, due to Nicolaus Steno (1638–1687), that govern the geologic cycle.

1. *Principle of original horizontality*, which states that sediments are deposited in layers parallel to the earth's surface.
2. *Principle of original continuity*, which states that depositions are sheetlike and are only terminated in contact with existing solid surfaces. Deformities occur from subsequent earth's forces.

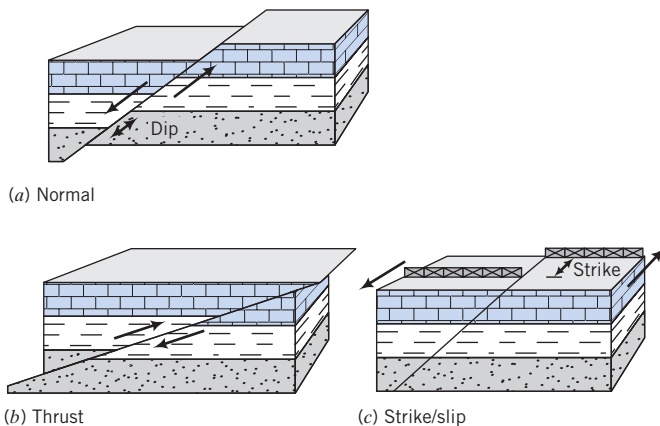


FIGURE 2.4 Three types of faults: (a) normal, (b) thrust, and (c) strike/slip.



FIGURE 2.5 Layered sediments as seen in the Grand Canyon. The youngest layer is the topmost layer. The deformation of the layers depends on, among other factors, the material properties, confinement pressures, strain rate, and temperatures.

3. *Principle of superposition*, which states that the age of a deposition is directly related to the order of deposition. Older layers are generally below younger layers.

Evidence of these principles is clearly seen in the Grand Canyon (Fig. 2.5).

Geological time is the dating of past events. The ages of earth’s materials are measured by radioactive methods. Potassium-Argon dating (Potassium is found in igneous rocks and it is transformed into Argon from radioactivity) and Rubidium-Strontium dating (Rubidium is found in metamorphic rocks and is transformed into Strontium from radioactivity) are the popular and the most useful radioactive dating methods. The time periods (million years), Fig. 2.6, have been assigned based on past bioactivity, but mainly on Carbon 14 (C_{14}) dating. Geological dating provides estimates of the frequency of occurrence of volcanic eruptions, earthquakes, landslides, floods, erosion, and temperature variations.

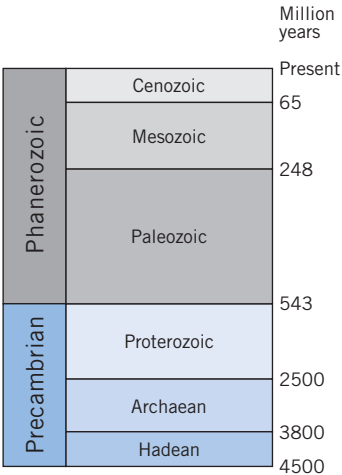


FIGURE 2.6 Geological time.

The essential points are:

- 1. Knowledge of geology is important for the successful practice of geotechnical engineering.*
- 2. The earth's surface (lithosphere) is fractured into about 20 mobile plates. Interaction of these plates causes volcanic activities and earthquakes.*
- 3. The three groups of rocks are igneous, sedimentary, and metamorphic. Igneous rocks are formed from magma (molten rock materials) emitted from volcanoes that have cooled and solidified. Sedimentary rocks are formed from sediments, animals, and plant materials that are deposited in water or on land on the earth's surface and then subjected to pressures and heat. Metamorphic rocks are formed deep within the earth's crust from the transformation of igneous and sedimentary rocks into denser rocks. They are foliated, have weak minerals and cleavage planes.*
- 4. Sedimentary rocks are of particular importance to geotechnical engineers because they cover about 75% of the earth's surface area.*
- 5. Rock masses are inhomogeneous and discontinuous.*

What's next. . . Now that you have a basic knowledge of geology, we will begin our study of engineering soils.

2.4 COMPOSITION OF SOILS



Interactive Concept Learning and Self-Assessment

Access Chapter 2, Section 2.4 on the CD for an interactive lesson with visualizations to learn about the composition of soils. Take Quiz 2.4 to test your understanding.

2.4.1 Soil Formation

Soils are formed from the physical and chemical weathering of rocks. Physical weathering involves reduction of size without any change in the original composition of the parent rock. The main agents responsible for this process are exfoliation, unloading, erosion, freezing, and thawing. Chemical weathering causes both reductions in size and chemical alteration of the original parent rock. The main agents responsible for chemical weathering are hydration, carbonation, and oxidation. Often, chemical and physical weathering take place in concert.

Soils that remain at the site of weathering are called residual soils. These soils retain many of the elements that comprise the parent rock. Alluvial soils, also called fluvial soils, are soils that were transported by rivers and streams. The composition of these soils depends on the environment under which they were transported and is often different from the parent rock. The profile of alluvial soils usually consists of layers of different soils. Much of our construction activity has been and is occurring in and on alluvial soils. Glacial soils are soils that were transported and deposited by glaciers. Marine soils are soils deposited in a marine environment.

2.4.2 Soil Types

Common descriptive terms such as gravels, sands, silts, and clays are used to identify specific textures in soils. We will refer to these soil textures as soil types; that is, sand is one soil type, clay is another.

Texture refers to the appearance or feel of a soil. Sands and gravels are grouped together as coarse-grained soils. Clays and silts are fine-grained soils. Coarse-grained soils feel gritty and hard. Fine-grained soils feel smooth. The coarseness of soils is determined from knowing the distribution of particle sizes, which is the primary means of classifying coarse-grained soils. To characterize fine-grained soils, we need further information on the types of minerals present and their contents. The response of fine-grained soils to loads, known as the mechanical behavior, depends on the type of predominant minerals present.

Currently, many soil descriptions and soil types are in usage. A few of these are listed below.

- *Alluvial soils* are fine sediments that have been eroded from rock, transported by water, and have settled on river and stream beds.
- *Calcareous soil* contains calcium carbonate and effervesces when treated with hydrochloric acid.
- *Caliche* consists of gravel, sand, and clay cemented together by calcium carbonate.
- *Colloidal soils* are soils found at the base of mountains that have been eroded by the combination of water and gravity.
- *Eolian* soils are sand-sized particles deposited by wind.
- *Expansive soils* are clays that undergo large volume changes from cycles of wetting and drying.
- *Glacial soils* are mixed soils consisting of rock debris, sand, silt, clays, and boulders.
- *Glacial till* is a soil that consists mainly of coarse particles.
- *Glacial clays* are soils that were deposited in ancient lakes and subsequently frozen. The thawing of these lakes revealed soil profiles of neatly stratified silt and clay, sometimes called varved clay. The silt layer is light in color and was deposited during summer periods while the thinner, dark clay layer was deposited during winter periods.
- *Gypsum* is calcium sulphate formed under heat and pressure from sediments in ocean brine.
- *Lacustrine* soils are mostly silts and clays deposited in glacial lake waters.
- *Lateritic* soils are residual soils that are cemented with iron oxides and are found in tropical regions.
- *Loam* is a mixture of sand, silt, and clay that may contain organic material.
- *Loess* is a wind blown, uniform fine-grained soil.
- *Marine soils* are sand, silts, and clays deposited in salt or brackish water.
- *Mud* is clay and silt mixed with water into a viscous fluid.

2.4.3 Clay Minerals

Minerals are crystalline materials and make up the solids constituent of a soil. The mineral particles of fine-grained soils are platy. Minerals are classified according to chemical composition and structure. Most minerals of interest to geotechnical engineers are composed of oxygen and silicon—two of the most abundant elements on earth. Silicates are a group of minerals with a structural unit called the silica tetrahedron. A central silica cation (positively charged ion) is surrounded by four oxygen anions (negatively charged ions), one at each corner of the tetrahedron (Fig. 2.7a). The charge on a single tetrahedron is -4 and to achieve a neutral charge, cations must be added or single tetrahedrons must be linked to each other sharing oxygen ions. Silicate minerals are formed by addition of cations and interactions of tetrahedrons. Silica tetrahedrons combine to form sheets, called silicate sheets, which are thin layers of silica tetrahedrons in which three oxygen ions are shared between adjacent tetrahedrons (Fig. 2.7b). Silicate sheets may contain other structural units such as alumina sheets.

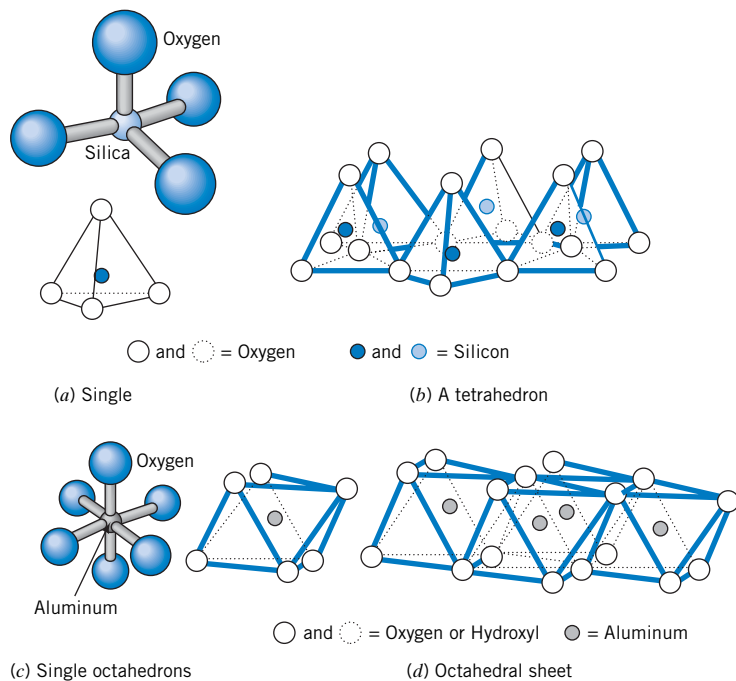


FIGURE 2.7 (a) Silica tetrahedrons, (b) silica sheets, (c) single aluminum octahedrons, and (d) aluminum sheets.

Alumina sheets are formed by combination of alumina minerals, which consists of an aluminum ion surrounded by six oxygen or hydroxyl atoms in an octahedron (Fig. 2.7c,d).

The main groups of crystalline materials that make up clays are the minerals: kaolinite, illite, and montmorillonite. Kaolinite has a structure that consists of one silica sheet and one alumina sheet bonded together into a layer about 0.72 nm thick and stacked repeatedly (Fig. 2.8a). The layers are held together by hydrogen bonds. Tightly stacked layers results from numerous hydrogen bonds. Kaolinite is common in clays in humid tropical regions. Illite consists of repeated layers of one alumina sheet sandwiched by two silicate sheets (Fig. 2.8b). The layers, each of thickness 0.96 nm, are held together by potassium ions.

Montmorillonite has a structure similar to illite, but the layers are held together by weak van der Waals forces and exchangeable ions (Fig. 2.8c). Water can easily enter the bond and separate the layers in montmorillonite, causing swelling. Montmorillonite is often called a swelling or expansive clay.

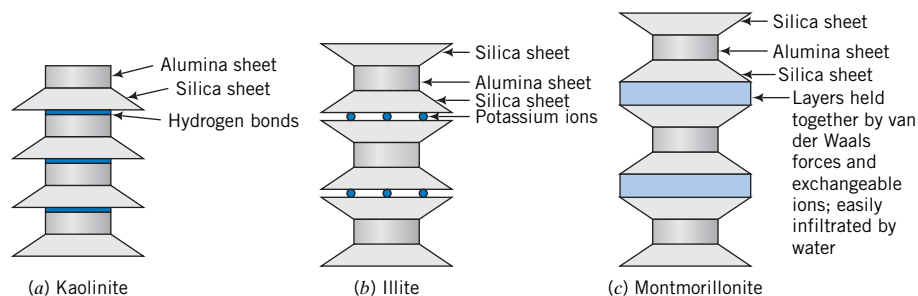


FIGURE 2.8 Structure of kaolinite, illite, and montmorillonite.

2.4.4 Surface Forces and Adsorbed Water

If we subdivide a body, the ratio of its surface area to its volume increases. For example, a cube of sides 1 cm has a surface area of 6 cm^2 . If we subdivide this cube into smaller cubes of sides 1 mm, the original volume is unchanged but the surface area increases to 60 cm^2 . The surface area per unit mass (specific surface) of sands is typically 0.01 m^2 per gram, while for clays, it is as high as 1000 m^2 per gram (montmorillonite). The specific surface of kaolinite ranges from 10 to 20 m^2 per gram while that of illite ranges from 65 to 100 m^2 per gram. The surface area of 45 grams of illite is equivalent to the area of a football field. Because of their large surfaces, surface forces significantly influence the behavior of fine-grained soils compared to coarse-grained soils.

The surface charges on fine-grained soils are negative (anions). These negative surface charges attract cations and the positively charged side of water molecules from surrounding water. Consequently, a thin film or layer of water, called adsorbed water, is bonded to the mineral surfaces. The thin film or layer of water is known as the diffuse double layer (Fig. 2.9). The largest concentration of cations occurs at the mineral surface and decreases exponentially with distance away from the surface (Fig. 2.9).

Surface forces on clay particles are of two types. One type, called attracting forces, is due to London–van der Waals forces. These forces are far reaching and decrease inversely proportional to l^3 (l is the distance between two particles). The other type, called repelling forces, is due to the diffuse double layer. Around each particle is an ionic cloud. When two particles are far apart, the electric charge on each is neutralized by equal and opposite charge of the ionic cloud around it. When the particles move closer together, such that the clouds mutually penetrate each other, the negative charges on the particles cause repulsion. Access Section 2.4 on the CD to view an animation of surface forces.

Drying of most soils, with the exception of gypsum, using an oven for which the standard temperature is $105 \pm 5^\circ\text{C}$, cannot remove the adsorbed water. The adsorbed water influences the way a soil behaves. For example, plasticity, which we will deal with in Section 3.5, in soils is attributed to the adsorbed water. Toxic chemicals that seep into the ground contaminate soil and groundwater. The surface chemistry of fine-grained soils is important in understanding the migration, sequestration, re-release, and ultimate removal of toxic compounds from soils.

Our main concern in this book is on the physical and mechanical properties of soils. Accordingly, we will not deal with the surface chemistry of fine-grained soils. You may refer to Mitchell (1993) for further information on the surface chemistry of fine-grained soils that are of importance to geotechnical and geoenvironmental engineers.

2.4.5 Soil Fabric

Soil particles are assumed rigid. During deposition, the mineral particles are arranged into structural frameworks that we call soil fabric (Fig. 2.10). Each particle is in random contact with neighboring

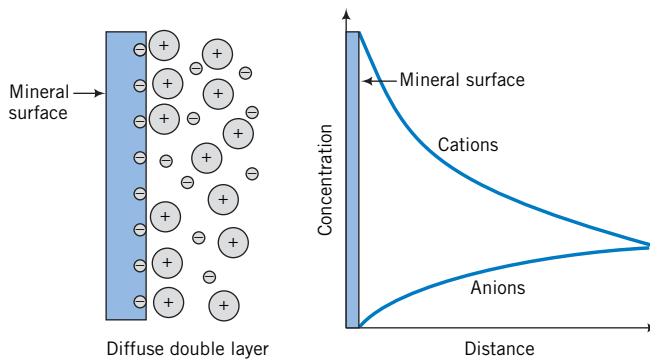


FIGURE 2.9 Diffuse double layer.

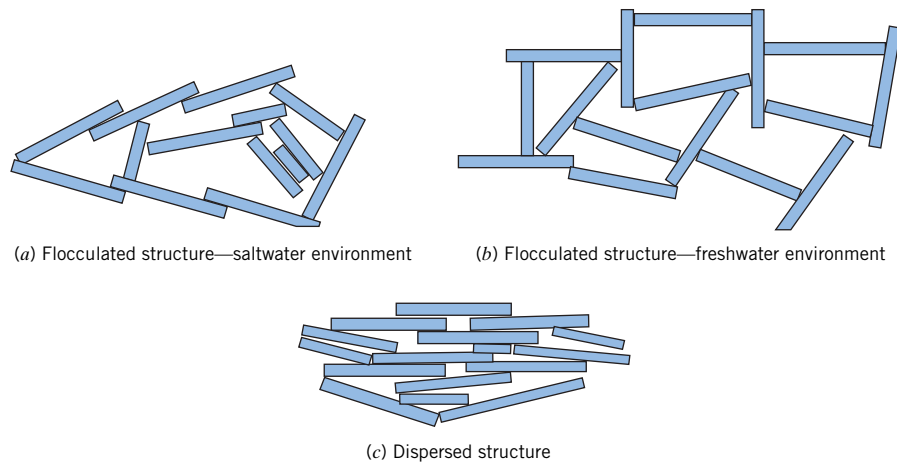


FIGURE 2.10 Soil fabric.

particles. The environment under which deposition occurs influences the structural framework that is formed. In particular, the electrochemical environment has the greatest influence on the kind of soil fabric that is formed during deposition of fine-grained soils.

Two common types of soil fabric—flocculated and dispersed—are formed during soil deposition of fine-grained soils as shown schematically in Fig. 2.10. A flocculated structure, formed under a saltwater environment, results when many particles tend to orient parallel to each other. A flocculated structure, formed under a freshwater environment, results when many particles tend to orient perpendicular to each other. A dispersed structure occurs when a majority of the particles orient parallel to each other.

Any loading (tectonic or otherwise) during or after deposition permanently alters the soil fabric or structural arrangement in a way that is unique to that particular loading condition. Consequently, the history of loading and changes in the environment is imprinted in the soil fabric. The soil fabric is the brain; it retains the memory of the birth of the soil and subsequent changes that occur.

The spaces between the mineral particles are called voids, which may be filled with liquids (essentially water), gases (essentially air), and cementitious materials (e.g., calcium carbonate). Voids occupy a large proportion of the soil volume. Interconnected voids form the passageway through which water flows in and out of soils. If we change the volume of voids, we will cause the soil to either compress (settle) or expand (dilate). Loads applied by a building, for example, will cause the mineral particles to be forced closer together, reducing the volume of voids and changing the orientation of the structural framework. Consequently, the building settles. The amount of settlement depends on how much we compress the volume of voids. The rate at which the settlement occurs depends on the interconnectivity of the voids. Free water, not the adsorbed water, and/or air trapped in the voids must be forced out for settlement to occur. The decrease in volume, which results in settlement of buildings and other structures, is usually very slow (almost ceaseless) in fine-grained soils because these soils have large surface areas compared with coarse-grained soils. The larger surface areas provide greater resistance to the flow of water through the voids.

If the rigid (mostly quartz) particles of coarse-grained soils can be approximated by spheres, then the loosest packing (maximum voids space) would occur when the spheres are stacked one on top of another (Fig. 2.11a). The densest packing would occur when the spheres are packed in a staggered pattern as shown in Fig. 2.11b. Real coarse-grained soils consist of an assortment of particle sizes and shapes and consequently the packing is random.

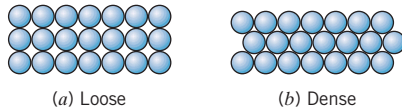


FIGURE 2.11 Loose and dense packing of spheres.

2.4.6 Comparison of Coarse-Grained and Fine-Grained Soils for Engineering Use

Coarse-grained soils have good load-bearing capacities and good drainage qualities, and their strength and volume change characteristics are not significantly affected by change in moisture conditions. They are practically incompressible when dense, but significant volume changes can occur when they are loose. Vibrations accentuate volume changes in loose coarse-grained soils by rearranging the soil fabric into a dense configuration.

Fine-grained soils have poor load-bearing capacities compared with coarse-grained soils. Fine-grained soils are practically impermeable, change volume and strength with variations in moisture conditions, and are frost susceptible. The engineering properties of coarse-grained soils are controlled mainly by the grain size of the particles and their structural arrangement. The engineering properties of fine-grained soils are controlled by mineralogical factors rather than grain size. Thin layers of fine-grained soils, even within thick deposits of coarse-grained soils, have been responsible for many geotechnical failures and therefore you need to pay special attention to fine-grained soils.

In this book, we will deal with soil as a construction and a foundation material. We will not consider soils containing organic material or the parent material of soils—rock. We will label our soils as engineering soils to distinguish our consideration of soils from geologists, agronomists, and soil scientists, who have additional interests in soils not related to construction activities.

The essential points are:

- 1. Soils are derived from the weathering of rocks and are commonly described by textural terms such as gravels, sands, silts, and clays.***
- 2. Physical weathering causes reduction in size of the parent rock without change in its composition.***
- 3. Chemical weathering causes reduction of size and chemical composition different from the parent rock.***
- 4. Particle size is used to distinguish various soil textures.***
- 5. Clays are composed of three main types of mineral—kaolinite, illite, and montmorillonite.***
- 6. The clay minerals consist of silica and alumina sheets that are combined to form layers. The bonds between layers play a very important role in the mechanical behavior of clays. The bond between the layers in montmorillonite is very weak compared with kaolinite and illite. Water can easily enter between the layers in montmorillonite, causing swelling.***
- 7. A thin layer of water, called adsorbed water, is bonded to the mineral surfaces of soils. This layer significantly influences the physical and mechanical characteristics of fine-grained soils.***
- 8. Fine-grained soils have much larger surface areas than coarse-grained soils and are responsible for the major physical and mechanical differences between coarse-grained and fine-grained soils.***
- 9. The engineering properties of fine-grained soils depend mainly on mineralogical factors.***

What's next... You should now have a general idea of engineering soils. Soils are observed and recovered during a soils investigation of a proposed site. A soils investigation is an essential part of the design and construction of a proposed structural system (buildings, dams, roads and highways, etc.). You will now be given a brief introduction to a site or soils investigation.

2.5 SOILS INVESTIGATION



Concept Learning

Access Chapter 2, Section 2.5 on the CD to interactively learn about soil investigation. You can view images of real soils, soil sampling tools, and a digital movie on soil sampling. Take Quiz 2.5 to test your understanding.

2.5.1 Purposes of a Soils Investigation

A soils investigation program is necessary to provide information for design and construction and for environmental assessment. The purposes of a soils investigation are:

1. To evaluate the general suitability of the site for the proposed project
2. To enable an adequate and economical design to be made
3. To disclose and make provision for difficulties that may arise during construction due to ground and other local conditions

2.5.2 Phases of a Soils Investigation

The scope of a soils investigation depends on the type, size, and importance of the structure, the client, the engineer's familiarity with the soils at the site, and local building codes. Structures that are sensitive to settlement such as machine foundations and high-use buildings usually require a thorough soils investigation compared to a foundation for a house. A client may wish to take a greater risk than normal to save money and set limits on the type and extent of the site investigation. If the geotechnical engineer is familiar with a site, he/she may undertake a very simple soils investigation to confirm his/her experience. Some local building codes have provisions that set out the extent of a site investigation. It is mandatory that a visit be made to the proposed site.

In the early stages of a project, the available information is often inadequate to allow a detailed plan to be made. A site investigation must be developed in phases.

Phase I. Collection of available information such as a site plan, type, size, and importance of the structure, loading conditions, previous geotechnical reports, topographic maps, air photographs, geologic maps, and newspaper clippings.

Phase II. Preliminary reconnaissance or a site visit to provide a general picture of the topography and geology of the site. It is necessary that you take with you on the site visit all the information gathered in Phase I to compare with the current conditions of the site.

Phase III. Detailed soils exploration. The objectives of a detailed soil exploration are:

1. To determine the geological structure, which should include the thickness, sequence, and extent of the soil strata.
2. To determine the groundwater conditions.

3. To obtain disturbed and undisturbed samples for laboratory tests.
4. To conduct in situ tests.

Phase IV. Write a report. The report must contain a clear description of the soils at the site, methods of exploration, soil profile, test methods and results, and the location of the groundwater. You should include information and/or explanations of any unusual soil, water-bearing stratum, and soil and groundwater condition that may be troublesome during construction.

The essential points are:

1. *A soils investigation is necessary to determine the suitability of a site for its intended purpose.*
2. *A soils investigation is conducted in phases. Each preceding phase affects the extent of the next phase.*

What's next... In the next section, we will study how a soils exploration program (Phase III) is normally conducted.

2.6 SOILS EXPLORATION PROGRAM



Concept Learning and Self-Assessment

Access Chapter 2, Section 2.6 to learn about soil exploration and view a movie on soil sampling. Take Quiz 2.6 to test your understanding.

A soils exploration program usually involves test pits and/or soil borings (boreholes). During the site visit (Phase II), you should work out most of the soils exploration program. A detailed soils exploration consists of:

1. Determining the need for and extent of geophysical exploration.
2. Preliminary location of each borehole and/or test pits.
3. Numbering of the boreholes or test pits.
4. Planned depth of each borehole or test pit.
5. Methods and procedures for advancing the boreholes.
6. Sampling instructions for at least the first borehole. The sampling instructions must include the number of samples and possible locations. Changes in the sampling instructions often occur after the first borehole.
7. Determining the need for and type of in situ tests.
8. Requirements for groundwater observations.

2.6.1 Soil Exploration Methods

Access to the soil may be obtained by the following methods:

- Geophysical methods—nondestructive techniques used to provide information on soils, rocks, hydrological, and environmental conditions. Popular methods are ground penetrating radar (GPR), seismic surveys, and electromagnetic surveys.

- Trial pits or test pits.
- Hand or powered augers.
- Wash boring.
- Rotary rigs.

The advantages and disadvantages of each of these methods are shown in Table 2.1.

2.6.2 Soil Identification in the Field

In the field, the predominant soil types based on texture are identified by inspection. Gravels and sands are gritty and the individual particles are visible. Silts easily crumble and water migrates to the surface on application of pressure. Clays fail this water migration test since water flows very slowly through clays. Clays feel smooth, greasy, and sticky to the touch when wet but are very hard and strong when dry.

Common descriptive terms and methods of identification are as follows.

Color: Color is not directly related to engineering properties of soils, but is related to soil mineralogy and texture.

Gray and bluish: unoxidized soils

White and cream: calcareous soils

Red and yellow: oxidized soil

Black and dark brown: soil containing organic matter

Moisture: Appearance due to water is described as wet, dry, and moist.

Structure:

Homogeneous: Color and texture feel the same throughout.

Nonhomogeneous: Color and texture vary.

Shape: Angular, subangular, subrounded, rounded, flaky.

Weathering: Fresh, decomposed, weathered.

Carbonate: Effervesces with acid. Add a small amount of hydrochloric acid and check if soil effervesces. If it does, it contains carbonate.

Smell: Organic soils give off a strong odor that intensifies with heat. Nonorganic soils have a subtle odor with the addition of water.

Feel: Use feel to distinguish between sand, silts and clays.

Sand has a gritty feel.

Silt has a rough feel similar to fine sand paper.

Clay feels smooth and greasy. It sticks to fingers and is powdery when dry.

Consistency: Very stiff: Finger pressure barely dents soil but cracks under significant pressure.

Stiff: Finger pressure dents soil.

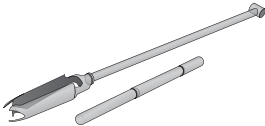
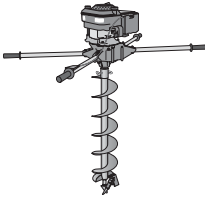
Firm: Soil can be molded using strong finger pressure.

Soft: Easily molded by finger.

Very soft: Soil flows between fingers when fist is closed.

Dilatancy: Place a small amount of the soil in your palm and shake horizontally. Then strike it with the other hand. If the surface is slurry and water appears, the soil probably has a large amount of silt.

TABLE 2.1 Advantages and Disadvantages of Soil Exploration Methods

Method	Advantages	Disadvantages
Geophysical methods Ground penetration radar Seismic surveys Electromagnetic surveys	• Nondestructive • Quick • Provide stratigraphy • Relatively inexpensive	• No soil samples • Limited design parameters
Test pits A pit is dug either by hand or by a backhoe.	 • Cost effective • Provide detailed information of stratigraphy • Large quantities of disturbed soils are available for testing • Large blocks of undisturbed samples can be carved out from the pits • Field tests can be conducted at the bottom of the pit	• Depth limited to about 6 m • Deep pits uneconomical • Excavation below groundwater and into rock difficult and costly • Too many pits may scar site and require backfill soils
Hand augers The auger is rotated by turning and pushing down on the handlebar.	 • Cost effective • Not dependent on terrain • Portable • Low headroom required • Used in uncased holes • Groundwater location can easily be identified and measured	• Depth limited to about 6 m • Labor intensive • Undisturbed samples can be taken only for soft clay deposit • Cannot be used in rock, stiff clays, dry sand, or caliche soils
Power augers Truck mounted and equipped with continuous flight augers that bore a hole 100 to 250 mm in diameter. Augers can have a solid or hollow stem.	 • Quick • Used in uncased holes • Undisturbed samples can be obtained quite easily • Drilling mud not used • Groundwater location can easily be identified	• Depth limited to about 15 m; At greater depth drilling becomes difficult and expensive • Site must be accessible to motorized vehicle
Wash boring Water is pumped to bottom of borehole and soil washings are returned to surface. A drill bit is rotated and dropped to produce a chopping action.	• Can be used in difficult terrain • Low equipment costs • Used in uncased holes	• Depth limited to about 30 m • Slow drilling through stiff clays and gravels • Difficulty in obtaining accurate location of groundwater level • Undisturbed soil samples cannot be obtained
Rotary drills A drill bit is pushed by the weight of the drilling equipment and rotated by a motor.	• Quick • Can drill through any type of soil or rock • Can drill to depths of 7500 m • Undisturbed samples can easily be recovered	• Expensive equipment • Terrain must be accessible to motorized vehicle • Difficulty in obtaining location of groundwater level • Additional time required for setup and cleanup

Packing: Coarse-grained soils are described as:

Very loose: collapses with slight disturbance, open structure

Loose: collapses upon disturbance, open structure

Medium dense: indents when pushed firmly

Dense: barely deforms when pushed by feet or by stomping

Very dense: impossible to depress with stomping

2.6.3 Depth of Boreholes

The depths of boreholes should cover the zone of soil that will be affected by the structural loads. There is no fixed rule to follow. In most cases, the depths of boreholes are governed by experience based on the geological character of the ground, the importance of the structure, the structural loads, and the availability of equipment. Some guidelines are:

1. In compressible soils such as clays, the borings should penetrate at least between 1 and 3 times the width of the proposed foundation or until the stress increment due to the heaviest foundation load is less than 10%, whichever is greater.
2. In very stiff clays, borings should penetrate 5–7 m to prove that the thickness of the stratum is adequate.
3. Borings should penetrate at least 3 m into rock.
4. Borings must penetrate below any fills or very soft deposits below the proposed structure.
5. The minimum depth of boreholes should be 6 m unless bedrock or very dense material is encountered.

2.6.4 Soil Sampling

The objective of soil sampling is to obtain soils of satisfactory size with minimum disturbance for observations and laboratory tests. Soil samples are usually obtained by attaching an open-ended thin-walled tube—called a Shelby tube or, simply, a sampling tube—to drill rods and forcing it down into the soil.

The tube is carefully withdrawn, hopefully, with the soil inside it. Soil disturbances occur from several sources during sampling, such as friction between the soil and the sampling tube, the wall thickness of the sampling tube, the sharpness of the cutting edge, and care and handling during transportation of the sample tube. To minimize friction, the sampling tube should be pushed instead of driven into the ground.

Sampling tubes that are in common use have been designed to minimize sampling disturbances. One measure of the effects of sampler wall thickness is the recovery ratio defined as L/z , where L is the length of the sample and z is the distance that the sampler was pushed. Higher wall thickness leads to a greater recovery ratio and greater sampling disturbance.

One common type of soil sampler is the “Shelby tube,” which is a thin-walled seamless steel tube of diameter 50 or 75 mm and length of 600–900 mm (Fig. 2.12a). Another popular sampler is the “standard” sampler, also known as the split spoon sampler, which has an inside diameter of 35 mm and an outside diameter of 51 mm (Fig. 2.12b). The sampler has a split barrel that is held together using a screw-on driving shoe at the bottom end and a cap at the upper end. In some countries, a steel liner is used inside the sampler. But in the United States, it is standard practice not to use this liner. Consequently, the soil sample has a greater diameter. The results of SPT (see Section 2.6.7a) are different for lined and unlined samplers. The thicker wall of the standard sampler permits higher

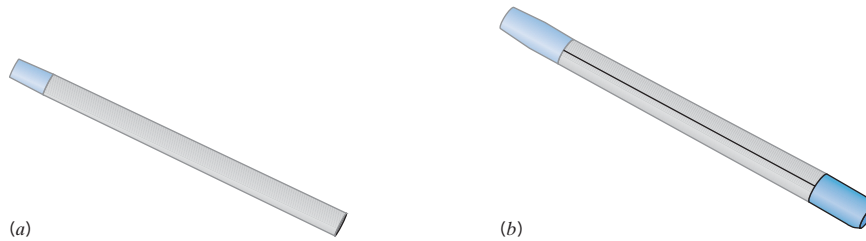


FIGURE 2.12 A thin-walled tube and a split tube sampler.

driving stresses than the Shelby tube but does so at the expense of higher levels of soil disturbances. Split spoon samples are disturbed. They are used for visual examination and for classification tests.

2.6.5 Groundwater Conditions

The upper surface of the groundwater is called the water table. Many construction failures, court battles, and construction cost overruns are due to the nonidentification or nondisclosure of groundwater conditions at a site. The water table invariably fluctuates depending on environmental conditions (e.g., rainfall patterns, winter rains, monsoons, drought), human activities (e.g., pumping groundwater from wells and draw down during construction), and geological conditions. When groundwater is trapped in a permeable soil layer overlaid by an impervious layer, the pressure of the groundwater could be considerable. If a borehole is made such that the reduction of soil pressure (due to soil removal) is greater than the pressure of the water trapped in the permeable stratum, the trapped groundwater may break out and shoot up above the ground surface. This is called artesian condition. If the water just reaches the ground surface, or stays below, we have a subartesian condition. It is particularly important to identify and disclose these conditions because, during construction, failure can occur with possible loss of lives and properties. You should identify not only the groundwater level and any special conditions (e.g., artesian condition) but also the possible range of groundwater level fluctuations.

2.6.6 Soils Laboratory Tests

Samples are normally taken from the field for laboratory tests to characterize the physical and mechanical (strength and deformation) properties. These parameters are used to design foundations and to determine the use of soils as a construction material. Disturbed samples such as from a standard sampler are usually used for visual inspection and for tests to determine the physical properties such as grain size and shape, and plasticity. Undisturbed samples such as from a thin-walled sampler are used for both physical and mechanical properties. Tests results, especially those that relate to the mechanical properties, are strongly affected by sampling, handling, transportation, and sample preparation disturbances. Care must therefore be exercised to protect the intact condition of the soil samples. Wax is often used to coat the soil samples to prevent moisture losses.

2.6.7 In Situ or Field Tests

Over the years, several in situ testing devices have emerged to characterize the soil and to measure strength and deformation properties. The most popular devices are:

1. Vane shear test (VST)
2. Standard penetration test (SPT)

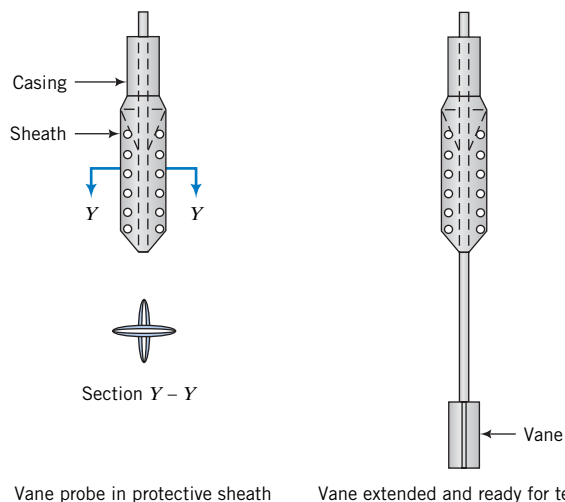


FIGURE 2.13 Shear vane tester.

3. Cone penetrometer test (CPT)
4. Pressuremeter test (PMT)
5. Flat plate dilatometer (DMT)

(a) Vane Shear Test (VST)—ASTM D 2573 The shear vane device consists of four thin metal blades welded orthogonally (90°) to a rod (Fig. 2.13). The vane is pushed, usually from the bottom of a borehole, to the desired depth. A torque is applied at a rate of 6° per minute by a torque head device located above the soil surface and attached to the shear vane rod.

After the maximum torque is obtained, the shear vane is rotated an additional 8 to 10 revolutions to measure the residual torque, T_{res} . The ratio of the maximum torque to the residual torque is the soil sensitivity, S_t , where

$$S_t = \frac{T_{\text{max}}}{T_r} \quad (2.1)$$

Sensitivity is a measure of the reduction of undrained shear strength due to soil disturbance. The results of a vane shear test are displayed as undrained or vane shear strength versus depth.

The VST is simple, inexpensive, quick to perform, and the equipment is widely available. The insertion of the vane causes soil remolding. Higher blade thickness results in greater remolding and lower soil strengths. The blade thickness should not exceed 5% of the vane diameter. Errors in the measurements of the torque include excessive friction, variable rotation, and calibration. The VST cannot be used for coarse-grained soils and very stiff clays.

(b) The Standard Penetration Test (SPT)—ASTM D 1586 The standard penetration test (SPT) was developed circa 1927 and it is perhaps the most popular field test. The SPT is performed by driving a standard split spoon sampler into the ground by blows from a drop hammer of mass 63.5 kg falling 760 mm (Fig. 2.14). The sampler is driven 152 mm (6 in.) into the soil at the bottom of a borehole, and the number of blows (N) required to drive it an additional 304 mm is counted. The number of blows (N) is called the standard penetration number.

The word “standard” is a misnomer for the standard penetration test. Several methods are used in different parts of the world to release the hammer. Also, different types of anvil, rods, and rod lengths are prevalent. Various corrections are applied to the N values to account for energy losses,

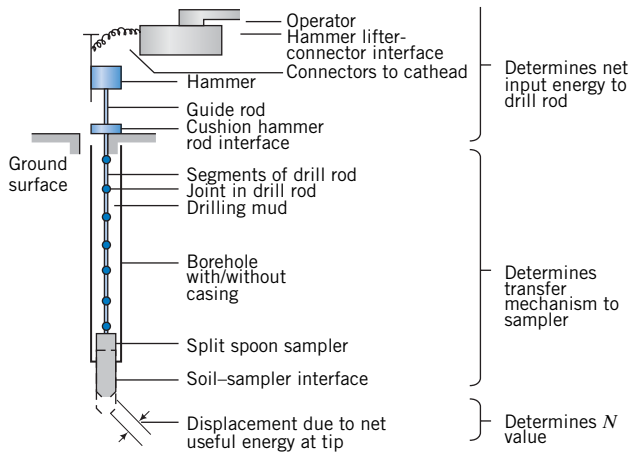


FIGURE 2.14 Standard penetration test.

overburden pressure, rod length, and so on. It is customary to correct the N values to a rod energy ratio of 60%. The rod energy ratio is the ratio of the energy delivered to the split spoon sampler to the free-falling energy of the hammer. The corrected N values are denoted as N_{60} , and given as

$$N_{60} = N \left(\frac{ER_r}{60} \right) \quad (2.2)$$

where ER_r is the energy ratio. Typical values for ER_r in North America are 45% for donut hammers released using two rope turns, 55% for safety hammers released using two rope turns, and between 55% and 83% for automatic hammers released using a trip mechanism. For the United Kingdom, $ER_r = 50\%$ for safety hammers and 60% for automatic hammers. Correction factors for rod lengths, sampler type, and borehole diameter as suggested by Skempton (1986) are given in Table 2.2.

We can write a composite correction factor, C_{rsd} , for the correction factors given in Table 2.2 as

$$C_{rsd} = C_r C_s C_d \quad (2.3)$$

TABLE 2.2 Correction Factors for Rod Length, Sampler Type, and Borehole Size

Correction Factor	Item	Correction factor
C_r	Rod length (below anvil)	
	≥10 m	1.0
	6–10 m	0.95
	4–6 m	0.85
C_s	3–4 m	0.70
	Standard sampler	1.0
	U.S. sampler without liners	1.2
C_d	Borehole diameter:	
	65–115 mm	1.0
	150 mm	1.05
	200 mm	1.15

Skempton (1986).

TABLE 2.3 Compactness of Coarse-Grained Soils Based on N Values

N	Compactness
0–4	Very loose
4–10	Loose
10–30	Medium
30–50	Dense
>50	Very dense

The corrected N value is

$$N_{\text{cor}} = C_{\text{rsd}} N \left(\frac{ER}{60} \right) \quad (2.4)$$

Eq. (2.4) gives only a partially corrected N value. Additional correction factors will be discussed in Chapter 7. Compactness of coarse-grained soils based on N values is given in Table 2.3.

The SPT is very useful for determining changes in stratigraphy and locating bedrock. Also, you can inspect the soil in the split spoon sampler to describe the soil profile and extract disturbed samples for laboratory tests.

The SPT is simple and quick to perform. The equipment is widely available and can penetrate dense materials. SPT results have been correlated with engineering properties of soils, bearing capacity, and settlement of foundations. Most of these correlations are, however, weak. There are multiple sources of errors including test performance and the use of nonstandard equipment. Test performance errors include the method of lifting and dropping the hammer, improper cleaning of the bottom of the borehole before the test commences, and not maintaining the groundwater level, if one is encountered. These errors give N values that are not representative of the soil. SPT tests are unreliable for coarse gravel, boulders, soft clays, silts, and mixed soils containing boulders, cobbles, clays, and silts.

EXAMPLE 2.1 Correcting SPT Values

The blow counts for an SPT test at a depth of 6 m in a coarse-grained soil at every 0.152 m are 8, 12, 15. A donut hammer and a standard sampler were used in a borehole 150 mm diameter. (1) Determine the N value; (2) correct the N value for rod length, sampler type, borehole size, and energy ratio to 60%; (3) make a preliminary description of the compactness of the soil.

Strategy The N value is the sum of the blow counts for the last 0.304 m of penetration. Just add the last two blow counts.

Solution 2.1

Step 1: Add the last two blow counts.

$$N = 12 + 15 = 27$$

Step 2: Apply correction factors.

From Table 2.2, $C_r = 0.95$ for rod length of 6 m, $C_s = 1.0$ for standard sampler, and $C_d = 1.05$ for a borehole of diameter 150 mm. For a donut hammer, $E_r = 45\%$.

$$N_{\text{cor}} = C_{\text{rsd}} N \left(\frac{ER_r}{60} \right) = 0.95 \times 1.0 \times 1.05 \times 27 \times \frac{45}{60} = 20$$

Step 3: Use Table 2.3 to describe the compactness.

For $N = 27$, the soil is medium dense.

(c) The Cone Penetrometer Test (CPT)—ASTM D 5778 The cone penetrometer is a cone with a base area of 10 cm^2 and cone angle 60° (Fig. 2.15a) that is attached to a rod. An outer sleeve encloses the rod. The thrusts required to drive the cone and the sleeve into the ground at a rate of 2 cm/s are measured independently so that the end resistance or cone resistance and side friction or sleeve resistance may be estimated separately. Although originally developed for the design of piles, the cone penetrometer has also been used to estimate the bearing capacity and settlement of foundations.

The piezocone (uCPT or CPTu) is a cone penetrometer that has porous elements inserted into the cone or sleeve to allow for pore water pressure measurements (Fig. 2.15b). The measured pore water pressure depends on the location of the porous elements. A load cell is often used to measure the force of penetration. The piezocone is a very useful tool for soil profiling. Researchers have claimed that the piezocone provides useful data to estimate the shear strength, bearing capacity, and consolidation characteristics of soils. Typical results from a piezocone are shown in Fig. 2.15c.

Other CPT variants include the seismic cone (SCPT) and the vision cone (VisCPT or VisCPTu). In the SCPT, geophones are installed inside the cone. Hammers on the surface are used to

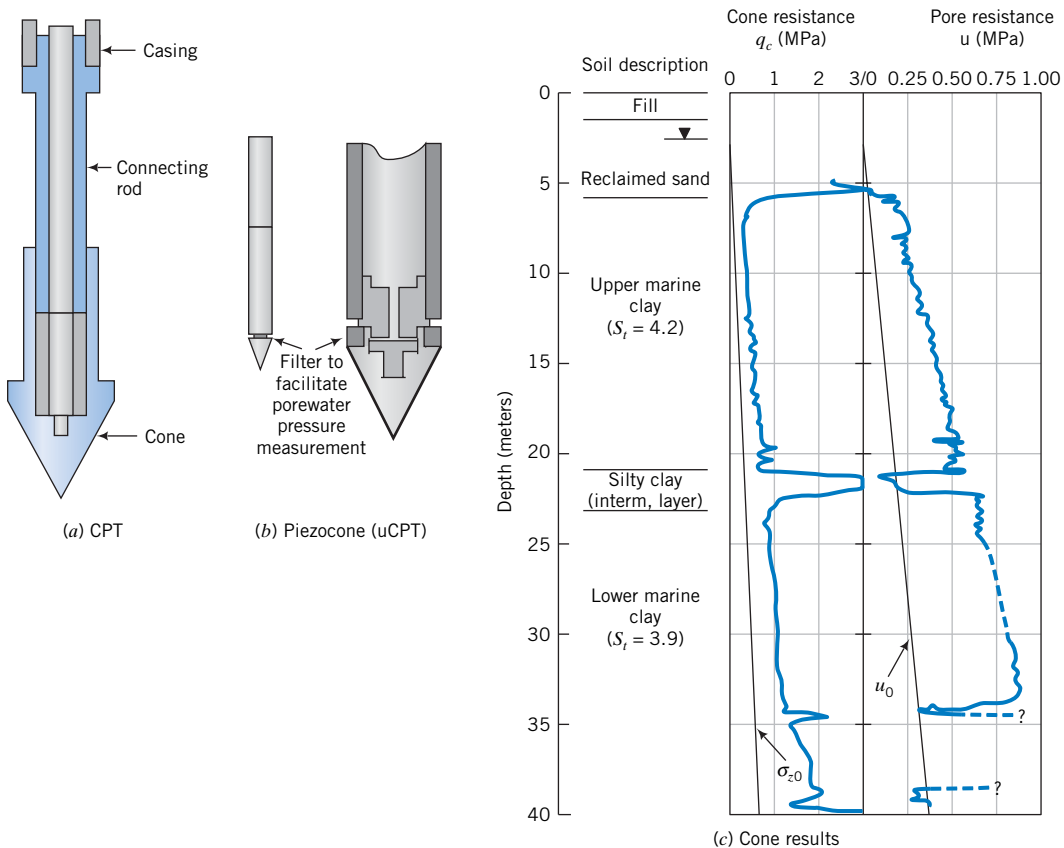


FIGURE 2.15 (a) CPT and (b) piezocone. (c) Piezocone results. (From Chang, 1988.)

produce surface disturbances and the resulting seismic waves are recorded by the geophones (usually three). The recorded data are then analyzed to give damping characteristics and soil strength parameters.

The VisCPT and VisCPTu have miniature cameras installed in the CPT probe that provide continuous images of the soil adjacent to the cone. Through image processing, the soil texture can be inferred. The VisCPTu can also be used to detect liquefiable soils.

Regardless of which CPT probe is used, the results are average values of the soil resistance over a length of about 10 cone diameters—about 5 diameters above the tip plus about 5 cone diameters below the tip. In layered soils, the soil resistances measured by the cone may not represent individual layers, especially thin layers (<5 cone diameters).

The cone resistance is influenced by several soil variables such as stress level, soil density, stratigraphy, soil mineralogy, soil type, and soil fabric. Results of CPT have been correlated with laboratory tests to build empirical relationships for strength and deformation parameters. Investigators have also related CPT results to other field tests, particularly SPT.

CPT is quick to perform with fewer performance errors compared with SPT. It can provide continuous records of soil conditions. CPT cannot be used in dense, coarse-grained soils (e.g., coarse gravel, boulders) and mixed soils containing boulders, cobbles, clays, and silts. The cone tip is prone to damage from contact with dense objects. The more sophisticated uCPT, SCPT, and VisCPT usually require specialists to perform and to interpret the results.

(d) Pressuremeters—ASTM D 4719-87 (1994) The Menard pressuremeter (Fig. 2.16a) is a probe that is placed at the desired depth in an unlined borehole and pressure is applied to a measuring cell of the probe. The stresses near the probe are shown in Fig. 2.16b. The pressure applied is analogous to the expansion of a cylindrical cavity. The pressure is raised in stages at constant time intervals and volume changes are recorded at each stage. A pressure-volume change curve is then drawn from which the elastic modulus, shear modulus, and the undrained shear strength may be estimated.

One of the disadvantages of the Menard pressuremeter is that it has to be inserted into a predrilled hole and consequently the soil is disturbed. The Cambridge Camkometer (Fig. 2.17) is a self-boring pressuremeter, which minimizes soil disturbances. A pressure is applied to radially expand a rubber membrane, which is built into the side wall of the Camkometer, and a feeler gauge

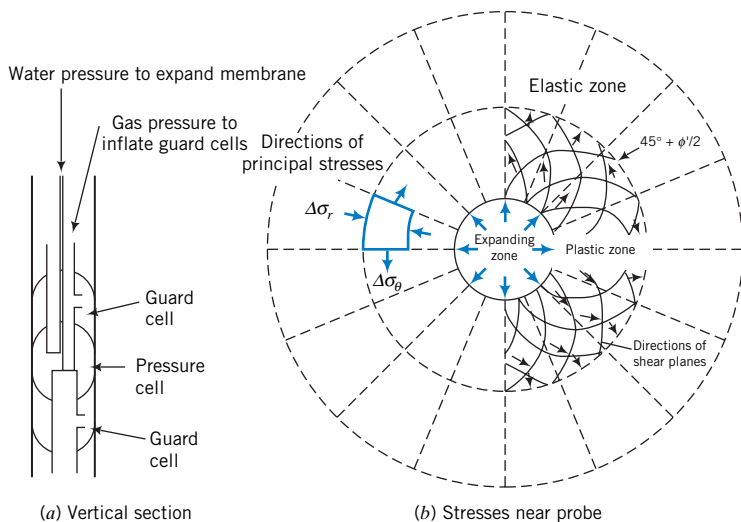


FIGURE 2.16 (a) Menard pressuremeter and (b) stresses near the probe.

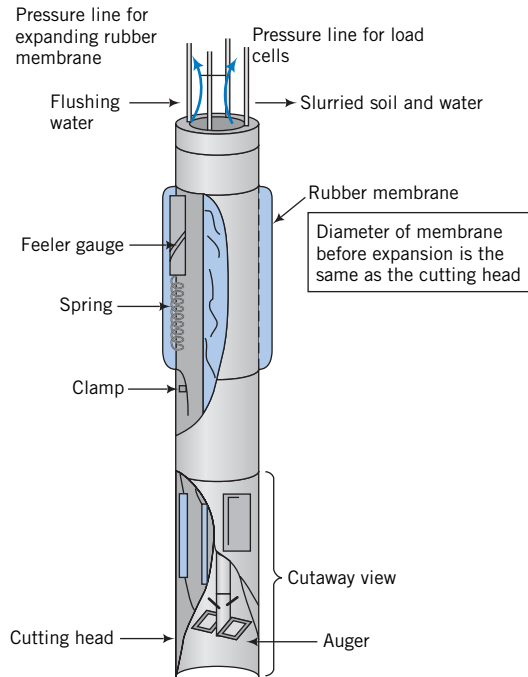


FIGURE 2.17 Schematic of Cambridge Camkometer. (Redrawn from Hughes and Wroth, 1972.)

measures the radial displacement. Thus, the stress–strain response of the soil can be obtained. Interpretation of the pressuremeter test is beyond the scope of this book.

Pressuremeter test provide measurement of horizontal stress and estimates of shear modulus and shear strength. Soil disturbance is small when self-boring pressuremeters are used. Pressuremeters are more costly than CPT and the flat plate dilatometer (Section 2.6.7e) and are not widely available. The drainage condition is unknown and consequently this leads to uncertainty in the interpretation of the test data to estimate the shear modulus and shear strength.

(e) Flat Plate Dilatometer (DMT) The flat plate dilatometer consists of a tapered blade 95 mm wide and 15 mm thick and 240 mm long (Fig. 2.18). On the flat face the dilatometer is a flexible steel membrane 60 mm in diameter that when inflated pushes the soil laterally. The blade is attached to drill rods and is pushed into the soil at a rate of 2 cm/s by a drill rig. Tests are normally conducted every 200 mm. The pneumatic pressures (a) to bring the membrane flush with the soil surface, (b) to push the soil laterally for a distance of 1.1 mm, and (c) at which the membrane returns to its original position are recorded.

Results from dilatometers have been related to undrained shear strength, lateral earth pressures, overconsolidation ratio, and elastic modulus. DMT is simple and quick to conduct. It provides reasonable estimates of horizontal stress and is less costly than the pressuremeter test. Dilatometers cause significant remolding of the soil before the test commences and the results obtained should be used with caution. The dilatometer test is best suited for clays and sands.

2.6.8 Boring Log

During soils exploration all pertinent details are recorded and presented in a boring log. A typical boring log is shown in Fig. 2.19. Additional information consisting mainly of laboratory and field test results is added to complete the boring log.

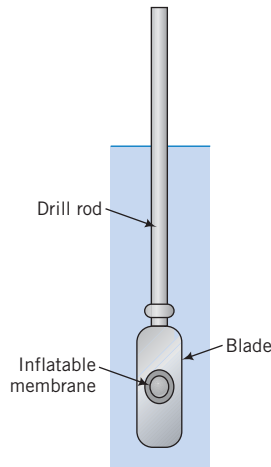


FIGURE 2.18 Flat plate dilatometer.

The essential points are:

- 1. A number of tools are available for soil exploration. You need to use judgment as to the type appropriate for a given project.*
- 2. The results of a soils exploration program are important in design and construction. Significant care and attention to details are necessary to make the results meaningful.*

2.7 SUMMARY

Geological understanding of soils is important for successful geotechnical engineering practice. Of the three main groups of rocks—igneous, sedimentary, and metamorphic—sedimentary rocks are of particular importance to geotechnical engineers because they compose nearly 75% of the earth's surface. Soils are derived from the weathering of rocks by physical and chemical processes. The main groups of soils for engineering purposes from these processes are coarse-grained soils—sand and gravels—and fine-grained soils—silts and clays. At a project site, the soils must be identified and characterized through a soils investigation. Such an investigation is done in phases and may include geophysical investigations, boreholes, and field and laboratory tests. At the completion of a soils investigation, the client normally requires a carefully written report.



Self-Assessment

Access Chapter 2 on the CD and take the end-of-chapter quiz to test your understanding of this chapter.

EXERCISES

- 2.1** In your area, choose a project under construction or a recently constructed project such as a road or a building. Describe the geology of the site.

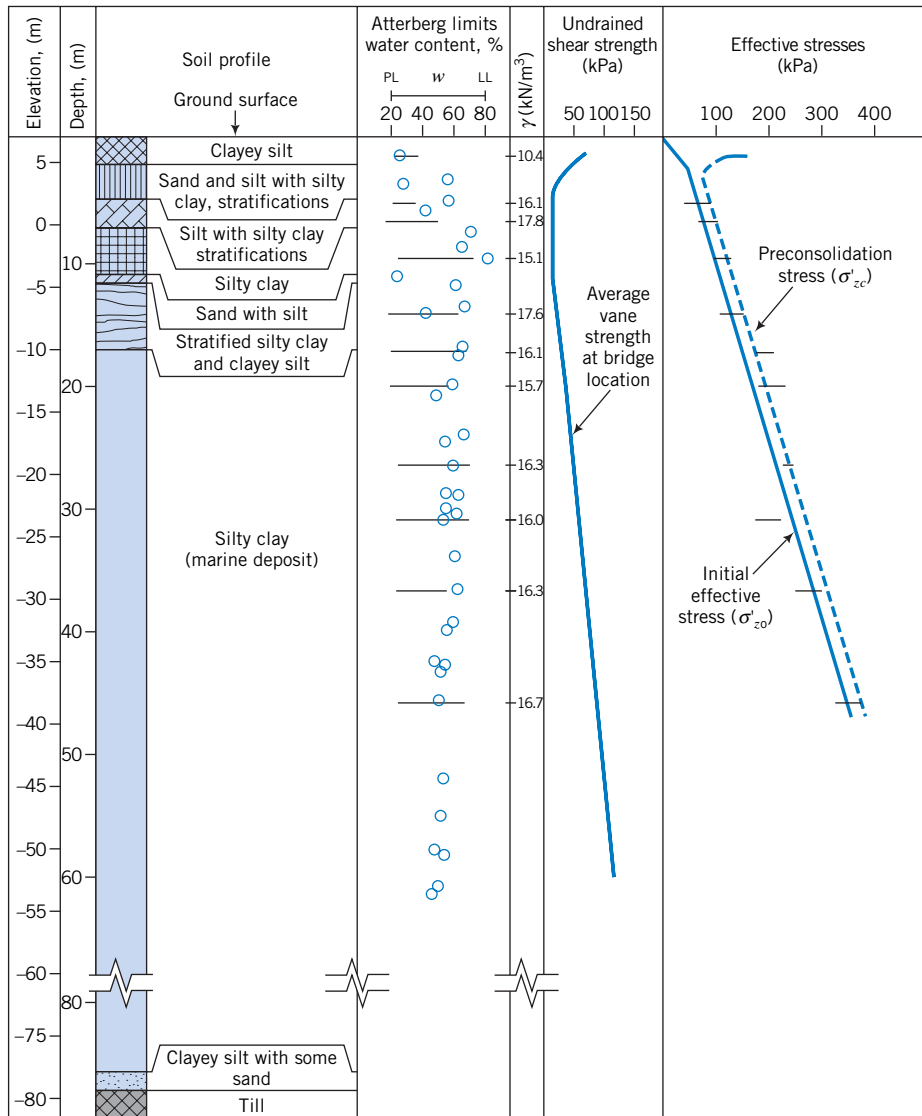


FIGURE 2.19 An example of a boring log. (Redrawn from Blanchet et al., 1980.)

- 2.2** Obtain borehole logs from a building site in your area. Describe the geology, the methods used in the soils exploration, and the type of field tests used, if any.
- 2.3** Collect soils and rocks in your neighborhood and write a description of each. The description should include color, hardness, stratifications, and joints (if any), etc.

CHAPTER

3

PHYSICAL SOIL PARAMETERS

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3.0 INTRODUCTION

In engineering, we disassemble complex systems into parts and then study each part and its relationship to the whole. We will do the same for soils. Soils will be dismantled into three constituents and we will examine how the proportions of each constituent characterize soils.

One of the primary tasks of a geotechnical engineer is to collect, classify, and investigate the physical properties of soils. In this chapter, we will deal with tests to determine the physical properties of soils, soil classification, and soil compaction.

When you complete this chapter, you should be able to:

- Determine the proportions of the main constituents in a soil.
- Determine particle size distribution in a soil mass.
- Classify soils.
- Determine index properties of soils.
- Determine maximum dry unit weight and optimum water content.

Sample Practical Situation A highway is proposed to link the city of Noscut to the village of Windsor Forest. The highway route will pass through a terrain that is relatively flat and is expected to be flooded by a 100 year storm event. The highway will be supported on an embankment constructed from soils trucked to the site from two possible pits. You, the geotechnical engineer, are to advise a stakeholders’ committee, consisting of engineers, farmers, community representatives, and lawyers, on the estimated cost of the embankment. In arriving at a cost, you must consider the suitability of the soil in each of the two pits for the embankment, the amount of soil, and the number of truckloads required. A highway under construction is shown in Fig. 3.1.

3.1 DEFINITIONS OF KEY TERMS

- Water content (w)** is the ratio of the weight of water to the weight of solids.
- Void ratio (e)** is the ratio of the volume of void space to the volume of solids.
- Porosity (n)** is the ratio of the volume of void to the total volume of soil.
- Degree of saturation (S)** is the ratio of the volume of water to the volume of void.
- Bulk unit weight (γ)** is the weight density, that is, the weight of a soil per unit volume.
- Saturated unit weight (γ_{sat})** is the weight of a saturated soil per unit volume.
- Dry unit weight (γ_d)** is the weight of a dry soil per unit volume.
- Effective unit weight (γ')** is the weight of soil solids in a submerged soil per unit volume.



FIGURE 3.1 A highway under construction.

Relative density (D_r) is an index that quantifies the degree of packing between the loosest and densest state of coarse-grained soils.

Effective particle size (D_{10}) is the average particle diameter of the soil at 10 percentile; that is, 10% of the particles are smaller than this size (diameter).

Average particle diameter (D_{50}) is the average particle diameter of the soil.

Liquid limit (LL) is the water content at which a soil changes from a plastic state to a liquid state.

Plastic limit (PL) is the water content at which a soil changes from a semisolid to a plastic state.

Shrinkage limit (SL) is the water content at which a soil changes from a solid to a semisolid state without further change in volume.

Maximum dry unit weight ($\gamma_{d(\max)}$) is the maximum unit weight that a soil can attain using a specified means of compaction.

Optimum water content (w_{opt}) is the water content required to allow a soil to attain its maximum dry unit weight.

3.2 QUESTIONS TO GUIDE YOUR READING

1. What are the physical parameters that characterize soils?
2. How is soil classified?
3. What are some of the basic soil tests required to characterize soils?
4. What are the Atterberg limits?
5. What is the purpose of classifying soils?
6. What is a grading curve?
7. What are the effects of water on the unit weight of soils?
8. What factors affect the compaction of soils?

3.3 PHASE RELATIONSHIPS



Interactive Concept Learning

Access Chapter 3, Section 3.3 to view an animation of soil phases and to interactively learn about phase relationships. Take Quiz 3.3 to test your understanding of the concepts. Develop problem-solving skills by solving problems interactively using the Problem Solver in the left sidebar.

Soil is composed of solids, liquids, and gases (Fig. 3.2a). The solid phase may be mineral, organic matter, or both. As mentioned before, we will not deal with organic matter in this textbook. The spaces between the solids (soil particles) are called voids. Water is often the predominant liquid and air is the predominant gas. We will use the terms water and air instead of liquids and gases. The soil water is commonly called porewater and it plays a very important role in the behavior of soils under load. If all the voids are filled by water, the soil is saturated. Otherwise, the soil is unsaturated. If all the voids are filled with air, the soil is said to be dry.

We can idealize the three phases of soil as shown in Fig. 3.2b. The physical properties of soils are influenced by the relative proportions of each of these phases. The total volume of the soil is the sum of the volume of solids (V_s), volume of water (V_w), and volume of air (V_a): that is,

$$V = V_s + V_w + V_a = V_s + V_v \quad (3.1)$$

where

$$V_v = V_w + V_a$$

is the volume of voids. The weight of the soil is the sum of the weight of solids (W_s) and the weight of water (W_w). The weight of air is negligible. Thus,

$$W = W_s + W_w \quad (3.2)$$

The following definitions have been established to describe the proportion of each constituent in soil. Each equation can be presented with different variables. The most popular and convenient forms are given. You should try to memorize these relationships. When you work on these relationships, think about a bread dough in which you have to reconstruct the amount of the constituent ingredients, for example, the amount of flour or water. If you add too much water to a bread dough it becomes softer and more malleable. The same phenomenon occurs in fine-grained soils.

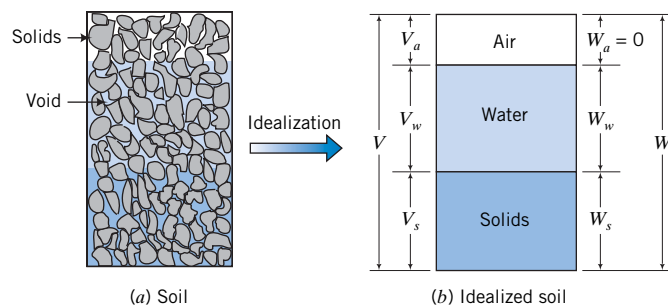


FIGURE 3.2 Soil phases.

1. Water content (w) is the ratio, often expressed as a percentage, of the weight of water to the weight of solids:

$$w = \frac{W_w}{W_s} \times 100\% \quad (3.3)$$

The water content of a soil is found by weighing a sample of the soil and then placing it in an oven at $110 \pm 5^\circ\text{C}$ until the weight of the sample remains constant; that is, all the absorbed water is driven out. For most soils, a constant weight is achieved in about 24 hours. The soil is removed from the oven, cooled, and then weighed. Example 3.2 illustrates the measurements and calculations required to determine the water content. It is a common mistake to use the total weight in the denominator. Remember it is the weight (or mass) of the solids.

2. Void ratio (e) is the ratio of the volume of void space to the volume of solids. Void ratio is usually expressed as a decimal quantity.

$$e = \frac{V_v}{V_s} \quad (3.4)$$

3. Specific volume (V') is the volume of soil per unit volume of solids:

$$V' = \frac{V}{V_s} = 1 + e \quad (3.5)$$

This equation is useful in relating volumes as shown in Example 3.3 and in the calculation of volumetric strains (Chapter 5).

4. Porosity (n) is the ratio of the volume of voids to the total volume. Porosity is usually expressed as a percentage.

$$n = \frac{V_v}{V} \quad (3.6)$$

Porosity and void ratio are related by the expression

$$n = \frac{e}{1 + e} \quad (3.7)$$

Let us prove Eq. (3.7). We will start with the basic definition [Eq. (3.6)] and then algebraically manipulate it to get Eq. (3.7). The total volume will be decomposed into the volume of solids and the volume of voids and then both the numerator and denominator will be divided by the volume of solids; that is,

$$n = \frac{V_v}{V} = \frac{V_v}{V_s + V_v} = \frac{V_v/V_s}{V_s/V_s + V_v/V_s} = \frac{e}{1 + e}$$

The porosity of soils can vary widely. If the particles of coarse-grained soils were spheres, the maximum and minimum porosities would be 48% and 26% respectively. This is equivalent to maximum and minimum void ratios of 0.91 and 0.35, respectively. The void ratios of real coarse-grained soils vary between 1 and 0.3. Clay soils can have void ratios greater than 1.

5. Specific gravity (G_s) is the ratio of the weight of the soil solids to the weight of water of equal volume:

$$G_s = \frac{W_s}{V_s \gamma_w} \quad (3.8)$$

where $\gamma_w = 9.81 \text{ kN/m}^3$ is the unit weight of water. We will use $\gamma_w = 9.8 \text{ kN/m}^3$ in this book. The specific gravity of soils ranges from approximately 2.6 to 2.8. For most problems, G_s can be assumed, with little error, to be equal to 2.7.

Two types of container are used to determine the specific gravity. One is a pycnometer, which is used for coarse-grained soils. The other is a 50 mL density bottle, which is used for fine-grained soils. The container is weighed and a small quantity of dry soil is placed in it. The mass of the container and the dry soil is determined. De-aired water is added to the soil in the container. The container is then agitated to remove air bubbles. When all air bubbles have been removed, the container is filled with de-aired water. The mass of container, soil, and water is determined. The contents of the container are discarded and the container is thoroughly cleaned. De-aired water is added to fill the container and the mass of the container and water is determined.

Let m_1 be the mass of the container; m_2 be the mass of the container and dry soil; m_3 be the mass of the container, soil, and water; and m_4 be the mass of the container and water. The mass of dry soil is $m_s = m_2 - m_1$, the mass of water displaced by the soil particles is $m_5 = m_4 - m_3 + m_s$, and $G_s = m_s/m_5$.

6. Degree of saturation (S) is the ratio, often expressed as a percentage, of the volume of water to the volume of voids:

$$S = \frac{V_w}{V_v} = \frac{wG_s}{e} \quad \text{or} \quad Se = wG_s \quad (3.9)$$

If $S = 1$ or 100%, the soil is saturated. If $S = 0$, the soil is bone dry. It is practically impossible to obtain a soil with $S = 0$.

7. Unit weight is the weight of a soil per unit volume. We will use the term **bulk unit weight**, γ , to denote unit weight:

$$\gamma = \frac{W}{V} = \left(\frac{G_s + Se}{1 + e} \right) \gamma_w \quad (3.10)$$

Special Cases

(a) Saturated unit weight ($S = 1$):

$$\gamma_{\text{sat}} = \left(\frac{G_s + e}{1 + e} \right) \gamma_w \quad (3.11)$$

(b) Dry unit weight:

$$\gamma_d = \frac{W_s}{V} = \left(\frac{G_s}{1 + e} \right) \gamma_w = \frac{\gamma}{1 + w} \quad (3.12)$$

(c) Effective or buoyant unit weight is the weight of a saturated soil, surrounded by water, per unit volume of soil:

$$\gamma' = \gamma_{\text{sat}} - \gamma_w = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w \quad (3.13)$$

TABLE 3.1 Typical Values of Unit Weight for Soils

Soil type	$\gamma_{\text{sat}} (\text{kN/m}^3)$	$\gamma_d (\text{kN/m}^3)$
Gravel	20–22	15–17
Sand	18–20	13–16
Silt	18–20	14–18
Clay	16–22	14–21

Typical values of unit weight of soils are given in Table 3.1.

8. Relative density (D_r) is an index that quantifies the degree of packing between the loosest and densest possible state of coarse-grained soils as determined by experiments:

$$D_r = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}} \quad (3.14)$$

where e_{max} is the maximum void ratio (loosest condition), e_{min} is the minimum void ratio (densest condition), and e is the current void ratio.

The relative density can also be written as

$$D_r = \frac{\gamma_d - (\gamma_d)_{\text{min}}}{(\gamma_d)_{\text{max}} - (\gamma_d)_{\text{min}}} \left\{ \frac{(\gamma_d)_{\text{max}}}{\gamma_d} \right\} \quad (3.15)$$

The maximum void ratio is found by pouring dry sand, for example, into a mold of volume (V) 2830 cm^3 using a funnel. The sand that fills the mold is weighed. If the weight of the sand is W , then by combining Eqs. (3.10) and (3.12) we get $e_{\text{max}} = G_s \gamma_w (V/W) - 1$. The minimum void ratio is determined by vibrating the sand with a weight imposing a vertical stress of 13.8 kPa on top of the sand. Vibration occurs for 8 minutes at a frequency of 3600 Hz and amplitude of 0.064 mm. From the weight of the sand (W_1) and the volume (V_1) occupied by it after vibration, we can calculate the minimum void ratio using $e_{\text{min}} = G_s \gamma_w (V_1/W_1) - 1$.

The relative density correlates very well with the strength of coarse-grained soils—denser soils being stronger than looser soils. A description of sand based on relative density is given in Table 3.2.

TABLE 3.2 Description Based on Relative Density

D_r (%)	Description
0–20	Very loose
20–40	Loose
40–70	Medium dense or firm
70–85	Dense
85–100	Very dense

What's next ... Four examples will be used to illustrate how to solve a variety of problems involving the constituents of soils. In the first example, we will derive some of the equations describing relationships among the soil constituents.



Interactive Problem Solving

Access Chapter 3 on the CD, click on Problem Solver on the sidebar to interactively solve problems on phase relationships. Each time you rerun a problem, the values in the problem change.

EXAMPLE 3.1 Deriving Soil Constituent Relationships

Prove the following relationships:

(a) $S = \frac{wG_s}{e}$

(b) $\gamma_d = \frac{\gamma}{1 + w}$

(c) $\gamma = \left(\frac{G_s + Se}{1 + e} \right) \gamma_w = \frac{G_s \gamma_w (1 + w)}{1 + e}$

Strategy The proofs of these equations are algebraic manipulations. Start with the basic definition and then manipulate the basic equation algebraically to get the desired form.

Solution 3.1

(a) For this relationship, we proceed as follows:

Step 1: Write down the basic equation,

$$S = \frac{V_w}{V_v}$$

Step 2: Manipulate the basic equation to get the desired equation.

You want to get e in the denominator and you have V_v . You know that $V_v = eV_s$ and V_w is the weight of water divided by the unit weight of water. From the definition of water content, the weight of water is wW_s . Here is the algebra:

$$\begin{aligned} V_v &= eV_s; & V_w &= \frac{W_w}{\gamma_w} = \frac{wW_s}{\gamma_w} \\ S &= \frac{wW_s}{e\gamma_w V_s} = \frac{G_s w}{e} \end{aligned}$$

(b) For this relationship, we proceed as follows:

Step 1: Write down the basic equation,

$$\gamma_d = \frac{W_s}{V}$$

Step 2: Manipulate the basic equation to get the new form of the equation.

$$\begin{aligned} \gamma_d &= \frac{W_s}{V} = \frac{W - W_w}{V} = \frac{W}{V} - \frac{wW_s}{V} = \gamma - w\gamma_d \\ \therefore \gamma_d + w\gamma_d &= \gamma \\ \gamma_d &= \frac{\gamma}{1 + w} \end{aligned}$$

(c) For this relationship, we proceed as follows:

Step 1: Start with the basic equation,

$$\gamma = \frac{W}{V}$$

Step 2: Manipulate the basic equation to get the new form of the equation.

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_v} = \frac{W_s + wW_s}{V_s + V_v}$$

Substituting $w = Se/G_s$ and $V_v = eV_s$, we obtain

$$\begin{aligned} \gamma &= \frac{W_s(1 + Se/G_s)}{V_s(1 + e)} \\ &= \frac{G_s \gamma_w (1 + Se/G_s)}{1 + e} = \frac{G_s \gamma_w (1 + w)}{1 + e} \end{aligned}$$

or

$$\gamma = \left(\frac{G_s + Se}{1 + e} \right) \gamma_w$$

EXAMPLE 3.2 Calculating Soil Constituents

A sample of saturated clay was placed in a container and weighed. The weight was 6 N. The clay in its container was placed in an oven for 24 hours at 105°C. The weight reduced to a constant value of 5 N. The weight of the container is 1 N. If $G_s = 2.7$, determine the (a) water content, (b) void ratio, (c) bulk unit weight, (d) dry unit weight, and (e) effective unit weight.

Strategy Write down what is given and then use the appropriate equations to find the unknowns. You are given the weight of the natural soil, sometimes called the wet weight, and the dry weight of the soil. The difference between these will give the weight of water and you can find the water content by using Eq. (3.3). You are also given a saturated soil, which means that $S = 1$.

Solution 3.2

Step 1: Write down what is given.

$$\text{Weight of sample + container} = 6 \text{ N}$$

$$\text{Weight of dry sample + container} = 5 \text{ N}$$

Step 2: Determine the weight of water and the weight of dry soil:

$$\text{Weight of water: } W_w = 6 - 5 = 1 \text{ N}$$

$$\text{Weight of dry soil: } W_s = 5 - 1 = 4 \text{ N}$$

Step 3: Determine the water content.

$$w = \frac{W_w}{W_s} \times 100 = \frac{1}{4} \times 100 = 25\%$$

Note: The denominator is the weight of solids, not the total weight.

Step 4: Determine the void ratio.

$$e = \frac{wG_s}{S} = \frac{0.25 \times 2.7}{1} = 0.675$$

Step 5: Determine the bulk unit weight.

$$\gamma = \frac{W}{V} = \frac{G_s \gamma_w (1 + w)}{1 + e} \quad (\text{see Example 3.1})$$

$$\gamma = \frac{2.7 \times 9.8(1 + 0.25)}{1 + 0.675} = 19.7 \text{ kN/m}^3$$

In this case the soil is saturated, so that the bulk unit weight is equal to the saturated unit weight.

Step 6: Determine the dry unit weight.

$$\gamma_d = \frac{W_s}{V} = \left(\frac{G_s}{1 + e} \right) \gamma_w = \frac{2.7}{1 + 0.675} \times 9.8 = 15.8 \text{ kN/m}^3$$

or

$$\gamma_d = \frac{\gamma}{(1 + w)} = \frac{19.7}{1 + 0.25} = 15.8 \text{ kN/m}^3$$

Step 7: Determine the effective unit weight.

$$\gamma' = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w = \left(\frac{2.7 - 1}{1 + 0.675} \right) \times 9.8 = 9.9 \text{ kN/m}^3$$

or

$$\gamma' = \gamma_{\text{sat}} - \gamma_w = 19.7 - 9.8 = 9.9 \text{ kN/m}^3$$

EXAMPLE 3.3 *Application of Soil Constituent Relationships to a Practical Problem*

An embankment for a highway is to be constructed from a soil compacted to a dry unit weight of 18 kN/m^3 . The clay has to be trucked to the site from a borrow pit. The bulk unit weight of the soil in the borrow pit is 17 kN/m^3 and its natural water content is 5%. Calculate the volume of clay from the borrow pit required for 1 cubic meter of embankment. Assume $G_s = 2.7$.

Strategy This problem can be solved in many ways. We will use two of these ways. One way is direct; the other a bit longer. In the first way, we are going to use the ratio of the dry unit weights of the compacted soil to the borrow pit soil to determine the volume. In the second way, we will use the specific volume. In this case, we need to find the void ratio for the borrow pit clay and the desired void ratio for the embankment. We can then relate the specific volumes of the embankment and the borrow pit clay.

Solution 3.3

Step 1: Find the dry unit weight of the borrow pit soil.

$$\gamma_d = \frac{\gamma}{1 + w} = \frac{17}{1 + 0.05} = 16.2 \text{ kN/m}^3$$

Step 2: Find the volume of borrow pit soil required.

$$\begin{aligned} &\text{Volume of borrow pit soil required per m}^3 \\ &= \frac{(\gamma_d)_{\text{compacted soil}}}{(\gamma_d)_{\text{borrow pit soil}}} = \frac{18}{16.2} = 1.11 \text{ m}^3 \end{aligned}$$

Alternatively:

Step 1: Define parameters for the borrow pit and embankment. Let

V'_1, e_1 = specific volume and void ratio, respectively, of borrow pit clay

V'_2, e_2 = volume and void ratio, respectively, of compacted clay

Step 2: Determine e_1 and e_2 .

$$\gamma_d = \frac{\gamma}{1 + w} = \frac{17}{1 + 0.05} = 16.2 \text{ kN/m}^3$$

But

$$\gamma_d = \frac{G_s}{1 + e_1} \gamma_w$$

and therefore

$$e_1 = G_s \frac{\gamma_w}{\gamma_d} - 1 = 2.7 \left(\frac{9.8}{16.2} \right) - 1 = 0.633$$

Similarly,

$$e_2 = G_s \frac{\gamma_w}{\gamma_d} - 1 = 2.7 \left(\frac{9.8}{18} \right) - 1 = 0.47$$

Step 3: Determine the volume of borrow pit material.

$$\frac{V'_1}{V'_2} = \frac{1 + e_1}{1 + e_2}$$

Therefore

$$V'_1 = V'_2 \frac{1 + e_1}{1 + e_2} = 1 \left(\frac{1 + 0.633}{1 + 0.47} \right) = 1.11 \text{ m}^3$$

EXAMPLE 3.4 Application of Soil Constituent Relationships to a Practical Problem

If the borrow soil in Example 3.3 were to be compacted to attain a dry unit weight of 18 kN/m³ at a water content of 7%, determine the amount of water required per cubic meter of embankment, assuming no loss of water during transportation.

Strategy Since water content is related to the weight of solids and not the total weight, we need to use the data given to find the weight of solids.

Solution 3.4

Step 1: Determine the weight of solids per unit volume of borrow pit soil.

$$W_s = \frac{\gamma}{1 + w} = \frac{17}{1 + 0.05} = 16.2 \text{ kN/m}^3$$

Step 2: Determine the amount of water required.

$$\text{Additional water} = 7 - 5 = 2\%$$

$$\text{Weight of water} = W_w = wW_s = 0.02 \times 16.2 = 0.32 \text{ kN}$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{0.32}{9.8} = 0.033 \text{ m}^3 = 33 \text{ liters}$$

What's next . . . In most soils, there is a distribution of particles with various sizes. The distribution of particle size influences the response of soils to loads and to flow of water. We will describe methods used in the laboratory to find particle sizes in soils.

3.4 DETERMINATION OF PARTICLE SIZE OF SOILS—ASTM D422-63 (1990)



Interactive Concept Learning and Self-Assessment

Access Chapter 3, Section 3.4 on the CD for an interactive lesson about particle size distribution. Take Quiz 3.4 to test your understanding.

3.4.1 Particle Size of Coarse-Grained Soils

The distribution of particle sizes or average grain diameter of coarse-grained soils—gravels and sands—is obtained by screening a known weight of the soil through a stack of sieves of progressively finer mesh size. A typical stack of sieves is shown in Fig. 3.3.



FIGURE 3.3 Stack of sieves.

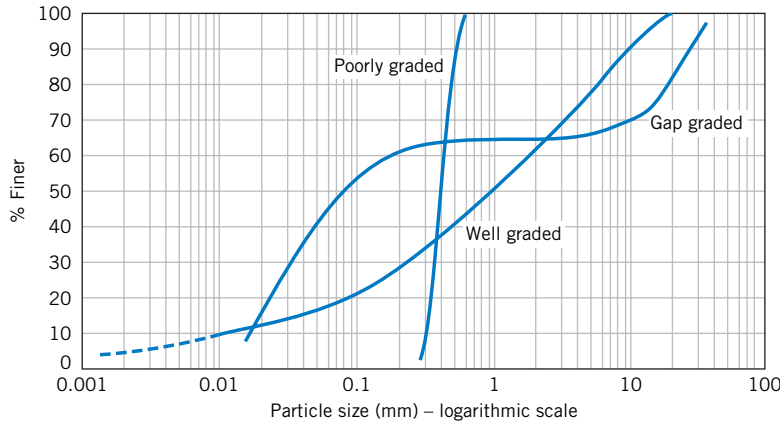


FIGURE 3.4 Particle size distribution curves.

Each sieve is identified by a number that corresponds to the number of square holes per linear inch of mesh. The particle diameter in the screening process, often called sieve analysis, is the maximum particle dimension to pass through the square hole of a particular mesh. A known weight of dry soil is placed on the largest sieve (the top sieve) and the nest of sieves is then placed on a vibrator, called a sieve shaker, and shaken. The nest of sieves is dismantled, one sieve at a time. The soil retained on each sieve is weighed and the percentage of soil retained on each sieve is calculated. The results are plotted on a graph of percent of particles finer than a given sieve (not the percent retained) as the ordinate versus the logarithm of the particle sizes as shown in Fig. 3.4. The resulting plot is called a particle size distribution curve(s) or, simply, the gradation curve(s). Engineers have found it convenient to use a logarithm scale for particle size because the ratio of particle sizes from the largest to the smallest in a soil can be greater than 10^4 .

Let W_i be the weight of soil retained on the i th sieve from the top of the nest of sieves and W be the total soil weight. The percent weight retained is

$$\% \text{ Retained on } i\text{th sieve} = \frac{W_i}{W} \times 100 \quad (3.16)$$

The percent finer is

$$\% \text{ Finer than } i\text{th sieve} = 100 - \sum_{i=1}^i (\% \text{ Retained on } i\text{th sieve}) \quad (3.17)$$

You can use mass instead of weight. The unit of mass is grams or kilograms.

3.4.2 Particle Size of Fine-Grained Soils

The screening process cannot be used for fine-grained soils—silts and clays—because of their extremely small size. The common laboratory method to determine the size distribution of fine-grained soils is a hydrometer test (Fig. 3.5). The hydrometer test involves mixing a small amount of soil into a suspension and observing how the suspension settles in time. Larger particles will settle quickly followed by smaller particles. When the hydrometer is lowered into the suspension, it will sink into the suspension until the buoyancy force is sufficient to balance the weight of the hydrometer.

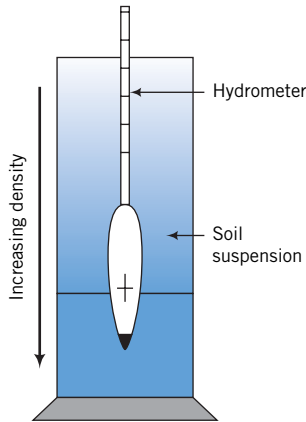


FIGURE 3.5 Hydrometer in soil-water suspension.

The length of the hydrometer projecting above the suspension is a function of the density, so it is possible to calibrate the hydrometer to read the density of the suspension at different times. The calibration of the hydrometer is affected by temperature and the specific gravity of the suspended solids. You must then apply a correction factor to your hydrometer reading based on the test temperatures.

Typically, a hydrometer test is conducted by taking a small quantity of a dry fine-grained soil (approximately 50 grams) and thoroughly mixing it with distilled water to form a paste. The paste is placed in a 1 liter glass cylinder and distilled water is added to bring the level to the 1 liter mark. The glass cylinder is then repeatedly shaken and inverted before being placed in a constant-temperature bath. A hydrometer is placed in the glass cylinder and a clock is simultaneously started. At different times, the hydrometer is read. The diameter D of the particle at time t_D is calculated from Stokes's law as

$$D = \sqrt{\frac{18\mu z}{(G_s - 1)\gamma_w t_D}} \quad (3.18)$$

where μ is the viscosity of water (10.09 millipoises at 20°C), z is the depth, γ_w is the unit weight of water, and G_s is the specific gravity.

There are several assumptions made in Stokes's law that are not fulfilled by the hydrometer test. In Stokes's law, the particles are assumed to be free-falling spheres with no collision, but the mineral particles of clays are plate-like and collision of particles during sedimentation is unavoidable. However, the results of this test suffice for most geotechnical engineering needs. For more accurate size distribution measurements in fine-grained soils, other more sophisticated methods are available (e.g., light scattering methods). The dashed line in Fig. 3.4 shows a typical particle size distribution for fine-grained soils.

3.4.3 Characterization of Soils Based on Particle Size

The grading curve is used for textural classification of soils. Various classification systems have evolved over the years to describe soils based on their particle size distribution. Each system was developed for a specific engineering purpose. In this book, we will use the Unified Soil Classification System (USCS). The USCS separates soils into two categories. One category is coarse-grained soils that are delineated if more than 50% of the soil is greater than 0.075 mm (No. 200 sieve). The other category is fine-grained soils that are delineated if more than 50% of the soil is finer than 0.075 mm. Coarse-grained soils are subdivided into gravels and sands while fine-grained soils are divided into

TABLE 3.3 Soil Types, Descriptions, and Average Grain Sizes According to USCS (ASTM D2487-93)

Soil type	Description	Average grain size
Gravel	Rounded and/or angular bulky hard rock	Coarse: 75 mm to 19 mm Fine: 19 mm to 4.75 mm
Sand	Rounded and/or angular bulky hard rock	Coarse: 4.75 mm to 2.0 mm (No. 10) Medium: 2.0 mm to 0.425 mm (No. 40) Fine: 0.425 mm to 0.075 mm (No. 200)
Silt	Particles smaller than 0.075 mm, exhibit little or no strength when dried	0.075 mm to 0.002 mm
Clay	Particles smaller than 0.002 mm, exhibit significant strength when dried; water reduces strength	<0.002 mm

silts and clays. Each soil type—gravel, sand, silt, and clay—is identified by grain size as shown in Table 3.3. The USCS does not differentiate silts from clays. Clays have particle sizes less than 0.002 mm. We will discuss the USCS in more detail in Section 3.7.

Real soils consist of a mixture of particle sizes. The selection of a soil for a particular use may depend on the assortment of particles it contains. Two coefficients have been defined to provide guidance on distinguishing soils based on the distribution of the particles. One of these is a numerical measure of uniformity, called the *uniformity coefficient*, C_u , defined as

$$C_u = \frac{D_{60}}{D_{10}} \quad (3.19)$$

where D_{60} is the diameter of the soil particles for which 60% of the particles are finer, and D_{10} is the diameter of the soil particles for which 10% of the particles are finer. Both of these diameters are obtained from the grading curve.

The other coefficient is the *coefficient of curvature*, C_c (other terms used are the coefficient of gradation and the coefficient of concavity), defined as

$$C_c = \frac{(D_{30})^2}{D_{10}D_{60}} \quad (3.20)$$

where D_{30} is the diameter of the soil particles for which 30% of the particles are finer. The average particle diameter is D_{50} .

A soil that has a uniformity coefficient of <4 contains particles of uniform size (approximately one size). The minimum value of C_u is 1 and corresponds to an assemblage of particles of the same size. The gradation curve for a poorly graded soil is almost vertical (Fig. 3.4). Humps in the gradation curve indicate two or more poorly graded soils. Higher values of uniformity coefficient (>4) indicate a wider assortment of particle sizes. A soil that has a uniformity coefficient of >4 is described as a well-graded soil and is indicated by a flat curve (Fig. 3.4). The coefficient of curvature is between 1 and 3 for well-graded soils. The absence of certain grain sizes, termed gap-graded, is diagnosed by a coefficient of curvature outside the range 1 to 3 and a sudden change of slope in the particle size distribution curve as shown in Fig. 3.4.

Poorly graded soils are sorted by water (e.g., beach sands) or by wind. Gap-graded soils are also sorted by water but certain sizes were not transported. Well-graded soils are produced by bulk transport processes (e.g., glacial till). The uniformity coefficient and the coefficient of concavity are strictly applicable to coarse-grained soils.

The diameter D_{10} is called the effective size of the soil and was established by Allen Hazen (1893) in connection with his work on soil filters. The effective size is the diameter of an artificial sphere that will approximately produce the same effect of an irregular shaped particle. The effective size is particularly important in regulating the flow of water through soils and can dictate the mechanical behavior of soils since the coarser fractions may not be in effective contact with each other; that is, they float in a matrix of finer particles. The higher the D_{10} value, the coarser the soil and the better the drainage characteristics.

Particle size analyses have many uses in engineering. They are used to select aggregates for concrete, soils for the construction of dams and highways, soils as filters, and material for grouting and chemical injection. In Section 3.7, you will learn about how the particle size distribution is used with other physical properties of soils in a classification system designed to help you select soils for particular applications.

The essential points are:

- 1. A sieve analysis is used to determine the grain size distribution of coarse-grained soils.***
- 2. For fine-grained soils, a hydrometer analysis is used to find the particle size distribution.***
- 3. Particle size distribution is represented on a semilogarithmic plot of % finer (ordinate, arithmetic scale) versus particle size (abscissa, logarithm scale).***
- 4. The particle size distribution plot is used to delineate the different soil textures (percentages of gravel, sand, silt, and clay) in a soil.***
- 5. The effective size, D_{10} , is the diameter of the particles of which 10% of the soil is finer. D_{10} is an important value in regulating flow through soils and can significantly influence the mechanical behavior of soils.***
- 6. D_{50} is the average grain size diameter of the soil.***
- 7. Two coefficients—the uniformity coefficient and the coefficient of curvature—are used to characterize the particle size distribution. Poorly graded soils have uniformity coefficients <4 and steep gradation curves. Well-graded soils have uniformity coefficients >4 , coefficients of curvature between 1 and 3, and flat gradation curves. Gap-graded soils have coefficients of curvature <1 or >3 , and one or more humps on the gradation curves.***

EXAMPLE 3.5 *Interpreting Sieve Analysis Data*

A sample of a dry coarse-grained material of mass 500 grams was shaken through a nest of sieves and the following results were obtained:

Sieve no.	Opening (mm)	Mass retained (grams)
4	4.75	0
10	2.00	14.8
20	0.85	98
40	0.425	90.1
100	0.15	181.9
200	0.075	108.8
Pan		6.1

- Plot the particle size distribution curve.
- Determine (1) the effective size, (2) the average particle size, (3) the uniformity coefficient, and (4) the coefficient of curvature.
- Determine the textural composition of the soil (i.e., the amount of gravel, sand, etc.).

Strategy The best way to solve this type of problem is to make a table to carry out the calculations and then plot a gradation curve. Total mass of dry sample (M) used is 500 grams but on summing the masses of the retained soil in column 2 we obtain 499.7 grams. The reduction in mass is due to losses mainly from a small quantity of soil that gets stuck in the meshes of the sieves. You should use the “after sieving” total mass of 499.7 grams in the calculations.

Solution 3.5

Step 1: Tabulate data to obtain % finer.

Sieve no.	Mass retained (grams) M_r	% Retained $(M_r/M) \times 100$	Σ (% Retained)	% Finer
4	0	0	add 0	$100 - 0 = 100$
10	14.8	3.0	3.0	$100 - 3.0 = 97.0$
20	98.0	19.6	22.6	$100 - 22.6 = 77.4$
40	90.1	18.0	40.6	$100 - 40.6 = 59.4$
100	181.9	36.4	77.0	$100 - 77 = 23.0$
200	108.8	21.8	98.8	$100 - 98.8 = 1.2$
Pan	6.1	1.2	check	
Total mass $M = 499.7$		100		

Step 2: Plot the gradation curve.

See Fig. E3.5 for a plot of the gradation curve.

Step 3: Extract the effective size.

$$\text{Effective size} = D_{10} = 0.1 \text{ mm}$$

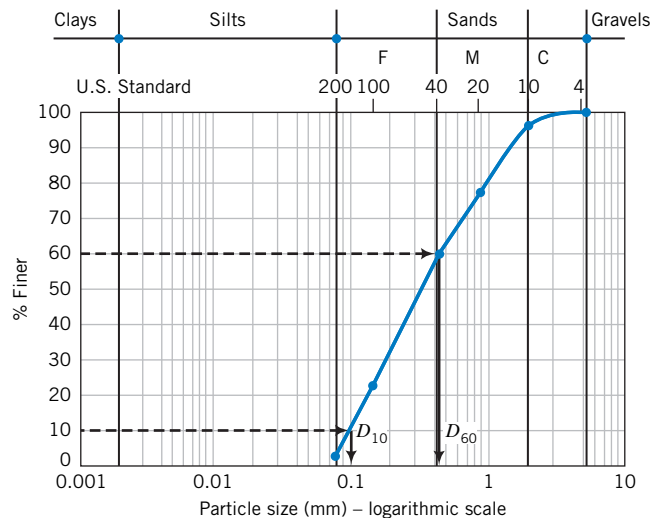


FIGURE E3.5 Particle size distribution curve.

Step 4: Extract percentages of gravel, sand, silt, and clay.

Gravel = 0%

Sand = 98.8%

Silt and clay = 1.2%

Step 5: Calculate C_u and C_c .

$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.45}{0.1} = 4.5$$

$$C_c = \frac{(D_{30})^2}{D_{10}D_{60}} = \frac{0.18^2}{0.1 \times 0.45} = 0.72$$

What's next . . . Particle size is adequate to classify coarse-grained soils. To classify fine-grained soils, we need additional information on the surface forces and mineral contents. We will discuss laboratory tests that are used to account for the surface forces and mineral content of a soil. In a conventional geotechnical course, you will be required to conduct the tests described next.

3.5 PHYSICAL STATES AND INDEX PROPERTIES OF FINE-GRAINED SOILS



Interactive Concept Learning and Self-Assessment

Access Chapter 3, Section 3.5 on the CD for an interactive lesson about the various soil states and take Quiz 3.5 to test your understanding of the key points.

The physical and mechanical behavior of fine-grained soils is linked to four distinct states: solid, semisolid, plastic, and liquid in order of increasing water content. Let us consider a soil initially in a liquid state that is allowed to dry uniformly. If we plot a diagram of volume versus water content as shown in Fig. 3.6, we can locate the original liquid state as point *A*. As the soil dries, its water content reduces and consequently its volume (see Fig. 3.2b).

At point *B*, the soil becomes so stiff that it can no longer flow as a liquid. The boundary water content at point *B* is called the liquid limit; it is denoted by *LL*. As the soil continues to dry, there is a range of water content at which the soil can be molded into any desired shape without rupture. The soil at this state is said to exhibit plastic behavior—the ability to deform continuously without rupture. But if drying is continued beyond the range of water content for plastic behavior, the soil becomes a semisolid. The soil cannot be molded now without visible cracks appearing. The water content at which the soil changes from a plastic to a semisolid is known as the plastic limit, denoted by

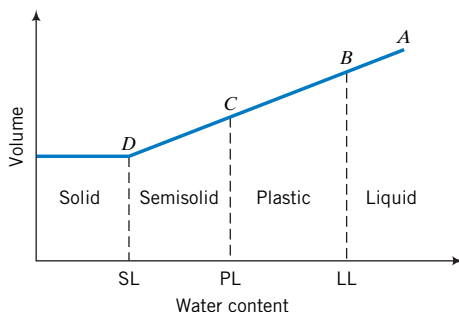


FIGURE 3.6 Changes in soil states as a function of soil volume and water content.

PL. The range of water contents over which the soil deforms plastically is known as the plasticity index, PI:

$$PI = LL - PL \quad (3.21)$$

As the soil continues to dry, it comes to a final state called the solid state. At this state, no further volume change occurs since nearly all the water in the soil has been removed. The water content at which the soil changes from a semisolid to a solid is called the shrinkage limit, denoted by SL. The shrinkage limit is useful for the determination of the swelling and shrinking capacity of soils. The liquid and plastic limits are called the Atterberg limits named after their originator, Swedish soil scientist, A. Atterberg (1911).

We have changed the states of fine-grained soils by changing the water content. Since engineers are interested in the strength and deformation of materials, we can associate specific strength characteristics to each of the soil states. At one extreme, the liquid state, the soil has the lowest strength and the largest deformation. At the other extreme, the solid state, the soil has the largest strength and the lowest deformation. A measure of soil strength using the Atterberg limits is known as the liquidity index (LI) and is expressed as

$$LI = \frac{w - PL}{PI} \quad (3.22)$$

The liquidity index is the ratio of the difference in water content between the natural or in situ water content of a soil and its plastic limit to its plasticity index. Table 3.4 shows a description of soil strength based on values of LI.

Typical values for the Atterberg limits for soils are shown in Table 3.5. The Atterberg limits depend on the type of predominant mineral in the soil. If montmorillonite is the predominant mineral, the liquid limit can exceed 100%. Why? Recall that the bond between the layers in montmorillonite is weak and large amounts of water can easily infiltrate the spaces between the layers. In the case of kaolinite, the layers are held relatively tightly and water cannot easily infiltrate between the layers in comparison with montmorillonite. Therefore, you can expect the Atterberg limits for kaolinite to be, in general, much lower than either montmorillonite or illite.

TABLE 3.4 Description of Soil Strength Based on Liquidity Index

Values of LI	Description of soil strength
LI < 0	Semisolid state — high strength, brittle, (sudden) fracture is expected
0 < LI < 1	Plastic state — intermediate strength, soil deforms like a plastic material
LI > 1	Liquid state — low strength, soil deforms like a viscous fluid

TABLE 3.5 Typical Atterberg Limits for Soils

Soil type	LL (%)	PL (%)	PI (%)
Sand		Nonplastic	
Silt	30–40	20–25	10–15
Clay	40–150	25–50	15–100

Skempton (1953) showed that for soils with a particular mineralogy, the plasticity index is linearly related to the amount of the clay fraction. He coined a term called activity (A) to describe the importance of the clay fractions on the plasticity index. The equation for A is

$$A = \frac{PI}{\text{Clay fraction (\%)}} \quad (3.23)$$

You should recall that the clay fraction is the amount of particles less than 2 μm .

3.6 DETERMINATION OF THE LIQUID, PLASTIC, AND SHRINKAGE LIMITS



Interactive Concept Learning and Self-Assessment

Access Chapter 3, Section 3.6 on CD to learn about the liquid, plastic, and shrinkage limits and to view digital movies on index tests. Take Quiz 3.6 to test your understanding.

3.6.1 Casagrande Cup Method

The liquid limit is determined from an apparatus (Fig. 3.7) that consists of a semispherical brass cup that is repeatedly dropped onto a hard rubber base from a height of 10 mm by a cam-operated mechanism. The apparatus was developed by A. Casagrande (1932) and the procedure for the test is called the Casagrande cup method.

A dry powder of the soil is mixed with distilled water into a paste and placed in the cup to a thickness of about 12.5 mm. The soil surface is smoothed and a groove is cut into the soil using a standard grooving tool. The crank operating the cam is turned at a rate of 2 revolutions per second and the number of blows required to close the groove over a length of 12.5 mm is counted and recorded. A specimen of soil within the closed portion is extracted for determination of the water content. The liquid limit is defined as the water content at which the groove cut into the soil will close over a distance of 12.5 mm following 25 blows. This is difficult to achieve in a single test. Four

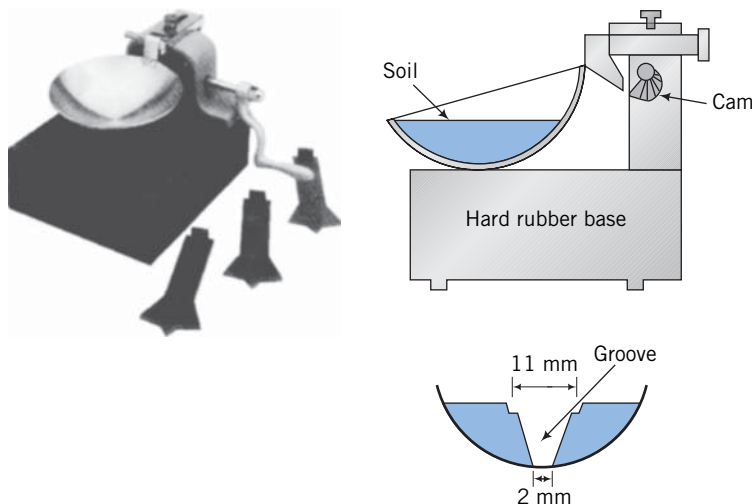


FIGURE 3.7 Cup apparatus for the determination of liquid limit. (Photo courtesy of Geotest.)

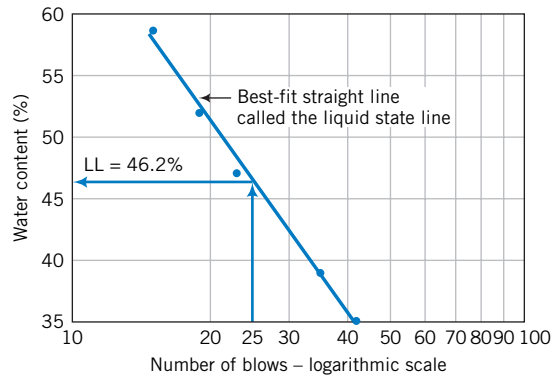


FIGURE 3.8 Typical liquid limit results from the Casagrande cup method.

or more tests at different water contents are usually required for terminal blows (number of blows to close the groove over a distance of 12.5 mm) ranging from 10 to 40. The results are presented in a plot of water content (ordinate, arithmetic scale) versus terminal blows (abscissa, logarithm scale) as shown in Fig. 3.8.

The best-fit straight line to the data points, usually called the flow line, is drawn. We will call this line the liquid state line to distinguish it from flow lines used in describing the flow of water through soils. The liquid limit is read from the graph as the water content on the liquid state line corresponding to 25 blows.

The cup method of determining the liquid limit has many shortcomings. Two of these are:

1. The tendency of soils of low plasticity to slide and to liquefy with shock in the cup rather than to flow plastically.
2. Sensitivity to operator and to small differences in apparatus.

3.6.2 Plastic Limit Test

The plastic limit is determined by rolling a small clay sample into threads and finding the water content at which threads approximately 3 mm in diameter will just start to crumble. Two or more determinations are made and the average water content is reported as the plastic limit.

3.6.3 Fall Cone Method to Determine Liquid and Plastic Limits

A fall cone test, popular in Europe and Asia, appears to offer a more accurate (less prone to operator's errors) method of determining both the liquid and plastic limits. In the fall cone test (Fig. 3.9), a cone with an apex angle of 30° and total mass of 80 grams is suspended above, but just in contact with, the soil sample. The cone is permitted to fall freely for a period of 5 seconds. The water content corresponding to a cone penetration of 20 mm defines the liquid limit.

The sample preparation is similar to the cup method except that the sample container in the fall cone test has a different shape and size (Fig. 3.9).

Four or more tests at different water contents are also required because of the difficulty of achieving the liquid limit from a single test. The results are plotted as water content (ordinate, logarithmic scale) versus penetration (abscissa, logarithm scale) and the best-fit straight line (liquid state line) linking the data points is drawn (Fig. 3.10). The liquid limit is read from the plot as the water content on the liquid state line corresponding to a penetration of 20 mm.

The plastic limit is found as follows.

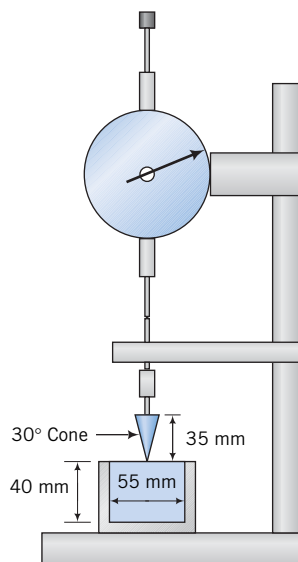


FIGURE 3.9 Fall cone apparatus.

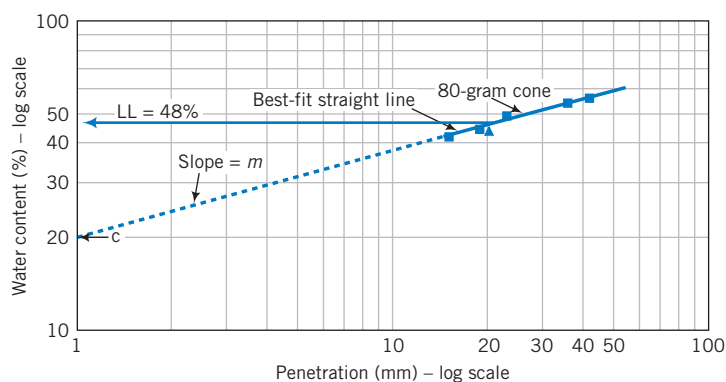


FIGURE 3.10 Typical fall cone test results.

Project the best fit-straight line backward to intersect the water content axis at a depth of penetration of 1 mm. The water content at this depth of penetration (1 mm) is c . The plastic limit is given as (Feng, 2000)

$$PL = c(2)^m \quad (3.24)$$

where m is the slope (taken as positive) of the best-fit straight line. If you use a spreadsheet program, you can obtain c and m from a power trend line function that gives the best fit equation.

3.6.4 Shrinkage Limit

The shrinkage limit is determined as follows. A mass of wet soil, m_1 , is placed in a porcelain dish 44.5 mm in diameter and 12.5 mm high and then oven-dried. The volume of oven-dried soil is determined by using mercury to occupy the vacant spaces caused by shrinkage. The mass of the mercury is determined and the volume decrease caused by shrinkage can be calculated from the known

density of mercury. The shrinkage limit is calculated from

$$SL = \left(\frac{m_1 - m_2}{m_2} - \frac{V_1 - V_2}{m_2} \frac{\gamma_w}{g} \right) \times 100 \quad (3.25)$$

where m_1 is the mass of the wet soil, m_2 is the mass of the oven-dried soil, V_1 is the volume of wet soil, V_2 is the volume of the oven-dried soil, and g is the acceleration due to gravity (9.8 m/s^2). The range of water content from the plastic to the shrinkage limits is called the shrinkage index (SI),

$$SI = PL - SL \quad (3.26)$$

The essential points are:

1. *Fine-grained soils can exist in one of four states: solid, semisolid, plastic, and liquid.*
2. *Water is the agent that is responsible for changing the states of soils.*
3. *A soil gets weaker if its water content increases.*
4. *Three limits are defined based on the water content that causes a change of state. These are the liquid limit—the water content that caused the soil to change from a liquid to a plastic state; the plastic limit—the water content that caused the soil to change from a plastic to a semisolid; and the shrinkage limit—the water content that caused the soil to change from a semisolid to a solid state. All these limiting water contents are found from laboratory tests.*
5. *The plasticity index defines the range of water content for which the soil behaves like a plastic material.*
6. *The liquidity index gives a measure of strength.*
7. *The soil strength is lowest at the liquid state and highest at the solid state.*

EXAMPLE 3.6 Interpreting Casagrande's Cup Data

A liquid limit test conducted on a soil sample in the cup device gave the following results:

Number of blows	10	19	23	27	40
Water content (%)	60.0	45.2	39.8	36.5	25.2

Two determinations for the plastic limit gave water contents of 20.3% and 20.8%. Determine (a) the liquid limit and plastic limit, (b) the plasticity index, (c) the liquidity index if the natural water content is 27.4%, and (d) the void ratio at the liquid limit, if $G_s = 2.7$. If the soil were to be loaded to failure, would you expect a brittle failure?

Strategy To get the liquid limit, you must make a semi-logarithm plot of water content versus number of blows. Use the data to make your plot, then extract the liquid limit (water content on the liquid state line corresponding to 25 blows). Two determinations of the plastic limit were made and the differences in the results are small. So, use the average value of water content as the plastic limit.

Solution 3.6

Step 1: Plot the data.
See Fig. E3.6.

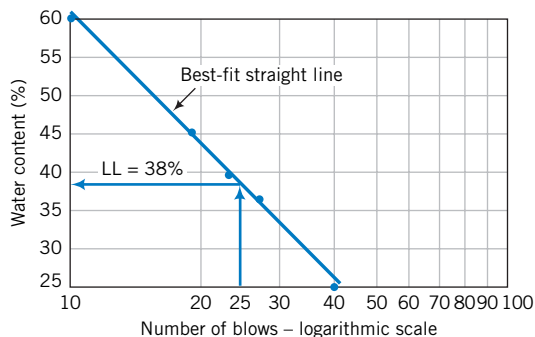


FIGURE E3.6 Plot of the liquid state line for the liquid limit by the Casagrande cup method.

Step 2: Extract the liquid limit.

The water content on the liquid state line corresponding to a terminal blow of 25 gives the liquid limit.

$$LL = 38\%$$

Step 3: Calculate plastic limit.

The plastic limit is

$$PL = \frac{20.3 + 20.8}{2} = 20.6\%$$

Step 4: Calculate PI

$$PI = LL - PL = 38 - 20.6 = 17.4\%$$

Step 5: Calculate LI

$$LI = \frac{(w - PL)}{PI} = \frac{27.4 - 20.6}{17.4} = 0.39$$

Step 6: Calculate the void ratio.

Assume the soil is saturated at the liquid limit. For a saturated soil, $e = wG_s$. Thus,

$$e_{LL} = LLG_s = 0.38 \times 2.7 = 1.03$$

Brittle failure is not expected as the soil is in a plastic state ($0 < LI < 1$). ■

EXAMPLE 3.7 Interpreting Fall Cone Data

The results of a fall cone test are shown in the table below.

Parameter	80 gram cone				
Penetration (mm)	5.5	7.8	14.8	22	32
Water content (%)	39.0	44.8	52.5	60.3	67

Determine (a) the liquid limit, (b) the plastic limit, (c) the plasticity index, and (d) the liquidity index if the natural water content is 46%.

Strategy Adopt the same strategy as in Example 3.6. Make a plot of water content versus penetration, both at logarithm scale. Use the data to make your plot, then extract the liquid limit (water content on the liquid state line corresponding to 20 mm). Find the water content difference between the two liquid state lines at any fixed penetration. Use this value to determine the plastic limit or use Eq. (3.24).

Solution 3.7

Step 1: Plot the data.
See Fig. E3.7.

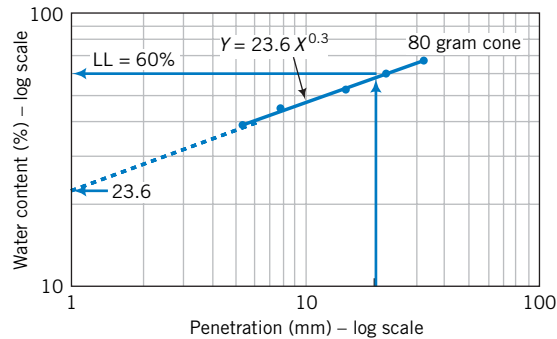


FIGURE E3.7 Plot of fall cone results.

Step 2: Extract the liquid limit.

$$LL = 60\%$$

Step 3: Determine the plastic limit.

The best-fit straight line for the 80-gram cone is $Y = 23.6X^{0.3}$ where Y is water content and X is penetration. Therefore, $C = 23.6$ and $m = 0.3$. From Eq. (3.25):

$$\begin{aligned} PL &= C(2)^m = 23.6(2)^{0.3} \\ &= 29\% \end{aligned}$$

Step 4: Determine PI.

$$PI = LL - PL = 60 - 29 = 31\%$$

Step 5: Determine LI.

$$LI = \frac{w - PL}{PI} = \frac{46 - 29}{31} = 0.55$$

What's next . . . We now know how to obtain some basic soil data—particle size and indices—from quick, simple tests. The question that arises is: What do we do with these data? Engineers would like to use the data to get a first impression on the use and possible performance of the soil for a particular purpose such as a construction material for an embankment. This is currently achieved by classification systems. Next, we will study two of these systems.

3.7 SOIL CLASSIFICATION SCHEMES



Interactive Concept Learning and Self-Assessment

Access Chapter 3, Section 3.7 on the CD to interactively learn about the unified classification system and then access Problem Solver on the left sidebar to interactively classify soils. Take Quiz 3.7 to test your understanding of the concepts.

A classification scheme provides a method of identifying soils in a particular group that would likely exhibit similar characteristics. Soil classification is used to specify a certain soil type that is best suitable for a given application. There are several classification schemes available. Each was devised for a specific use. For example, the American Association of State Highway and Transportation Officials (AASHTO) developed one scheme that classifies soils according to their usefulness in roads and highways while the Unified Soil Classification System (USCS) was originally developed for use in airfield construction but was later modified for general use.

Unified Soil Classification System The USCS is neither too elaborate nor too simplistic. The USCS uses symbols for the particle size groups. These symbols and their representations are: G—gravel, S—sand, M—silt, C—clay. These are combined with other symbols expressing gradation characteristics—W for well graded and P for poorly graded—and plasticity characteristics—H for high and L for low, and a symbol, O, indicating the presence of organic material. A typical classification of CL means a clay soil with low plasticity, while SP means a poorly graded sand. The flowcharts shown in Figs. 3.11a,b provide systematic means of classifying a soil according to the USCS. The American Society for Testing and Materials (ASTM) flowcharts linking the group symbols with group names are shown in Figs. 3.12a,b.

Experimental results from soils tested from different parts of the world were plotted on a graph of plasticity index (ordinate) versus liquid limit (abscissa). It was found that clays, silts, and organic soils lie in distinct regions of the graph. A line defined by the equation

$$PI = 0.73(LL - 20)\% \quad (3.27)$$

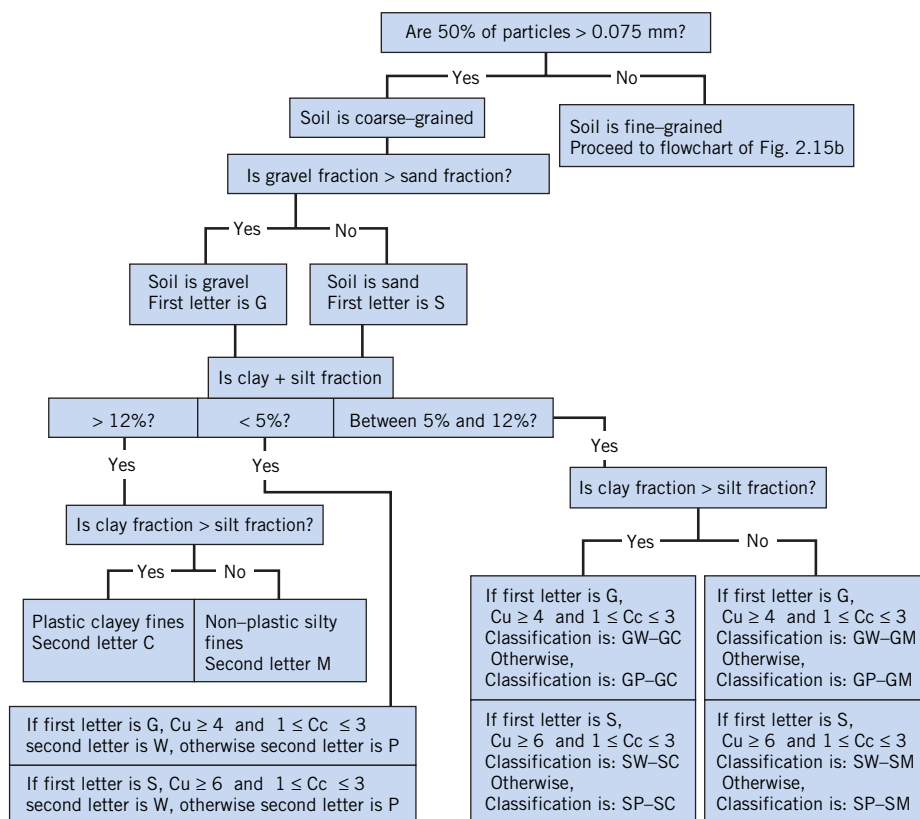


FIGURE 3.11a Unified Soil Classification flowchart for coarse-grained soils.

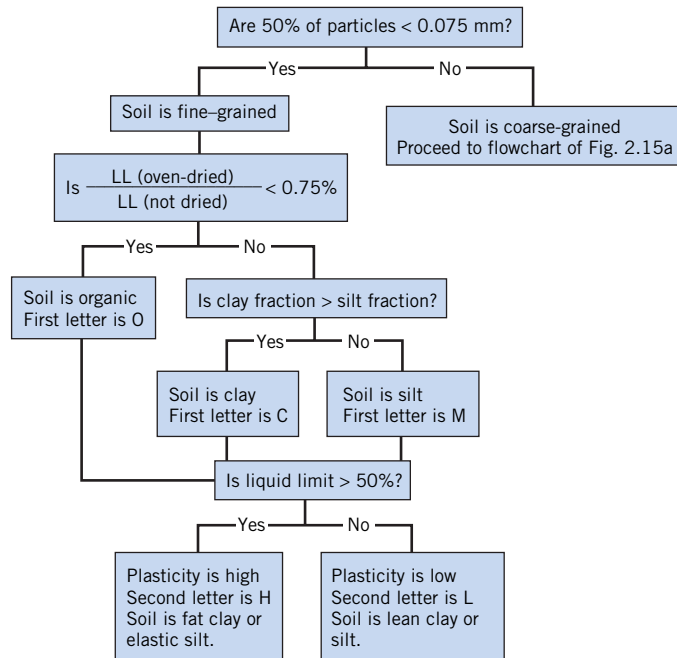
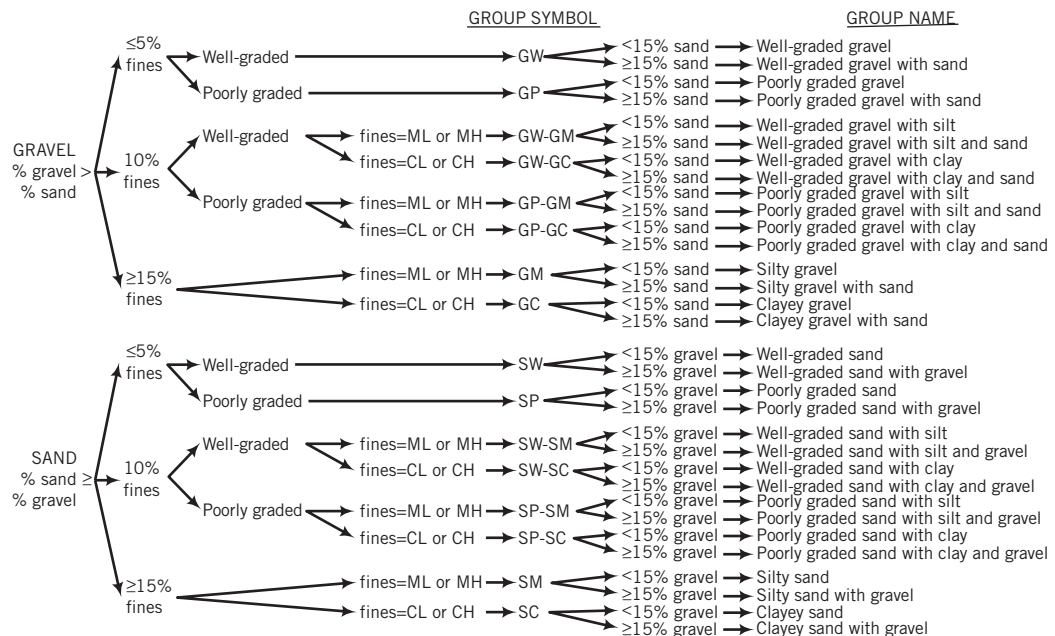


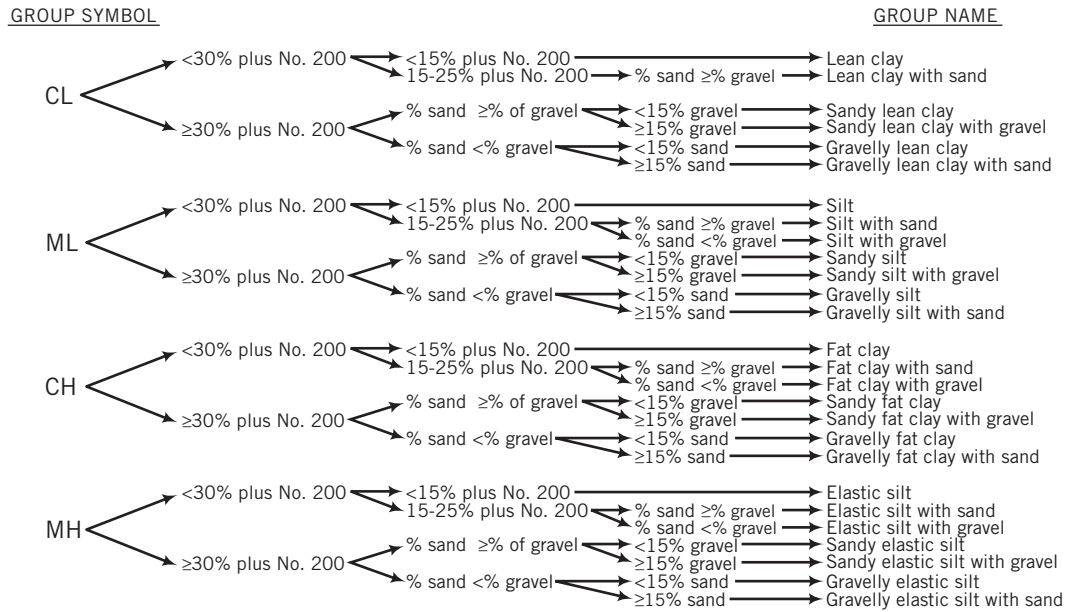
FIGURE 3.11b Unified Soil Classification flowchart for fine-grained soils.



Note—Percentages are based on estimating amounts of fines, sand, and gravel to the nearest 5%.

FIGURE 3.12a Flowchart for identifying coarse-grained soils (less than 50% fines). (Source: ASTM standards.)

called the “A-line,” delineates the boundaries between clays (above the line) and silts and organic soils (below the line) as shown in Fig. 3.13. A second line, the U-line expressed as $PI = 0.9(LL - 8)$, defines the upper limit of the correlation between plasticity index and liquid limit. If the results of your soil tests fall above the U-line, you should be suspicious of your results and repeat your tests.



Note—Percentages are based on estimating amounts of fines, sand, and gravel to the nearest 5%.

FIGURE 3.12b Flowchart for indentifying inorganic fine-grained soil (50% or more fines). (Source: ASTM standards.)

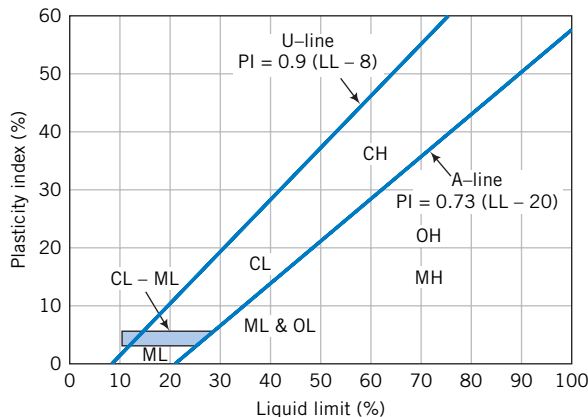


FIGURE 3.13 Plasticity chart.

AASHTO Soil Classification System The AASHTO soil classification is used to determine the suitability of soils for earthworks, embankments, and road bed materials (subgrade—natural material below a constructed pavement, subbase—a layer of soil above the subgrade, and base—a layer of soil above the subbase that offers high stability to distribute wheel loads). According to AASHTO, granular soils are soils in which 35% or less are finer than the No. 200 sieve (0.075 mm). Silt-clay soils are soils in which more than 35% are finer than the No. 200 sieve (Table 3.6).

The AASHTO system classifies soils into seven major groups, A-1 through A-7. The first three groups, A-1 through A-3, are granular (coarse-grained) soils while the last four groups, A-4 through A-7, are silt-clay (fine-grained) soils (Table 3.7).

TABLE 3.6 Soil Types, Average Grain Size, and Description According to AASHTO

Gravel	75 mm to 2 mm (No. 10 sieve)
Sand	2 mm (No. 10 sieve) to 0.075 mm (No. 200 sieve)
Silt & Clay	<0.075 mm (No. 200 sieve)
	Silty: PI < 10 %
	Clayey: PI > 11 %

TABLE 3.7A AASHTO Classification of Soils and Soil-Aggregate Mixtures

General Classification	Granular Materials (35% or less passing No. 200)			Silt-Clay Materials (More than 35% passing No. 200)			
Group Classification	A-1	A-3 ^A	A-2	A-4	A-5	A-6	A-7
Sieve analysis, % passing							
No. 10 (2.00 mm)	...	...	...	...	...	...	...
No. 40 (425 µm)	50 max	51 min	...	...	...	...	...
No. 200 (75 µm)	25 max	10 max	35 max	35 min	36 min	35 min	35 max
Characteristics of fraction passing No. 40 (425 µm)							
Liquid limit	...	...	^B	40 max	41 min	40 max	^B
Plasticity index	6 max	N.P.	^B	40 max	10 max	11 min	^B
General rating as subgrade	Excellent to Good			Fair to Poor			

^AThe placing of A-3 before A-2 is necessary in the "left to right elimination process" and does not indicate superiority of A-3 over A-2.

^BSee Table 3.7B for values.

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TABLE 3.7B AASHTO Classification of Soils and Soil-Aggregate Mixtures

General Classification			Granular Materials (35% or less passing No. 200)					Silt-Clay Materials (More than 35% passing No. 200)			
Group Classification	A-1		A-3	A-2				A-4	A-5	A-6	A-7 A-7-5, A-7-6
	A-1-a	A-1-b		A-2-4	A-2-5	A-2-6	A-2-7				
Sieve analysis, % passing											
No. 10 (2.00 mm)	50 max	...	...	...	...	...	...	...	...	...	...
No. 40 (425 µm)	30 max	50 max	51 min	...	...	...	...	...	...	...	...
No. 200 (75 µm)	15 max	25 max	10 max	35 max	35 max	35 max	35 max	36 min	36 min	36 min	35 min
Characteristics of fraction passing No. 40 (425 µm)											
Liquid limit	...		...	40 max	41 max	40 max	41 min	40 max	41 min	40 max	41 min
Plasticity index	6 max		N.P.	10 max	10 max	11 min	11 min	10 max	10 max	11 min	11 min ^A
Usual types of significant constituent materials	Stone Fragments, Gravel and Sand		Fine Sand	Silty or Clayey Gravel and Sand				Silty Soils		Clayey Soils	
General rating as subgrade			Excellent to Good					Fair to Poor			

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Silt and clay soils are located within the plasticity chart as shown in Fig. 3.14.

A group index (GI) value is appended in parentheses to the main group to provide a measure of quality of a soil as highway subgrade material. The group index is given as

$$\text{Group Index : GI} = (F - 35)[0.2 + 0.005(LL - 40)] + 0.01(F - 15)(PI - 10) \quad (3.28)$$

where F is percent passing No. 200 sieve and the other terms have been defined before. The GI index is reported to the nearest whole number (2.4 reported as 2, 2.5 reported as 3) and if $GI < 0$, it is set to 0.

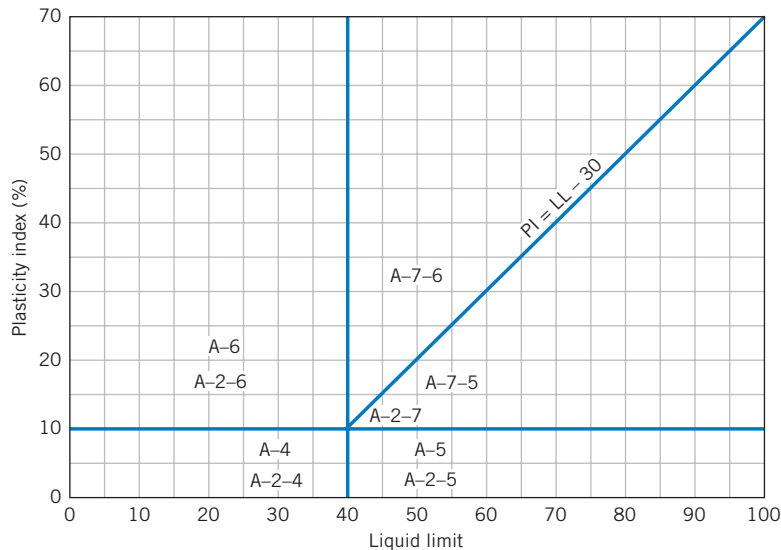


FIGURE 3.14 Silt and clay materials within the plasticity chart.

GI for groups A-1-a, A-1-b, A-2-4, A-2-5, and A-3 is zero. For groups A-2-6 and A-2-7, the partial group index equation

$$GI = 0.01(F - 15)(PI - 10) \quad (3.29)$$

is used. The higher the group index the lower the quality of the soil as a subgrade material. The GI should not exceed 20 for any of groups A-4 through A-7.

3.8 ENGINEERING USE CHART

You may ask: “How do I use a soil classification to select a soil for a particular type of construction, for example, a dam?” Geotechnical engineers have prepared charts based on experience to assist you in selecting a soil for a particular construction purpose. One such chart is shown in Table 3.8. The numerical values 1 to 9 are ratings with No. 1 the best. The chart should only be used to provide guidance and to make a preliminary assessment of the suitability of a soil for a particular use. You should not rely on such descriptions as “excellent” shear strength or “negligible” compressibility to make final design and construction decisions. We will deal later (Chapters 6 and 7) with more reliable methods to determine strength and compressibility properties.

EXAMPLE 3.8 Soil Classification

Particle size analyses were carried out on two soils—Soil A and Soil B—and the particle size distribution curves are shown in Fig. E3.8. The Atterberg limits for the two soils are:

Soil	LL	PL
A	26	18
B	Nonplastic	

- (a) Classify these soils according to the Unified Soil Classification Scheme.
- (b) Is either of the soils organic?
- (c) In a preliminary assessment, which of the two soils is a better material for the core of a rolled earth dam?

Strategy If you examine the flowcharts of Figs. 3.11a,b, you will notice that you need to identify the various soil types based on texture: for example, the percentage of gravel or sand. Use the particle size distribution curve to extract the different percentages of each soil type and then follow the flowchart. To determine whether your soil is organic or inorganic, plot your Atterberg limits on the plasticity chart and check whether the limits fall within an inorganic or organic soil region.

Solution 3.8

Step 1: Determine the percentages of each soil type from the particle size distribution curve.

Constituent	Soil A	Soil B
Percent of particle greater than 0.075 mm	12	80
Gravel fraction (%)	0	16
Sand fraction (%)	12	64
Silt fraction (%)	59	20
Clay fraction (%)	29	0

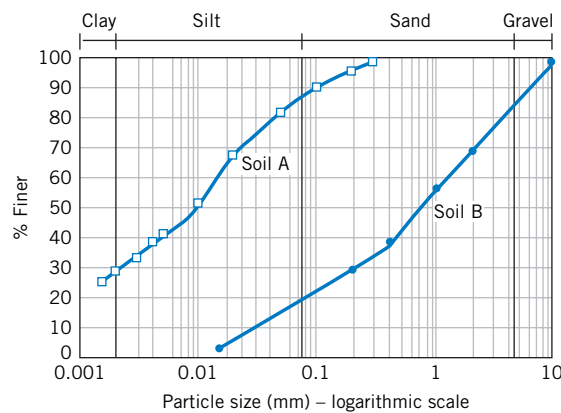


FIGURE E3.8

Step 2: Use the flowchart.

Following the flowchart (Figs. 3.11a,b), Soil A is ML and Soil B is SM.

Soil A: %<No. 200 = 59 + 29 = 88%, % sand > % gravel, gravel < 15%

Group symbol = ML, group name = sandy silt (Fig. 3.12)

Soil B: %<No. 200 = 20% (silt), % sand > 15%, fines = ML, gravel = 16%

Group symbol = SM, group name = silty sand with gravel

Step 3: Plot the Atterberg limits on the plasticity chart.

$$\text{Soil A: PI} = 26 - 18 = 8\%$$

TABLE 3.8 Engineering Use Chart (after Wagner, 1957)

Typical names of soil groups	Group symbols	Important properties			
		Permeability when compacted	Shearing strength when compacted and saturated	Compressibility when compacted and saturated	Workability as a construction material
Well-graded gravels, gravel–sand mixtures, little or no fines	GW	Pervious	Excellent	Negligible	Excellent
Poorly graded gravels, gravel–sand mixtures, little or no fines	GP	Very pervious	Good	Negligible	Good
Silty gravels, poorly graded gravel–sand–silt mixtures	GM	Semipervious to impervious	Good	Negligible	Good
Clayey gravels, poorly graded gravel–sand–clay mixtures	GC	Impervious	Good to fair	Very low	Good
Well-graded sands, gravelly sands, little or no fines	SW	Pervious	Excellent	Negligible	Excellent
Poorly graded sands, gravelly sands, little or no fines	SP	Pervious	Good	Very low	Fair
Silty sands, poorly graded sand–silt mixtures	SM	Semipervious to impervious	Good	Low	Fair
Clayey sands, poorly graded sand–clay mixtures	SC	Impervious	Good to fair	Low	Good
Inorganic silts and very fine sands, rock flour, silty or clayey fine sands with slight plasticity	ML	Semipervious to impervious	Fair	Medium	Fair
Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silky clays, lean clays	CL	Impervious	Fair	Medium	Good to fair
Organic silts and organic silt–clays of low plasticity	OL	Semipervious to impervious	Poor	Medium	Fair
Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts	MH	Semipervious to impervious	Fair to poor	High	Poor
Inorganic clays of high plasticity, fat clays	CH	Impervious	Poor	High	Poor
Organic clays of medium to high plasticity	OH	Impervious	Poor	High	Poor
Peat and other highly organic soils	Pt	—	—	—	—

The point (26, 8) falls above the A-line; the soil is inorganic.

Soil B: Nonplastic and inorganic

Step 4: Use Table 3.8 to make a preliminary assessment.

Soil B with a rating of 5 is better than Soil B with a rating of 6 for the dam core. ■

Relative desirability for various uses									
Rolled earth dams			Canal sections		Foundations		Roadways		
							Fills		Surfacing
Homogeneous embankment	Core	Shell	Erosion resistance	Compacted earth lining	Seepage important	Seepage not important	Frost heave not possible	Frost have possible	
—	—	1	1	—	—	1	1	1	3
—	—	2	2	—	—	3	3	3	—
2	4	—	4	4	1	4	4	9	5
1	1	—	3	1	2	6	5	5	1
—	—	3 if gravelly	6	—	—	2	2	2	4
—	—	4 if gravelly	7 if gravelly	—	—	5	6	4	—
4	5	—	8 if gravelly	5 erosion critical	3	7	8	10	6
3	2	—	5	2	4	8	7	6	2
6	6	—	—	6 erosion critical	6	9	10	11	—
5	3	—	9	3	5	10	9	7	7
8	8	—	—	7 erosion critical	7	11	11	12	—
9	9	—	—	—	8	12	12	13	—
7	7	—	10	8 volume change critical	9	13	13	8	—
10	10	—	—	—	10	14	14	14	—
—	—	—	—	—	—	—	—	—	—

EXAMPLE 3.9 AASHTO Soil Classification

Classify Soil A and B in Example 3.8 according to the AASHTO system. Which soil is better for a subgrade?

Strategy Determine if 35% or more of the particles are passing the No. 200 sieves. If so, the soil is one of A-4 through A-7. If not, it is one of A-1 through A-3. Extract the amount passing No. 10 and No. 40 and then use Table 3.7. You should also calculate GI.

Solution 3.9

Step 1: Determine % passing No. 200 sieve.

Soil A: 88% passing No. 200, i.e., >35% passing No. 200; Soil B: 20% passing No. 200, i.e., <35% passing No. 200.

Soil A is silt-clay; Soil B is granular.

Step 2: Make a table of values according to Table 3.7.

	Soil A	Soil B
No. 10	100	70
No. 40	100	40
No. 200	88	20
LL (%)	26	18
PI (%)	8	NP

Soil A: $GI = (F - 35)[0.2 + 0.005(LL - 40)] + 0.01(F - 15)(PI - 10) = (88 - 35)[0.2 + 0.005(26 - 40)] + 0.01(88 - 15)(8 - 10) = 5.4 = 5$.

Soil B is nonplastic. Therefore, $GI = 0$.

Step 3: Use Table 3.7 with the values in Step 2 to classify the soils.

Soil A is A-4 (5). Note: The value in parentheses is GI.

Soil B is A-1-b [stone fragments, gravel and sand].

Step 4: Decide which soil is better for a subgrade material.

According to Table 3.7, Soil B (A-1-b) is an excellent material for a subgrade. Soil A is fair to poor. Soil B is then the preferable material. ■

3.9 DRY UNIT WEIGHT–WATER CONTENT RELATIONSHIP**Concept Learning and Self-Assessment**

Access Chapter 3, Section 3.9, to learn about soil compaction. Take Quiz 3.9 to test your understanding.

3.9.1 Basic Concept

Let's examine Eq. (3.12) for dry unit weight, that is,

$$\gamma_d = \left(\frac{G_s}{1 + e} \right) \gamma_w = \frac{\gamma}{1 + w} = \left(\frac{G_s}{1 + wG_s/S} \right) \gamma_w \quad (3.30)$$

The extreme right-hand-side term was obtained by replacing e by $e = wG_s/S$. How can we increase the dry unit weight? Examination of Eq. (3.30) reveals that we have to reduce the void ratio; that is, w/S must be reduced. The theoretical maximum dry unit weight is obtained when $S = 1$; that is,

$$e_{\min} = wG_s \quad (3.31)$$

3.9.2 Proctor Compaction Test

A laboratory test, called the Proctor test, was developed to deliver a standard amount of mechanical energy (compactive effort) to determine the maximum dry unit weight of a soil. In the standard Proctor

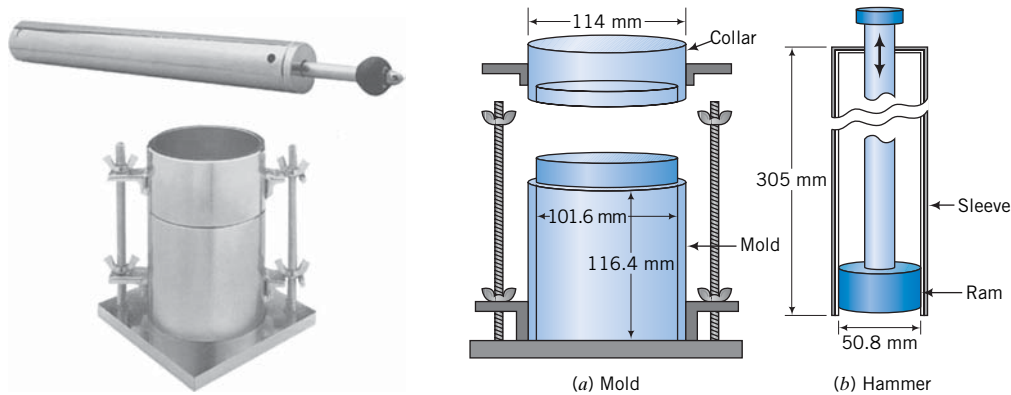


FIGURE 3.15 Compaction apparatus. (Photo courtesy of Geotest.)

test, a dry soil specimen is mixed with water and compacted in a cylindrical mold of volume of $9.44 \times 10^{-4} \text{ m}^3$ (standard Proctor mold) by repeated blows from the mass of a hammer, 2.5 kg, falling freely from a height of 305 mm (Fig. 3.15). The soil is compacted in three layers, each of which is subjected to 25 blows.

A modified Proctor test was developed for compaction of airfields to support heavy aircraft loads. In the modified Proctor test, a hammer with a mass 4.54 kg falls freely from a height of 457 mm. The soil is compacted in five layers with 25 blows per layer in the standard Proctor mold.

Four or more tests are conducted on the soil using different water contents. The last test is identified when additional water causes the bulk unit weight of the soil to decrease. The results are plotted as dry unit weight (ordinate) versus water content (abscissa). Typical dry unit weight–water content plots are shown in Fig. 3.16.

Clays usually yield bell-shaped curves. Sands do not often yield a clear bell; rather, they sometimes show an initial decrease in dry unit weight, attributed to capillary tension that restrains the free movement of soil particles, and then yield the bell-shaped curve. Some soils, those with liquid limit less than 30%, may produce one or more humps before the maximum dry unit weight is achieved.

The water content at which the maximum dry unit weight, $(\gamma_d)_{\max}$, is achieved is called the optimum water content (w_{opt}). At water contents below optimum (dry of optimum), air is expelled and

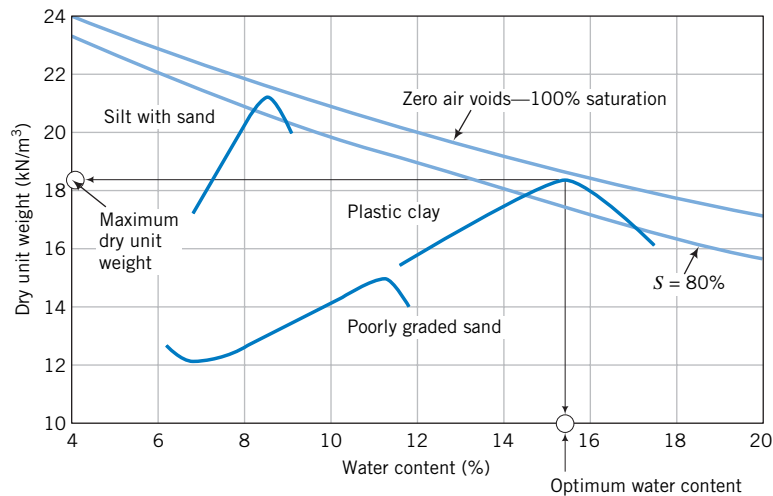


FIGURE 3.16 Dry unit weight–water content curves.

water facilitates the rearrangement of soil grains into a denser configuration—the number of soil grains per unit volume of soil increases. At water contents just above optimum (wet of optimum), the compactive effort cannot expel more air and additional water displaces soil grains, thus decreasing the number of soil grains per unit volume of soil. Consequently, the dry unit weight decreases.

3.9.3 Zero Air Voids Curve

The soil is invariably unsaturated at the maximum dry unit weight, that is, $S < 1$. We can determine the degree of saturation at the maximum dry unit weight using Eq. (3.30). We know $\gamma_d = (\gamma_d)_{\max}$ and $w = w_{\text{opt}}$ from our Proctor test results. If G_s is known, we can solve Eq. (3.30) for S . If G_s is not known, you can substitute a value of 2.7 with little resulting error in most cases. Equation (3.30) can be used to plot a series of theoretical curves of dry unit weight versus water content for different degrees of saturation (lines of constant degree of saturation) as shown in Fig. 3.16 for $S = 100\%$ and $S = 80\%$. You plot these curves as follows:

1. Assume a fixed value of S , say, $S = 1$ (100% saturation).
2. Substitute arbitrarily chosen values of w , approximately within the range of water content on your graph.
3. With the fixed value of S and either an estimated value of $G_s (= 2.7)$ or a known value, find γ_d for each value of w using Eq. (3.30) and plot the results of γ_d versus w .
4. Repeat for a different value of S .

The curve corresponding to $S = 1$ is known as the saturation line or zero air voids line. This line represents the minimum void ratio ($e_{\min}$) attainable at a given water content [Eq. (3.31)].

The achievement of zero air voids by compaction is rare. The modified Proctor test, using higher levels of compaction energy, achieves a higher maximum dry unit weight at a lower optimum water content than the standard test (Fig. 3.17). The degree of saturation is also lower at higher levels of compaction than the standard compaction test.

3.9.4 Importance of Compaction

Knowledge of the optimum water content and the maximum dry unit weight of soils is very important for construction specifications of soil improvement by compaction. Specifications for earth structures (embankments, footings, etc.) usually call for a minimum of 95% of Proctor maximum dry unit weight. This level of compaction can be attained at two water contents: one before the attainment of the maximum dry unit weight or dry of optimum, the other after attainment of the maximum dry unit weight or wet of optimum. Normal practice is to compact the soil at the lower water content value except for swelling (expansive) soils.

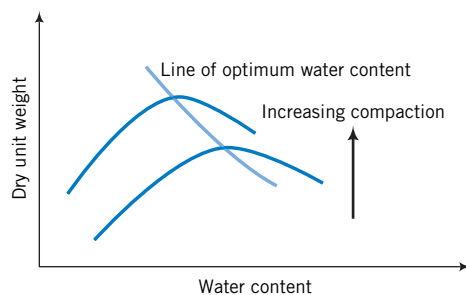


FIGURE 3.17 Effect of increasing compaction efforts on the dry unit weight–water content relationship.

Compaction increases the strength, lowers the compressibility, and reduces the flow rate of water (permeability) of a soil by rearranging its fabric. The soil fabric is forced into a dense configuration by the mechanical effort used in compaction. Compaction is the most popular technique of improving soils.

Although soil strength increases and the flow rate of water decreases with soil compaction, you need to be careful when shearing stresses are applied to the compacted soil. When a heavily compacted soil mass (near to maximum dry unit weight) is sheared, it tends to expand (dilate) and gets looser. Usually this expansion is not uniform; some parts of the soil mass are looser than other parts. The flow rate of water in the soil will increase as water can easily (compared to the intact one) flow through the looser parts possibly leading to catastrophic failure. Heavily compacted soils tend to show sudden decrease in strength when sheared. In some earth structures, for example, earth dams, you should try to achieve a level of compaction that would cause the soil to behave ductile (ability to deform without rupture).

3.9.5 Field Compaction

A variety of mechanical equipment is used to compact soils in the field. You may have seen various types of rollers being used in road construction. Each type of roller has special mechanical systems to effectively compact a particular soil type. For example, a sheepfoot roller (Fig. 3.18a) is generally used to compact fine-grained soils while a drum type roller (Fig. 3.18b) is generally used to compact coarse-grained soils.

3.9.6 Compaction Quality Control

A geotechnical engineer needs to check that field compaction meets specifications. Various types of equipment are available to check the amount of compaction achieved in the field. Three popular apparatuses are (1) the sand cone, (2) the balloon, and (3) nuclear density meters.

3.9.6.1 Sand Cone A sand cone apparatus is shown in Fig. 3.19. It consists of a glass or plastic jar with a funnel attached to the neck of the jar.

The procedure for a sand cone test is as follows:

1. Fill the jar with a standard sand—a sand with known density—and determine the weight of the sand cone apparatus with the jar filled with sand (W_1). The American Society for Testing and Materials (ASTM) recommends Ottawa sand as the standard.
2. Determine the weight of sand to fill the cone (W_2).
3. Excavate a small hole in the soil and determine the weight of the excavated soil (W_3).
4. Determine the water content of the excavated soil (w).
5. Fill the hole with the standard sand by inverting the sand cone apparatus over the hole and opening the valve.
6. Determine the weight of the sand cone apparatus with the remaining sand in the jar (W_4).
7. Calculate the unit weight of the soil follows:

$$\text{Weight of sand to fill hole: } W_s = W_1 - (W_2 + W_4)$$

$$\text{Volume of hole: } V = \frac{W_s}{(\gamma_d)_{\text{Ottawa sand}}}$$

$$\text{Weight of dry soil: } W_d = \frac{W_3}{1 + w}$$

$$\text{Dry unit weight: } \gamma_d = \frac{W_d}{V}$$

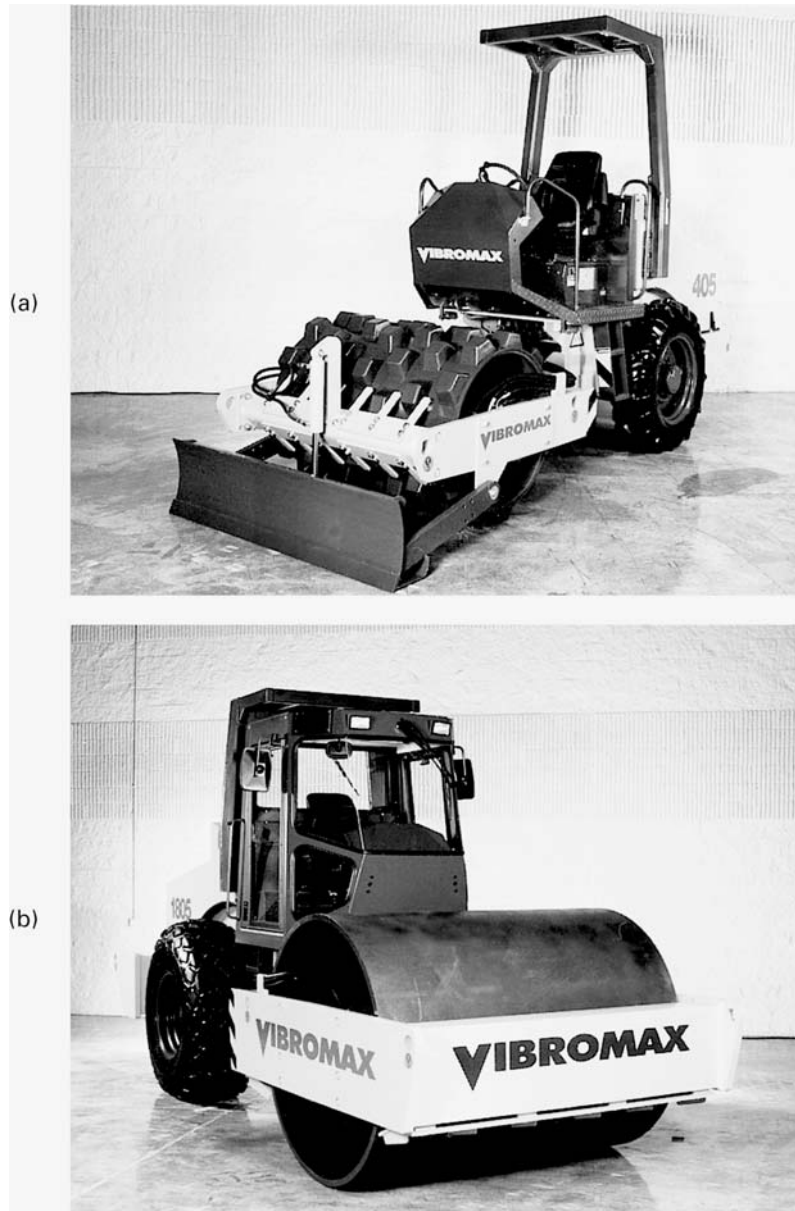


FIGURE 3.18 Two types of machinery for field compaction. (Photos courtesy of Vibromax America, Inc.)

3.9.6.2 Balloon Test The balloon test apparatus (Fig. 3.20) consists of a graduated cylinder with a centrally placed balloon. The cylinder is filled with water. The procedure for the balloon test is as follows:

1. Fill the cylinder with water and record its volume, V_1 .
2. Excavate a small hole in the soil and determine the weight of the excavated soil (W).
3. Determine the water content of the excavated soil (w).

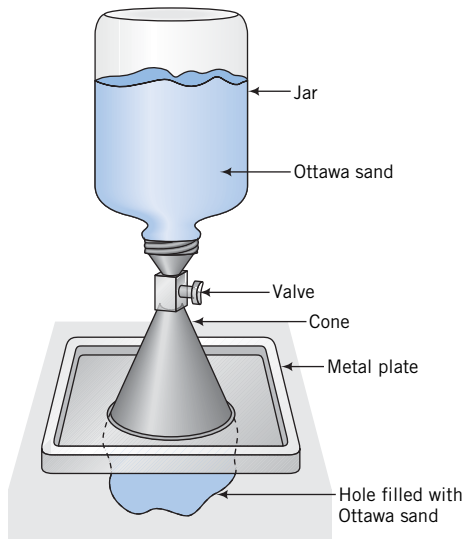


FIGURE 3.19 A sand cone apparatus.

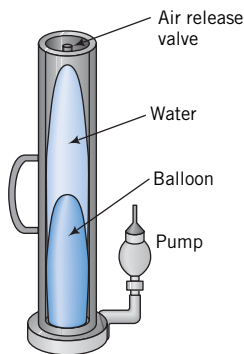


FIGURE 3.20 Balloon test device.

4. Use the pump to invert the balloon to fill the hole.
5. Record the volume of water remaining in the cylinder, V_2 .
6. Calculate the unit weight of the soil as follows:

$$\gamma = \frac{W}{V_1 - V_2}; \quad \gamma_d = \frac{\gamma}{1 + w}$$

3.9.6.3 Nuclear Density Meter The nuclear density apparatus (Fig. 3.21) is a versatile device to rapidly obtain the unit weight and water content of the soil nondestructively. Soil particles cause radiation to scatter to a detector tube and the amount of scatter is counted. The scatter count rate is inversely proportional to the unit weight of the soil. If water is present in the soil, the hydrogen in water scatters the neutrons and the amount of scatter is proportional to the water content. The radiation source is either radium or radioactive isotopes of cesium and americium. The nuclear density apparatus is first calibrated using the manufacturer's reference blocks. This calibration serves as a reference to determine the unit weight and water content of a soil at a particular site.



FIGURE 3.21 Nuclear density meter. (Photo courtesy of Seaman Nuclear Corp.)

The essential points are:

1. *Compaction is the densification of a soil by the expulsion of air and the rearrangement of soil particles.*
2. *The Proctor test is used to determine the maximum dry unit weight and the optimum water content and serves as the reference for field specifications of compaction.*
3. *Higher compactive effort increases the maximum dry unit weight and reduces the optimum water content.*
4. *Compaction increases strength, lowers compressibility, and reduces the permeability of soils.*
5. *A variety of field equipment is used to check the dry unit weights achieved in the field. Popular field equipment includes the sand cone apparatus, the balloon apparatus, and the nuclear density meter.*

EXAMPLE 3.10 Interpreting Compaction Data

The results of a standard compaction test are shown in the table below. Determine the maximum dry unit weight and optimum water content.

Water content (%)	6.2	8.1	9.8	11.5	12.3	13.2
Bulk unit weight (kN/m ³)	16.9	18.7	19.5	20.5	20.4	20.1

- (a) What is the dry unit weight and water content at 95% standard compaction?
- (b) Determine the degree of saturation at the maximum dry density.
- (c) Plot the zero air voids line.

Strategy Compute γ_d and then plot the results of γ_d versus w (%). Then extract the required information.

Solution 3.10

Step 1: Use a table or a spreadsheet program to tabulate γ_d .

Use Eq. (3.12) to find the dry unit weight and Eq. (3.30) to calculate the dry unit weight for $S = 1$.

Water content	Bulk unit weight (kN/m ³)	Dry unit weight (kN/m ³) $\gamma_d = \frac{\gamma}{1+w}$	Zero air voids	
			Water content (%)	Dry unit weight (kN/m ³) $\gamma_d = \left(\frac{G_s}{1 + wG_s/S} \right) \gamma_w$ $S = 1$
6.2	16.9	15.9	6	22.8
8.1	18.7	17.3	8	21.8
9.8	19.5	17.8	10	20.8
11.5	20.5	18.4	12	20.0
12.3	20.4	18.2	14	19.2
13.2	20.1	17.8		

Step 2: Plot graphs as shown in Fig. E3.10

Step 3: Extract the desired values.

$$(\gamma_d)_{\max} = 18.4 \text{ kN/m}^3, \quad w_{\text{opt}} = 11.5\%$$

At 95% compaction, $\gamma_d = 18.4 \times 0.95 = 17.5 \text{ kN/m}^3$ and $w = 9.2\%$ (from graph).

Step 4: Calculate the degree of saturation at maximum dry unit weight.

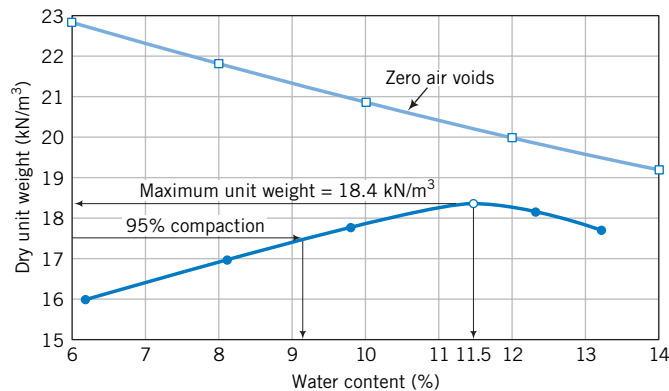


FIGURE E3.10 Compaction test results.

Algebraically manipulate Eq. (3.30) to find S as

$$S = \frac{wG_s(\gamma_d)_{\max}/\gamma_w}{G_s - (\gamma_d)_{\max}/\gamma_w} = \frac{0.115 \times 2.7 \times (18.4/9.8)}{2.7 - 18.4/9.8} = 0.71 = 71\%$$

3.10 SUMMARY

We have dealt with a large body of basic information on the physical properties of soils. Soils are conveniently idealized as three-phase materials: solids, water, and air. The physical properties of a soil depend on the relative proportion of these constituents in a given mass. Soils are classified into groups through their particle sizes and Atterberg limits. Soils within the same group are likely to have similar mechanical behavior and construction use. A soil mass can be made denser by removing the air constituents through mechanical effort (compaction). Some of the main physical parameters for soils are the particle sizes, void ratio, liquid limit, plastic limit, shrinkage limit, and plasticity and liquidity indices. Water can significantly change the characteristics of soils.

Practical Examples

EXAMPLE 3.11 Calculating Soil Quantities for a Highway Embankment

An embankment for a highway 30 m wide and 1.5 m in compacted thickness is to be constructed from a sandy soil trucked from a borrow pit. The water content of the sandy soil in the borrow pit is 15% and its void ratio is 0.69. The specification requires the embankment be compacted to a dry unit weight of 18 kN/m³. Determine, for 1 km length of embankment, the following:

- (a) The weight of sandy soil from the borrow pit required to construct the embankment.
- (b) The number of 10.0 m³ truckloads of sandy soil required for the construction.
- (c) The weight of water per truckload of sandy soil.
- (d) The degree of saturation of the sandy soil in situ.

Strategy The strategy is similar to that adopted in Example 3.3.

Solution 3.11

Step 1: Calculate γ_d for the borrow pit material.

$$\gamma_d = \frac{G_s \gamma_w}{1 + e} = \frac{2.7 \times 9.8}{1 + 0.69} = 15.7 \text{ kN/m}^3$$

Step 2: Determine the volume of borrow pit soil required.

$$\text{Volume of finished embankment: } V = 30 \times 1.5 \times 1 = 45 \text{ m}^3$$

$$\begin{aligned} \text{Volume of borrow pit soil required: } \frac{(\gamma_d)_{\text{required}}}{(\gamma_d)_{\text{borrow pit}}} \times V &= \frac{18}{15.7} \times 45 \times 10^3 \\ &= 51.6 \times 10^3 \text{ m}^3 \end{aligned}$$

Step 3: Determine the number of trucks required.

$$\text{Number of trucks} = \frac{51.6 \times 10^3}{10} = 5160$$

Step 4: Determine the weight of water required.

$$\text{Weight of dry soil in one truckload: } W_d = 10 \times 15.7 = 157 \text{ kN}$$

$$\text{Weight of water: } wW_d = 0.15 \times 157 = 23.6 \text{ kN}$$

Step 5: Determine the degree of saturation.

$$S = \frac{wG_s}{e} = \frac{0.15 \times 2.7}{0.69} = 0.59 = 59\%$$

EXAMPLE 3.12 *Calculating Soil Quantities and Cost for a Dam*

An earth dam requires 1 million cubic meters of soil compacted to a void ratio of 0.8. In the vicinity of the proposed dam, three borrow pits were identified as having suitable materials. The cost of purchasing the soil and the cost of excavation are the same for each borrow pit. The only cost difference is transportation cost. The table below provides the void ratio and the transportation cost for each borrow pit. Which borrow pit would be the most economical?

Borrow pit	Void ratio	Transportation cost (\$/m ³)
#1	1.8	\$0.60
#2	0.9	\$1.00
#3	1.5	\$0.75

Strategy The specific volume is very useful in this problem to find the desired volume of borrow pit material.

Solution 3.12

Step 1: Define parameters and set up relevant equations. Let V_o, e_o be the specific volume and void ratio of the compacted soil in the dam; and V_i, e_i be the specific volume and void ratio of the soil from the borrow pits, where $i = 1, 2, 3$. Now

$$\frac{V_i}{V_o} = \frac{1 + e_i}{1 + e_o}$$

and

$$V_i = V_o \left(\frac{1 + e_i}{1 + e_o} \right) = \frac{1 \times 10^6}{1 + 0.8} (1 + e_i)$$

Step 2: Determine the volume of soil from each borrow pit. Substituting the void ratio from the table into the last equation, we obtain

$$V_1 = 1,555,555 \text{ m}^3$$

$$V_2 = 1,055,555 \text{ m}^3$$

$$V_3 = 1,388,888 \text{ m}^3$$

Step 3: Determine transportation costs.

$$\text{Transportation cost} = \text{Volume} \times \$/\text{m}^3$$

$$\text{Pit 1} = \$933,333$$

$$\text{Pit 2} = \$1,055,555$$

$$\text{Pit 3} = \$1,041,667$$

Borrow pit 1 is therefore the most economical.

EXERCISES

Theory

3.1 Prove the following relations:

$$(a) \gamma_d = \frac{G_s \gamma_w}{1 + e}$$

$$(b) S = \frac{w G_s (1 - n)}{n}$$

3.2 Show that

$$D_r = \frac{\gamma_d - (\gamma_d)_{\min}}{(\gamma_d)_{\max} - (\gamma_d)_{\min}} \left\{ \frac{(\gamma_d)_{\max}}{\gamma_d} \right\}$$

3.3 Tests on a soil gave the following results: $G_s = 2.7$ and $e = 1.96$. Make a plot of degree of saturation versus water content for this soil.

3.4 Assuming soil particles to be spheres, derive equations for the maximum and minimum porosities, and maximum and minimum void ratios.

3.5 A spherical soil particle of radius r is settling in water, which is at rest, at a constant velocity, v . If the drag force for a sphere is $6\pi r\eta v$, show that

$$r = \sqrt{\frac{9\eta}{2(\gamma_s - \gamma_w)}} \sqrt{v}$$

where γ_s is the unit weight of the spherical soil particle, γ_w is the unit weight of water, and η is the dynamic or absolute viscosity of water. [Hint: Draw a free body diagram (see Fig. P3.5) and use statics to solve the problem. You are actually deriving Stokes's equation.]

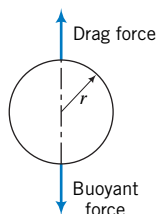


FIGURE P3.5

Problem Solving



Interactive Problem Solving

Access Chapter 3, click on Problem Solver to interactively solve problems similar to some in this chapter.

Assume $G_s = 2.7$, where necessary, for solving the following problems.

- 3.6 A cylinder has 500 cm³ of water. After a mass of 100 grams of sand is poured into the cylinder and all air bubbles are removed by a vacuum pump, the water level rises to 537.5 cm³. Determine the specific gravity of the sand.
- 3.7 The mass of a sample of saturated soil is 500 grams. The dry mass, after oven drying, is 400 grams. Determine the (a) water content, (b) void ratio, (c) saturated unit weight, and (d) effective unit weight.

- 3.8** A soil sample has a bulk unit weight of 19.6 kN/m^3 at a water content of 10%. Determine the void ratio, percentage air in the voids (air voids), and the degree of saturation of this sample.
- 3.9** A saturated silty clay encountered in a deep excavation is found to have a water content of 23%. Determine its porosity and bulk unit weight.
- 3.10** A wet sand sample has a volume of $4.64 \times 10^{-4} \text{ m}^3$ and weighs 8 N. After oven drying, the weight reduces to 7.5 N. Calculate the water content, void ratio, and degree of saturation.
- 3.11** A sandy soil has a natural water content of 10% and bulk unit weight of 18.5 kN/m^3 . The void ratios corresponding to the densest and loosest state of this soil are 0.51 and 0.87. Find the relative density and degree of saturation for this soil.
- 3.12** The void ratio of a soil is 1.2. Determine the bulk and effective unit weights for the following degrees of saturation: (a) 75%, (b) 95%, and (c) 100%. What is the percentage error in the bulk unit weight if the soil were 95% saturated but assumed to be 100% saturated?
- 3.13** A particle size analysis on a soil sample yields the following data. Plot the particle size distribution curve and classify the soil using the Unified Soil Classification System. Clay to silt ratio is 3:1.

Sieve no.	4	10	20	60	200	Pan
Sieve size (mm)	4.75	2.0	0.84	0.25	0.074	—
Weight retained (N)	3.1	5.8	3.8	2.6	6.8	2.1

- 3.14** The following results were obtained from sieve analyses and Atterberg limits of three soils—A, B, and C. Hydrometer tests were conducted on the material that passed the 200 sieve for Soil A and Soil C. The ratio of clay to silt is 4:1 for Soil A and 1.5:1 for Soil C. Classify each of these soils according to the USCS.

Sieve no.	Opening (mm)	Mass retained (grams)		
		Soil A	Soil B	Soil C
4	4.75	0	0	0
10	2.00	20.2	48.2	15
20	0.85	25.7	19.6	98
40	0.425	40.4	60.3	90
100	0.15	18.1	37.2	182
200	0.075	27.2	22.1	109
Pan		68.2		6
LL		23%	non-plastic	non-plastic
PL		8%		

- 3.15** Repeat Problem 3.14 but use the AASHTO classification system.
- 3.16** The following results were obtained from a liquid limit test on a clay using the Casagrande cup device.

Number of blows	6	12	20	28	32
Water content (%)	52.5	47.1	43.2	38.6	37.0

- (a) Determine the liquid limit of this clay.
- (b) If the natural water content is 38% and the plastic limit is 23%, calculate the liquidity index.
- (c) Do you expect a brittle type of failure for this soil? Why?

- 3.17** A fall cone test was carried out on a soil to determine its liquid and plastic limits using a cone of mass 80 grams. The following results were obtained:

80 gram cone				
Penetration (mm)	8	15	19	28
Water content (%)	43.1	52.0	56.1	62.9

Determine (a) the liquid and plastic limits and (b) the plasticity index.

- 3.18** The following data were obtained from a Proctor test:

Water content (%)	12	13	14	16	19
Weight of wet sample (N)	17	18	18.7	19.3	18.9

The volume of the mold was $9.44 \times 10^{-4} \text{ m}^3$.

- (a) Determine the optimum water content and maximum dry unit weight.
 - (b) Plot the zero air voids line.
 - (c) Determine the degree of saturation at the maximum dry unit weight.
- 3.19** A 200,000 m³ clay embankment is to be constructed at an optimum water content of 16%. It is expected that 95% saturation will be attained. Calculate the volume of soil that will be needed if the void ratio of the fill material is 1.3.

Practical

- 3.20** The water contents of soil samples taken at different depths are given in the table below.

Depth (m)	1	2	3	4	5	6
w (%)	21.3	23.6	6.1	32.7	41.5	42.0

The ground water level is at the surface. Assume $G_s = 2.7$.

- (a) Plot on the same graph (a) depth versus water content and (b) depth versus saturated unit weight.
 - (b) Are there any questionable water content value(s)? If so, at what depth?
 - (c) If the water contents are all correct, what type or types of soils (e.g., clay, sand) are probably at the site?
- 3.21** A Proctor compaction test in the laboratory on a borrow pit soil gives a maximum dry unit weight of 19 kN/m³ and optimum water content of 11.5%. The bulk unit weight and water content of the soil in the borrow pit are 17.2 kN/m³ and 8.2%, respectively. A highway fill is to be constructed using this soil. The specifications require the fill to be compacted to 95% Proctor compaction.
- (a) How many cubic meters of borrow pit material is needed for 1 cubic meter of highway fill?
 - (b) The water content at 95% compaction is 11% on the dry side and 12% on the wet side of the compaction curve. What is the minimum additional weight of water per unit volume to achieve 95% Proctor compaction?
 - (c) If the soil were expansive, what is the additional weight of water per unit volume to achieve 95% Proctor compaction?
 - (d) How many truckloads of soil will be required for a 100,000 m³ highway embankment? Each truck has a capacity of 10 m³.

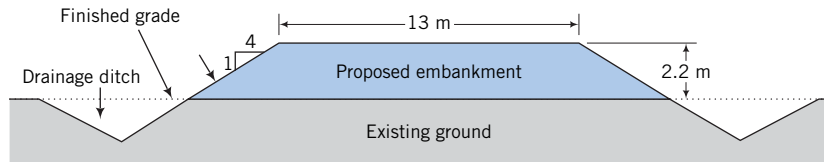


FIGURE P3.22a

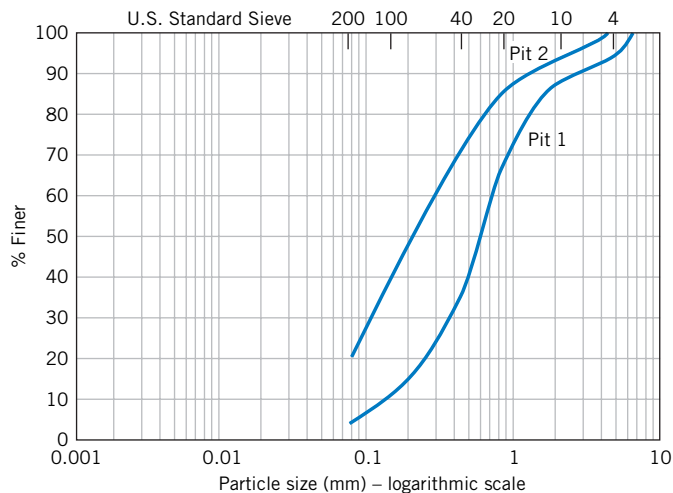


FIGURE P3.22b

(e) Determine the cost for 100,000 m³ of compacted soil based on the following:

purchase and load borrow pit material at site, haul 2 km round trip, and spread with 200 HP dozer = \$15/m³; extra mileage charge for each km = \$0.5/m³; round trip distance = 10 km; compaction = \$1.02/m³.

3.22 The highway embankment from Noscut to Windsor Forest, described in the sample practical situation, is 10 km long. The average cross section of the embankment is shown in Fig. P3.22a. The gradation curves for the soils at the two borrow pits are shown in Fig. P3.22b. Pit 1 is located 5 km from the start of the embankment while pit 2 is 3 km away. Estimated costs for various earthmoving operations are shown in the table below. You are given 10 minutes by the stakeholder's committee to present your recommendations. Prepare your presentation. The available visual aid equipment consists of an overhead projector and an LCD projector.

Operation	Cost	
	Pit 1	Pit 2
Purchase and load borrow pit material at site, haul 2 km round trip, and spread with 200 HP dozer	\$10/m ³	\$12/m ³
Extra mileage charge for each km	\$0.50/m ³	\$0.55/m ³
Compaction	\$1.02/m ³	\$1.26
Miscellaneous	\$1.10/m ³	\$0.85/m ³

CHAPTER

4

ONE-DIMENSIONAL FLOW OF WATER THROUGH SOILS

ABET	ES	ED
	90	10

4.0 INTRODUCTION

We have discussed particle sizes and index properties, and used these to classify soils. You know that water changes the soil states in fine-grained soils; the greater the water content in a soil the weaker it is. Soils are porous materials much like sponges. Water can flow between the interconnected voids. Particle sizes and the structural arrangement of the particles influence the rate of flow.

Water can cause instability and many geotechnical structures (e.g., roads, bridges, dams, and excavations) have failed because of instability from flow of water. The key physical property that governs flow of water in soils is permeability. In this chapter, we will discuss soil permeability by considering one-dimensional flow of water through soils. We will discuss instability from two-dimensional flow of water in Chapter 11.

When you complete this chapter, you should be able to:

- Determine the rate of flow of water through soils.
- Determine the hydraulic conductivity of soils.

Sample Practical Situation An excavation is required to construct the basement of a building. During construction, the base of the excavation needs to be free of water. The engineer decides to use a retaining wall around the excavation to keep it dry. Water from outside the excavation will flow under the wall and to determine the length of the wall to keep the excavation dry, the soil permeability must be known.

4.1 DEFINITIONS OF KEY TERMS

Groundwater is water under gravity that fills the soil pores.

Head (H) is the mechanical energy per unit weight.

Hydraulic conductivity (sometimes called the coefficient of permeability) (k) is a proportionality constant to determine the flow velocity of water through soils.

4.2 QUESTIONS TO GUIDE YOUR READING

1. What causes flow of water through soils?
2. What law describes the flow of water through soils?
3. What is permeability and how is it determined?
4. What are the values of hydraulic conductivities for coarse-grained and fine-grained soils?

4.3 GROUNDWATER



Interactive Concept Learning and Self-Assessment

Access Chapter 4, Section 4.3 for an interactive lesson on one-dimensional flow of water through soils. Take Quiz 4.3 to evaluate your understanding of the key ideas in Section 4.3.

We will be discussing gravitational flow of water under a steady state condition. You may ask: “What is a steady state condition?” Gravitational flow can only occur if there is a gradient. Flow takes place downhill. The steady state condition occurs if neither the flow nor the porewater pressures change with time. porewater pressure is the water pressure within the voids.

If you dig a hole into a soil mass that has all the voids filled with water (fully saturated), you will observe water in the hole up to a certain level. This water level is called groundwater level or groundwater table and exists under a hydrostatic condition. A hydrostatic condition occurs when the flow is zero. The top of the groundwater level is under atmospheric pressure. We will denote groundwater level by the symbol ▼.

4.4 HEAD



Interactive Concept Learning and Self-Assessment

Access Chapter 4, Section 4.4 to learn about hydraulic head. Take Quiz 4.4 to test your understanding.

Darcy’s law governs the flow of water through soils. But before we delve into Darcy’s law, we will discuss an important principle in fluid mechanics—Bernoulli’s principle—which is essential in understanding flow through soils.

If you cap one end of a tube, fill the tube with water, and then rest it on your table (Fig. 4.1), the height of water with reference to your table is called the pressure head (h_p). Head refers to the mechanical energy per unit weight. If you raise the tube above the table, the mechanical energy or total head increases.

You now have two components of total head—the pressure head (h_p) and the elevation head (h_z). If water were to flow through the tube with a velocity v , under steady state condition, then we have an additional head due to the velocity given as $v^2/2g$. The total head, H , according to Bernoulli’s principle is

$$H = h_z + h_p + \frac{v^2}{2g} \quad (4.1)$$

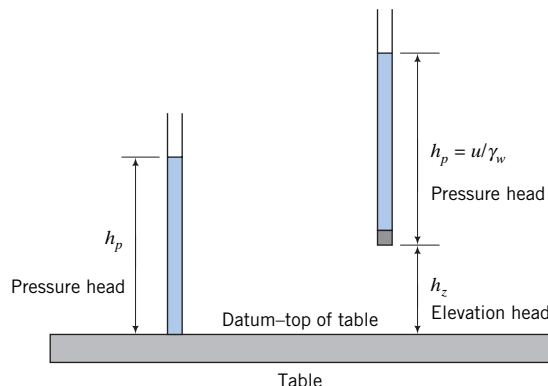


FIGURE 4.1 Illustration of elevation and pressure heads.

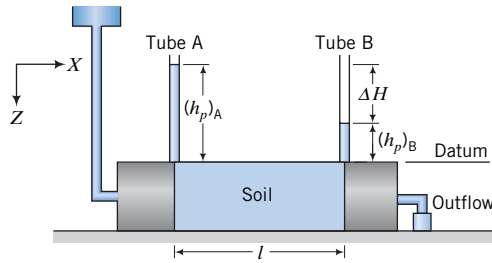


FIGURE 4.2 Head loss due to flow of water through soil.

The elevation or potential head is referenced to an arbitrary datum and the total head will change depending on the choice of the datum position. Therefore, it is essential that you identify your datum position in solutions to flow problems. Pressures are defined relative to atmospheric pressure. The velocity of flow through soils is generally small (<1 cm/s) and we usually neglect the velocity head. The total head in soils is then

$$H = h_z + h_p = h_z + \frac{u}{\gamma_w} \quad (4.2)$$

where $u = h_p \gamma_w$ is the porewater pressure.

Consider a cylinder containing a soil mass with water flowing through it at a constant rate as depicted in Fig. 4.2. If we connect two tubes, A and B, called piezometers, at a distance l apart, the water will rise to different heights in each of the tubes. The height of water in tube B near the exit is lower than A. Why? As the water flows through the soil, energy is dissipated through friction with the soil particles, resulting in a loss of head. The head loss between A and B, assuming decrease in head is positive and our datum is arbitrarily selected at the top of the cylinder, is $\Delta H = |(h_p)_B - (h_p)_A|$.

EXAMPLE 4.1 Determination of Hydraulic Heads

Determine (a) the variations of the elevation, pressure, and total heads through the soil when the pressure gage in the experimental setup shown in Fig. E4.1a has a pressure of 19.6 kPa, and (b) the elevation, pressure, and total heads in the middle of the soil.

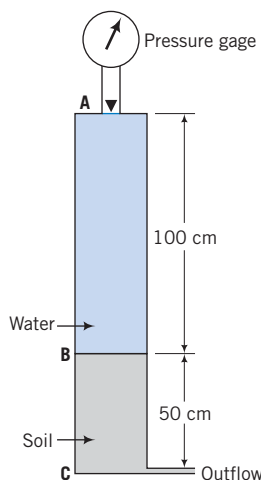


FIGURE E4.1a

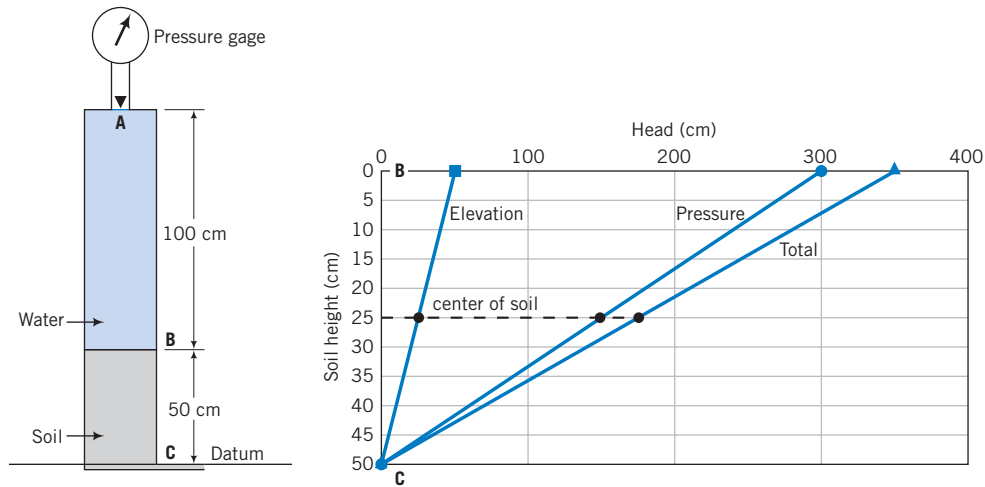


FIGURE E4.1b

Strategy The first thing you need to do is to define your datum. Outflows or exits are good choices; the pressure head there is zero. The pressure from the pressure gage should be converted to a pressure head by dividing by the unit weight of water (9.8 kN/m^3). To determine the pressure head at a point, assume that you connect a small tube at that point and then figure out how high the water will rise in the tube.

Solution 4.1

Step 1: Define the datum position. Choose C as datum (Fig. E4.1b).

Step 2: Determine the heads.

Top of soil—B

Pressure gage = 19.6 kPa; equivalent pressure head = pressure/unit weight of water = $19.6/9.8 = 2 \text{ m} = 200 \text{ cm}$

Elevation head = 50 cm; pressure head of water above B + pressure gage head = $100 + 200 = 300 \text{ cm}$; total head = elevation head + pressure head = $50 + 300 = 350 \text{ cm}$

Bottom of soil—C

Elevation head = 0 cm; pressure head = 0 cm; total head = $0 + 0 = 0 \text{ cm}$

Step 3: Determine the heads at the center of the soil.

The heads are linearly distributed through the soil. Therefore, the heads are proportional to the soil height.

At the center of the soil, elevation head = 25 cm, pressure head = $300/2 = 150 \text{ cm}$, and the total head = $25 + 150 = 175 \text{ cm}$. ■

4.5 DARCY'S LAW



Interactive Concept Learning and Self-Assessment

Access Chapter 4, Section 4.5 to learn about Darcy's law. Take Quiz 4.5 to test your understanding.

Darcy (1856) proposed that average flow velocity through soils is proportional to the gradient of the total head. The flow in any direction, j , is

$$v_j = k_j \frac{dH}{dx_j} \quad (4.3)$$

where v is the average flow velocity, k is a coefficient of proportionality called the hydraulic conductivity (sometimes called the coefficient of permeability), and dH is the change in total head over a distance dx . The unit of measurement for k is length/time, that is, cm/s. With reference to Fig. 4.2, Darcy's law becomes

$$v_x = k_x \frac{\Delta H}{l} = k_x i \quad (4.4)$$

where $i = \Delta H/l$ is the hydraulic gradient. Darcy's law is valid for all soils if the flow is laminar.

The average velocity, v , calculated from Eq. (4.4) is for the cross-sectional area normal to the direction of flow. Flow through soils, however, occurs only through the interconnected voids. The velocity through the void spaces is called seepage velocity (v_s) and is obtained by dividing the average velocity by the porosity of the soil:

$$v_s = \frac{k_j}{n} i \quad (4.5)$$

The volume rate of flow, q_z , or, simply, flow rate is the product of the average velocity and the cross-sectional area:

$$q_z = v_j A = A k_j i \quad (4.6)$$

The unit of measurement for q_z is m^3/s or cm^3/s . The conservation of flow (law of continuity) stipulates that the volume rate of inflow $(q_z)_{\text{in}}$ into a soil element must equal the volume rate of outflow, $(q_z)_{\text{out}}$, or, simply, inflow must equal outflow: $(q_z)_{\text{in}} = (q_z)_{\text{out}}$.

The hydraulic conductivity depends on

1. *Soil type*: Coarse-grained soils have higher hydraulic conductivities than fine-grained soils. The water in the double layer in fine-grained soils significantly reduces the seepage pore space.
2. *Particle size*: Hydraulic conductivity depends on D_{50}^2 (or D_{10}^2) for coarse-grained soils.
3. *Pore fluid properties, particularly viscosity*: $k_1 : k_2 \approx \mu_2 : \mu_1$, where μ is viscosity and the subscripts 1 and 2 denote two types of pore fluids in a given soil.
4. *Void ratio*: $k_1 : k_2 \approx e_1^2 : e_2^2$, where subscripts 1 and 2 denote two types of soil fabric for coarse-grained soils. This ratio is useful in comparing the hydraulic conductivities of similar soils with different void ratios.
5. *Pore size*: The greater the pore size, the higher the hydraulic conductivity.
6. *Homogeneity, layering, and fissuring*: Water tends to seep quickly through loose layers, through fissures and along the interface of layered soils. Catastrophic failures can occur from such seepage.
7. *Entrapped gases*: Entrapped gases tend to reduce the hydraulic conductivity. It is often very difficult to get gas-free soils. Even soils that are under groundwater level and are assumed to be saturated may still have some entrapped gases.

TABLE 4.1 Hydraulic Conductivity for Common Soil Types

Soil type	k_z (cm/s)
Clean gravel	>1.0
Clean sands, clean sand and gravel mixtures	1.0 to 10^{-3}
Fine sands, silts, mixtures comprising sands, silts, and clays	10^{-3} to 10^{-7}
Homogeneous clays	$<10^{-7}$

8. *Validity of Darcy's Law:* Darcy's Law is valid only for laminar flow (Reynold's number less than 2000). Fancher et al. (1933) gave the following criterion for the applicability of Darcy's law,

$$\frac{vD_s\gamma_w}{\mu g} \leq 1 \quad (4.7)$$

where v is velocity, D_s is the diameter of a sphere of equivalent volume to the soil, μ is viscosity of water, and g is the acceleration due to gravity. Typical ranges of k_z for various soil types are shown in Table 4.1.

Homogeneous clays are practically impervious. Two popular uses of "impervious" clays are in dam construction to curtail the flow of water through the dam and as barriers in landfills to prevent migration of effluent to the surrounding area. Clean sands and gravels are pervious and can be used as drainage materials or soil filters.

4.6 EMPIRICAL RELATIONSHIPS FOR k

For a homogeneous soil, the hydraulic conductivity depends predominantly on its void ratio and pore size. You should recall that the void ratio is dependent on the soil fabric or structural arrangement of the soil grains. Taylor (1948) proposed a relationship linking k with void ratio as

$$k_z = D_{50}^2 \frac{\gamma_w}{\mu} \frac{C_1 e^3}{1 + e} \quad (4.8)$$

where C_1 is a constant that can be obtained from laboratory experiments. A number of empirical relationships have been proposed linking k to void ratio and grain size for coarse-grained soils. Hazen (1930) proposed one of the early relationships as

$$k_z = CD_{10}^2 \text{ cm/s} \quad (4.9)$$

where C is a constant varying between 0.4 and 1.4 if the unit of measurement of D_{10} is mm. Typically, $C = 1.0$. Hazen's tests were done on sands with D_{10} ranging from 0.1 mm to 3 mm and $Cu < 5$. Other relationships were proposed for coarse- and fine-grained soils by Samarasinghe et al. (1982), Kenny et al. (1984), and others. One has to be extremely cautious in using empirical relationships for k_z because it is very sensitive to changes in void ratio, pore size, and the homogeneity of your soil mass.

The essential points are:

1. *The flow of water through soils is governed by Darcy's law, which states that the average flow velocity is proportional to the hydraulic gradient.*
2. *The proportionality coefficient in Darcy's law is called the hydraulic conductivity, k .*

3. The value of k_z is influenced by the void ratio, pore size, particle size distribution, the homogeneity of the soil mass, the properties of the pore fluid, and the amount of undissolved gas in the pore fluid.
4. Homogeneous clays are practically impervious while sands and gravels are pervious.

EXAMPLE 4.2 Calculating Flow Parameters

A soil sample 10 cm in diameter is placed in a tube 1 m long. A constant supply of water is allowed to flow into one end of the soil at A and the outflow at B is collected by a beaker (Fig. E4.2). The average amount of water collected is 1 cm^3 for every 10 seconds. The tube is inclined as shown in Fig. E4.2. Determine the (a) hydraulic gradient, (b) flow rate, (c) average velocity, (d) seepage velocity, if $e = 0.6$, and (e) the hydraulic conductivity.

Strategy In flow problems, you must define a datum position. So your first task is to define the datum position and then find the difference in total head between A and B. Use the head difference to calculate the hydraulic gradient and use Eqs. (4.4) to (4.6) to solve the problem.

Solution 4.2

Step 1: Define the datum position. Select the top of the table as the datum.

Step 2: Find the total heads at A (inflow) and B (outflow).

$$H_A = (h_p)_A + (h_z)_A = 1 + 1 = 2 \text{ m}$$

$$H_B = (h_p)_B + (h_z)_B = 0 + 0.8 = 0.8 \text{ m}$$

Step 3: Find the hydraulic gradient.

$$\Delta H = |H_B - H_A| = |0.8 - 2| = 1.2 \text{ m}$$

$$l = 1 \text{ m}$$

$$i = \frac{\Delta H}{l} = \frac{1.2}{1} = 1.2$$

If you were to select the outflow, point B, as the datum, then $H_A = 1 \text{ m} + 0.2 \text{ m} = 1.2 \text{ m}$ and $H_B = 0$. The head loss is $\Delta H = 1.2 \text{ m}$, which is the same value we obtained using the table's top as the datum. It is often simpler, for calculation purposes, to select the exit flow position as the datum.

Step 4: Determine the flow rate.

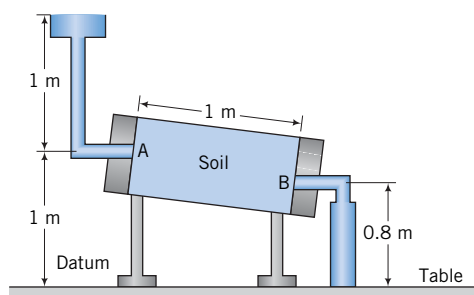


FIGURE E4.2

Volume of water collected, $Q = 1 \text{ cm}^3$, $t = 10$ seconds

$$q_z = \frac{Q}{t} = \frac{1}{10} = 0.1 \text{ cm}^3/\text{s}$$

Step 5: Determine the average velocity.

$$q_z = Av$$

$$A = \frac{\pi \times (\text{diam})^2}{4} = \frac{\pi \times 10^2}{4} = 78.5 \text{ cm}^2$$

$$v = \frac{q_z}{A} = \frac{0.1}{78.5} = 0.0013 \text{ cm/s}$$

Step 6: Determine seepage velocity.

$$v_s = \frac{v}{n}$$

$$n = \frac{e}{1+e} = \frac{0.6}{1+0.6} = 0.38$$

$$v_s = \frac{0.0013}{0.38} = 0.0034 \text{ cm/s}$$

Step 7: Determine the hydraulic conductivity. From Darcy's law $v = k_z i$.

$$\therefore k_z = \frac{v}{i} = \frac{0.0013}{1.2} = 10.8 \times 10^{-4} \text{ cm/s}$$

EXAMPLE 4.3 Calculating Hydraulic Heads and Application to a Practical Scenario

A drainage pipe (Fig. E4.3a) became completely blocked during a storm by a plug of sand, 1.5 m long, followed by another plug of a mixture of clays, silts, and sands, 0.5 m long. When the storm was over, the water level above ground was 1 m. The hydraulic conductivity of the sand is 2 times that of the mixture of clays, silts, and sands.

(a) Plot the variation of pressure, elevation, and total head over the length of the pipe.

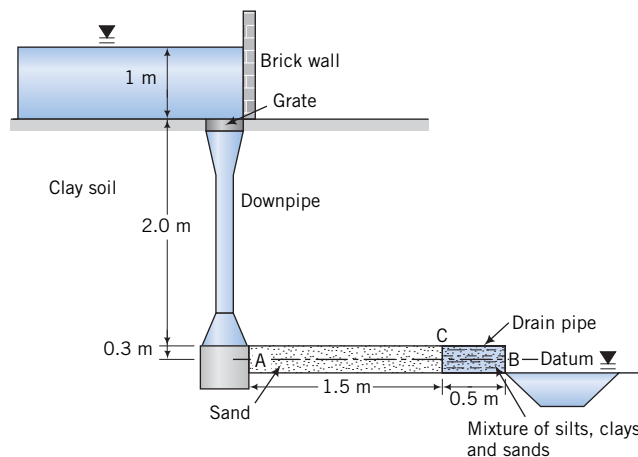


FIGURE E4.3a Illustration of blocked drainage pipe

- (b) Calculate the porewater pressure at (1) the center of the sand plug and (2) the center of the mixture of clays, silts, and sands.
- (c) Find the average hydraulic gradients in the sand and in the mixture of clays, silts, and sands.

Strategy You need to select a datum. From the information given, you can calculate the total head at A and B. The difference in head is the head loss over both plugs but you do not know how much head is lost in the sand and in the mixture of clays, silts, and sands. The continuity equation provides the key to finding the head loss over each plug.

Solution 4.3

Step 1: Select a datum.

Select the exit at B along the centerline of the drainage pipe as the datum.

Step 2: Determine heads at A and B.

$$(h_z)_A = 0 \text{ m}, (h_p)_A = 0.3 + 2 + 1 = 3.3 \text{ m}, H_A = 0 + 3.3 = 3.3 \text{ m}$$

$$(h_z)_B = 0 \text{ m}, (h_p)_B = 0 \text{ m}, H_B = 0 \text{ m}$$

Step 3: Determine the head loss in each plug.

Head loss between A and B = $|H_B - H_A| = 3.3 \text{ m}$ (decrease in head taken as positive). Let $\Delta H_1, L_1, k_1$, and q_1 be the head loss, length, hydraulic conductivity, and flow in the sand; let $\Delta H_2, L_2, k_2$, and q_2 be the head loss, length, hydraulic conductivity, and flow in the mixture of clays, silts, and sands. Now,

$$q_1 = Ak_1 \frac{\Delta H_1}{L_1} = A \times 2k_2 \frac{\Delta H_1}{L_1}$$

$$q_2 = Ak_2 \frac{\Delta H_2}{L_2} = A \times k_2 \frac{\Delta H_2}{L_2}$$

From the continuity equation, $q_1 = q_2$.

$$\therefore A \times 2k_2 \frac{\Delta H_1}{L_1} = Ak_2 \frac{\Delta H_2}{L_2}$$

Solving, we get

$$\frac{\Delta H_1}{\Delta H_2} = \frac{L_1}{2L_2} = \frac{1.5}{2 \times 0.5} = 1.5$$

$$\Delta H_1 = 1.5 \Delta H_2 \quad (1)$$

However, we know that

$$\Delta H_1 + \Delta H_2 = \Delta H = 3.3 \text{ m} \quad (2)$$

Solving for ΔH_1 and ΔH_2 from Eqs. (1) and (2), we obtain

$$\Delta H_1 = 1.98 \text{ m} \quad \text{and} \quad \Delta H_2 = 3.3 - 1.98 = 1.32 \text{ m}$$

Step 4: Calculate heads at the junction of the two plugs.

$$\text{Total head at } C = H_C = H_A - \Delta H_1 = 3.3 - 1.98 = 1.32 \text{ m}$$

$$(h_z)_C = 0$$

$$(h_p)_C = H_C - (h_z)_C = 1.32 \text{ m}$$

Step 5: Plot distribution of heads.

See Fig. E4.3b.

Step 6: Calculate porewater pressures.

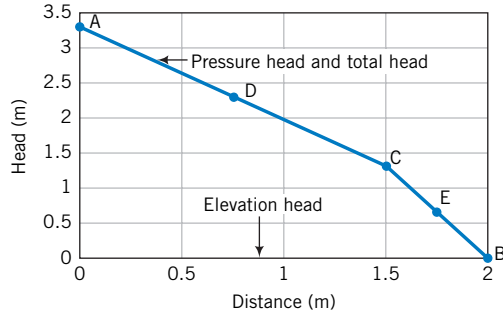


FIGURE E4.3b Variation of elevation, pressure, and total heads along pipe.

Let D be the center of the sand.

$$(h_p)_D = \frac{(h_p)_A + (h_p)_C}{2} = \frac{3.3 + 1.32}{2} = 2.31 \text{ m}$$

$$u_D = 2.31 \times \gamma_w = 2.31 \times 9.8 = 22.6 \text{ kPa}$$

Let E be the center of the mixture of clays, silts, and sands.

$$(h_p)_E = \frac{(h_p)_C + (h_p)_B}{2} = \frac{1.32 + 0}{2} = 0.66 \text{ m}$$

$$u_E = 0.66 \times 9.8 = 6.5 \text{ kPa}$$

Step 7: Find the average hydraulic gradients.

$$i_1 = \frac{\Delta H_1}{l_1} = \frac{1.98}{1.5} = 1.32$$

$$i_2 = \frac{\Delta H_2}{l_2} = \frac{1.32}{0.5} = 2.64$$

What's next... We have considered flow only through homogeneous soils. In reality, soils are stratified or layered with different soil types. In calculating flow through layered soils, an average or equivalent hydraulic conductivity representing the whole soil mass is determined from the permeability of each layer. Next, we will consider flow of water through layered soil masses: one flow occurs parallel to the layers, the other flow occurs normal to the layers.

4.7 FLOW PARALLEL TO SOIL LAYERS



Interactive Concept Learning and Self-Assessment

Access Chapter 4, Section 4.7 to learn about flow parallel to soil layers. Take Quiz 4.7 to test your understanding.

When the flow is parallel to the soil layers (Fig. 4.3), the hydraulic gradient is the same at all points. The flow through the soil mass as a whole is equal to the sum of the flow through each of the layers. There is a parallel here with the flow of electricity through resistors in parallel. If we consider a unit width (in the y direction) of flow and use Eq. (4.6), we obtain

$$q_x = Av = (1 \times H_o)k_{x(\text{eq})}i = (1 \times z_1)k_{x1}i + (1 \times z_2)k_{x2}i + \cdots + (1 \times z_n)k_{xn}i \quad (4.10)$$

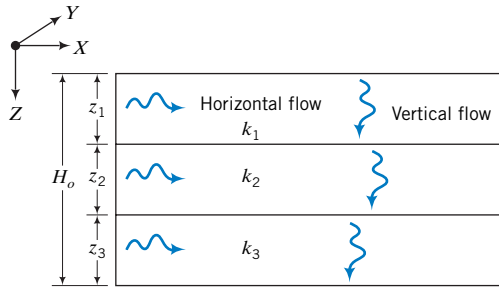


FIGURE 4.3 Flow through stratified layers.

where H_o is the total thickness of the soil mass, $k_{x(eq)}$ is the equivalent permeability in the horizontal (x) direction, z_1 to z_n are the thickness of the first to the n th layers, and k_{x1} to k_{xn} are the horizontal hydraulic conductivities of the first to the n th layer. Solving Eq. (4.10) for $k_{x(eq)}$, we get

$$k_{x(eq)} = \frac{1}{H_o} (z_1 k_{x1} + z_2 k_{x2} + \cdots + z_n k_{xn}) \quad (4.11)$$

4.8 FLOW NORMAL TO SOIL LAYERS



Interactive Concept Learning and Self-Assessment

Access Chapter 4, Section 4.8 to learn about flow normal to soil layers. Take Quiz 4.8 to test your understanding.

For flow normal to the soil layers, the head loss in the soil mass is the sum of the head losses in each layer:

$$\Delta H = \Delta h_1 + \Delta h_2 + \cdots + \Delta h_n \quad (4.12)$$

where ΔH is the total head loss, and Δh_1 to Δh_n are the head losses in each of the n layers. The velocity in each layer is the same. The analogy to electricity is flow of current through resistors in series. From Darcy's law, we obtain

$$k_{z(eq)} \frac{\Delta H}{H_o} = k_{z1} \frac{\Delta h_1}{z_1} = k_{z2} \frac{\Delta h_2}{z_2} = \cdots = k_{zn} \frac{\Delta h_n}{z_n} \quad (4.13)$$

where $k_{z(eq)}$ is the equivalent hydraulic conductivity in the vertical (z) direction and k_{z1} to k_{zn} are the vertical hydraulic conductivities of the first to the n th layer. Solving Eqs. (4.12) and (4.13) leads to

$$k_{z(eq)} = \frac{H_o}{\frac{z_1}{k_{z1}} + \frac{z_2}{k_{z2}} + \cdots + \frac{z_n}{k_{zn}}} \quad (4.14)$$

Values of $k_{z(eq)}$ are generally less than $k_{x(eq)}$ —sometimes as much as 10 times less.

4.9 EQUIVALENT HYDRAULIC CONDUCTIVITY



Interactive Concept Learning and Self-Assessment

Access Chapter 4, Section 4.9 to learn more about equivalent hydraulic conductivity. Take Quiz 4.9 to test your understanding.

The equivalent hydraulic conductivity for flow parallel and normal to soil layers is

$$k_{eq} = \sqrt{k_{x(eq)} k_{z(eq)}} \quad (4.15)$$

EXAMPLE 4.4 Vertical and Horizontal Flows in Layered Soils

A canal is cut into a soil with a stratigraphy shown in Fig. E4.4. Assuming flow takes place laterally and vertically through the sides of the canal and vertically below the canal, determine the equivalent hydraulic conductivity in the horizontal and vertical directions. The vertical and horizontal hydraulic conductivities for each layer is assumed to be the same. Calculate the ratio of the equivalent horizontal hydraulic conductivity to the equivalent vertical hydraulic conductivity for flow through the sides of the canal.

Strategy Use Eq. (4.11) to find the equivalent horizontal hydraulic conductivity over the depth of the canal (3.0 m) and then use Eq. (4.14) to find the equivalent vertical hydraulic conductivity below the canal. To make the calculations easier, convert all exponential quantities to a single exponent.

Solution 4.4

Step 1: Find $k_{x(eq)}$ and $k_{z(eq)}$ for flow through the sides of the canal.

$$H_o = 3 \text{ m}$$

From Eq. (4.11),

$$\begin{aligned} k_{x(eq)} &= \frac{1}{H_o} (z_1 k_{x1} + z_2 k_{x2} + \cdots + z_n k_{xn}) \\ &= \frac{1}{3} (1 \times 0.23 \times 10^{-6} + 1.5 \times 5.2 \times 10^{-6} + 0.5 \times 2 \times 10^{-6}) \\ &= 3 \times 10^{-6} \text{ cm/s} \end{aligned}$$

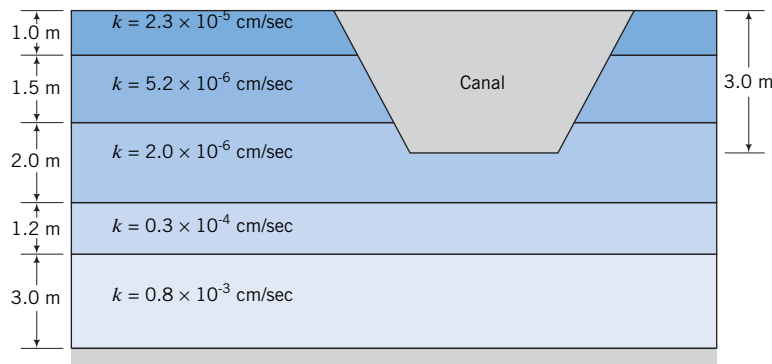


FIGURE E4.4

From Eq. (4.14),

$$k_{z(\text{eq})} = \frac{H_o}{\frac{z_1}{k_{z1}} + \frac{z_2}{k_{z2}} + \cdots + \frac{z_n}{k_{zn}}} = \frac{3}{\frac{1}{10^{-6}} \left(\frac{1}{0.23} + \frac{1.5}{5.2} + \frac{0.5}{2} \right)} = 0.61 \times 10^{-6} \text{ cm/s}$$

Step 2: Find $k_{x(\text{eq})}/k_{z(\text{eq})}$ ratio.

$$\frac{k_{x(\text{eq})}}{k_{z(\text{eq})}} = \frac{3 \times 10^{-6}}{0.61 \times 10^{-6}} = 4.9$$

Step 3: Find $k_{z(\text{eq})}$ below the bottom of the canal.

$$H_o = 1.5 + 1.2 + 3.0 = 5.7 \text{ m}$$

$$k_{z(\text{eq})} = \frac{H_o}{\frac{z_1}{k_{z1}} + \frac{z_2}{k_{z2}} + \cdots + \frac{z_n}{k_{zn}}} = \frac{5.7}{\frac{1.5}{2 \times 10^{-6}} + \frac{1.2}{30 \times 10^{-6}} + \frac{3}{800 \times 10^{-6}}} = 7.2 \times 10^{-6} \text{ cm/s}$$

What's next... In order to calculate flow, we need to know the hydraulic conductivity k_z . We will discuss how this is determined in the laboratory and in the field.

4.10 DETERMINATION OF THE HYDRAULIC CONDUCTIVITY



Virtual Laboratory and Self-Assessment

Access Chapter 4 on the CD, click on Virtual Lab, and select constant-head test to conduct an interactive virtual constant-head permeability test. After you complete the virtual constant-head test, select the falling-head test and conduct a virtual falling-head permeability test. Take Quiz 4.10 to test your understanding.

4.10.1 Constant-Head Test

The constant-head test is used to determine the hydraulic conductivity of coarse-grained soils. A typical constant-head apparatus is shown in Fig. 4.4. Water is allowed to flow through a cylindrical

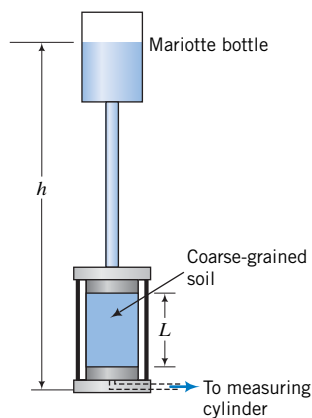


FIGURE 4.4 A constant-head apparatus.

sample of soil under a constant head (h). The outflow (Q) is collected in a graduated cylinder at a convenient duration (t).

With reference to Fig. 4.4,

$$\Delta H = h \quad \text{and} \quad i = \frac{\Delta H}{L} = \frac{h}{L}$$

The flow rate through the soil is $q_z = Q/t$, where Q is the total quantity of water collected in the measuring cylinder over time t .

From Eq. (4.6),

$$k_z = \frac{q_z}{Ai} = \frac{QL}{tAh} \quad (4.16)$$

where k_z is the hydraulic conductivity in the vertical direction and A is the cross-sectional area.

The viscosity of the fluid, which is a function of temperature, influences the value of k_z . The experimental value ($k_{T^\circ\text{C}}$) is corrected to a baseline temperature of 20°C using

$$k_{20^\circ\text{C}} = k_{T^\circ\text{C}} \frac{\mu_{T^\circ\text{C}}}{\mu_{20^\circ\text{C}}} = k_{T^\circ\text{C}} R_T \quad (4.17)$$

where μ is the viscosity of water, T is the temperature in $^\circ\text{C}$ at which the measurement was made, and $R_T = \mu_{T^\circ\text{C}}/\mu_{20^\circ\text{C}}$ is the temperature correction factor that can be calculated from

$$R_T = 2.42 - 0.475 \ln(T) \quad (4.18)$$

4.10.2 Falling-Head Test

The falling-head test is used for fine-grained soils because the flow of water through these soils is too slow to get reasonable measurements from the constant-head test. A compacted soil sample or a sample extracted from the field is placed in a metal or acrylic cylinder (Fig. 4.5). Porous stones are positioned at the top and bottom faces of the sample to prevent its disintegration and to allow water to percolate through it. Water flows through the sample from a standpipe attached to the top of the cylinder. The head of water (h) changes with time as flow occurs through the soil. At different times, the head of water is recorded. Let dh be the drop in head over a time period dt . The velocity or rate of head loss in the tube is

$$v = -\frac{dh}{dt}$$

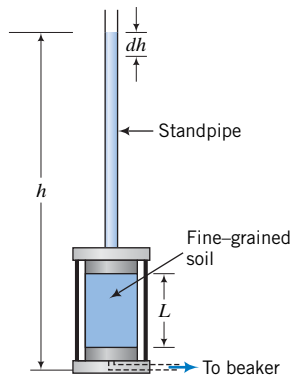


FIGURE 4.5 A falling-head apparatus.

and the inflow of water to the soil is

$$(q_z)_{\text{in}} = av = -a \frac{dh}{dt}$$

where a is the cross-sectional area of the tube. We now appeal to Darcy's law to get the outflow:

$$(q_z)_{\text{out}} = Aki = Ak \frac{h}{L}$$

where A is the cross-sectional area, L is the length of the soil sample, and h is the head of water at any time t . The continuity condition requires that $(q_z)_{\text{in}} = (q_z)_{\text{out}}$. Therefore,

$$-a \frac{dh}{dt} = Ak \frac{h}{L}$$

By separating the variables (h and t) and integrating between the appropriate limits, the last equation becomes

$$\frac{Ak}{aL} \int_{t_1}^{t_2} dt = - \int_{h_1}^{h_2} \frac{dh}{h}$$

and the solution for k in the vertical direction is

$$k = k_z = \frac{aL}{A(t_2 - t_1)} \ln \left(\frac{h_1}{h_2} \right) \quad (4.19)$$

The hydraulic conductivity is corrected using Eq. (4.17).

The essential points are:

1. The constant-head test is used to determine the hydraulic conductivity of coarse-grained soils.
2. The falling-head test is used to determine the hydraulic conductivity of fine-grained soils.

EXAMPLE 4.5 Interpretation of Constant-Head Test Data

A sample of sand, 5 cm in diameter and 15 cm long, was prepared at a porosity of 60% in a constant-head apparatus. The total head was kept constant at 30 cm and the amount of water collected in 5 seconds was 40 cm³. The test temperature was 20°C. Calculate the hydraulic conductivity and the seepage velocity.

Strategy From the data given, you can readily apply Darcy's law to find k_z .

Solution 4.5

Step 1: Calculate the sample cross-sectional area, hydraulic gradient, and flow.

$$\begin{aligned} D &= 5 \text{ cm} \\ A &= \frac{\pi \times D^2}{4} = \frac{\pi \times 5^2}{4} = 19.6 \text{ cm}^2 \\ \Delta H &= 30 \text{ cm} \\ i &= \frac{\Delta H}{L} = \frac{30}{15} = 2 \\ Q &= 40 \text{ cm}^3 \\ q_z &= \frac{Q}{t} = \frac{40}{5} = 8 \text{ cm}^3/\text{s} \end{aligned}$$

Step 2: Calculate k_z .

$$k_z = \frac{q_z}{Ai} = \frac{8}{19.6 \times 2} = 0.2 \text{ cm/s}$$

Step 3: Calculate the seepage velocity.

$$v_s = \frac{k_z i}{n} = \frac{0.2 \times 2}{0.6} = 0.67 \text{ cm/s}$$

EXAMPLE 4.6 Interpretation of Falling-Head Test Data

The data from a falling-head test on a silty clay are:

Cross-sectional area of soil = 80 cm^2

Length of soil = 10 cm

Initial head = 90 cm

Final head = 84 cm

Duration of test = 15 minutes

Diameter of tube = 6 mm

Temperature = 22°C

Determine k .

Strategy Since this is a falling-head test, you should use Eq. (4.19). Make sure you are using consistent units.

Solution 4.6

Step 1: Calculate the parameters required in Eq. (4.19).

$$a = \frac{\pi \times (6/10)^2}{4} = 0.28 \text{ cm}^2$$

$$A = 80 \text{ cm}^2 \text{ (given)}$$

$$t_2 - t_1 = 15 \times 60 = 900 \text{ seconds}$$

Step 2: Calculate k_z .

$$k_z = \frac{aL}{A(t_2 - t_1)} \ln\left(\frac{h_1}{h_2}\right) = \frac{0.28 \times 10}{80 \times 900} \ln\left(\frac{90}{84}\right) = 2.7 \times 10^{-6} \text{ cm/s}$$

$$\text{From Eq. (4.18), } R_T = 2.42 - 0.475 \ln(22) = 0.95$$

$$k_{20^\circ\text{C}} = k_z R_T = 2.7 \times 10^{-6} \times 0.95 = 2.6 \times 10^{-6} \text{ cm/s}$$

What's next... In the constant-head test and the falling-head test, we determined the hydraulic conductivity of only a small volume of soil at a specific location in a soil mass. In some cases, we have to use remolded or disturbed soil samples. In addition, if field samples are used, they are invariably disturbed by sampling processes (see Section 2.6). The hydraulic conductivity is sensitive to alteration in the fabric of the soil and, consequently, there are doubts about the accuracy of representing the in situ soil conditions using laboratory permeability tests. There are several field methods to determine the hydraulic conductivity. Next, we will discuss one popular method.

Completing the integration leads to

$$k = \frac{q_v \ln(r_2/r_1)}{\pi(h_2^2 - h_1^2)} \quad (4.21)$$

With measurements of r_1 , r_2 , h_1 , h_2 , and q_v (flow rate of the pump), k can be calculated from Eq. (4.21). This test is only practical for coarse-grained soils.

Pumping tests lower the groundwater, which then causes stress changes in the soil. Since the groundwater is not lowered uniformly as shown by the drawdown curve in Fig. 4.6, the stress changes in the soil will not be even. Consequently, pumping tests near existing structures can cause them to settle unevenly. You should consider the possibility of differential settlement on existing structures when you plan a pumping test.

EXAMPLE 4.7 Interpretation of Pumping Test Data

A pumping test was carried out in a soil bed of thickness 15 m and the following measurements were recorded. Rate of pumping was $10.6 \times 10^{-3} \text{ m}^3/\text{s}$; drawdowns in observation wells located at 15 m and 30 m from the center of the pumping well were 1.6 m and 1.4 m, respectively, from the initial groundwater level. The initial groundwater level was located at 1.9 m below ground level. Determine k .

Strategy You are given all the measurements to directly apply Eq. (4.21) to find k . You should draw a sketch of the pump test to identify the values to be used in Eq. (4.21).

Solution 4.7

Step 1: Draw a sketch of the pump test with the appropriate dimensions—see Fig. E4.7.

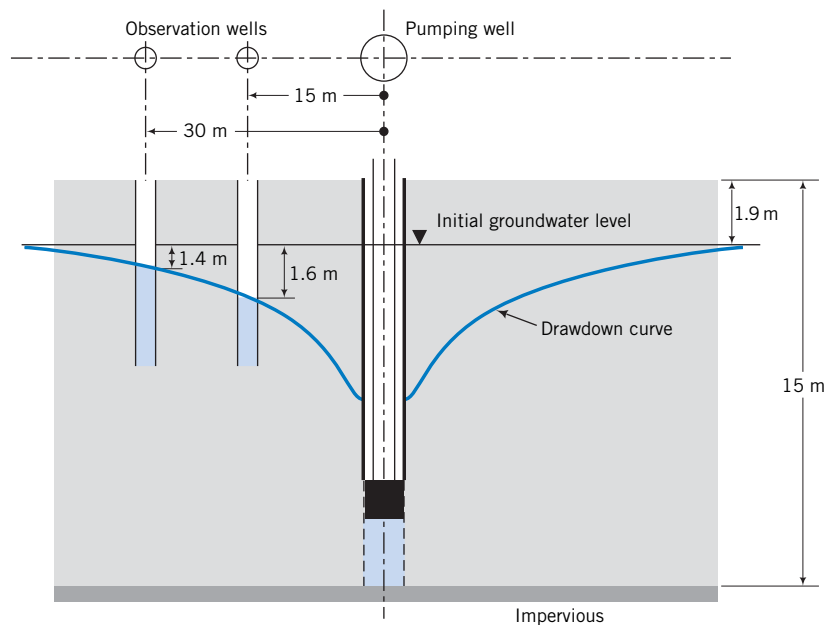


FIGURE E4.7

Step 2: Substitute given values in Eq. (4.21) to find k .

$$r_2 = 30 \text{ m}, r_1 = 15 \text{ m}, h_2 = 15 - (1.9 + 1.4) = 11.7 \text{ m},$$

$$h_1 = 15 - (1.9 + 1.6) = 11.5 \text{ m}$$

$$k = \frac{q_z \ln(r_2/r_1)}{\pi(h_2^2 - h_1^2)} = \frac{10.6 \times 10^{-3} \ln(30/15)}{\pi(11.7^2 - 11.5^2)10^4} = 5.0 \times 10^{-2} \text{ cm/s}$$

4.11 GROUNDWATER LOWERING BY WELLPOINTS



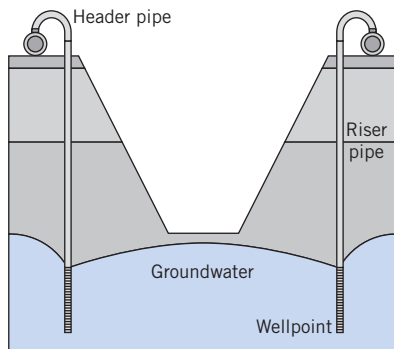
Concept Learning, Computer Program Utility, and Self-Assessment

Access Chapter 4, Section 4.11 to learn about wellpoints. Take Quiz 4.11 to test your understanding.

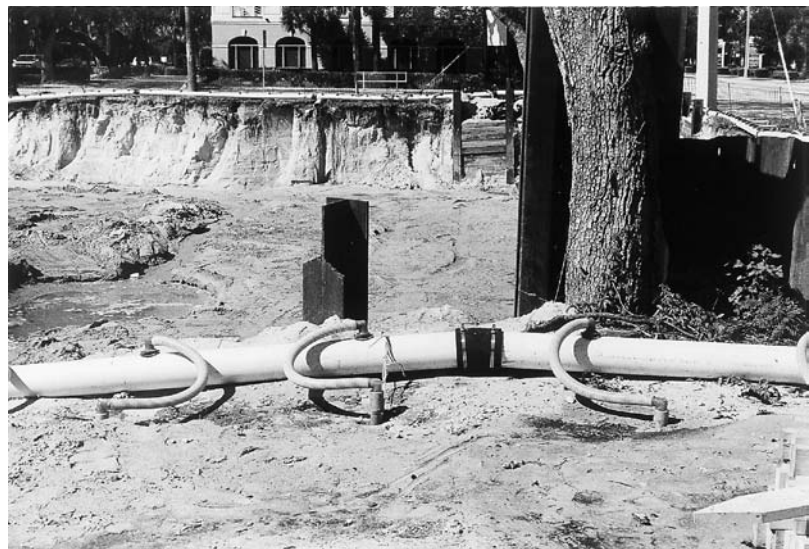
Sometimes it is necessary to temporarily lower the groundwater level for construction of foundations. The process of lowering the groundwater is called dewatering and it is accomplished by inserting wellpoints around the excavation for the foundations. A wellpoint system consists of an interconnected network of wells (pipes) installed around the perimeter of an excavation (Fig. 4.7). The wells are installed in rows and the spacing depends on the soil type and the hydraulic conductivity. The spacing in clean sands with water depth about 5 m is about 1 m to 1.5 m.

Reconsidering Fig. 4.6, the drawdown, d , is

$$d = H - h \quad (4.22)$$



(a)



(b)

FIGURE 4.7 Wellpoint system for an excavation.

The radius of influence, R , of the depression cone is the radius at which the drawdown is zero. The flow rate or discharge between the limits r and R , and h and H can be found from Eq. (4.21) as

$$q_w = \frac{k\pi(H^2 - h^2)}{\ln\left(\frac{R}{r}\right)} \quad (4.23)$$

Solving for h , we get

$$h = \pm \sqrt{H^2 - \frac{q_w \ln\left(\frac{R}{r}\right)}{k\pi}} \quad (4.24)$$

Therefore, at any point with coordinates (r, z) , the drawdown is

$$d = H \pm \sqrt{H^2 - \frac{q_w \ln\left(\frac{R}{r}\right)}{k\pi}} \quad (4.25)$$

The positive sign is used when water is pumped into the well and the negative sign is used when water is pumped from the well. The maximum drawdown, $d_{\max}$, occurs at the well face, i.e., $r = r_o$ and from Eq. (4.25)

$$d_{\max} = H \pm \sqrt{H^2 - \frac{q_w \ln\left(\frac{R}{r_o}\right)}{k\pi}} \quad (4.26)$$

The radius of influence of the depression cone is found from experience and can be estimated (Slichter, 1899) from

$$R = 3000d_{\max}\sqrt{k}; d \text{ is in m and } k \text{ is in m/s.} \quad (4.27)$$

Eq. (4.27) does not have a theoretical basis and is not dimensionally correct. However, it has been satisfactorily applied in practice. An equation (Kozeny, 1933) that is dimensionally correct is

$$R = \sqrt{12 \frac{t}{n} \sqrt{\frac{q_w k}{\pi}}} \quad (4.28)$$

where t (sec) is duration for a discharge q_w (m^3/s), n is porosity and k is the average hydraulic conductivity (m/s). The predictions of R from Eq. (4.27) and Eq. (4.28) are normally significantly different. However, the discharge is not very sensitive to the accuracy of R because the changes in $\ln\left(\frac{R}{r}\right)$ are small for large changes in $\left(\frac{R}{r}\right)$. However, the accuracy of R has significant impact for drawdown near existing buildings. For closely spaced wellpoints, a two-dimensional flow analysis (Chapter 11) is required.

EXAMPLE 4.8 Interpretation of Wellpoint Data

A wellpoint, 0.1 m radius, in a permeable soil layer 7 m thick has a constant discharge of $0.05 \text{ m}^3/\text{s}$ in a 24-hour operation. The soil parameters are $k = 0.004 \text{ m/s}$ and $e = 0.5$. Determine the radius of influence and the maximum drawdown.

Strategy Use Kozeny (1933) equation for R and then use Eq. (4.26) to calculate $d_{\max}$.

Solution 4.8

Step 1: Calculate R .

$$n = \frac{e}{1+e} = \frac{0.5}{1+0.5} = 0.33$$

$$R = \sqrt{12 \frac{t}{n} \sqrt{\frac{q_w k}{\pi}}} = R = \sqrt{12 \frac{24 \times 3600}{0.33} \sqrt{\frac{0.05 \times 0.004}{\pi}}} = 158.3 \text{ m}$$

Compare R using Slichter's equation.

$$R = 3000 d_{\max} \sqrt{k} = 3000 \times 2.56 \sqrt{0.004} = 485.7 \text{ m}$$

Step 2: Calculate $d_{\max}$.

$$d_{\max} = H - \sqrt{H^2 - \frac{q_w \ln\left(\frac{R}{r_o}\right)}{k\pi}} = 7 - \sqrt{7^2 - \frac{0.05 \ln\left(\frac{158.3}{0.1}\right)}{0.004\pi}} = 2.56 \text{ m}$$

4.12 SUMMARY

Flow of water through soils is governed by Darcy's law, which states that the velocity is proportional to the hydraulic gradient. The proportionality constant is the hydraulic conductivity. The hydraulic conductivity depends on soil type, particle size, pore fluid properties, void ratio, pore size, homogeneity, layering and fissuring, and entrapped gases. In coarse-grained soils, the hydraulic conductivity is determined using a constant-head test while for fine-grained soils a falling-head test is used. In the field, a pumping test is used to determine the hydraulic conductivity. Wellpoints are used at a construction site to lower the groundwater level.

Practical Example

EXAMPLE 4.9 Application of Flow Data to a Canal

A ditch is required for a utility line near an ephemeral canal, which at the time of excavation was filled with water as shown in Fig. E4.9. The average vertical and horizontal hydraulic conductivities are 1×10^{-5} cm/s and 2×10^{-4} cm/s respectively. Assume a 1 m length of ditch, determine the flow rate of water into it.

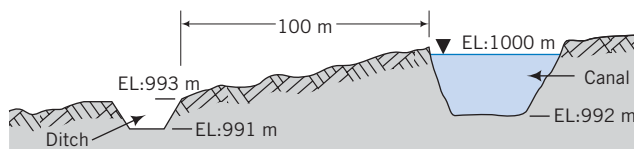


FIGURE E4.9

Strategy You need to determine an equivalent hydraulic conductivity and then calculate the flow rate using Darcy's law. However, to calculate the flow rate, you need to determine the hydraulic gradient. To do so, find the difference in total head between the canal and the ditch, and then divide by the length of the flow.

Solution 4.9

Step 1: Calculate an equivalent hydraulic conductivity.

$$k_{eq} = \sqrt{k_z k_x} = \sqrt{10^{-5} \times 2 \times 10^{-4}} = 4.5 \times 10^{-5} \text{ cm/s}$$

Step 2: Determine the hydraulic gradient.

Take datum as the bottom of the ditch.

Elevation head at base of ditch = 0, pressure head at base of ditch = 0; total head at ditch = 0

Elevation head at base of canal = 1 m, pressure head at base of canal = 8 m; total head at canal = 9 m

Head difference, $\Delta h = 9 \text{ m}$

$$\text{Slope} \cong \tan^{-1} \left(\frac{1000 - 993}{100} \right) = 4^\circ$$

$$\text{Average length of flow path, } l = \frac{100}{\cos(4^\circ)} = 100.2 \text{ m}$$

$$i = \frac{\Delta h}{l} = \frac{9}{100.2} = 0.09$$

Step 3: Calculate the flow rate.

Assume flow parallel to the slope and consider a vertical section of the ditch.

$$A = (993 - 991) \times 1 = 2 \text{ m}^2$$

$$q_i = A k_{eq} i = 2 \times (4.5 \times 10^{-5} / 100) \times 0.09 = 0.81 \times 10^{-7} \text{ m}^3/\text{s}$$

EXERCISES

Theory

- 4.1** A pump test is carried out to determine the hydraulic conductivity of a confined water-bearing stratum shown in Fig. P4.1. Show that the equation for k is

$$k = \frac{q \ln(r_1/r_2)}{2\pi H(h_1 - h_2)}$$

Problem Solving

- 4.2** Determine the pressure head, elevation head, and total head at A, B, and C for the arrangement shown in Fig. P4.2. Take the water level at exit as datum. *Hint:* You need to convert the pressure 10 kPa to head.
- 4.3** In a constant-head permeability test, a sample of sand 12 cm long and 8 cm in diameter discharged $1.5 \times 10^{-3} \text{ m}^3$ of water in 10 minutes. The head difference in two manometers A and B located at 10 mm and 110 mm, respectively, from the bottom of the sample is 20 mm. Determine the hydraulic conductivity of the sand.

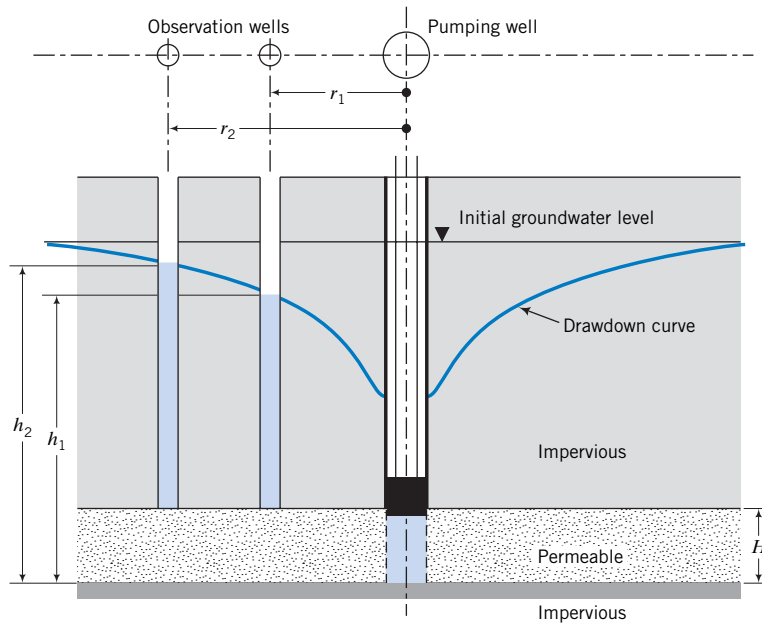


FIGURE P4.1

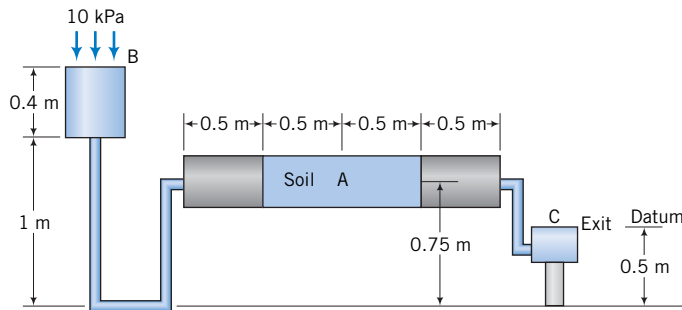


FIGURE P4.2

- 4.4** A constant-head test was conducted on a sample of soil 15 cm long and 60 cm^2 in cross-sectional area. The quantity of water collected was 40.5 cm^3 in 15 seconds under a head difference of 24 cm. Calculate the hydraulic conductivity. If the porosity of the sand were 55%, calculate the average velocity and the seepage velocity. Estimate the hydraulic conductivity of a similar soil with a porosity of 35% from the results of this test.
- 4.5** A falling-head permeability test was carried out on a clay soil of diameter 100 mm and length 150 mm. In 4 minutes the head in the standpipe of diameter 5 mm dropped from 680 to 530 mm. Calculate the hydraulic conductivity of this clay.
- 4.6** Calculate the equivalent hydraulic conductivity for the soil profile shown in Fig. P4.6.
- 4.7** A pumping test was carried out to determine the average hydraulic conductivity of a sand deposit 20 m thick overlying impermeable clay. The discharge from the pumping well was $12 \times 10^{-3} \text{ m}^3/\text{s}$. Drawdowns in the observation wells located 15 m and 30 m from the centerline of the pumping well were 2.1 m and 1.6 m, respectively. Groundwater table was reached at 3.2 m below the ground surface. Determine the hydraulic conductivity of the sand. Estimate the effective grain size using Hazen's equation.

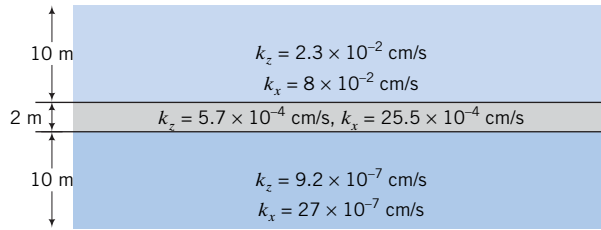


FIGURE P4.6

Practical

- 4.8** An excavation is proposed for a square area near the bend of a river as shown in Fig. P4.8. It is expected that the flow of water into the excavation will come through the silt layer. Pumping tests reveal an average horizontal hydraulic conductivity of 5.2×10^{-5} cm/s in the silt layer. The excavation has to be kept dry. Determine the flow (q_i) into the excavation.

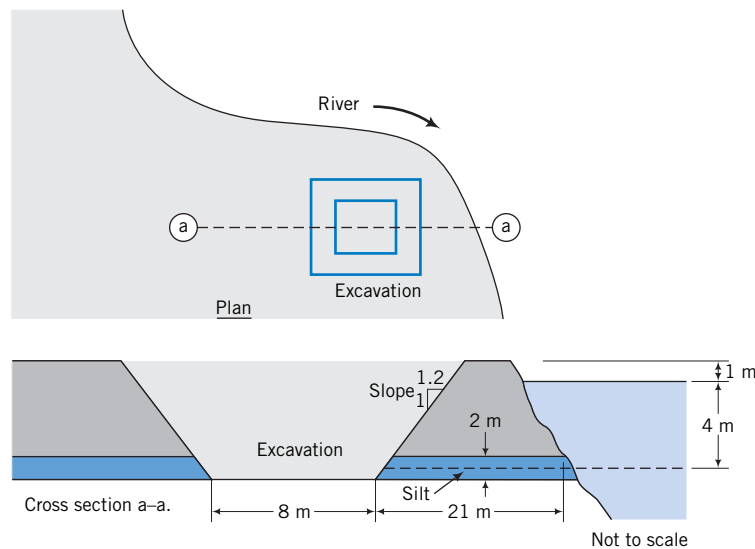


FIGURE P4.8

- 4.9** Groundwater is pumped for domestic use from a water-bearing sand layer under subartesian conditions. The thickness of the clay layer above the sand layer is 20 m and its initial porosity is 40%. After 10 years of pumping, the porosity is reduced to 30%. Determine the subsidence of the clay surface.
- 4.10** A canal is dug parallel to a river as shown in Fig. P4.10. A sandy-silt seam of average thickness 0.5 m cuts across the otherwise impermeable clay. The average vertical and horizontal hydraulic conductivities are

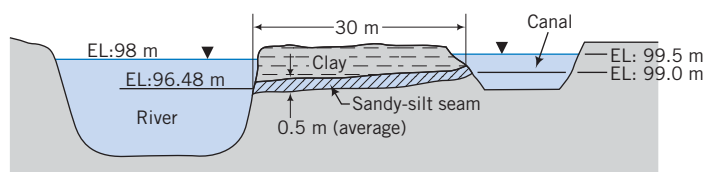


FIGURE P4.10

1.5×10^{-5} cm/s and 1.5×10^{-5} cm/s respectively. Assume a 1 m length of canal, determine the flow rate of water from the canal to the river.

- 4.11** A well, 0.1 m radius, is part of a wellpoint network to keep an excavation dry (Fig. P4.11). The groundwater at the far edge of the excavation must be 0.5 m below the base.

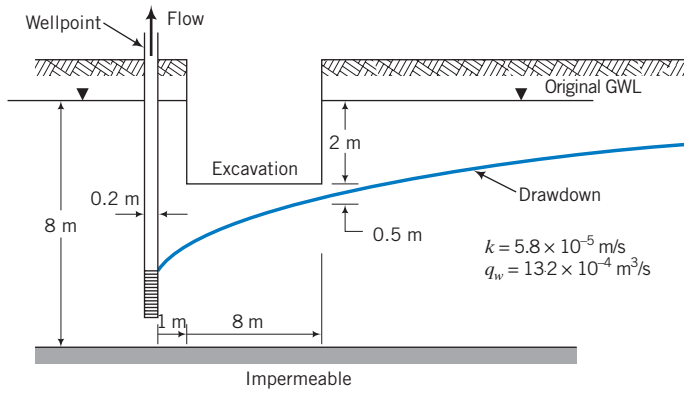


FIGURE P4.11

- Calculate the radius of influence.
- Calculate the maximum drawdown.
- Plot the drawdown curve.
- For the radius of influence in (a), (i) calculate the discharge if the well radius increases to 0.2 m and (ii) compare it to the discharge for the 0.1 m radius well.

CHAPTER 5

STRESSES, STRAINS, AND ELASTIC DEFORMATIONS OF SOILS

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5.0 INTRODUCTION

You would have studied in mechanics the stresses imposed on homogeneous, elastic, rigid bodies by external forces. Soils are not homogeneous, elastic, rigid bodies, so the determination of stresses and strains in soils is a particularly difficult task. You may ask: “If soils are not elastic materials, then why do I have to study elastic methods of analysis?” Here are some reasons why a knowledge of elastic analysis is advantageous.

An elastic analysis of an isotropic material involves only two constants—Young’s modulus and Poisson’s ratio—and thus if we assume that soils are isotropic elastic materials then we have a powerful, but simple, analytical tool to predict a soil’s response under loading. We will have to determine only the two elastic constants from our laboratory or field tests.

A geotechnical engineer must ensure that a geotechnical structure must not collapse under any anticipated loading condition and that settlement under working load (a fraction of the collapse load) must be within tolerable limits. We would prefer the settlement under working loads to be elastic so that no permanent settlement would occur. To calculate the elastic settlement, we have to use an elastic analysis. For example, in designing foundations on coarse-grained soils, we normally assume that the settlement is elastic and we then use elastic analysis to calculate the settlement.

An important task of a geotechnical engineer is to determine the stresses and strains that are imposed on a soil mass by external loads. It is customary to assume that the strains in the soils are small and this assumption allows us to apply our knowledge of mechanics of elastic bodies to soils. Small strains mean infinitesimal strains. For a realistic description of soils, elastic analysis is not satisfactory. We need soil models that can duplicate the complexity of soil behavior. However, even for complex soil models, an elastic analysis is a first step.

In this chapter, we will review some fundamental principles of mechanics and strength of materials and apply these principles to soils as elastic porous materials. This chapter contains a catalog of a large number of equations for soil stresses and strains. You may become weary of these equations but they are necessary for the analyses of the mechanical behavior of soils. You do not have to memorize these equations except the fundamental ones.

When you complete this chapter, you should be able to:

- Calculate stresses and strains in soils (assuming elastic behavior) from external loads.
- Determine stress states.
- Determine effective stresses.
- Determine stress paths.
- Calculate elastic settlement.



FIGURE 5.1 The “kissing” silos (Bozozuk, 1976, permission from National Research Council of Canada).

You will use the following principles learned from statics and strength of materials:

- Stresses and strains
- Mohr’s circle
- Elasticity—Hooke’s law

Sample Practical Situation Two storage tanks are to be founded on a deep layer of stiff saturated clay. Your client and the mechanical engineer, who is designing the pipe works, need an estimate of the settlement of the tanks when they are completely filled. Because of land restrictions, your client desires that the tanks be as close as possible to each other. If two separate foundations are placed too close to each other, the stresses in the soil induced by each foundation overlap and cause intolerable tilting of the structures and their foundations. An example of tilting of structures caused by stress overlap is shown in Fig. 5.1.

These silos tilted toward each other at the top because stresses in the soil overlap at and near the internal edges of their foundations. The foundations are too close to each other.

5.1 DEFINITIONS OF KEY TERMS

Stress or intensity of loading is the load per unit area. The fundamental definition of stress is the ratio of the force ΔP acting on a plane ΔS to the area of the plane ΔS when ΔS tends to zero; Δ denotes a small quantity.

Effective stress (σ') is the stress carried by the soil particles.

Total stress (σ) is the stress carried by the soil particles and the liquids and gases in the voids.

Strain or intensity of deformation is the ratio of the change in a dimension to the original dimension or the ratio of change in length to the original length.

Stress (strain) state at a point is a set of stress (strain) vectors corresponding to all planes passing through that point. Mohr's circle is used to graphically represent stress (strain) state for two-dimensional bodies.

Mean stress, p , is the average stress on a body or the average of the orthogonal stresses in three dimensions.

Deviatoric stress, q , is the shear or distortional stress or stress difference on a body.

Porewater pressure, u , is the pressure of the water held in the soil pores.

Stress path is a graphical representation of the locus of stresses on a body.

Isotropic means the same material properties in all directions and also the same loading in all directions.

Anisotropic means the material properties are different in different directions and also the loadings are different in different directions.

Elastic materials are materials that return to their original configuration on unloading and obey Hooke's law.

Plastic materials do not return to their original configuration on unloading.

5.2 QUESTIONS TO GUIDE YOUR READING

1. What are normal and shear stresses?
2. What is stress state and how is it determined?
3. Is soil an elastic material?
4. What are the limitations in analyzing soils based on the assumption that they (soils) are elastic materials?
5. What are shear strains, vertical strains, volumetric strains, and deviatoric strains?
6. How do I use elastic analysis to estimate the elastic settlement of soils and what are the limitations?
7. What are mean and deviatoric stresses?
8. What are the differences between plane strain and axisymmetric conditions?
9. How do I determine the stresses and strains/displacements imposed on a soil mass by external loads?
10. What is effective stress?
11. Is deformation a function of effective or total stress?
12. What is a stress path and its significance in practical problems?

5.3 STRESSES AND STRAINS



Concept Learning and Self-Assessment

Access Chapter 5, Section 5.3 on the CD to learn about stresses and strains. Take Quiz 5.3 to test your understanding.

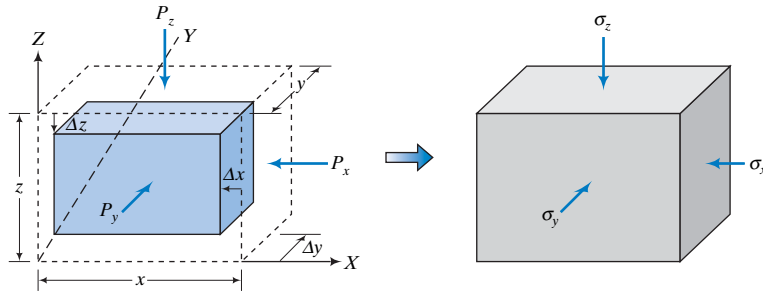


FIGURE 5.2 Stresses and displacements due to applied loads.

5.3.1 Normal Stresses and Strains

Consider a cube of dimensions $x = y = z$ that is subjected to forces P_x, P_y, P_z , normal to three adjacent sides as shown in Fig. 5.2. The normal stresses are

$$\sigma_z = \frac{P_z}{xy}, \quad \sigma_x = \frac{P_x}{yz}, \quad \sigma_y = \frac{P_y}{xz} \quad (5.1)$$

Let us assume that under these forces the cube compressed by $\Delta x, \Delta y$, and Δz in the X, Y , and Z directions. The strains in these directions, assuming they are small (infinitesimal), are

$$\epsilon_z = \frac{\Delta z}{z}, \quad \epsilon_x = \frac{\Delta x}{x}, \quad \epsilon_y = \frac{\Delta y}{y} \quad (5.2)$$

5.3.2 Volumetric Strain

The volumetric strain is

$$\epsilon_p = \epsilon_x + \epsilon_y + \epsilon_z \quad (5.3)$$

5.3.3 Shear Stresses and Shear Strains

Let us consider, for simplicity, the XZ plane and apply a force F that causes the square to distort into a parallelogram as shown in Fig. 5.3. The force F is a shearing force and the shear stress is

$$\tau = \frac{F}{xy} \quad (5.4)$$

Simple shear strain is a measure of the angular distortion of a body by shearing forces. If the horizontal displacement is Δx , the shear strain or simple shear strain, γ_{zx} , is

$$\gamma_{zx} = \tan^{-1} \frac{\Delta x}{z}$$

For small strains, $\tan \gamma_{zx} = \gamma_{zx}$ and therefore

$$\gamma_{zx} = \frac{\Delta x}{z} \quad (5.5)$$

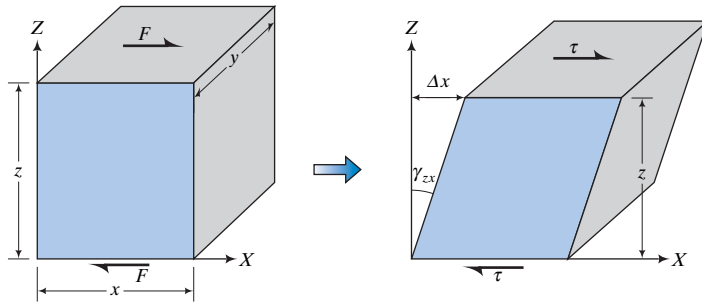


FIGURE 5.3 Shear stresses and shear strains.

If the shear stress on a plane is zero, the normal stress on that plane is called a principal stress. We will discuss principal stresses later. In geotechnical engineering, compressive stresses in soils are assumed to be positive. Soils cannot sustain any appreciable tensile stresses and we normally assume that the tensile strength of soils is negligible. Strains can be compressive or tensile.

The essential points are:

1. A normal stress is the load per unit area on a plane normal to the direction of the load.
2. A shear stress is the load per unit area on a plane parallel to the direction of the shear force.
3. Normal stresses compress or elongate a material; shear stresses distort a material.
4. A normal strain is the change in length divided by the original length in the direction of the original length.
5. Principal stresses are normal stresses on planes of zero shear stress.
6. Soils can only sustain compressive stresses.

What's next... What happens when we apply stresses to a deformable material? From the last section, you may answer that the material deforms, and you are absolutely correct. Different materials respond differently to applied loads. Next, we will examine some typical responses of deformable materials to applied loads to serve as a base to characterize the loading responses of soils.

5.4 IDEALIZED STRESS-STRAIN RESPONSE AND YIELDING



Interactive Concept Learning and Self-Assessment

Access Chapter 5, Section 5.4 to learn about stress-strain response, elastic and shear moduli, and yielding through interactive animations. Take Quiz 5.4 to test your understanding.

5.4.1 Material Responses to Normal Loading and Unloading

If we apply an incremental vertical load, ΔP , to a deformable cylinder (Fig. 5.4) of cross-sectional area A , the cylinder will compress by, say, Δz and the radius will increase by Δr . The loading condition we apply here is called uniaxial loading. The change in vertical stress is

$$\Delta\sigma_z = \frac{\Delta P}{A} \quad (5.6)$$

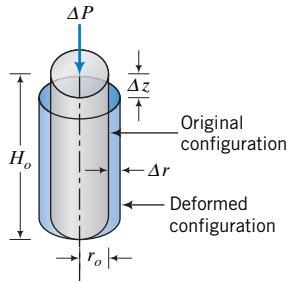


FIGURE 5.4 Forces and displacements on a cylinder.

The vertical and radial strains are, respectively,

$$\Delta\epsilon_z = \frac{\Delta z}{H_o} \quad (5.7)$$

$$\Delta\epsilon_r = \frac{\Delta r}{r_o} \quad (5.8)$$

where H_o is the original length and r_o is the original radius. The ratio of the radial (or lateral) strain to the vertical strain is called Poisson's ratio, ν , defined as

$$\nu = \frac{-\Delta\epsilon_r}{\Delta\epsilon_z} \quad (5.9)$$

Typical values of Poisson's ratio for soil are listed in Table 5.1.

We can plot a graph of $\sigma_z = \Sigma\Delta\sigma_z$ versus $\epsilon_z = \Sigma\Delta\epsilon_z$. If for equal increments of ΔP , we get the same value of Δz , then we will get a straight line in the graph of σ_z versus ϵ_z as shown by OA in Fig. 5.5. If at some stress point, say, at A (Fig. 5.5), we unload the cylinder and it returns to its original configuration, the material comprising the cylinder is called a *linearly elastic* material. Suppose for equal increments of ΔP we get different values of Δz , but on unloading the cylinder it returns to its original configuration. Then a plot of the stress-strain relationship will be a curve as illustrated by OB in Fig. 5.5. In this case, the material comprising the cylinder is called a *nonlinearly elastic* material. If we apply a load P_1 that causes a displacement Δz_1 on an elastic material and a second load P_2 that causes a displacement Δz_2 , then the total displacement is $\Delta z = \Delta z_1 + \Delta z_2$. Elastic materials obey the principle of superposition. The order in which the load is applied is not important; we could apply P_2 first and then P_1 but the final displacement would be the same.

Some materials, soil is one of them, do not return to their original configurations after unloading. They exhibit a stress-strain relationship similar to that depicted in Fig. 5.6, where OA is the loading response, AB the unloading response, and BC the reloading response. The strains that occur during loading, OA , consist of two parts—an elastic or recoverable part, BD , and a plastic or unrecoverable part, OB . Such material behavior is called *elastoplastic*. Part of the loading response is elastic, the other plastic.

TABLE 5.1 Typical Values of Poisson's Ratio

Soil type	Description	ν^a
Clay	Soft	0.35–0.40
	Medium	0.30–0.35
	Stiff	0.20–0.30
Sand	Loose	0.15–0.25
	Medium	0.25–0.30
	Dense	0.25–0.35

^a These values are effective values, ν' (see discussions later in this chapter, Sections 5.12 and 5.13).

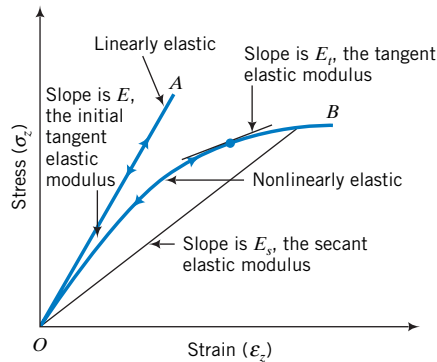


FIGURE 5.5 Linear and nonlinear stress-strain curves of an elastic material.

As engineers, we are particularly interested in the plastic strains since these are the result of permanent deformations of the material. But to calculate the permanent deformation, we must know the elastic deformation. Here, elastic analyses become useful. The stress at which permanent deformation initiates is called the yield stress.

The *elastic modulus* or *initial tangent elastic modulus* (E) is the slope of the stress-strain line for linear isotropic material (Fig. 5.5). For a nonlinear elastic material either the tangent modulus (E_t) or the secant modulus (E_s) or both are determined from the stress-strain relationship (Fig. 5.5). The *tangent elastic modulus* is the slope of the tangent to the stress-strain point under consideration. The *secant elastic modulus* is the slope of the line joining the origin (0,0) to some desired stress-strain point. For example, some engineers prefer to determine the secant modulus by using a point on the stress-strain curve corresponding to the maximum stress while others prefer to use a point on the stress-strain curve corresponding to a certain level of strain, for example, 1%. The tangent elastic modulus and the secant elastic modulus are not constants. These moduli tend to decrease as shear strains increase. It is customary to determine the *initial tangent elastic modulus* for an elastoplastic material by unloading it and calculating the initial slope of the unloading line as the initial tangent elastic modulus (Fig. 5.6).

5.4.2 Material Response to Shear Forces

Shear forces distort materials. A typical response of an elastoplastic material to simple shear is shown in Fig. 5.7. The initial shear modulus (G_i) is the slope of the initial straight portion of the τ_{zx} versus γ_{zx} curve. The secant shear modulus (G) is the slope of a line from the desired shear stress-shear strain point to the origin of the τ_{zx} versus γ_{zx} plot (Fig. 5.7).

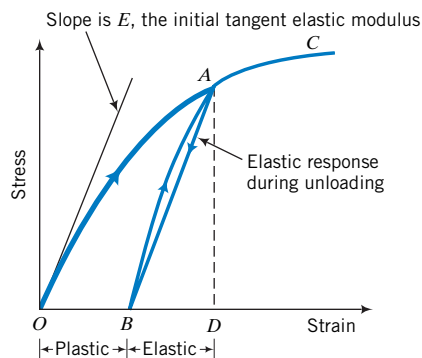


FIGURE 5.6 Idealized stress-strain curves of an elastoplastic material.

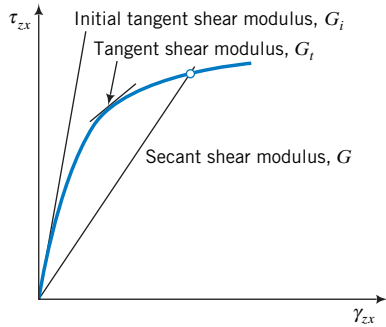


FIGURE 5.7 Shear stress–shear strain response of elastoplastic material.

5.4.3 Yield Surface

Let us consider a more complex situation than the uniaxial loading of a cylinder (Fig. 5.8a). In this case, we are going to apply increments of vertical and radial stresses. Since we are not applying any shear stresses, the axial stresses and radial stresses are principal stresses: $\sigma_z = \sigma_1 = \Sigma \Delta \sigma_z$ and $\sigma_r = \sigma_3 = \Sigma \Delta \sigma_r$, respectively. Let us, for example, set σ_3 to zero and increase σ_1 . The material will yield at some value of σ_1 , which we will call $(\sigma_1)_y$, and plots as point *A* in Fig. 5.8b. If, alternatively, we set $\sigma_1 = 0$ and increase σ_3 , the material will yield at $(\sigma_3)_y$ and is represented by point *B* in Fig. 5.8b. We can then subject the cylinder to various combinations of σ_1 and σ_3 and plot the resulting yield points. Linking the yield points results in a curve, *AB*, which is called the *yield curve* or *yield surface* as shown in Fig. 5.8b. A material subjected to a combination of stresses that lies below this curve will respond elastically (recoverable deformation). If loading is continued beyond the yield stress, the material will respond elastoplastically (irrecoverable or permanent deformations occur). If the material is isotropic, the yield surface will be symmetrical about the σ_1, σ_3 axes.

The essential points are:

1. An elastic material recovers its original configuration on unloading; an elastoplastic material undergoes both elastic (recoverable) and plastic (permanent) deformation during loading.
2. Soils are elastoplastic materials.
3. At small strains, soils behave like an elastic material and thereafter like an elastoplastic material.
4. The locus of the stresses at which a soil yields is called a yield surface. Stresses below the yield stress cause the soil to respond elastically; stresses beyond the yield stress cause the soil to respond elastoplastically.

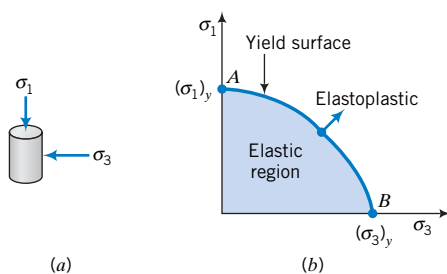


FIGURE 5.8 Elastic, yield, and elastoplastic stress states.

What's next... In the next two sections, we will write the general expression for Hooke's law, which is the fundamental law for linear elastic materials, and then consider two loading cases appropriate to soils.

5.5 HOOKE'S LAW



Interactive Concept Learning and Self-Assessment

Access Chapter 5, Section 5.5 to learn two-dimensional stresses and strains and principal stresses and strains through interactive animations. Take Quiz 5.5 to test your understanding.

5.5.1 General State of Stress

Stresses and strains for a linear, isotropic, elastic soil are related through Hooke's law. For a general state of stress (Fig. 5.9), Hooke's law is

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \quad (5.10)$$

where E is the elastic or Young's modulus and ν is Poisson's ratio. Equation (5.10) is called the elastic equations or elastic stress–strain constitutive equations. From Eq. (5.10), we have, for example,

$$\gamma_{zx} = \frac{2(1+\nu)}{E} \tau_{zx} = \frac{\tau_{zx}}{G} \quad (5.11)$$

where

$$G = \frac{E}{2(1+\nu)} \quad (5.12)$$

is the shear modulus. We will call E , G , and ν the elastic parameters. Only two of these parameters—either E or G and ν —are required to solve problems dealing with isotropic, elastic materials. We can calculate G from Eq. (5.12), if E and ν are known. Poisson's ratio for soils is not easy to determine and a direct way to obtain G is to subject the material to shearing forces as described in Section 5.4.2. For nonlinear elastic materials, the tangent modulus or the secant modulus is used in Eq. (5.10) and the calculations are done incrementally for small increments of stress.

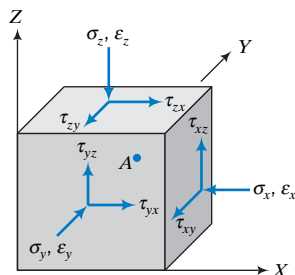


FIGURE 5.9 General state of stress.

TABLE 5.2 Typical Values of E and G

Soil type	Description	E^a (Mpa)	G^a (Mpa)
Clay	Soft	1–15	0.5–5
	Medium	15–30	5–15
	Stiff	30–100	15–40
Sand	Loose	10–20	5–10
	Medium	20–40	10–15
	Dense	40–80	15–35

^aThese are average secant elastic moduli for the drained condition (see discussions later in this chapter, Sections 5.12 and 5.13).

The elastic and shear moduli for soils depend on the stress history, direction of loading, and the magnitude of the applied strains. In Chapter 7, we will study a few tests that are used to determine E and G and, in Chapter 8, explore the details of the use of E and G in soil analyses. Typical values of E and G are shown in Table 5.2.

5.5.2 Principal Stresses

If the stresses applied to a soil are principal stresses, then Hooke's law reduces to

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} \quad (5.13)$$

The matrix on the right-hand side of Eq. (5.13) is called the compliance matrix. The inverse of Eq. (5.13) is

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} \quad (5.14)$$

The matrix on the right-hand side is called the stiffness matrix. If you know the stresses and the material parameters E and ν , you can use Eq. (5.13) to calculate the strains; or if you know the strains, E , and ν , you can use Eq. (5.14) to calculate the stresses.

5.5.3 Displacements from Strains and Forces from Stresses

The displacements and forces are obtained by integration. For example, the vertical displacement, Δz , is

$$\Delta z = \int \varepsilon_z dz \quad (5.15)$$

and the axial force is

$$P_z = \int \Delta \sigma_z dA \quad (5.16)$$

where dz is the height or thickness of the element and dA is the elemental area.

The essential points are:

1. Hooke's law applies to a linearly elastic material.
2. As a first approximation, you can use Hooke's law to calculate stresses, strains, and elastic settlement of soils.

3. For nonlinear materials, Hooke's law is used with an approximate elastic modulus (tangent modulus or secant modulus) and the calculations are done for incremental increases in stresses or strains.

What's next... The stresses and strains in three dimensions become complicated when applied to real problems. For practical purposes, many geotechnical problems can be solved using two-dimensional stress and strain parameters. In the next section, we will discuss two conditions that simplify the stress and strain states of soils.

5.6 PLANE STRAIN AND AXIAL SYMMETRIC CONDITIONS



Interactive Concept Learning and Self-Assessment

Access Chapter 5, Section 5.6 to learn about plane strain and axisymmetric conditions. Take Quiz 5.6 to test your understanding.

5.6.1 Plane Strain Condition

There are two conditions of stresses and strains that are common in geotechnical engineering. One is the *plane strain* condition in which the strain in one direction is zero. As an example of a plane strain condition, let us consider an element of soil, *A*, behind a retaining wall (Fig. 5.10). Because the displacement that is likely to occur in the *Y* direction (Δy) is small compared with the length in this direction, the strain tends to zero; that is, $\epsilon_y = \Delta y/y \cong 0$. We can then assume that soil element *A* is under a plane strain condition. Since we are considering principal stresses, we will map the *X*, *Y*, and *Z* directions as 3, 2, and 1 directions. In the case of the retaining wall the *Y* direction (2 direction) is the zero strain direction and therefore $\epsilon_2 = 0$ in Eq. (5.13).

Hooke's law for a plane strain condition is

$$\epsilon_1 = \frac{1+\nu}{E} [(1-\nu)\sigma_1 - \nu\sigma_3] \quad (5.17)$$

$$\epsilon_3 = \frac{1+\nu}{E} [(1-\nu)\sigma_3 - \nu\sigma_1] \quad (5.18)$$

and

$$\sigma_2 = \nu(\sigma_1 + \sigma_3) \quad (5.19)$$

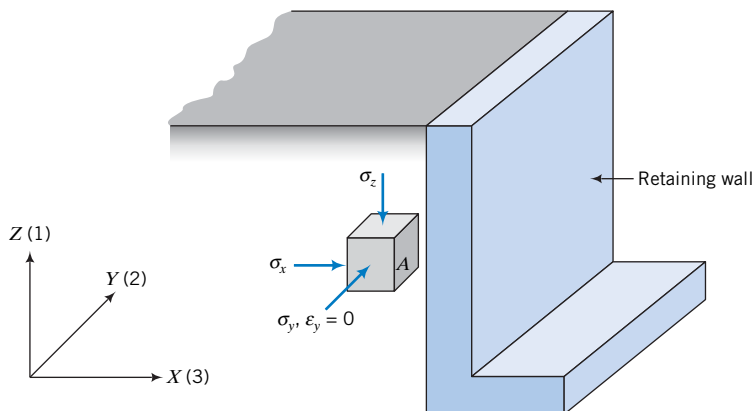


FIGURE 5.10 Plane strain condition in a soil element behind a retaining wall.

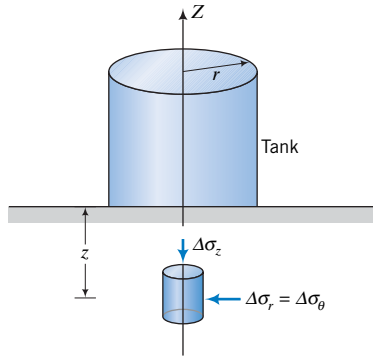


FIGURE 5.11 Axisymmetric condition on a soil element under the center of a tank.

In matrix form, Eqs. (5.17) and (5.18) become

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_3 \end{Bmatrix} = \frac{1+\nu}{E} \begin{bmatrix} 1-\nu & -\nu \\ -\nu & 1-\nu \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix} \quad (5.20)$$

The inverse of Eq. (5.20) gives

$$\begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu \\ \nu & 1-\nu \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_3 \end{Bmatrix} \quad (5.21)$$

5.6.2 Axisymmetric Condition

The other condition that occurs in practical problems is *axial symmetry* or the *axisymmetric* condition where two stresses are equal. Let us consider a water tank or an oil tank founded on a soil mass as illustrated in Fig. 5.11.

The radial stresses (σ_r) and circumferential stresses (σ_θ) on a cylindrical element of soil directly under the center of the tank are equal because of axial symmetry. The oil tank will apply a uniform vertical (axial) stress at the soil surface and the soil element will be subjected to an increase in axial stress, $\Delta\sigma_z = \Delta\sigma_1$, and an increase in radial stress, $\Delta\sigma_r = \Delta\sigma_\theta = \Delta\sigma_3$. Will a soil element under the edge of the tank be under an axisymmetric condition? The answer is no, since the stresses at the edge of the tank are all different; there is no symmetry.

Hooke's law for the axisymmetric condition is

$$\varepsilon_1 = \frac{1}{E} [\sigma_1 - 2\nu\sigma_3] \quad (5.22)$$

$$\varepsilon_3 = \frac{1}{E} [(1-\nu)\sigma_3 - \nu\sigma_1] \quad (5.23)$$

or, in matrix form,

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_3 \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -2\nu \\ -\nu & 1-\nu \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix} \quad (5.24)$$

The inverse of Eq. (5.24) gives

$$\begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & 2\nu \\ \nu & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_3 \end{Bmatrix} \quad (5.25)$$

The essential points are:

1. A plane strain condition is one in which the strain in one or more directions is zero or small enough to be neglected.
2. An axisymmetric condition is one in which two stresses are equal.

EXAMPLE 5.1 Application of Hooke's Law for Plane Strain Condition

A retaining wall moves outward causing a lateral strain of 0.1% and a vertical strain of 0.05% on a soil element located 3 m below ground level. Assuming the soil is a linear, isotropic, elastic material with $E = 5000 \text{ kPa}$ and $\nu = 0.3$, calculate the increase in stresses imposed. If the retaining wall is 6 m high and the stresses you calculate are the average stresses, determine the lateral force increase per unit length of wall.

Strategy You will have to make a decision whether to use the plane strain or axisymmetric condition and then use the appropriate equation. You are asked to find the increase in stresses, so it is best to write the elastic equations in terms of increment. The retaining wall moves outward, so the lateral strain is tensile (–) while the vertical strain is compressive (+). The increase in lateral force is found by integration of the average lateral stress increase.

Solution 5.1

Step 1: Determine the appropriate stress condition and write the appropriate equation.

The soil element is likely to be under the plane strain condition ($\epsilon_2 = 0$); use Eq. (5.21).

$$\begin{Bmatrix} \Delta\sigma_1 \\ \Delta\sigma_3 \end{Bmatrix} = \frac{5000}{(1+0.3)(1-2 \times 0.3)} \begin{bmatrix} 1-0.3 & 0.3 \\ 0.3 & 1-0.3 \end{bmatrix} \begin{Bmatrix} 0.0005 \\ -0.001 \end{Bmatrix}$$

Step 2: Solve the equation.

$$\Delta\sigma_1 = 9615.4\{(0.7 \times 0.0005) + [0.3 \times (-0.001)]\} = 0.5 \text{ kPa}$$

$$\Delta\sigma_3 = 9615.4\{(0.3 \times 0.0005) + [0.7 \times (-0.001)]\} = -5.3 \text{ kPa;}$$

the negative sign means reduction

Step 3: Calculate the lateral force per unit length.

$$\Delta\sigma_3 = \Delta\sigma_x$$

$$\Delta P_x = \int_0^6 \Delta\sigma_x dA = - \int_0^6 5.3(dx \times 1) = -[5.3x]_0^6 = -31.8 \text{ kN/m}$$

EXAMPLE 5.2 Application of Hooke's Law for Axisymmetric Condition

An oil tank is founded on a layer of medium sand 5 m thick underlain by a deep deposit of dense sand. The geotechnical engineer assumed, based on experience, that the settlement of the tank would occur from settlement in the medium sand. The vertical and lateral stresses at the middle of the medium sand directly under the center of the tank are 50 kPa and 20 kPa, respectively. The values of E and ν are 20 MPa and 0.3, respectively. Assuming a linear, isotropic, elastic material behavior, calculate the strains imposed in the medium sand and the vertical settlement.

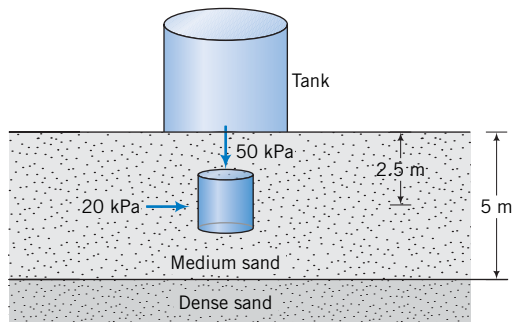


FIGURE E5.2

Strategy You have to decide on the stress conditions on the soil element directly under the center of the tank. Once you make your decision, use the appropriate equations to find the strains and then integrate the vertical strains to calculate the settlement. Draw a diagram illustrating the problem.

Solution 5.2

Step 1: Draw a diagram of the problem—see Fig. E5.2.

Step 2: Decide on a stress condition.

The element is directly under the center of the tank, so the axisymmetric condition prevails.

Step 3: Choose the appropriate equations and solve.

Use Eq. (5.24).

$$\begin{Bmatrix} \Delta\epsilon_1 \\ \Delta\epsilon_3 \end{Bmatrix} = \frac{1}{20 \times 10^3} \begin{bmatrix} 1 & -0.6 \\ -0.3 & 0.7 \end{bmatrix} \begin{Bmatrix} 50 \\ 20 \end{Bmatrix}$$

Using algebra, we get

$$\Delta\epsilon_1 = \frac{1}{20 \times 10^3} [1 \times 50 - 0.6 \times 20] = 1.9 \times 10^{-3}$$

$$\Delta\epsilon_3 = \frac{1}{20 \times 10^3} [-0.3 \times 50 + 0.7 \times 20] = -5 \times 10^{-5}$$

Step 4: Calculate vertical displacement.

$$\Delta\epsilon_1 = \Delta\epsilon_z$$

$$\Delta z = \int_0^5 \Delta\epsilon_z dz = [1.9 \times 10^{-3} z]_0^5 = 9.5 \times 10^{-3} \text{ m} = 9.5 \text{ mm}$$

What's next... We have used the elastic equations to calculate stresses, strains, and displacements in soils assuming that soils are linear, isotropic, elastic materials. Soils, in general, are not linear, isotropic, elastic materials. We will briefly discuss anisotropic, elastic materials in the next section.

5.7 ANISOTROPIC ELASTIC STATES



Interactive Concept Learning and Self-Assessment

Access Chapter 5, Section 5.7 to learn about anisotropy. Take Quiz 5.7 to test your understanding.

Anisotropic materials have different elastic parameters in different directions. Anisotropy in soils results from essentially two causes.

1. The manner in which the soil is deposited. This is called structural anisotropy and it is the result of the kind of soil fabric that is formed during deposition. You should recall (Chapter 2) that the soil fabric produced is related to the history of the environment in which the soil is formed. A special form of structural anisotropy occurs when the horizontal plane is a plane of isotropy. We call this form of structural anisotropy, transverse anisotropy.
2. The difference in stresses in the different directions. This is known as stress-induced anisotropy.

Transverse anisotropy, also called cross anisotropy, is the most prevalent type of anisotropy in soils. If we were to load the soil in the vertical direction (Z direction) and repeat the same loading in the horizontal direction, say, the X direction, the soil will respond differently; its stress-strain characteristics and strength would be different in these directions. However, if we were to load the soil in the Y direction, the soil's response would be similar to the response obtained in the X direction. The implication is that a soil mass will, in general, respond differently depending on the direction of the load. For transverse anisotropy, the elastic parameters are the same in the lateral directions (X and Y directions) but are different from the vertical direction.

To fully describe anisotropic soil behavior, we need 21 elastic constants (Love, 1927) but for transverse anisotropy, we need only 5 elastic constants; these are E_z , E_x , ν_{xx} , ν_{zx} , and ν_{zz} . The first letter in the double subscripts denotes the direction of loading and the second letter denotes the direction of measurement. For example, ν_{zx} means Poisson's ratio determined from the ratio of the strain in the lateral direction (X direction) to the strain in the vertical direction (Z direction) with the load applied in the vertical direction (Z direction).

In the laboratory, the direction of loading of soil samples taken from the field is invariably vertical. Consequently, we cannot determine the five desired elastic parameters from conventional laboratory tests. Graham and Houlsby (1983) suggested a method to overcome the lack of knowledge of the five desired elastic parameters in solving problems on transverse anisotropy. However, their method is beyond the scope of this book.

For axisymmetric conditions, the transverse anisotropic, elastic equations are

$$\begin{Bmatrix} \Delta \varepsilon_z \\ \Delta \varepsilon_r \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_z} & \frac{-2\nu_{rz}}{E_r} \\ \frac{-\nu_{zr}}{E_z} & \frac{(1-\nu_{rr})}{E_r} \end{bmatrix} \begin{Bmatrix} \Delta \sigma_z \\ \Delta \sigma_r \end{Bmatrix} \quad (5.26)$$

where the subscript z denotes vertical and r denotes radial. By superposition, $\nu_{rz}/\nu_{zr} = E_r/E_z$.

The essential points are:

1. *Two forms of anisotropy are present in soils. One is structural anisotropy, which is related to the history of loading and environmental conditions during deposition, and the other is stress-induced anisotropy, which results from differences in stresses in different directions.*
2. *The prevalent form of structural anisotropy in soils is transverse anisotropy; the soil properties and the soil response in the lateral directions are the same but are different from those in the vertical direction.*
3. *You need to find the elastic parameters in different directions of a soil mass to determine elastic stresses, strains, and displacements.*

EXAMPLE 5.3 *Application of Hooke's Law for Transverse Anisotropic Soils*

Redo Example 5.2 but now the soil under the oil tank is an anisotropic elastic material with $E_z = 20$ MPa, $E_r = 25$ MPa, $\nu_{rz} = 0.15$, and $\nu_{rr} = 0.3$.

Strategy The solution of this problem is a straightforward application of Eq. (5.26).

Solution 5.3

Step 1: Determine ν_{zr} (by superposition).

$$\begin{aligned}\frac{\nu_{rz}}{\nu_{zr}} &= \frac{E_r}{E_z} \\ \nu_{zr} &= \frac{20}{25} \times 0.15 = 0.12\end{aligned}$$

Step 2: Find the strains.

Use Eq. (5.26).

$$\begin{Bmatrix} \Delta \varepsilon_z \\ \Delta \varepsilon_r \end{Bmatrix} = 10^{-3} \begin{bmatrix} \frac{1}{20} & \frac{-2 \times 0.15}{25} \\ \frac{-0.12}{20} & \frac{(1 - 0.3)}{25} \end{bmatrix} \begin{Bmatrix} 50 \\ 20 \end{Bmatrix}$$

The solution is $\varepsilon_z = 2.26 \times 10^{-3} = 0.23\%$ and $\varepsilon_r = 0.26 \times 10^{-3} = 0.03\%$.

Step 3: Determine vertical displacement.

$$\Delta z = \int_0^5 \varepsilon_z dz = [2.26 \times 10^{-3} z]_0^5 = 11.3 \times 10^{-3} \text{ m} = 11.3 \text{ mm}$$

The vertical displacement in the anisotropic case is about 19% more than in the isotropic case (Example 5.2). Also, the radial strain is tensile for the isotropic case but compressive in the anisotropic case for this problem. ■

What's next... We now know how to calculate stresses and strains in soils if we assume soils are elastic, homogeneous materials. One of the important tasks for engineering works is to determine strength or failure of materials. We can draw an analogy of the strength of materials with the strength of a chain. The chain is only as strong as its weakest link. For soils, failure may be initiated at a point within a soil mass and then propagate through it; this is known as progressive failure. The stress state at a point in a soil mass due to applied boundary forces may be equal to the strength of the soil, thereby initiating failure. Therefore, as engineers, we need to know the stress state at a point due to applied loads. We will discuss stress state next using your knowledge in strength of materials.

5.8 STRESS AND STRAIN STATES



Interactive Concept Learning Computer Program Utility, and Self-Assessment

Access Chapter 5, Section 5.8 on the CD to learn through interactive animation about stress and strain states using Mohr's circle. Access Mohr's circle on the sidebar to plot, interpret, and explore a variety of stress states interactively. Take Quiz 5.8 to test your understanding.

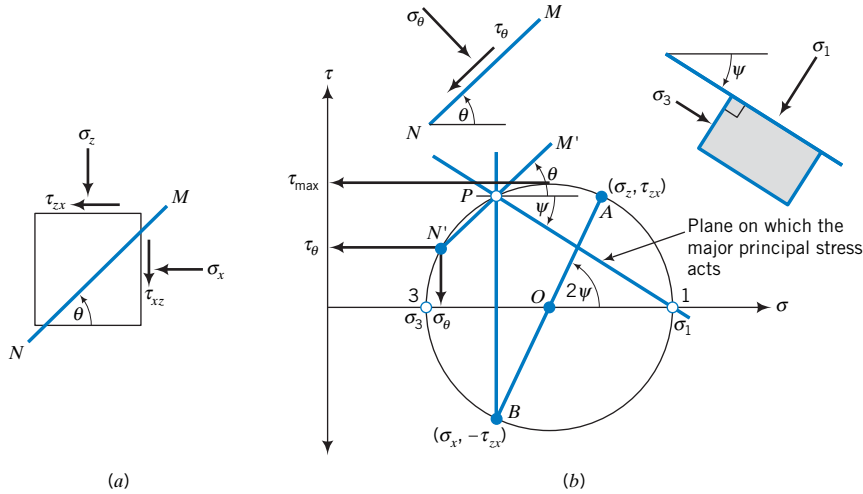


FIGURE 5.12 Stresses on a two-dimensional element and Mohr's circle.

5.8.1 Mohr's Circle for Stress States

Suppose a cuboidal sample of soil is subjected to the stresses shown in Fig. 5.9. We would like to know what the stresses are at a point, say, A , within the sample due to the applied stresses. One approach to find the stresses at A , called the stress state at A , is to use Mohr's circle. The stress state at a point is the set of stress vectors corresponding to all planes passing through that point. For simplicity, we will consider a two-dimensional element with stresses as shown in Fig. 5.12a. Let us draw Mohr's circle. First, we have to choose a sign convention. We have already decided that compressive stresses are positive for soils. We will assume counterclockwise shear is positive and $\sigma_z > \sigma_x$. The two coordinates of the circle are (σ_z, τ_{zx}) and (σ_x, τ_{xz}) . Recall from your strength of materials course that, for equilibrium, $\tau_{xz} = -\tau_{zx}$; these are called complementary shear stresses and are orthogonal to each other. Plot these two coordinates on a graph of shear stress (ordinate) and normal stress (abscissa) as shown by A and B in Fig. 5.12b. Draw a circle with AB as the diameter. The circle crosses the normal stress axis at 1 and 3. The stresses at these points are the major principal stress, σ_1 , and the minor principal stress, σ_3 .

The principal stresses are related to the stress components σ_z , σ_x , τ_{zx} by

$$\sigma_1 = \frac{\sigma_z + \sigma_x}{2} + \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} \quad (5.27)$$

$$\sigma_3 = \frac{\sigma_z + \sigma_x}{2} - \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} \quad (5.28)$$

The angle between the major principal stress plane and the horizontal plane (ψ) is

$$\tan \psi = \frac{\tau_{zx}}{\sigma_1 - \sigma_x} \quad (5.29)$$

The stresses on a plane oriented at an angle θ from the major principal stress plane are

$$\sigma_\theta = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta \quad (5.30)$$

$$\tau_\theta = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \quad (5.31)$$

The stresses on a plane oriented at an *angle* θ from the horizontal plane are

$$\sigma_{\theta} = \frac{\sigma_z + \sigma_x}{2} + \frac{\sigma_z - \sigma_x}{2} \cos 2\theta + \tau_{zx} \sin 2\theta \quad (5.32)$$

$$\tau_{\theta} = \tau_{zx} \cos 2\theta - \frac{\sigma_z - \sigma_x}{2} \sin 2\theta \quad (5.33)$$

In the above equations, θ is positive for counterclockwise orientation.

The maximum shear stress is at the top of the circle with magnitude

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \quad (5.34)$$

For the stresses shown in Fig. 5.9, we would get three circles but we have simplified the problem by plotting one circle for stresses on all planes perpendicular to one principal direction.

The stress σ_z acts on the horizontal plane and the stress σ_x acts on the vertical plane for our case. If we draw these planes in Mohr's circle, they intersect at a point, P . Point P is called the pole of the stress circle. It is a special point because any line passing through the pole will intersect Mohr's circle at a point that represents the stresses on a plane parallel to the line. Let us see how this works. Suppose we want to find the stresses on a plane inclined at an angle θ from the horizontal plane as depicted by MN in Fig. 5.12a. Once we locate the pole, P , we can draw a line parallel to MN through P as shown by $M'N'$ in Fig. 5.12b. The line $M'N'$ intersects the circle at N' and the coordinates of N' , $(\sigma_{\theta}, \tau_{\theta})$, represent the normal and shear stresses on MN .

5.8.2 Mohr's Circle for Strain States

So far, we have studied stress states. The strain state is found in a similar manner to the stress state. With reference to Fig. 5.13, the principal strains are

Major principal strain:
$$\epsilon_1 = \frac{\epsilon_z + \epsilon_x}{2} + \sqrt{\left(\frac{\epsilon_z - \epsilon_x}{2}\right)^2 + \left(\frac{\gamma_{zx}}{2}\right)^2} \quad (5.35)$$

Minor principal strain:
$$\epsilon_3 = \frac{\epsilon_z + \epsilon_x}{2} - \sqrt{\left(\frac{\epsilon_z - \epsilon_x}{2}\right)^2 + \left(\frac{\gamma_{zx}}{2}\right)^2} \quad (5.36)$$

where γ_{zx} is called the engineering shear strain or simple shear strain.

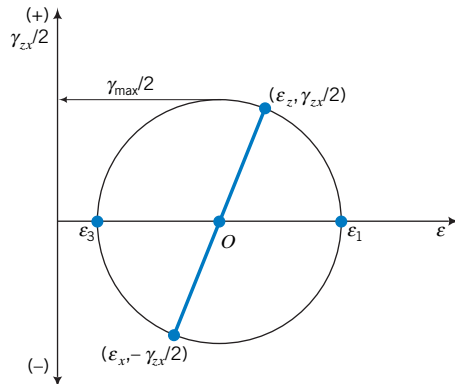


FIGURE 5.13 Mohr's circle of strain.

The maximum simple shear strain is

$$\gamma_{\max} = \varepsilon_1 - \varepsilon_3 \quad (5.37)$$

In soils, strains can be compressive or tensile. There is no absolute reference strain. For stresses, we can select atmospheric pressure as the reference but not so for strains. Usually, we deal with changes or increments of strains resulting from stress changes.

The essential points are:

1. *Mohr's circle is used to find the stress state or strain state from a two-dimensional set of stresses or strains on a soil.*
2. *The pole on a Mohr's circle identifies a point through which any plane passing through it will intersect the Mohr's circle at a point that represents the stresses on that plane.*

EXAMPLE 5.4 Mohr Circle for Stress State

A sample of soil ($0.1 \text{ m} \times 0.1 \text{ m}$) is subjected to the forces shown in Fig. E5.4a. Determine (a) σ_1 , σ_3 , and Ψ ; (b) the maximum shear stress, and (c) the stresses on a plane oriented at 30° counterclockwise from the major principal stress plane.

Strategy There are two approaches to solve this problem. You can either use Mohr's circle or the appropriate equations. Both approaches will be used here.

Solution 5.4

Step 1: Find the area.

$$\text{Area : } A = 0.1 \times 0.1 = 10^{-2} \text{ m}^2$$

Step 2: Calculate the stresses.

$$\sigma_z = \frac{\text{Force}}{\text{Area}} = \frac{5}{10^{-2}} = 500 \text{ kPa}$$

$$\sigma_x = \frac{3}{10^{-2}} = 300 \text{ kPa}$$

$$\tau_{zx} = \frac{1}{10^{-2}} = 100 \text{ kPa}; \quad \tau_{xz} = -\tau_{zx} = -100 \text{ kPa}$$

Step 3: Draw Mohr's circle and extract σ_1 , σ_3 , and $\tau_{\max}$.

Mohr's circle is shown in Fig. E5.4b.

$$\sigma_1 = 540 \text{ kPa}, \quad \sigma_3 = 260 \text{ kPa}, \quad \tau_{\max} = 140 \text{ kPa}$$

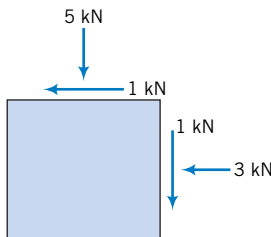


FIGURE E5.4a

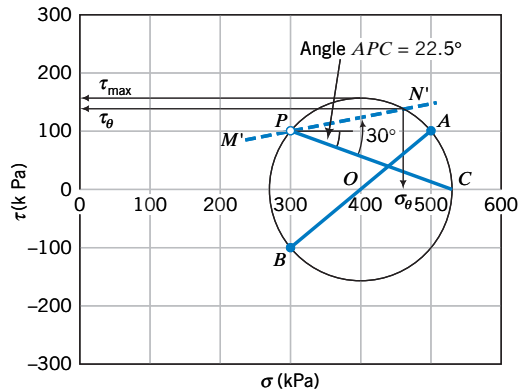


FIGURE E5.4b

Step 4: Draw the pole on Mohr's circle. The pole of Mohr's circle is shown by point P in Fig. E5.4b.

Step 5: Determine Ψ .

Draw a line from P to σ_1 and measure the angle between the horizontal plane and this line.

$$\Psi = 22.5^\circ$$

Alternatively, the angle $AOC = 2\Psi = 45^\circ$

$$\therefore \Psi = 22.5^\circ$$

Step 6: Determine the stresses on a plane inclined at 30° from the major principal stress plane.

Draw a line M^1N^1 through P with an inclination of 30° from the major principal stress plane, angle CPN' . The coordinate at point N' is (470, 120).

Alternatively:

Step 1: Use Eqs. (5.27) to (5.29) and (5.34) to find σ_1 , σ_3 , Ψ , and $\tau_{\max}$.

$$\sigma_1 = \frac{500 + 300}{2} + \sqrt{\left(\frac{500 - 300}{2}\right)^2 + 100^2} = 541.4 \text{ kPa}$$

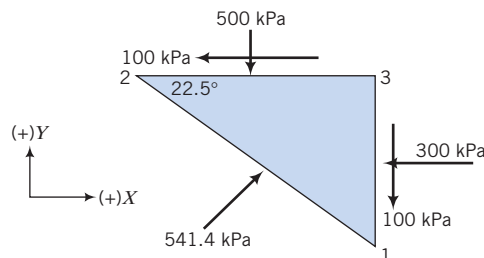
$$\sigma_3 = \frac{500 + 300}{2} - \sqrt{\left(\frac{500 - 300}{2}\right)^2 + 100^2} = 258.6 \text{ kPa}$$

$$\tan \Psi = \frac{\tau_{yx}}{\sigma_1 - \sigma_x} = \frac{100}{541.4 - 300} = 0.414$$

$$\therefore \Psi = 22.5^\circ$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{541.4 - 258.6}{2} = 141.4 \text{ kPa}$$

Check Equilibrium



Length of 2-3 = 0.1 m

Length of 3-1 = $0.1 \times (\tan 22.5^\circ) = 0.0414$ m

Length of 1-2 = $0.1/(\cos 22.5^\circ) = 0.1082$ m

$$\sum F_x = 0 : -300 \times 0.0414 - 100 \times 0.1 + 541.4 \times 0.1082 \times \cos(22.5^\circ) = 0$$

$$\sum F_y = 0 : -100 \times 0.0414 - 500 \times 0.1 + 541.4 \times 0.1082 \times \sin(22.5^\circ) = 0$$

Step 2: Use Eqs. (5.30) and (5.31) to find σ_θ and τ_θ .

$$\sigma_\theta = \frac{541.4 + 258.6}{2} + \frac{541.4 - 258.6}{2} \cos(2 \times 30) = 470.7 \text{ kPa}$$

$$\tau_\theta = \frac{541.4 - 258.6}{2} \sin(2 \times 30) = 122.5 \text{ kPa}$$

What's next... The stresses we have calculated are for soils as solid elastic materials. We have not accounted for the pressure within the soil pore spaces. In the next section, we will discuss the principle of effective stresses that accounts for the pressures within the soil pores. This principle is the most important principle in soil mechanics.

5.9 TOTAL AND EFFECTIVE STRESSES



Interactive Concept Learning and Self-Assessment

Access Chapter 5, Section 5.9 on the CD to learn the principles of effective stress through interactive visualizations. Explore interactive examples to learn problem-solving techniques. You can explore the effects of different groundwater conditions on the effective stresses. Take Quiz 5.9 to test your understanding.

5.9.1 The Principle of Effective Stress

The deformations of soils are similar to the deformations of structural framework such as a truss. The truss deforms from changes in loads carried by each member. If the truss is loaded in air or submerged in water, the deformations under a given load will remain unchanged. Deformations of the truss are independent of hydrostatic pressure. The same is true for soils.

Let us consider an element of a *saturated soil* subjected to a normal stress, σ , applied on the horizontal boundary as shown in Fig. 5.14. The stress σ is called the *total stress* and for equilibrium (Newton's third law) the stresses in the soil must be equal and opposite to σ . The resistance or reaction

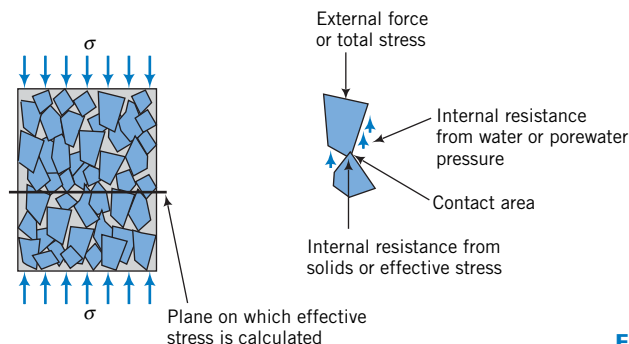


FIGURE 5.14 Effective stress.

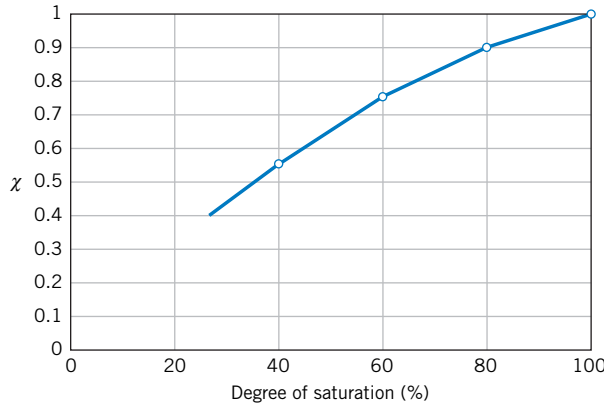


FIGURE 5.15 Values of χ for a silt at different degrees of saturation.

to σ is provided by a combination of the stresses from the solids, called *effective stress* (σ'), and from water in the pores, called *porewater pressure* (u). We will denote effective stresses by a prime (') following the symbol for normal stress, usually σ . The equilibrium equation is

$$\sigma = \sigma' + u \quad (5.38)$$

so that

$$\sigma' = \sigma - u \quad (5.39)$$

Equation (5.39) is called the *principle of effective stress* and was first recognized by Terzaghi (1883–1963) in the mid-1920s during his research into soil consolidation (Chapter 6). **The principle of effective stress is the most important principle in soil mechanics. Deformations of soils are a function of effective stresses not total stresses. The principle of effective stresses applies only to normal stresses and not to shear stresses.** The porewater cannot sustain shear stresses and therefore the soil solids must resist the shear forces. Thus $\tau = \tau'$, where τ is the total shear stress and τ' is the effective shear stress. The effective stress is not the contact stress between the soil solids. Rather, it is the average stress on a plane through the soil mass.

Soils cannot sustain tension. Consequently, the effective stress cannot be less than zero. Porewater pressures can be positive or negative. The latter is sometimes called suction or suction pressure.

For unsaturated soils, the effective stress (Bishop et al., 1960) is

$$\sigma' = \sigma - u_a + \chi(u_a - u_w) \quad (5.40)$$

where u_a is the pore air pressure, u_w is the porewater pressure, and χ is a factor depending on the degree of saturation. For dry soil, $\chi = 0$; for saturated soil, $\chi = 1$. Values of χ for a silt are shown in Fig. 5.15.

To determine the effective stress in a soil mass, the porewater pressure must be known. The porewater pressure at a particular point in a soil mass is the depth of water above that point multiplied by the unit weight of water. Porewater pressures are measured by porewater pressure transducers (Fig. 5.16) or by piezometers (Fig. 5.17). In a porewater pressure transducer, water passes through a

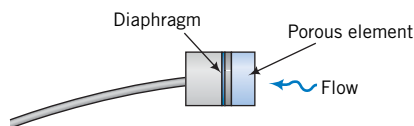


FIGURE 5.16 Schematic of a porewater pressure transducer.

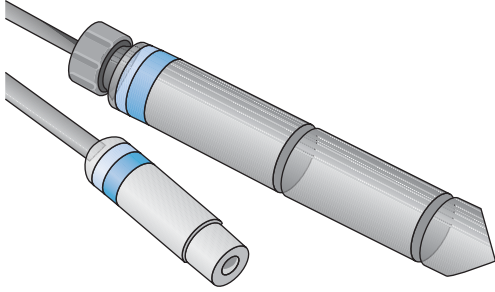


FIGURE 5.17 Piezometers.

porous material and pushes against a metal diaphragm to which a strain gauge is attached. The strain gauge is usually wired into a Wheatstone bridge. The porewater pressure transducer is calibrated by applying known pressures and measuring the electrical voltage output from the Wheatstone bridge. Piezometers are porous tubes that allow the passage of water. In a simple piezometer, you can measure the height of water in the tube from a fixed elevation and then calculate the porewater pressure by multiplying the height of water by the unit weight of water (Chapter 4). A borehole cased to a certain depth acts like a piezometer. Modern piezometers are equipped with porewater pressure transducers for electronic reading and data acquisition.

5.9.2 Effective Stresses Due to Geostatic Stress Fields

The effective stress in a soil mass not subjected to external loads is found from the unit weight of the soil and the depth of groundwater. Consider a soil element at a depth z below the ground surface and the groundwater level (GWL) is at ground surface (Fig. 5.18a). The total vertical stress is

$$\sigma = \gamma_{\text{sat}} z \quad (5.41)$$

The porewater pressure is

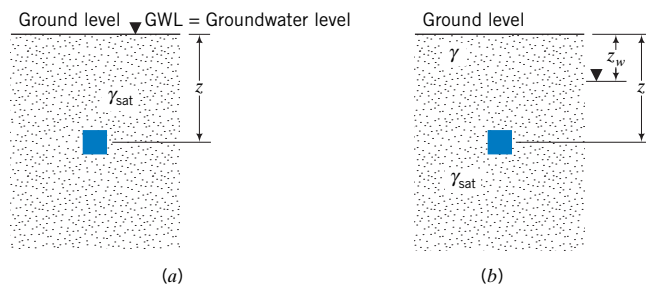
$$u = \gamma_w z \quad (5.42)$$

and the effective stress is

$$\sigma' = \sigma - u = \gamma_{\text{sat}} z - \gamma_w z = (\gamma_{\text{sat}} - \gamma_w) z = \gamma' z \quad (5.43)$$

If the GWL is at a depth z_w below ground level (Fig. 5.18b), then

$$\sigma = \gamma z_w + \gamma_{\text{sat}}(z - z_w) \quad \text{and} \quad u = \gamma_w(z - z_w)$$

FIGURE 5.18 Soil element at a depth z with groundwater level (a) at ground level and (b) below ground level.

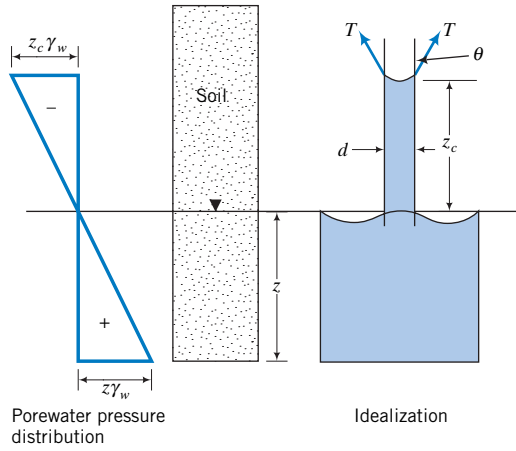


FIGURE 5.19 Capillary simulation in soils.

The effective stress is

$$\begin{aligned}\sigma' &= \sigma - u = \gamma z_w + \gamma_{\text{sat}}(z - z_w) - \gamma_w(z - z_w) \\ &= \gamma z_w + (\gamma_{\text{sat}} - \gamma_w)(z - z_w) = \gamma z_w + \gamma'(z - z_w)\end{aligned}$$

5.9.3 Effects of Capillarity

In silts and fine sands, the soil above the groundwater can be saturated by capillary action. You would have encountered capillary action in your physics course when you studied menisci. We can get an understanding of capillarity in soils by idealizing the continuous void spaces as capillary tubes. Consider a single idealized tube as shown in Fig. 5.19. The height at which water will rise in the tube can be found from statics. Summing forces vertically (upward forces are negative), we get

$$\Sigma F_z = \text{weight of water} - \text{the tension forces from capillary action}$$

that is,

$$\frac{\pi d^2}{4} z_c \gamma_w - \pi d T \cos \theta = 0 \quad (5.44)$$

Solving for z_c , we get

$$z_c = \frac{4T \cos \theta}{d \gamma_w} \quad (5.45)$$

where T is the surface tension (force per unit length), α is the contact angle, z_c is the height of capillary rise, and d is the diameter of the tube representing the diameter of the void space. The surface tension of water is 0.073 N/m and the contact angle of water with a clean glass surface is 0. Since T , θ , and γ_w are constants,

$$z_c \propto \frac{1}{d} \quad (5.46)$$

For soils, d is assumed to be equivalent to $0.1 D_{10}$ where D_{10} is the effective size. The interpretation of Eq. (5.46) is that the smaller the soil pores, the higher the capillary zone. The capillary zone in fine sands will be larger than for medium or coarse sands.

The porewater pressure due to capillarity is negative (suction) as shown in Fig. 5.19 and is a function of the size of the soil pores and the water content. At the groundwater level, the porewater pressure is zero and decreases (becomes negative) as you move up the capillary zone. The effective

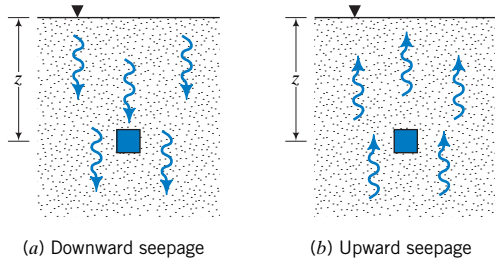


FIGURE 5.20 Seepage in soils.

stress increases because the porewater pressure is negative. For example, for the capillary zone, z_c , the porewater pressure at the top is $-z_c\gamma_w$ and the effective stress is $\sigma' = \sigma - (-z_c\gamma_w) = \sigma + z_c\gamma_w$.

The approach we have taken to interpret capillary action in soils is simple but it is sufficient for most geotechnical applications. For a comprehensive treatment of capillary action you can refer to Adamson (1982).

5.9.4 Effects of Seepage

In Chapter 4, we discussed one-dimensional flow of water through soils. As water flows through soil it exerts a frictional drag on the soil particles resulting in head losses. The frictional drag is called seepage force in soil mechanics. It is often convenient to define seepage as the seepage force per unit volume (it has units similar to unit weight), which we will denote by j_s . If the head loss over a flow distance, L , is Δh , the seepage force is

$$j_s = \frac{\Delta h \gamma_w}{L} = i \gamma_w \quad (5.47)$$

If seepage occurs downward (Fig. 5.20a), then the seepage stresses are in the same direction as the gravitational effective stresses. From static equilibrium the resultant vertical effective stress is

$$\sigma'_z = \gamma'z + iz\gamma_w = \gamma'z + j_s z \quad (5.48)$$

If seepage occurs upwards (Fig. 5.20b), then the seepage stresses are in the opposite direction to the gravitational effective stresses. From static equilibrium the resultant vertical effective stress is

$$\sigma'_z = \gamma'z - iz\gamma_w = \gamma'z - j_s z \quad (5.49)$$

Seepage forces play a very important role in destabilizing geotechnical structures. For example, a cantilever retaining wall, shown in Fig. 5.21, depends on the depth of embedment for its stability. The retained soil (left side of wall) applies an outward lateral pressure to the wall, which is resisted by an inward lateral resistance from the soil on the right side of the wall. If a steady quantity of water is available on the left side of the wall, for example, from a busted water pipe, then water will flow from the left side to the right side of the wall. The path followed by a particle of water is depicted by AB in Fig. 5.21 and as water flows from A to B , head loss occurs. The seepage stresses on the left side of the wall are in the direction of the gravitational stresses. The effective stress increases and, consequently, an additional outward lateral force is applied on the left side of the wall. On the right side of the wall, the seepage stresses are upward and the effective stress decreases. The lateral resistance provided by

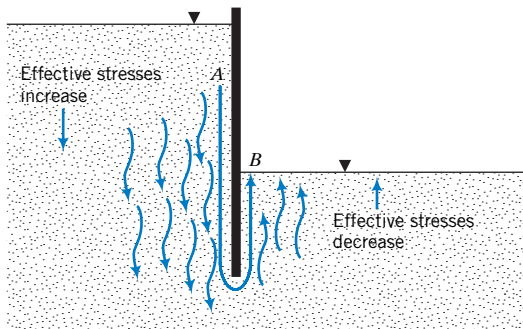


FIGURE 5.21 Effects of seepage on the effective stresses near a retaining wall.

the embedment is reduced. Seepage stresses in this problem play a double role (increase the lateral disturbing force and reduce the lateral resistance) in reducing the stability of a geotechnical structure. In Chapters 11 through 13, you will study the effects of seepage on the stability of several types of geotechnical structures.

The essential points are:

1. *The effective stress represents the average stress carried by the soil solids and is the difference between the total stress and the porewater pressure.*
2. *The effective stress principle applies only to normal stresses and not to shear stresses.*
3. *Deformations of soils are due to effective not total stress.*
4. *Soils, especially silts and fine sands, can be affected by capillary action.*
5. *Capillary action results in negative porewater pressures and increases the effective stresses.*
6. *Downward seepage increases the resultant effective stress; upward seepage decreases the resultant effective stress.*

EXAMPLE 5.5 *Calculating Vertical Effective Stress*

Calculate the effective stress for a soil element at depth 5 m in a uniform deposit of soil as shown in Fig. E5.5.

Strategy You need to get unit weights from the given data and you should note that the soil above the groundwater level is not saturated.

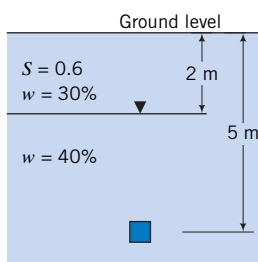


FIGURE E5.5

Solution 5.5**Step 1:** Calculate unit weights.**Above groundwater level**

$$\gamma = \left(\frac{G_s + Se}{1 + e} \right) \gamma_w = \frac{G_s(1 + w)}{1 + e} \gamma_w$$

$$Se = wG_s, \quad \therefore e = \frac{0.3 \times 2.7}{0.6} = 1.35$$

$$\gamma = \frac{2.7(1 + 0.3)}{1 + 1.35} \times 9.8 = 14.6 \text{ kN/m}^3$$

Below groundwater levelSoil is saturated, $S = 1$

$$e = wG_s = 0.4 \times 2.7 = 1.08$$

$$\gamma_{\text{sat}} = \left(\frac{G_s + e}{1 + e} \right) \gamma_w = \left(\frac{2.7 + 1.08}{1 + 1.08} \right) 9.8 = 17.8 \text{ kN/m}^3$$

Step 2: Calculate the effective stress.

$$\text{Total stress: } \sigma_z = 2\gamma + 3\gamma_{\text{sat}} = 2 \times 14.6 + 3 \times 17.8 = 82.6 \text{ kPa}$$

$$\text{Porewater pressure: } u = 3\gamma_w = 3 \times 9.8 = 29.4 \text{ kPa}$$

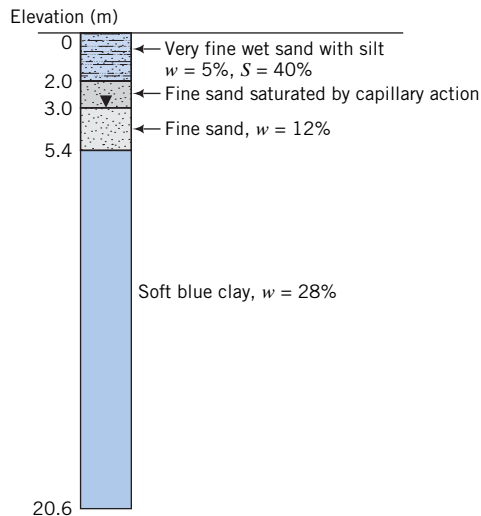
$$\text{Effective stress: } \sigma'_z = \sigma_z - u = 82.6 - 29.4 = 53.2 \text{ kPa}$$

Alternatively:

$$\sigma'_z = 2\gamma + 3(\gamma_{\text{sat}} - \gamma_w) = 2\gamma + 3\gamma' = 2 \times 14.6 + 3(17.8 - 9.8) = 53.2 \text{ kPa}$$

EXAMPLE 5.6 Calculating and Plotting Vertical Effective Stress Distribution

A borehole at a site reveals the soil profile shown in Fig. E5.6a. Plot the distribution of vertical total and effective stresses with depth.

**FIGURE 5.6a**

Strategy From the data given, you will have to find the unit weight of each soil layer to calculate the stresses. You are given that the 1.0 m of fine sand above the groundwater level is saturated by capillary action. Therefore, the porewater pressure in this 1.0 m zone is negative.

Solution 5.6

Step 1: Calculate the unit weights.

0–2 m

$$S = 40\% = 0.4; \quad w = 0.05$$

$$e = \frac{wG_s}{S} = \frac{0.05 \times 2.7}{0.4} = 0.34$$

$$\gamma = \frac{G_s(1+w)}{1+e} \gamma_w = \frac{2.7(1+0.05)}{1+0.34} 9.8 = 20.7 \text{ kN/m}^3$$

2–5.4 m

$$S = 1; \quad w = 0.12$$

$$e = wG_s = 0.12 \times 2.7 = 0.32$$

$$\gamma_{\text{sat}} = \left(\frac{G_s + e}{1 + e} \right) \gamma_w = \left(\frac{2.7 + 0.32}{1 + 0.32} \right) 9.8 = 22.4 \text{ kN/m}^3$$

5.4–20.6 m

$$S = 1; \quad w = 0.28$$

$$e = wG_s = 0.28 \times 2.7 = 0.76$$

$$\gamma_{\text{sat}} = \left(\frac{2.7 + 0.76}{1 + 0.76} \right) 9.8 = 19.3 \text{ kN/m}^3$$

Step 2: Calculate the stresses using a table or use a spreadsheet program.

Depth (m)	Thickness (m)	σ_z (kPa)	u (kPa)	$\sigma'_z = \sigma - u$ (kPa)
0	0	0	0	0
2	2	$20.7 \times 2 = 41.4$	$-1 \times 9.8 = -9.8$	51.6
3	1	$41.4 + 22.4 \times 1 = 63.8$	0	63.8
5.4	2.4	$63.8 + 22.4 \times 2.4 = 117.6$	$2.4 \times 9.8 = 23.5$	94.1
20.6	15.2	$117.6 + 19.3 \times 15.2 = 411$	$23.5 + 15.2 \times 9.8 = 172.5$ or $17.6 \times 9.8 = 172.5$	238.5

Step 3: Plot the stresses versus depth—see Fig. E5.6b.

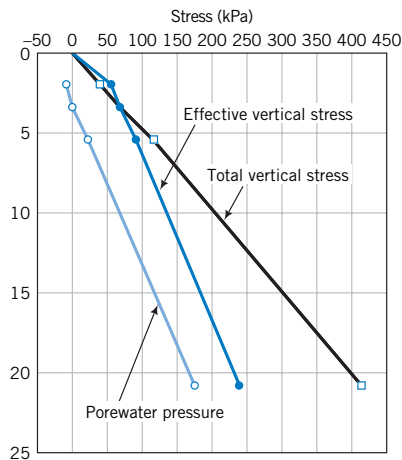


FIGURE E5.6b

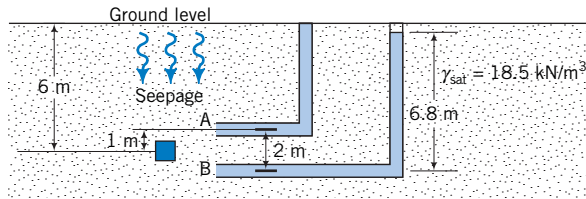


FIGURE E5.7

EXAMPLE 5.7 Effects of Seepage on Effective Stress

Water is seeping downward through a soil layer as shown in Fig. E5.7. Two piezometers (A and B) located 2 m apart (vertically) showed a head loss of 0.2 m. Calculate the resultant vertical effective stress for a soil element at a depth of 6 m as shown in Fig. E5.7.

Strategy You have to calculate the seepage stress. But to obtain this you must know the hydraulic gradient, which you can find from the data given.

Solution 5.7

Step 1: Find the hydraulic gradient.

$$\Delta H = 0.2 \text{ m}; L = 2 \text{ m}; i = \frac{\Delta H}{L} = \frac{0.2}{2} = 0.1$$

Step 2: Determine the effective stress.

Assume the hydraulic gradient is the average for the soil mass; then

$$\sigma'_z = (\gamma_{\text{sat}} - \gamma_w)z + i\gamma_w z = (18.5 - 9.8)6 + 0.1 \times 9.8 \times 6 = 58.1 \text{ kPa}$$

What's next... We have only considered vertical stresses. But an element of soil in the ground is also subjected to lateral stresses. Next, we will introduce an equation that relates the vertical and lateral effective stresses.

5.10 LATERAL EARTH PRESSURE AT REST**Concept Learning and Self-Assessment**

Access Chapter 5, Section 5.10, and take Quiz 5.10 to test your understanding of lateral earth pressure at rest.

The ratio of the horizontal principal effective stress to the vertical principal effective stress is called the lateral earth pressure coefficient at rest (K_o), that is,

$$K_o = \frac{\sigma'_3}{\sigma'_1} \quad (5.50)$$

The at-rest condition implies that no deformation occurs. We will revisit the at-rest coefficient in Chapters 7, 8, and 12. You must remember that K_o applies only to effective not total stresses. To find the lateral total stress, you must add the porewater pressure. Remember that the porewater pressure is hydrostatic and, at any given depth, the porewater pressures in all directions are equal.

For a soil that was never subjected to effective stresses higher than its current effective stress (normally consolidated soil) $K_o = K_o^{\text{nc}}$ is reasonably predicted by an equation suggested by Jaky (1944) as

$$K_o^{\text{nc}} \approx 1 - \sin \phi'_{\text{cs}} \quad (5.51)$$

where ϕ'_{cs} is a fundamental frictional soil constant that will be discussed in Chapter 7.

The value of K_o^{nc} is constant. During unloading or reloading, the soil stresses must adjust to be in equilibrium with the applied stress. This means that stress changes take place not only vertically but also horizontally. For a given surface stress, the changes in horizontal total stresses and vertical total stresses are different but the porewater pressure changes in every direction are the same. Therefore, the current effective stresses are different in different directions. A soil in which the current effective stress is lower than the past maximum stress is called an overconsolidated soil (to be discussed further in Chapter 6). The K_o values for overconsolidated soils are not constants. We will denote K_o for overconsolidated soils as K_o^{oc} . Various equations have been suggested linking K_o^{oc} to K_o^{nc} . One equation that is popular and found to match test data reasonably well is an equation proposed by Meyerhof (1976) as

$$K_o^{\text{oc}} = K_o^{\text{nc}}(\text{OCR})^{1/2} = (1 - \sin \phi'_{\text{cs}})(\text{OCR})^{1/2} \quad (5.52)$$

where OCR is the overconsolidation ratio (see Chapter 6 for more information) defined as the ratio of the past vertical effective stress to the current vertical effective stress.

EXAMPLE 5.8 *Calculating Horizontal Effective and Total Stresses*

Calculate the horizontal effective stress and the horizontal total stress for the soil element at 5 m in Example 5.5 if $K_o = 0.5$.

Strategy The stresses on the horizontal and vertical planes on the soil element are principal stresses (no shear stress occurs on these planes). You need to apply K_o to the effective stress and then add the porewater pressure to get the lateral total stress.

Solution 5.8

Step 1: Calculate the horizontal effective stress.

$$K_o = \frac{\sigma'_3}{\sigma'_1} = \frac{\sigma'_x}{\sigma'_z}; \quad \sigma'_x = K_o \sigma'_z = 0.5 \times 53.2 = 26.6 \text{ kPa}$$

Step 2: Calculate the horizontal total stress.

$$\sigma_x = \sigma'_x + u = 26.6 + 29.4 = 56 \text{ kPa}$$

EXAMPLE 5.9 *Horizontal Total and Effective Stresses*

Determine the horizontal effective and total stresses on a normally consolidated soil sample for:

(a) time: $t = t_1$, $\Delta u = 20 \text{ kPa}$

(b) time: $t \rightarrow \infty$, $\Delta u = 0$

The vertical total stress is 100 kPa and the frictional constant, $\phi'_{\text{cs}} = 30^\circ$.

Strategy The horizontal earth pressure coefficient must be applied to the vertical effective stress, not the vertical total stress. You need to calculate the vertical effective stress, then the horizontal effective stress. Add the excess porewater pressure to the horizontal effective stress to find the horizontal total stress.

Solution 5.9

Step 1: Calculate the vertical effective stresses.

$$\sigma'_z = \sigma_z - \Delta u$$

(a) $\sigma'_z = 100 - 20 = 80 \text{ kPa}$

(b) $\sigma'_z = 100 - 0 = 100 \text{ kPa}$

Step 2: Calculate the horizontal effective stress.

$$K_0^{\text{nc}} = 1 - \sin \phi'_{\text{cs}} = 1 - \sin 30^\circ = 0.5$$

$$\sigma'_x = K_0^{\text{nc}} \sigma'_z$$

(a) $\sigma'_x = 0.5 \times 80 = 40 \text{ kPa}$

(b) $\sigma'_x = 0.5 \times 100 = 50 \text{ kPa}$

Step 3: Calculate the total horizontal stresses.

$$\sigma_x = \sigma'_x + \Delta u$$

(a) $\sigma_x = 40 + 20 = 60 \text{ kPa}$

(b) $\sigma_x = 50 + 0 = 50 \text{ kPa}$

What's next... The stresses we considered so far are called geostatic stresses and when we considered elastic deformation of soils, the additional stresses imposed on the soil were given. But in practice, we have to find these additional stresses from applied loads located either on the ground surface or within the soil mass. We will use elastic analysis to find these additional stresses. Next, we will consider increases in stresses from a number of common surface loads. You will encounter myriad equations. You are not expected to remember these equations but you are expected to know how to use them.

5.11 STRESSES IN SOIL FROM SURFACE LOADS



Computer Program Utility and Self-Assessment

Access Chapter 5 and click on surface stresses on the sidebar to run a computer program to obtain the stress increases and displacements due to surface loads. You can use this program to explore stress changes due to different types of loads, and prepare and print Newmark charts for vertical stresses beneath arbitrarily shaped loads (described in Section 5.11.8). This computer program will also be helpful in solving problems in later chapters. Take Quiz 5.11 to test your problem-solving skills to find stress increases due to surface loads.

Various types of surface loads are applied to soils. For example, an oil tank will impose a uniform, circular, vertical stress on the surface of the soil while an unsymmetrical building may impose a

nonuniform vertical stress. We would like to know how the surface stresses are distributed within the soil mass and the resulting deformations. The distribution of surface stresses within a soil is determined by assuming that the soil is a semi-infinite, homogeneous, linear, isotropic, elastic material. A semi-infinite mass is bounded on one side and extends infinitely in all other directions; this is also called an “elastic half-space.” For soils, the horizontal surface is the bounding side. Equations and charts for several types of flexible surface loads based on the above assumptions are presented. A computer program to compute stresses from flexible surface loads is available in the CD-ROM. For rigid surface loads, the increase in vertical stresses may be 15% to 30% less for clays and 20% to 30% less for sands.

Most soils exist in layers with finite thicknesses. The solution based on a semi-infinite soil mass will not be accurate for these layered soils. In Appendix C, you will find selected graphs and tables for vertical stress increases in one-layer and two-layer soils. A comprehensive set of equations for a variety of loading situations is available in Poulos and Davis (1974).

5.11.1 Point Load

Boussinesq (1885) presented a solution for the distribution of stresses for a point load applied on the soil surface. An example of a point load is the vertical load transferred to the soil from an electric power line pole.

The increases in stresses on a soil element located at point *A* (Fig. 5.22a) due to a point load, *Q*, are

$$\Delta\sigma_z = \frac{3Q}{2\pi z^2 \left[1 + \left(\frac{r}{z} \right)^2 \right]^{5/2}} \quad (5.53)$$

$$\Delta\sigma_r = \frac{Q}{2\pi} \left(\frac{3r^2 z}{(r^2 + z^2)^{5/2}} - \frac{1 - 2\nu}{r^2 + z^2 + z(r^2 + z^2)^{1/2}} \right) \quad (5.54)$$

$$\Delta\sigma_\theta = -\frac{Q}{2\pi} (1 - 2\nu) \left(\frac{z}{(r^2 + z^2)^{3/2}} - \frac{1}{r^2 + z^2 + z(r^2 + z^2)^{1/2}} \right) \quad (5.55)$$

$$\Delta\tau_{rz} = \frac{3Q}{2\pi} \left[\frac{rz^2}{(r^2 + z^2)^{5/2}} \right] \quad (5.56)$$

where ν is Poisson's ratio. Most often, the increase in vertical stress is needed in practice. Equation (5.53) can be written as

$$\Delta\sigma_z = \frac{Q}{z^2} I \quad (5.57)$$

where *I* is an influence factor, and

$$I = \frac{3}{2\pi} \frac{1}{\left[1 + \left(\frac{r}{z} \right)^2 \right]^{5/2}} \quad (5.58)$$

The distributions of the increase in vertical stress from Eqs. 5.57 and 5.58 reveal that the increase in vertical stress decreases with depth (Fig. 5.22b) and radial distance (Fig. 5.22c).

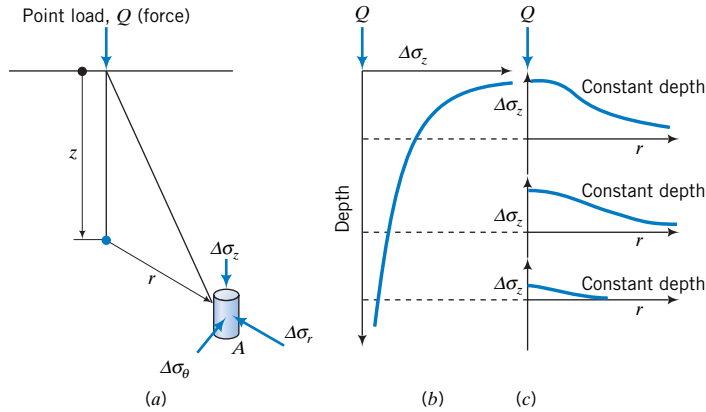


FIGURE 5.22 Point load and vertical stress distribution with depth and radial distance.

The vertical displacement is

$$\Delta z = \frac{Q(1+\nu)}{2\pi Ez} \left[1 + \left(\frac{r}{z} \right)^2 \right]^{1/2} \left[2(1-\nu) + \frac{1}{1 + \left(\frac{r}{z} \right)^2} \right] \quad (5.59)$$

and the radial displacement is

$$\Delta r = \frac{Q(1+\nu)}{2\pi Ez} \left[1 + \left(\frac{r}{z} \right)^2 \right]^{1/2} \left[\frac{\left(\frac{r}{z} \right)}{\left\{ 1 + \left(\frac{r}{z} \right)^2 \right\}} - \frac{(1-2\nu)\left(\frac{r}{z} \right)}{\left\{ 1 + \left(\frac{r}{z} \right)^2 \right\}^{1/2} + 1} \right] \quad (5.60)$$

where E is Young's modulus.

5.11.2 Line Load

With reference to Fig. 5.23a, the increases in stresses due to a line load, Q (force/length), are

$$\Delta \sigma_z = \frac{2Qz^3}{\pi(x^2 + z^2)^2} \quad (5.61)$$

$$\Delta \sigma_x = \frac{2Qx^2z}{\pi(x^2 + z^2)^2} \quad (5.62)$$

$$\Delta \tau_{zx} = \frac{2Qxz^2}{\pi(x^2 + z^2)^2} \quad (5.63)$$

A practical example of a line load is the load from a long brick wall.

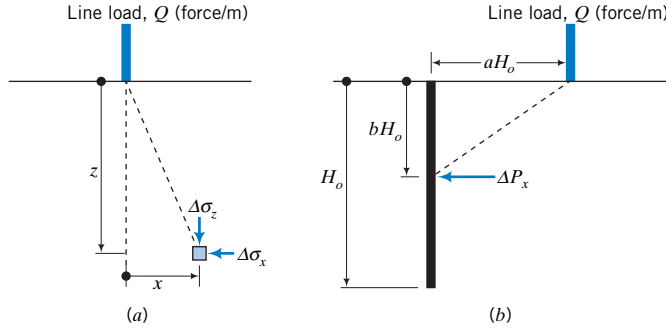


FIGURE 5.23 (a) Line load and (b) line load near a retaining wall.

5.11.3 Line Load Near a Buried Earth Retaining Structure

The increase in lateral stress on a buried earth retaining structure (Fig. 5.23b) due to a line load of intensity Q (force/length) is

$$\Delta\sigma_x = \frac{4Qa^2b}{\pi H_o(a^2 + b^2)^2} \quad (5.64)$$

The increase in lateral force is

$$\Delta P_x = \frac{2Q}{\pi(a^2 + 1)} \quad (5.65)$$

5.11.4 Strip Load

A strip load is the load transmitted by a structure of finite width and infinite length on a soil surface. Two types of strip loads are common in geotechnical engineering. One is a load that imposes a uniform stress on the soil, for example, the middle section of a long embankment (Fig. 5.24a). The other is a load that induces a triangular stress distribution over an area of width B (Fig. 5.24b). An example of a strip load with a triangular stress distribution is the stress under the side of an embankment.

The increases in stresses due to a surface stress q_s (force/area) are as follows:

(a) Area transmitting a uniform stress (Fig. 5.24a)

$$\Delta\sigma_z = \frac{q_s}{\pi} [\alpha + \sin \alpha \cos(\alpha + 2\beta)] \quad (5.66)$$

$$\Delta\sigma_x = \frac{q_s}{\pi} [\alpha - \sin \alpha \cos(\alpha + 2\beta)] \quad (5.67)$$

$$\Delta\tau_{zx} = \frac{q_s}{\pi} [\sin \alpha \sin(\alpha + 2\beta)] \quad (5.68)$$

where q_s is the applied surface stress.

The vertical displacement due to a strip loading is useful only as relative displacement between two points not located at infinity. The relative vertical displacement between the center of the strip load (0,0) and a point at the surface ($x, 0$) is

$$\Delta z(x, 0) - \Delta z(0, 0) = \frac{2q_s(1 - \nu^2)}{\pi E} \left\{ \left(x - \frac{B}{2} \right) \ln \left| x - \frac{B}{2} \right| - \left(x + \frac{B}{2} \right) \ln \left| x + \frac{B}{2} \right| + B \ln \left(\frac{B}{2} \right) \right\} \quad (5.69)$$

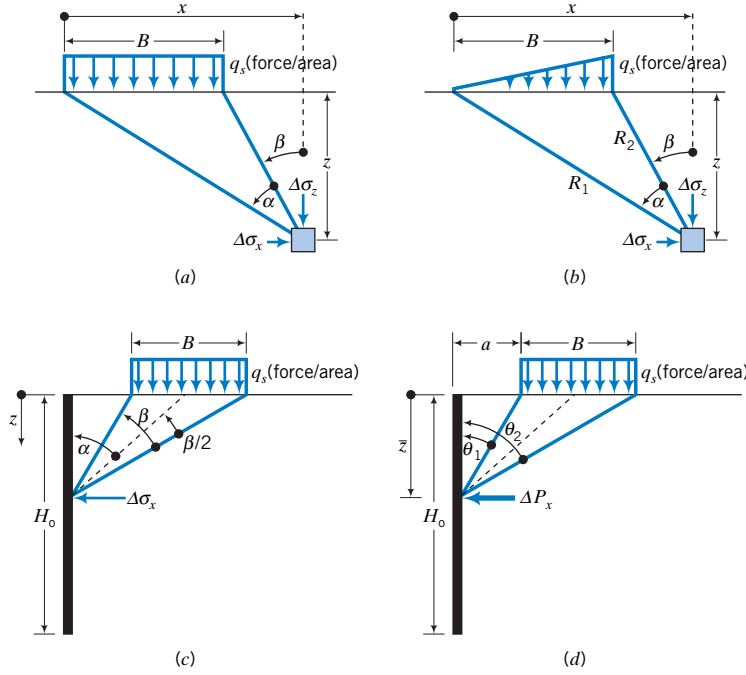


FIGURE 5.24 Strip load imposing (a) a uniform stress and (b) a linear varying stress. (c) Strip load near a retaining wall and (d) lateral force on a retaining wall from a strip load.

(b) Area transmitting a triangular stress (Fig. 5.24b)

$$\Delta\sigma_z = \frac{q_s}{\pi} \left(\frac{x}{B} \alpha - \frac{1}{2} \sin 2\beta \right) \quad (5.70)$$

$$\Delta\sigma_x = \frac{q_s}{\pi} \left(\frac{x}{B} \alpha - \frac{z}{B} \ln \frac{R_1^2}{R_2^2} + \frac{1}{2} \sin 2\beta \right) \quad (5.71)$$

$$\Delta\tau_{zx} = \frac{q_s}{2\pi} \left(1 + \cos 2\beta - 2 \frac{z}{B} \alpha \right) \quad (5.72)$$

The relative vertical displacement between the center of the strip load (0,0) and a point at the surface ($x, 0$) is

$$\Delta z(x, 0) - \Delta(0, 0) = \frac{q_s(1 - \nu^2)}{\pi E(\frac{B}{2})} \left\{ \frac{B^2}{2} \ln B - \frac{x^2}{2} \ln x + \left(\frac{x^2}{2} - \frac{B^2}{2} \right) \ln |B - x| + \frac{B}{2} x \right\} \quad (5.73)$$

(c) Area transmitting a uniform stress near a retaining wall (Fig. 5.24c,d)

$$\Delta\sigma_x = \frac{2q_s}{\pi} (\beta - \sin \beta \cos 2\alpha) \quad (5.74)$$

The lateral force and its location were derived by Jarquio (1981) and are

$$\Delta P_x = \frac{q_s}{90} [H_0(\theta_2 - \theta_1)] \quad (5.75)$$

$$\bar{z} = \frac{H_0^2(\theta_2 - \theta_1) - (R_1 - R_2) + 57.3BH_0}{2H_0(\theta_2 - \theta_1)} \quad (5.76)$$

where

$$\theta_1 = \tan^{-1}\left(\frac{a}{H_o}\right), \quad \theta_2 = \tan^{-1}\left(\frac{a+B}{H_o}\right),$$

$$R_1 = (a+B)^2(90 - \theta_2), \quad \text{and} \quad R_2 = a^2(90 - \theta_1)$$

5.11.5 Uniformly Loaded Circular Area

An example of a circular area that transmits stresses to a soil mass is a circular foundation of an oil or water tank. The increases of vertical and radial stresses under the center of a circular area of radius r_o are

$$\Delta\sigma_z = q_s \left[1 - \left(\frac{1}{1 + (r_o/z)^2} \right)^{3/2} \right] = q_s I_c \quad (5.77)$$

where

$$I_c = \left[1 - \left(\frac{1}{1 + (r_o/z)^2} \right)^{3/2} \right]$$

is an influence factor and

$$\Delta\sigma_r = \Delta\sigma_\theta = \frac{q_s}{2} \left[(1 + 2\nu) - \frac{2(1 + \nu)}{[1 + (r_o/z)^2]^{1/2}} + \frac{1}{[1 + (r_o/z)^2]^{3/2}} \right] \quad (5.78)$$

The vertical elastic settlement at the surface due to a circular flexible loaded area is

Below center of loaded area:

$$\Delta z = \frac{q_s D(1 - \nu^2)}{E} \quad (5.79)$$

Below edge:

$$\Delta z = \frac{2}{\pi} \frac{q_s D(1 - \nu^2)}{E} \quad (5.80)$$

where $D = 2r_o$ is the diameter of the loaded area. Appendix B gives the vertical stress increases and vertical elastic settlements at all points in the soil mass.

5.11.6 Uniformly Loaded Rectangular Area

Many structural foundations are rectangular or approximately rectangular in shape. The increases in stresses below the *corner* of a rectangular area of width B and length L are

$$\Delta\sigma_z = \frac{q_s}{2\pi} \left[\tan^{-1} \frac{LB}{zR_3} + \frac{LBz}{R_3} \left(\frac{1}{R_1^2} + \frac{1}{R_2^2} \right) \right] \quad (5.81)$$

$$\Delta\sigma_x = \frac{q_s}{2\pi} \left[\tan^{-1} \frac{LB}{zR_3} - \frac{LBz}{R_1^2 R_3} \right] \quad (5.82)$$

$$\Delta\sigma_y = \frac{q_s}{2\pi} \left[\tan^{-1} \frac{LB}{zR_3} - \frac{LBz}{R_2^2 R_3} \right] \quad (5.83)$$

$$\Delta\tau_{zx} = \frac{q_s}{2\pi} \left[\frac{B}{R_2} - \frac{z^2 B}{R_1^2 R_3} \right] \quad (5.84)$$

where $R_1 = (L^2 + z^2)^{1/2}$, $R_2 = (B^2 + z^2)^{1/2}$, and $R_3 = (L^2 + B^2 + z^2)^{1/2}$.

These equations can be written as

$$\Delta\sigma_z = q_s I_z \quad (5.85)$$

$$\Delta\sigma_x = q_s I_x \quad (5.86)$$

$$\Delta\sigma_y = q_s I_y \quad (5.87)$$

$$\tau_{zx} = q_s I_\tau \quad (5.88)$$

where I denotes the influence factor. The influence factor for the vertical stress is

$$I_z = \frac{1}{4\pi} \left[\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2 n^2 + 1} \left(\frac{m^2 + n^2 + 2}{m^2 + n^2 + 1} \right) + \tan^{-1} \left(\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 - m^2 n^2 + 1} \right) \right] \quad (5.89)$$

where $m = B/z$ and $n = L/z$. You can program your calculator or use a spreadsheet program to find I_z . You must be careful in the last term ($\tan^{-1}$) in programming. If $m^2 + n^2 + 1 < m^2 n^2$, then you have to add π to the bracketed quantity in the last term. The distribution of vertical stress below a uniformly loaded square foundation is shown in Fig. 5.25. The increase in vertical stress is about 10% below a depth of $2B$; B is the diameter of the foundation. The vertical stress decreases from the center of the foundation outward, reaching a value of about 10% at a horizontal distance of $B/2$ from the edge at a depth of B . A chart for I_z for the corner of rectangular loaded area is shown in Fig. 5.26. You would have to calculate $m = B/z$ and $n = L/z$ and read I_z from the chart; m and n are interchangeable. In general, the vertical stress increase is less than 10% of the surface stress when $z > 2B$.

The vertical elastic settlement at the ground surface under a rectangular flexible surface load is

$$\Delta z = \frac{q_s B (1 - \nu^2)}{E} I_s \quad (5.90)$$

where I_s is a settlement influence factor that is a function of the L/B ratio (L is length and B is width). Setting $\xi_s = L/B$, the equations for I_s are

At center of a rectangle (Giroud, 1968):

$$I_s = \frac{2}{\pi} \left[\ln(\xi_s + \sqrt{1 + \xi_s^2}) + \xi_s \ln \frac{1 + \sqrt{1 + \xi_s^2}}{\xi_s} \right] \quad (5.91)$$

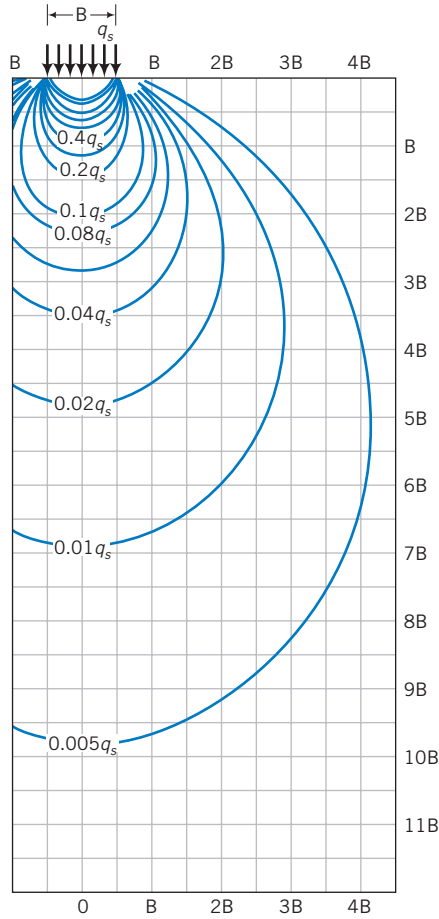


FIGURE 5.25 Vertical stress contour below a square foundation.

At corner of a rectangle (Giroud, 1968):

$$I_s = \frac{1}{\pi} \left[\ln(\xi_s + \sqrt{1 + \xi_s^2}) + \xi_s \ln \frac{1 + \sqrt{1 + \xi_s^2}}{\xi_s} \right] \quad (5.92)$$

The above equations can be simplified to the following for $\xi_s \geq 1$:

At center of a rectangle: $I_s \cong 0.62 \ln(\xi_s) + 1.12$ (5.93)

At corner of a rectangle: $I_s \cong 0.31 \ln(\xi_s) + 0.56$ (5.94)

5.11.7 Approximate Method for Rectangular Loads

In preliminary analyses of vertical stress increases under the center of rectangular loads, geotechnical engineers often use an approximate method (sometimes called the 2:1 method). The surface load on an

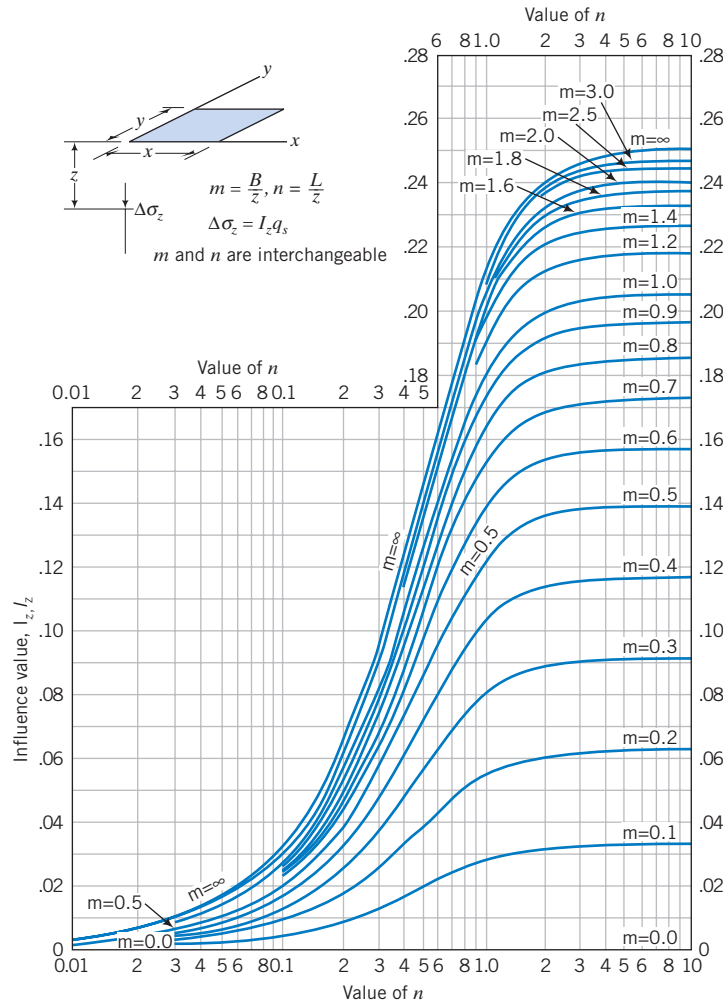


FIGURE 5.26 Influence factor for calculating the vertical stress increase under the corner of a rectangle. (Source: NAVFAC - DM 7.1.)

area, $B \times L$, is dispersed at a depth z over an area $(B + z) \times (L + z)$ as illustrated in Fig. 5.27. The vertical stress increase under the center of the load is

$$\Delta\sigma_z = \frac{q_s BL}{(B + z)(L + z)} \quad (5.95)$$

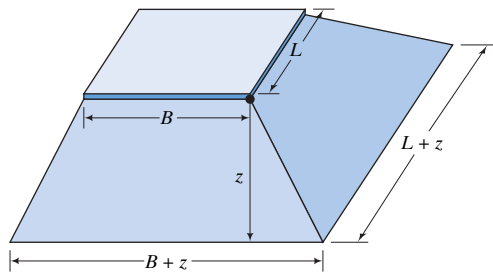


FIGURE 5.27 Dispersion of load for approximate increase in vertical stress under a rectangle.

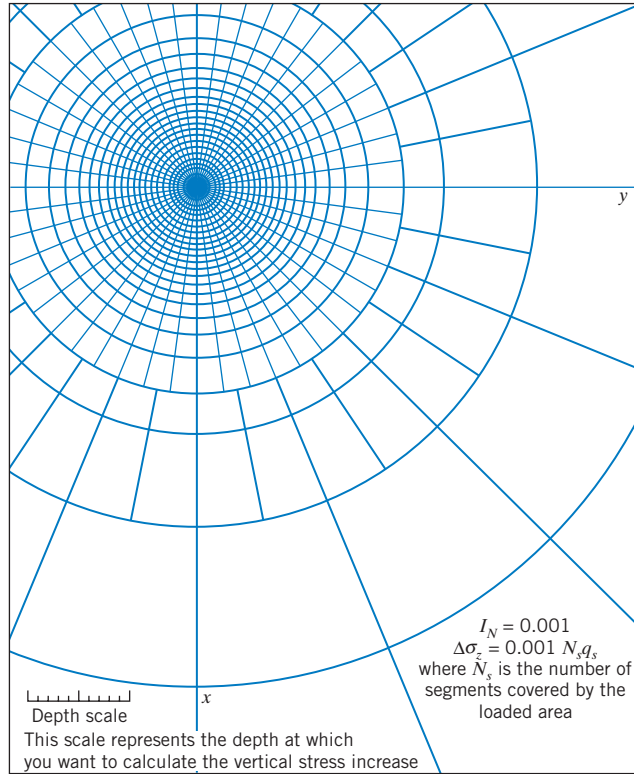


FIGURE 5.28 Newmark's chart for increase in vertical stress.

The approximate method is reasonably accurate (compared with Boussinesq's elastic solution) when $z > B$.

5.11.8 Vertical Stress Below Arbitrarily Shaped Areas

Newmark (1942) developed a chart to determine the increase in vertical stress due to a uniformly loaded area of any shape. The chart consists of concentric circles divided by radial lines (Fig. 5.28). The area of each segment represents an equal proportion of the applied surface stress at a depth z below the surface. If there are 10 concentric circles and 20 radial lines, the stress on each circle is $q_s/10$ and on each segment is $q_s/(10 \times 20)$. The radius to depth ratio of the first (inner) circle is found by setting $\Delta\sigma_z = 0.1q_s$ in Eq. (5.77), that is,

$$0.1q_s = q_s \left[1 - \left\{ \frac{1}{1 + (r_0/z)^2} \right\}^{3/2} \right]$$

from which $r/z = 0.27$. For the other circles, substitute the appropriate value for $\Delta\sigma_z$; for example, for the second circle $\Delta\sigma_z = 0.2q_s$, and find r/z . The chart is normalized to the depth; that is, all dimensions are scaled by a factor initially determined for the depth. Every chart should show a scale and an influence factor I_N , which for our case is $1/(10 \times 20) = 0.005$.

The procedure for using Newmark's chart is as follows:

1. Set the scale, shown on the chart, equal to the depth at which the increase in vertical stress is required. We will call this the depth scale.

2. Identify the point on the loaded area below which the stress is required. Let us say this point is A .
3. Plot the loaded area using the depth scale with point A at the center of the chart.
4. Count the number of segments (N_s) covered by the scaled loaded area. If certain segments are not fully covered, you can estimate what fraction is covered.
5. Calculate the increase in vertical stress as $\Delta\sigma_z = q_s I_N N_s$.

The essential points are:

1. *The increases in stresses below a surface load are found by assuming the soil is an elastic, semi-infinite mass.*
2. *Various equations are available for the increases in stresses from surface loading.*
3. *The stress increase at any depth depends on the shape and distribution of the surface load.*
4. *A stress applied at the surface of a soil mass by a loaded area decreases with depth and lateral distance away from the center of the loaded area.*
5. *The vertical stress increases are generally less than 10% of the surface stress when the depth to width ratio is greater than 2.*

EXAMPLE 5.10 Vertical Stress Increase Due to a Point Load

A pole carries a vertical load of 200 kN. Determine the vertical stress increase at a depth 5 m (a) directly below the pole and (b) at a radial distance of 2 m.

Strategy The first step is to determine the type of surface load. The load carried by the pole can be approximated to a point load. You can then use the equation for the vertical stress increase for a point load.

Solution 5.10

Step 1: Determine the load type.

Assume the load from the pole can be approximated by a point load.

Step 2: Use the equation for a point load. Use Eq. (5.57):

$$z = 5 \text{ m}, \quad Q = 200 \text{ kN}; \quad \text{Under load, } r = 0, \therefore \frac{r}{z} = 0$$

$$\text{From Eq. (5.58): } \frac{r}{z} = 0, \quad I = 0.48$$

$$\Delta\sigma_z = \frac{Q}{z^2} I = \frac{200}{5^2} \times 0.48 = 3.8 \text{ kPa}$$

Step 3: Determine the vertical stress at the radial distance.

$$r = 2 \text{ m}, \quad \frac{r}{z} = \frac{2}{5} = 0.4, \quad I = 0.33$$

$$\Delta\sigma_z = \frac{200}{5^2} \times 0.33 = 2.6 \text{ kPa}$$

EXAMPLE 5.11 Vertical Stress Increase Due to a Rectangular Load

A rectangular concrete slab, $3 \text{ m} \times 4.5 \text{ m}$, rests on the surface of a soil mass. The load on the slab is 2025 kN. Determine the vertical stress increase at a depth of 3 m (a) under the center of the slab, point

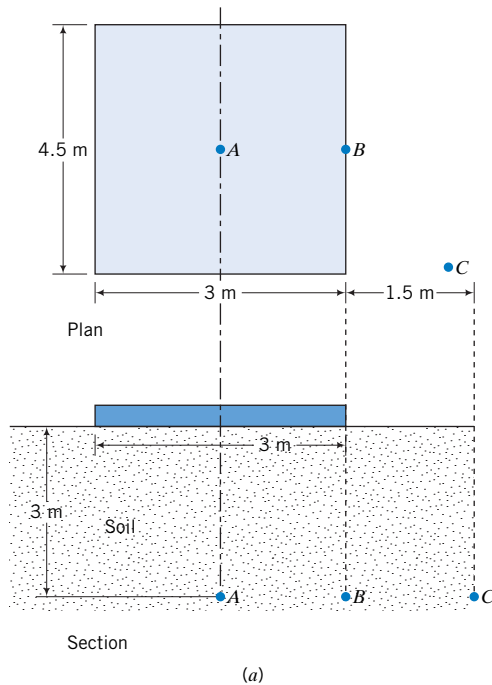


FIGURE E5.11a

A (Fig. E5.11a), (b) under point B (Fig. E5.11a), and (c) at a distance of 1.5 m from a corner, point C (Fig. E5.11a).

Strategy The slab is rectangular and the equations for a uniformly loaded rectangular area are for the corner of the area. You should divide the area so that the point of interest is a corner of a rectangle(s). You may have to extend the loaded area if the point of interest is outside it (loaded area). The extension is fictitious so you have to subtract the fictitious increase in vertical stress for the extended area.

Solution 5.11

Step 1: Identify the loading type.

It is a uniformly loaded rectangle.

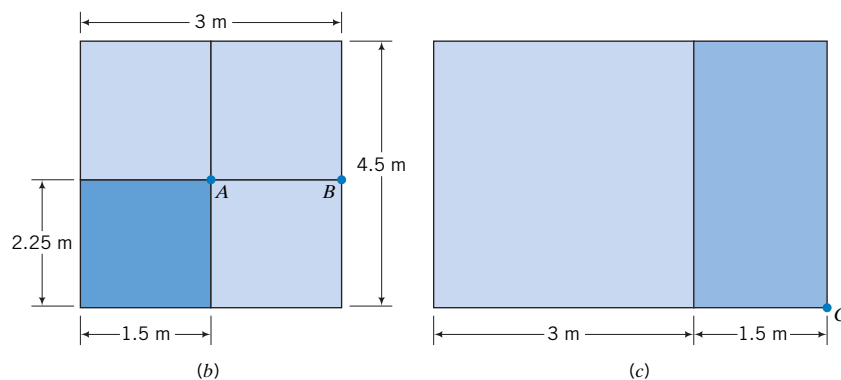


FIGURE E5.11b,c

Step 2: Divide the rectangle so that the center is a corner.

In this problem, all four rectangles after the subdivision are equal (Fig. E5.11b; point *C* is excluded for simplicity), so you only need to find the vertical stress increase for one rectangle of size $B = 1.5$ m, $L = 2.25$ m and multiply the results by 4.

$$m = \frac{B}{z} = \frac{1.5}{3} = 0.5; \quad n = \frac{L}{z} = \frac{2.25}{3} = 0.75$$

From the chart in Fig. 5.26, $I_z = 0.105$.

Step 3: Find the vertical stress increase at the center of the slab (point *A*, Fig. E5.11b).

$$q_s = \frac{2025}{3 \times 4.5} = 150 \text{ kPa}$$

$$\Delta\sigma_z = 4q_s I_z = 4 \times 150 \times 0.105 = 63 \text{ kPa}$$

Note: The approximate method [Eq. (5.95)] gives

$$\Delta\sigma_z = \frac{q_s B L}{(B + z)(L + z)} = \frac{2025}{(3 + 3)(4.5 + 3)} = 45 \text{ kPa}$$

which is about 30% less than the elastic solution.

Step 4: Find the vertical stress increase for point *B*.

Point *B* is at the corner of two rectangles each of width 3 m and length 2.25 m. You need to find the vertical stress increase for one rectangle and multiply the result by 2.

$$m = \frac{3}{3} = 1; \quad n = \frac{2.25}{3} = 0.75$$

From the chart in Fig. 5.26, $I_z = 0.158$.

$$\Delta\sigma_z = 2 \times 150 \times 0.158 = 47.4 \text{ kPa}$$

You should note that the vertical stress increase at *B* is lower than at *A*, as expected.

Step 5: Find the stress increase for point *C*.

Stress point *C* is outside the rectangular slab. You have to extend the rectangle to *C* (Fig. E5.11c) and find the stress increase for the large rectangle of width $B = 4.5$, length $L = 4.5$ m and then subtract the stress increase for the smaller rectangle of width $B = 1.5$ m and length $L = 4.5$ m.

Large rectangle

$$m = \frac{4.5}{3} = 1.5, \quad n = \frac{4.5}{3} = 1.5; \quad \text{from chart in Fig. 5.26, } I_z = 0.22$$

Small rectangle

$$m = \frac{1.5}{3} = 0.5, \quad n = \frac{4.5}{3} = 1.5; \quad \text{from chart in Fig. 5.26, } I_z = 0.13$$

$$\Delta\sigma_z = 150 \times (0.22 - 0.13) = 13.5 \text{ kPa}$$

EXAMPLE 5.12 Vertical Stress Increase Due to an Irregular Loaded Area

The plan of a foundation of uniform thickness for a building is shown in Fig. E5.12a. Determine the vertical stress increase at a depth of 4 m below the centroid. The foundation applies a vertical stress of 200 kPa on the soil surface.

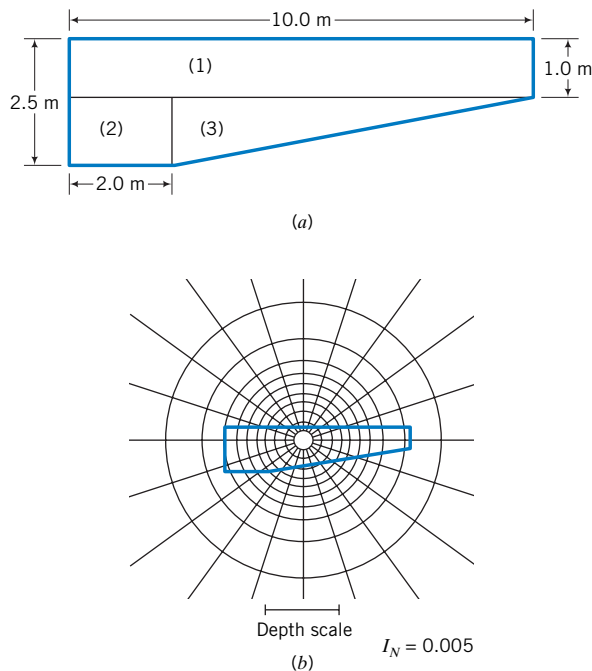


FIGURE E5.12a,b

Strategy You need to locate the centroid of the foundation, which you can find using the given dimensions. The shape of the foundation does not fit neatly into one of the standard shapes (e.g., rectangles or circles) discussed. The convenient method to use for this (odd) shape foundation is Newmark's chart.

Solution 5.12

Step 1: Find the centroid.

$$\begin{aligned}\bar{x} &= \frac{1.0 \times 10.0 \times 5.0 + 1.5 \times 2.0 \times 1.0 + \frac{1}{2} \times 8.0 \times 1.5 \times (20 + \frac{1}{3} \times 8.0)}{(1.0 \times 10.0) + (1.5 \times 2.0) + \frac{1}{2} \times 8.0 \times 1.5} = \frac{81}{19} = 4.26 \text{ m} \\ \bar{y} &= \frac{1.0 \times 10.0 \times 0.5 + 1.5 \times 2.0 \times (1.0 + 1.5/2) + \frac{1}{2} \times 8.0 \times 1.5 \times (1.0 + 1.5/3)}{(1.0 \times 10.0) + (1.5 \times 2.0) + \frac{1}{2} \times 8.0 \times 1.5} = \frac{16.25}{19} = 0.86 \text{ m}\end{aligned}$$

Step 2: Scale and plot the foundation on a Newmark's chart.

The scale on the chart is set equal to the depth. The centroid is located at the center of the chart and the foundation is scaled using the depth scale (Fig. E5.12b).

Step 3: Count the number of segments covered by the foundation.

$$N_s = 61$$

Step 4: Calculate the vertical stress increase.

$$\Delta\sigma_z = 200 \times 0.005 \times 61 = 61 \text{ kPa}$$

What's next... The stresses and strains discussed in previous sections are all dependent on the axis system chosen. We have arbitrarily chosen the Cartesian coordinate and the cylindrical coordinate systems. We could, however, define a set of stresses and strains that are independent of the axis system. Such a system, which we will discuss next, will allow us to use generalized stress and strain parameters to analyze and interpret soil behavior. In particular, we would be able to represent a three-dimensional system of stresses and strains by a two-dimensional system.

5.12 STRESS AND STRAIN INVARIANTS



Interactive Concept Learning and Self-Assessment

Access Chapter 5, Section 5.12 to learn stress and strain invariants through interactive visualizations. Take Quiz 5.12 to test your understanding.

Stress and strain invariants are measures that are independent of the axis system. We will define stress invariants that provide measures of (1) mean stress and (2) deviatoric or distortional or shear stress, and strain invariants that provide measures of (1) volumetric strains and (2) deviatoric or distortional or shear strains.

5.12.1 Mean Stress

$$p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} \quad (5.96)$$

On a graph with orthogonal principal stress axes, σ_1 , σ_2 , σ_3 , the mean stress is the space diagonal (Fig. 5.29). Mean stress causes volume changes.

5.12.2 Deviatoric or Shear Stress

$$q = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \quad (5.97)$$

A line normal to the mean stress as illustrated in Fig. 5.29 represents the deviatoric stress. Deviatoric stress causes distortions or shearing of a soil mass.

5.12.3 Volumetric Strain

$$\varepsilon_p = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_x + \varepsilon_y + \varepsilon_z \quad (5.98)$$

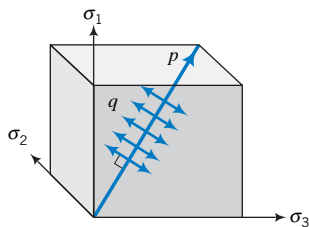


FIGURE 5.29 Mean and deviatoric stresses.

5.12.4 Deviatoric or Distortional or Shear Strain

$$\varepsilon_q = \frac{\sqrt{2}}{3} [(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2]^{1/2} \quad (5.99)$$

In this book, we will be dealing with two conditions—axisymmetric and plane strain. For these two conditions, the stress and strain invariants are as follows.

5.12.5 Axisymmetric Condition, $\sigma'_2 = \sigma'_3$ or $\sigma_2 = \sigma_3$; $\varepsilon_2 = \varepsilon_3$

$$p' = \frac{\sigma'_1 + 2\sigma'_3}{3} \quad \text{and} \quad p = \frac{\sigma_1 + 2\sigma_3}{3} \quad (5.100)$$

$$p' = p - u$$

$$q = \sigma_1 - \sigma_3; \quad q' = \sigma'_1 - \sigma'_3 = (\sigma_1 - \Delta u) - (\sigma_3 - \Delta u) = \sigma_1 - \sigma_3 \quad (5.101)$$

Therefore, $q = q'$; shear is unaffected by porewater pressures.

$$\varepsilon_p = \varepsilon_1 + 2\varepsilon_3 \quad (5.102)$$

$$\varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \quad (5.103)$$

5.12.6 Plane Strain, $\varepsilon_2 = 0$

$$p' = \frac{\sigma'_1 + \sigma'_2 + \sigma'_3}{3} \quad \text{and} \quad p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad (5.104)$$

$$p' = p - u \quad (5.105)$$

$$q' = q = \frac{1}{\sqrt{2}} [(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2]^{1/2} \quad (5.106)$$

or

$$q = q' = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

$$\varepsilon_p = \varepsilon_1 + \varepsilon_3 \quad (5.107)$$

$$\varepsilon_q = \frac{2}{3}(\varepsilon_1^2 + \varepsilon_3^2 - \varepsilon_1\varepsilon_3)^{1/2} \quad (5.108)$$

5.12.7 Hooke's Law Using Stress and Strain Invariants

The stress and strain invariants for an elastic material are related as follows:

$$\varepsilon_p^e = \frac{1}{K'} p' \quad (5.109)$$

where

$$K' = \frac{p'}{\varepsilon_p^e} = \frac{E'}{3(1 - 2\nu')} \quad (5.110)$$

is the effective bulk modulus and the superscript e denotes elastic.

$$\varepsilon_q^e = \frac{1}{3G} q \quad (5.111)$$

where

$$G = G' = \frac{E'}{2(1 + \nu')} \quad (5.112)$$

is called the shear modulus. Hooke's law in terms of the stress and strain invariants is

$$\begin{Bmatrix} p' \\ q \end{Bmatrix} = \begin{bmatrix} K' & 0 \\ 0 & 3G \end{bmatrix} \begin{Bmatrix} \varepsilon_p^e \\ \varepsilon_q^e \end{Bmatrix} \quad (5.113)$$

Equation (5.113) reveals that for a linear, isotropic, elastic material, shear stresses do not cause volume changes and mean effective stresses do not cause shear deformation.

We can generate a generalized Poisson's ratio by eliminating E' from Eqs. (5.110) and (5.112) as follows:

$$\text{Equation (5.110): } E' = 3K'(1 - 2\nu')$$

$$\text{Equation (5.112): } E' = 2G(1 + \nu')$$

$$\therefore \frac{3K'(1 - 2\nu')}{2G(1 + \nu')} = 1$$

and

$$\nu' = \frac{3K' - 2G}{2G + 6K'} \quad (5.114)$$

The essential points are:

1. *Stress and strain invariants are independent of the chosen axis system.*
2. *Stress and strain invariants are convenient measures to determine the effects of a general state of stresses and strains on soils.*
3. *Mean stress represents the average stress on a soil while deviatoric stress represents the average shear or distortional stress.*

EXAMPLE 5.13 *Calculating Stress and Strain Invariants*

A cylindrical sample of soil 50 mm in diameter and 100 mm long is subjected to an axial effective principal stress of 400 kPa and a radial effective principal stress of 100 kPa. The axial and radial displacements are 0.5 mm and -0.04 mm, respectively. Assuming the soil is an isotropic, elastic material, calculate (a) the mean and deviatoric stresses, (b) the volumetric and shear (distortional) strains, and (c) the shear, bulk, and elastic moduli.

Strategy This is a straightforward problem. You only need to apply the equations given in the previous section. The negative sign for the radial displacement indicates an expansion.

Solution 5.13

Step 1: Calculate the mean and deviatoric stresses.

$$\sigma'_1 = \sigma'_z = 400 \text{ kPa}, \quad \sigma'_3 = \sigma'_r = 100 \text{ kPa}$$

$$(a) \quad p' = \frac{\sigma'_z + 2\sigma'_r}{3} = \frac{400 + 2 \times 100}{3} = 200 \text{ kPa}$$

$$q = q' = \sigma'_z - \sigma'_r = 400 - 100 = 300 \text{ kPa}$$

Step 2: Calculate the volumetric and shear strains.

$$\Delta z = 0.5 \text{ mm}, \quad \Delta r = -0.04 \text{ mm}, \quad r = 50/2 = 25 \text{ mm}, \quad L = 100 \text{ mm}$$

$$(b) \quad \varepsilon_z = \varepsilon_1 = \frac{\Delta z}{L} = \frac{0.5}{100} = 0.005$$

$$\varepsilon_r = \varepsilon_3 = \frac{\Delta r}{r} = \frac{-0.04}{25} = -0.0016$$

$$\varepsilon_p^e = \varepsilon_z + 2\varepsilon_r = 0.005 - 2 \times 0.0016 = 0.0018 = 0.18\%$$

$$\varepsilon_q^e = \frac{2}{3}(\varepsilon_z - \varepsilon_r) = \frac{2}{3}(0.005 + 0.0016) = 0.0044 = 0.44\%$$

Step 3: Calculate the moduli.

$$(c) \quad K' = \frac{p'}{\varepsilon_p^e} = \frac{200}{0.0018} = 111,111 \text{ kPa}$$

$$G = \frac{q}{3\varepsilon_q^e} = \frac{300}{3 \times 0.0044} = 22,727 \text{ kPa}$$

but

$$G = \frac{E'}{2(1 + \nu')} \quad \text{and} \quad \nu' = \frac{3K' - 2G}{2G + 6K'} = \frac{3 \times 111,111 - 2 \times 22,727}{2 \times 22,727 + 6 \times 111,111} = 0.4$$

$$\therefore E' = 2G(1 + \nu') = 2 \times 22,727(1 + 0.4) = 63,636 \text{ kPa}$$

EXAMPLE 5.14 *Strain Invariants in Simple Shear*

A sample of soil was subjected to simple shear, which consists of deforming a cuboidal sample into a parallelepiped (Fig. E5.14). The strains in the X and Y directions are zero. The sample size is $100 \text{ mm} \times 100 \text{ mm} \times 20 \text{ mm}$ high. The maximum lateral displacement applied at the top is 0.2 mm and

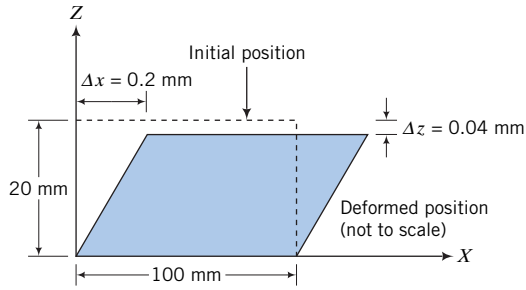


FIGURE E5.14

results in a vertical displacement of 0.04 mm and a shear force of 2 kN. Calculate the principal, volumetric, and deviatoric strains.

Strategy You are given the displacement. You can use this information to calculate strains. The sample is deformed under the plane strain condition. Therefore, the appropriate equations to use are Eqs. (5.107) and (5.108).

Solution 5.14

Step 1: Calculate ε_1 and ε_3 .

$$\Delta z = 0.04, \quad h = 20 \text{ mm}, \quad \Delta x = 0.2 \text{ mm}$$

$$\varepsilon_z = \frac{\Delta z}{h} = \frac{0.04}{20} = 0.002, \quad \varepsilon_x = \varepsilon_y = 0, \quad \gamma_{zx} = \frac{\Delta x}{h} = \frac{0.2}{20} = 0.01$$

$$\text{Equation (5.35): } \varepsilon_1 = \frac{0.002 + 0}{2} + \sqrt{\left(\frac{0.002 - 0}{2}\right)^2 + \left(\frac{0.01}{2}\right)^2} = 0.006$$

$$\text{Equation (5.36): } \varepsilon_3 = \frac{0.002 + 0}{2} - \sqrt{\left(\frac{0.002 - 0}{2}\right)^2 + \left(\frac{0.01}{2}\right)^2} = -0.004$$

Step 2: Calculate the volumetric and deviatoric strains.

$$\varepsilon_p = \varepsilon_1 + \varepsilon_3 = 0.006 - 0.004 = 0.002$$

Also,

$$\varepsilon_p = \varepsilon_x + \varepsilon_y + \varepsilon_z = 0 + 0 + 0.002 = 0.002$$

$$\varepsilon_q = \frac{2}{3}(\varepsilon_1^2 + \varepsilon_3^2 - \varepsilon_1\varepsilon_3)^{1/2} = \frac{2}{3}[0.006^2 + (-0.004)^2 - (0.006)(-0.004)]^{1/2} = 0.0058$$

What's next... We have examined how applied surface stresses are distributed in soils as if soils were linear, isotropic, elastic materials. Different structures will impose different stresses and cause the soil to respond differently. For example, an element of soil under the center of an oil tank will experience a continuous increase/decrease in vertical stress while the tank is being filled/emptied. However, the soil near a retaining earth structure will suffer a reduction in lateral stress if the wall moves out. If the soil were the same for both structures, the different loading conditions would cause the soil to respond differently. Therefore, we need to trace the history of stress increases/decreases in soils to evaluate possible soil responses and to conduct tests that replicate the loading history of the in situ soil. In the next section, a method of keeping track of the loading history of a soil is described.

5.13 STRESS PATHS



Interactive Concept Learning and Self-Assessment

Access Chapter 5, Section 5.13 on the CD to learn about stress paths interactively. Take Quiz 5.13 to assess your understanding.

5.13.1 Basic Concept

Consider two marbles representing two particles of a coarse-grained soil. Let us fix one marble in a hemispherical hole and stack the other on top of it (Fig. 5.30a). We are constructing a one-dimensional system in which relative displacement of the two marbles will occur at the contact. Let us incrementally apply a vertical, concentric force, F_z , on the top marble. We will call this loading “A”. The forces at the contact are equal to the applied loads and the marbles are forced together vertically. No relative displacement between the marbles occurs. For the system to become unstable or fail, the applied forces must crush the marbles. We can make a plot of our loading by arbitrarily choosing an axis system. Let us choose a Cartesian system with the X axis representing the horizontal force and the Z axis representing the vertical forces. We can represent loading “A” by a line OA as shown in Fig. 5.30c. The line OA is called a load path or a force path.

Let us now apply the same force at an angle θ to the X axis in the ZX plane (Fig. 5.30b) and call this loading “B”. There are now two components of force. One component is $F_x = F \cos \theta$ and the other is $F_z = F \sin \theta$. If the frictional resistance at the contacts of the two marbles is less than the horizontal force, the top marble will slide relative to the bottom. You should recall from your mechanics or physics course that the frictional resistance is μF_z (Coulomb’s law), where μ is the coefficient of friction at the contact between the two marbles. Our one-dimensional system now has two modes of instability or failure—one due to relative sliding and the other due to crushing of the marbles. The force path for loading “B” is represented by OB in Fig. 5.30c. The essential point or principle is that the response, stability, and failure of the system depend on the force path.

Soils, of course, are not marbles but the underlying principle is the same. The soil fabric can be thought of as a space frame with the soil particles representing the members of the frame and the particle contacts representing the joints. The response, stability, and failure of the soil fabric or the space frame depend on the stress path.

Stress paths are presented in a plot showing the relationship between stress parameters and provide a convenient way to allow a geotechnical engineer to study the changes in stresses in a soil caused by loading conditions. We can, for example, plot a two-dimensional graph of σ_1 versus σ_3 or σ_2 , which will give us a relationship between these stress parameters. However, the stress invariants, being independent of the axis system, are more convenient to use.

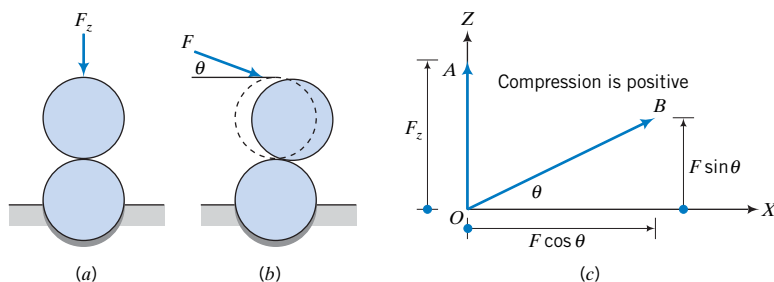


FIGURE 5.30 Effects of force paths on a one-dimensional system of marbles.

5.13.2 Plotting Stress Paths

We will explore the stress paths for a range of loading conditions. We will use a cylindrical soil sample for illustrative purposes and subject it to several loading conditions. Let us apply equal increments of axial and radial stresses ($\Delta\sigma_z = \Delta\sigma_r = \Delta\sigma$) to an initially stress-free sample as illustrated in the inset figure labeled “1” in Fig. 5.31. Since we are not applying any shearing stresses on the horizontal and vertical boundaries, the axial and radial stresses are principal stresses: that is, $\Delta\sigma_z = \Delta\sigma_1$ and $\Delta\sigma_r = \Delta\sigma_3$.

The loading condition we are applying is called isotropic compression; that is, the stresses in all directions are equal ($\Delta\sigma_1 = \Delta\sigma_2 = \Delta\sigma_3$). We will call this loading condition, loading “1.” It is often convenient to work with increments of stresses in determining stress paths. Consequently, we are going to use the incremental form of the stress invariants. The stress invariants for isotropic compression are

$$\Delta p_1 = \frac{\Delta\sigma_1 + 2\Delta\sigma_3}{3} = \frac{\Delta\sigma_1 + 2\Delta\sigma_1}{3} = \Delta\sigma_1$$

$$\Delta q_1 = \Delta\sigma_1 - \Delta\sigma_3 = \Delta\sigma_1 - \Delta\sigma_1 = 0$$

The subscript 1 on p and q denotes loading “1.”

Let us now prepare a graph with axes p (abscissa) and q (ordinate), as depicted in Fig. 5.31. We will call this graph the q - p plot. The initial stresses on the soil sample are zero; that is, $p_0 = 0$ and $q_0 = 0$. The stresses at the end of loading “1” are

$$p_1 = p_0 + \Delta p_1 = 0 + \Delta\sigma_1 = \Delta\sigma_1$$

$$q_1 = q_0 + \Delta q_1 = 0 + 0 = 0$$

and are shown as coordinate A in Fig. 5.31. The line OA is called the stress path for isotropic compression. The slope of OA is

$$\frac{\Delta q_1}{\Delta p_1} = 0$$

Let us now apply loading “2” by keeping σ_3 constant, that is, $\Delta\sigma_3 = 0$, but continue to increase σ_1 , that is, $\Delta\sigma_1 > 0$ (insert figure labeled “2” in Fig. 5.31). Increases in the stress invariants for loading “2” are

$$\Delta p_2 = \frac{\Delta\sigma_1 + 2 \times 0}{3} = \frac{\Delta\sigma_1}{3}$$

$$\Delta q_2 = \Delta\sigma_1 - 0 = \Delta\sigma_1$$

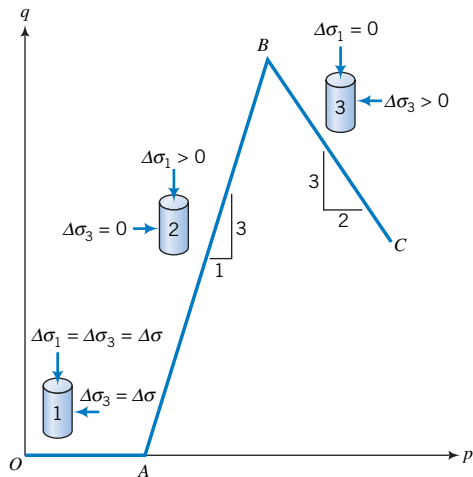


FIGURE 5.31 Stress paths.

and the stress invariants at the end of loading “2” are

$$\begin{aligned} p_2 &= p_1 + \Delta p_2 = \Delta\sigma_1 + \frac{\Delta\sigma_1}{3} = \frac{4}{3}\Delta\sigma_1 \\ q_2 &= q_1 + \Delta q_2 = 0 + \Delta\sigma_1 = \Delta\sigma_1 \end{aligned}$$

Point B in Fig. 5.31 represents (q_2, p_2) and the line AB is the stress path for loading “2.” The slope of AB is

$$\frac{\Delta q_2}{\Delta p_2} = \frac{\Delta\sigma_1}{(\Delta\sigma_1/3)} = 3$$

Let us make another change to the loading conditions. We will now keep σ_1 constant ($\Delta\sigma_1 = 0$) and then increase σ_3 ($\Delta\sigma_3 > 0$) as illustrated by the inset figure labeled “3” in Fig. 5.31. The increases in stress invariants are

$$\begin{aligned} \Delta p_3 &= \frac{0 + 2\Delta\sigma_3}{3} = \frac{2\Delta\sigma_3}{3} \\ \Delta q_3 &= 0 - \Delta\sigma_3 = -\Delta\sigma_3 \end{aligned}$$

The stress invariants at the end of loading “3” are

$$\begin{aligned} p_3 &= p_2 + \Delta p_3 = \frac{4}{3}\Delta\sigma_1 + \frac{2}{3}\Delta\sigma_3 \\ q_3 &= q_2 + \Delta q_3 = \Delta\sigma_1 - \Delta\sigma_3 \end{aligned}$$

The stress path for loading “3” is shown as BC in Fig. 5.31. The slope of BC is

$$\frac{\Delta q_3}{\Delta p_3} = \frac{-\Delta\sigma_3}{\frac{2}{3}\Delta\sigma_3} = -\frac{3}{2}$$

You should note that q decreases but p increases for stress path BC .

So far, we have not discussed whether the soil was allowed to drain or not. You will recall that the soil solids and the porewater (Section 5.9) must carry the applied increase in stresses in a saturated soil. If the soil porewater is allowed to drain from the soil sample, the increase in stress carried by the porewater, called excess porewater pressure (Δu), will continuously decrease to zero and the soil solids will have to support all of the increase in applied stresses. We will assume that during loading “1,” the excess porewater was allowed to drain—this is called the drained condition in geotechnical engineering. The type of loading imposed by loading “1” is called isotropic consolidation. In Chapter 7, we will discuss isotropic consolidation further. Since the excess porewater pressure (Δu_1) dissipates as the porewater drains from the soil, the mean effective stress at the end of each increment of loading “1” is equal to the mean total stress; that is,

$$\Delta p'_1 = \Delta p_1 - \Delta u_1 = \Delta p_1 - 0 = \Delta p$$

The effective stress path (ESP) and the total stress path (TSP) are the same and represented by OA in Fig. 5.32. You should note that we have used dual labels, p' , p , for the horizontal axis in Fig. 5.32. This dual labeling allows us to use one plot to represent both the effective and total stress paths.

We will assume that for loadings “2” and “3” the excess porewater pressures were prevented from draining out of the soil. In geotechnical engineering, the term undrained is used to denote a loading situation in which the excess porewater cannot drain from the soil. The implication is that the volume of our soil sample remains constant. In Chapter 7, we will discuss drained and undrained loading conditions in more detail. For loading “2,” the total stress path is AB . In this book, we will represent total stress paths by dashed lines.

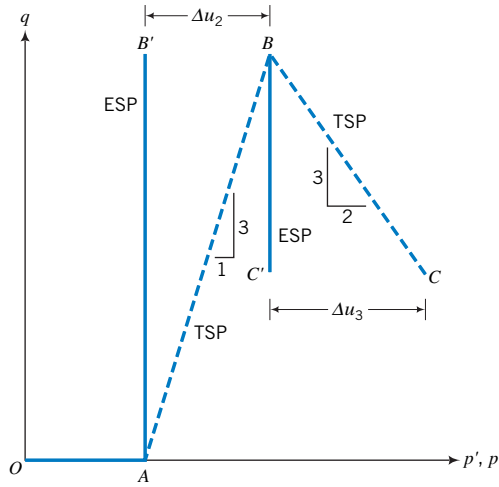


FIGURE 5.32 Total and effective stress paths.

If our soil were an isotropic, elastic material, then according to Eq. (5.109), written in incremental form

$$\Delta \epsilon_p^e = \frac{\Delta p'}{K'} = 0 \quad (5.115)$$

The solution of Eq. (5.115) leads to either $\Delta p' = 0$ or $K' = \infty$. There is no reason why K' should be ∞ . The act of preventing the drainage of the excess porewater cannot change the (effective) bulk modulus of the soil solids. Remember the truss analogy we used for effective stresses. The same analogy is applicable here. The only tenable solution is $\Delta p' = 0$. We can also write Eq. (5.115) in terms of total stresses; that is,

$$\Delta \epsilon_p^e = \frac{\Delta p}{K} = 0 \quad (5.116)$$

where $K = E_u/3(1 - 2\nu_u)$ and the subscript u denotes undrained condition. In this case, Δp cannot be zero since this is the change in mean total stress from the applied loading. Therefore, the only tenable solution is $K = K_u = \infty$, which leads to $\nu_u = 0.5$. The implications of Eqs. (5.115) and (5.116) for a linear, isotropic, elastic soil under undrained conditions are:

1. The change in mean effective stress is zero and, consequently, the effective stress path is vertical.
2. The undrained bulk modulus is ∞ and $\nu_u = 0.5$.

The deviatoric stress is unaffected by porewater pressure changes. We can write Eq. (5.112) in terms of total stress parameters as

$$G = G_u = \frac{E_u}{2(1 + \nu_u)}$$

Since $G = G_u = G'$ then

$$\frac{E_u}{2(1 + \nu_u)} = \frac{E'}{2(1 + \nu')}$$

and, by substituting $\nu_u = 0.5$, we obtain

$$E_u = \frac{1.5E'}{(1 + \nu')} \quad (5.117)$$

For many soils, $\nu' \cong \frac{1}{3}$ and, as a result, $E_u \cong 1.1E'$; that is, the undrained elastic modulus is about 10% greater than the effective elastic modulus.

The effective stress path for loading “2,” assuming our soil sample behaves like an isotropic, elastic material, is represented by AB' (Fig. 5.32); the coordinates of B' are

$$\begin{aligned} p'_2 &= p'_1 + \Delta p'_2 = p'_1 + 0 = \Delta\sigma_1 \\ q_2 &= q_1 + \Delta q_2 = 0 + \Delta\sigma_1 = \Delta\sigma_1 \end{aligned}$$

The difference in mean stress between the TSP and the ESP at a fixed value of q is the change in excess porewater pressure. That is, the magnitude of a horizontal line between the TSP and ESP is the change in excess porewater pressure. The maximum change in excess porewater pressure at the end of loading “2” is

$$\Delta u_2 = p_2 - p'_2 = \frac{4}{3}\Delta\sigma_1 - \Delta\sigma_1 = \frac{1}{3}\Delta\sigma_1$$

For loading “3,” the ESP for an elastic soil is BC' and the maximum change in excess porewater pressure is denoted by CC' (Fig. 5.32).

Soils only behave as elastic materials over a small range of strains and therefore the condition $\Delta p' = 0$ under undrained loading has only limited application. Once the soil yields, the ESP tends to bend. In Chapter 8, we will discuss how soil yielding affects the ESP.

You can use the above procedure to determine the stress paths for any loading condition. For example, let us confine our soil sample laterally, that is, we are keeping the diameter constant, $\Delta\epsilon_r = 0$, and incrementally increase σ_1 under drained conditions (Fig. 5.33). The loading condition we are imposing on our sample is called one-dimensional compression.

The increase in lateral effective stress for an increment of vertical stress $\Delta\sigma_1$ under the drained condition is given by Eq. (5.50) as $\Delta\sigma_3 = \Delta\sigma'_3 = K_o\Delta\sigma'_1$. The stress invariants are

$$\begin{aligned} \Delta p' &= \frac{\Delta\sigma'_1 + 2\Delta\sigma'_3}{3} = \frac{\Delta\sigma'_1 + 2K_o\Delta\sigma'_1}{3} = \Delta\sigma'_1 \left(\frac{1 + 2K_o}{3} \right) \\ \Delta q &= \Delta q' = \Delta\sigma'_1 - \Delta\sigma'_3 = \Delta\sigma'_1 - K_o\Delta\sigma'_1 = \Delta\sigma'_1(1 - K_o) \end{aligned}$$

The slope of the TSP is equal to the slope of the ESP; that is

$$\frac{\Delta q}{\Delta p} = \frac{\Delta q}{\Delta p'} = \frac{3(1 - K_o)}{1 + 2K_o}$$

The one-dimensional compression stress path is shown in Fig. 5.33.

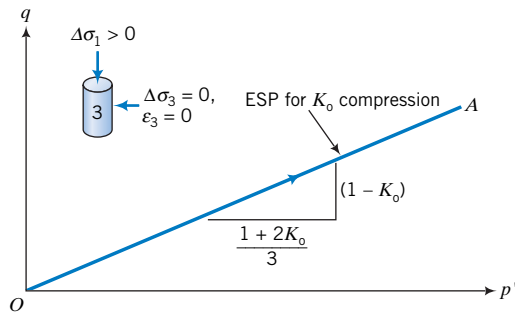


FIGURE 5.33 One-dimensional compression stress path.

The essential points are:

1. *A stress path is a graphical representation of stresses in stress space. For convenience, stress paths are plotted as deviatoric stress (q) on the ordinate versus mean effective stress (p') and/or mean total stress (p) on the abscissa.*
2. *The effective stress path for a linear, elastic soil under the undrained condition is vertical; that is, $\Delta p' = 0$.*
3. *The mean stress difference between the total stress path and the effective stress path is the excess porewater pressure.*
4. *The response, stability, and failure of soils depend on stress paths.*

5.13.3 Procedure for Plotting Stress Paths

A summary of the procedure for plotting stress paths is as follows:

1. Determine the loading conditions drained or undrained or both.
2. Calculate the initial loading values of p'_o , p_o , and q_o .
3. Set up a graph of p' (and p , if you are going to also plot the total stress path) as the abscissa and q as the ordinate. Plot the initial values of (p'_o, q_o) and (p_o, q_o) .
4. Determine the increase in stresses, $\Delta\sigma_1$, $\Delta\sigma_2$, and $\Delta\sigma_3$. These stresses can be negative.
5. Calculate the increase in stress invariants, $\Delta p'$, Δp , and Δq . These stress invariants can be negative.
6. Calculate the current stress invariants as $p' = p'_o + \Delta p'$, $p = p_o + \Delta p$, and $q = q_o + \Delta q$. The current value of p' cannot be negative but q can be negative.
7. Plot the current stress invariants (p', q) and (p, q) .
8. Connect the points identifying effective stresses and do the same for total stresses.
9. Repeat items 4 to 8 for the next loading condition.
10. The excess porewater pressure at a desired level of deviatoric stress is the mean stress difference between the total stress path and the effective stress path.

Remember that for a drained loading condition, $ESP = TSP$, and for an undrained condition, the ESP for a linear, elastic soil is vertical.

EXAMPLE 5.15 Stress Paths Due to Axisymmetric Loading (Triaxial Test)

Two cylindrical specimens, A and B, of a soil were loaded as follows. Both specimens were isotropically loaded by a stress of 200 kPa under drained conditions. Subsequently, the radial stress applied on specimen A was held constant and the axial stress was incrementally increased to 440 kPa under undrained conditions. The axial stress on specimen B was held constant and the radial stress incrementally reduced to 50 kPa under drained conditions. Plot the total and effective stress paths for each specimen assuming the soil is a linear, isotropic, elastic material. Calculate the maximum excess porewater pressure in specimen A.

Strategy The loading conditions on both specimens are axisymmetric. The easiest approach is to write the mean stress and deviatoric stress equations in terms of increments and make the necessary substitutions.

Solution 5.15**Step 1:** Determine loading condition.

Loading is axisymmetric and both drained and undrained conditions are specified.

Step 2: Calculate initial stress invariants for isotropic loading path.For axisymmetric, isotropic loading under drained conditions, $\Delta u = 0$,

$$\Delta p' = \frac{\Delta \sigma'_a + 2\Delta \sigma'_r}{3} = \frac{\Delta \sigma'_1 + 2\Delta \sigma'_1}{3} = \Delta \sigma'_1 = 200 \text{ kPa}$$

$p_o = p'_o = 200 \text{ kPa}$, since the soil specimens were loaded from a stress-free state under drained conditions.

$$q_o = q'_o = 0$$

Step 3: Set up graph and plot initial stress points.Create a graph with axes p' and p as the abscissa and q as the ordinate and plot the isotropic stress path with coordinates $(0, 0)$ and $(200, 0)$ as shown by OA in Fig. E5.15.**Step 4:** Determine the increases in stresses.**Specimen A**We have (1) an undrained condition, Δu is not zero and (2) no change in the radial stress but the axial stress is increased to 440 kPa. Therefore,

$$\Delta \sigma_3 = 0, \quad \Delta \sigma_1 = 440 - 200 = 240 \text{ kPa}$$

Specimen BDrained loading ($\Delta u = 0$); therefore, TSP = ESP.Axial stress held constant, $\Delta \sigma_1 = \Delta \sigma'_1 = 0$; radial stress decreases to 50 kPa; that is,

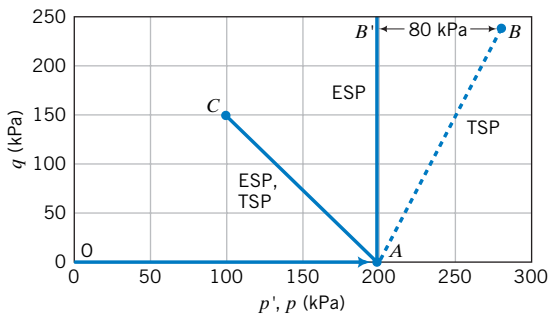
$$\Delta \sigma_3 = \Delta \sigma'_3 = 50 - 200 = -150 \text{ kPa}$$

Step 5: Calculate the increases in stress invariants.**Specimen A**

$$\Delta p = \frac{\Delta \sigma_1 + 2\Delta \sigma_3}{3} = \frac{240 + 2 \times 0}{3} = 80 \text{ kPa}$$

$$\Delta q = \Delta \sigma_1 - \Delta \sigma_3 = 240 - 0 = 240 \text{ kPa}$$

$$\text{Slope of total stress path} = \frac{\Delta q}{\Delta p} = \frac{240}{80} = 3$$

**FIGURE E5.15**

Specimen B

$$\Delta p = \Delta p' = \frac{\Delta \sigma'_1 + 2\Delta \sigma'_3}{3} = \frac{0 + 2 \times (-150)}{3} = -100 \text{ kPa}$$

$$\Delta q = \Delta \sigma_1 - \Delta \sigma_3 = 0 - (-150) = 150 \text{ kPa}$$

$$\text{Slope of ESP (or TSP)} = \frac{\Delta q}{\Delta p'} = \frac{150}{-100} = -1.5$$

Step 6: Calculate the current stress invariants.

Specimen A

$$p = p_o + \Delta p = 200 + 80 = 280 \text{ kPa}, \quad q = q' = q_o + \Delta q = 0 + 240 = 240 \text{ kPa}$$

$$p' = p_o + \Delta p' = 200 + 0 = 200 \text{ kPa} \quad (\text{elastic soil})$$

Specimen B

$$p = p' = p_o + \Delta p = 200 - 100 = 100 \text{ kPa}$$

$$q = q_o + \Delta q = 0 + 150 = 150 \text{ kPa}$$

Step 7: Plot the current stress invariants.

Specimen A

Plot point B as (280, 240); plot point B' as (200, 240).

Specimen B

Plot point C as (100, 150).

Step 8: Connect the stress points.

Specimen A

AB in Fig. E5.15 shows the total stress path and AB' shows the effective stress path.

Specimen B

AC in Fig. E5.15 shows the ESP and TSP.

Step 9: Determine the excess porewater pressure.

Specimen A

BB' shows the maximum excess porewater pressure. The mean stress difference is $280 - 200 = 80 \text{ kPa}$.

5.14 SUMMARY

Elastic theory provides a simple, first approximation to calculate the deformation of soils at small strains. You are cautioned that the elastic theory cannot adequately describe the behavior of most soils and more involved theories are required. The most important principle in soil mechanics is the principle of effective stress. Soil deformation is due to effective not total stresses. Applied surface stresses are distributed such that their magnitudes decrease with depth and distance away from their points of application.

Stress paths provide a useful means through which the history of loading of a soil can be followed. The mean effective stress changes for a linear, isotropic, elastic soil are zero under undrained loading and the effective stress path is a vector parallel to the deviatoric stress axis with the q ordinate equal to the corresponding state on the total stress path. The difference in mean stress between the total stress path and the effective stress path gives the excess porewater pressure at a desired value of deviatoric stress.

Practical Example**EXAMPLE 5.16 Vertical Stress Increase Due to a Foundation**

A building foundation of width 10 m and length 40 m transmits a load of 80 MN to a deep deposit of stiff saturated clay (Fig. E5.16a). The elastic modulus of the clay varies with depth (Fig. E5.16b) and $\nu' = 0.32$. Estimate the elastic settlement of the clay under the center of the foundation assuming (1) drained condition and (2) undrained condition.

Strategy The major decision in this problem is what depth to use to determine an appropriate elastic modulus. One option is to use an average elastic modulus over a depth of $3B$. Beyond a depth of $3B$, the vertical stress increase is less than 10%.

Solution 5.16

Step 1: Find the applied vertical surface stress.

$$q_s = \frac{80 \times 10^3}{10 \times 40} = 200 \text{ kPa}$$

Step 2: Determine the elastic modulus.

Assume an effective depth of $3B = 3 \times 10 = 30 \text{ m}$.

The average value of E' is 34.5 MPa.

From Eq. (5.117) with $\nu_u = 0.5$, $E_u = 1.1 E' = 1.1 \times 34.5 = 38 \text{ MPa}$

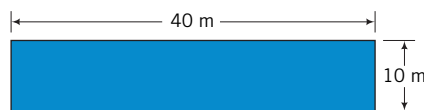
Step 3: Calculate the vertical settlement.

$$\text{Use Eq. (5.90): } \Delta z = \frac{q_s B (1 - \nu^2)}{E} I_s$$

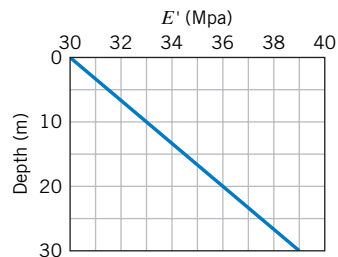
$$\frac{L}{B} = \frac{40}{10} = 4, \quad I_s = 0.62 \ln\left(\frac{L}{B}\right) + 1.12 = 0.62 \ln(4) + 1.12 = 1.98$$

(1) Drained condition.

$$\Delta z = \frac{200 \times 5 \times (1 - 0.32^2)}{34.5 \times 10^6} 1.98 = 51.5 \times 10^{-6} \text{ m} = 51.5 \times 10^{-3} \text{ mm}$$



(a) Foundation plan and section



(b) Variation of elastic modulus with depth

FIGURE E5.16

- (2) Undrained condition.

$$\Delta z = \frac{200 \times 5 \times (1 - 0.5^2)}{38 \times 10^6} 1.98 = 39.1 \times 10^{-6} \text{ m} = 39.1 \times 10^{-3} \text{ mm}$$

EXERCISES

Theory

- 5.1** An elastic soil is confined laterally and is axially compressed under drained conditions. In soil mechanics, the loading imposed on the soil is called K_o compression or consolidation. Show that under the K_o condition,

$$\frac{\sigma'_x}{\sigma'_z} = \frac{\nu'}{1 - \nu'}$$

- 5.2** The increase in porewater pressure in a saturated soil is given by $\Delta u = \Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)$. Show that if the soil is a linear, isotropic, elastic material, $A = \frac{1}{3}$ for the axisymmetric condition.
- 5.3** If the axial stress on a cylindrical sample of soil is decreased and the radial stress is increased by twice the decrease in axial stress, show that the stress path has a slope $q/p = -3$.
- 5.4** The initial mean effective stress on a soil is p'_0 and the deviatoric stress $q = 0$. If the soil is a linear, isotropic, elastic material, plot the total and effective stress paths for the following axisymmetric undrained loading condition: (a) $\Delta \sigma_3 = \frac{1}{2} \Delta \sigma_1$, and (b) $\Delta \sigma_3 = -\frac{1}{2} \Delta \sigma_1$.

Problem Solving

Interactive Problem Solving

Access Chapter 5 and click on Problem Solver on the sidebar to interactively solve problems similar to some in this chapter.

- 5.5** A long embankment is located on a soil profile consisting of 4 m of medium clay followed by 8 m of medium to dense sand on top of bedrock. A vertical settlement of 5 mm at the center of the embankment was measured during construction. Assuming all the settlement is elastic and occurs in the medium clay, determine the average stresses imposed on the medium clay under the center of the embankment using the elastic equations. The elastic parameters are $E = 15 \text{ MPa}$ and $\nu = 0.3$ [Hint: Assume the lateral strain is zero.]
- 5.6** An element of soil (sand) behind a retaining wall is subjected to an increase in vertical stress of 60 kPa and an increase in lateral stress of 25 kPa. Determine the increase in vertical and lateral strains, assuming the soil is a linearly elastic material with $E = 20 \text{ MPa}$ and $\nu = 0.3$.
- 5.7** A cylindrical specimen of soil is compressed by an axial principal stress of 500 kPa and a radial principal stress of 200 kPa. Plot Mohr's circle of stress and determine (a) the maximum shear stress and (b) the normal and shear stresses on a plane inclined at 30° counterclockwise from the horizontal.
- 5.8** A soil specimen ($100 \text{ mm} \times 100 \text{ mm} \times 100 \text{ mm}$) is subjected to the forces shown in Fig. P5.8. Determine (a) the magnitude of the principal stresses, (b) the orientation of the principal stress plane to the horizontal, (c) the maximum shear stress, and (d) the normal and shear stresses on a plane inclined at 20° clockwise to the horizontal.

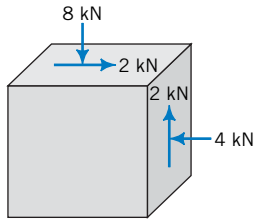


FIGURE P5.8

- 5.9** The initial principal stresses at a certain depth in a clay soil are 200 kPa on the horizontal plane and 100 kPa on the vertical plane. Construction of a surface foundation induces additional stresses consisting of a vertical stress of 45 kPa, a lateral stress of 20 kPa, and a counterclockwise (with respect to the horizontal plane) shear stress of 40 kPa. Plot Mohr's circle (1) for the initial state of the soil and (2) after construction of the foundation. Determine (a) the change in magnitude of the principal stresses, (b) the change in maximum shear stress, and (c) the change in orientation of the principal stress plane resulting from the construction of the foundation.
- 5.10** Plot the distribution of total stress, effective stress, and porewater pressure with depth for the soil profile shown in Fig. P5.10. Neglect capillary action.

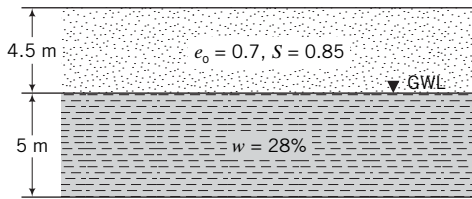


FIGURE P5.10

- 5.11** If the groundwater in 5.10 were (a) to rise to the surface, (b) to rise 2 m above the surface, and (c) to rapidly decrease from 2 m above the surface to 1 m below its present level, determine and plot the distributions of effective and total stress and porewater pressure with depth.
- 5.12** At what depth would the vertical effective stress in a deep deposit of clay be 100 kPa, if $e = 1.1$; groundwater level is at 1 m below ground surface and $S = 95\%$ above the groundwater level?
- 5.13** A culvert is to be constructed in a bed of sand ($e = 0.5$) for drainage purposes. The roof of the culvert will be located 3 m below ground surface. Currently, the groundwater level is at ground surface. But, after installation of the culvert, the groundwater level is expected to drop to 2 m below ground surface. Calculate the change in vertical effective stress on the roof of the culvert after installation. You can assume the sand above the groundwater level is saturated.
- 5.14** A soil profile consists of 10 m thick fine sand of effective size 0.15 mm above a very thick layer of clay. Groundwater level is at 3 m below the ground surface. (a) Determine the height of capillary rise assuming that the equivalent capillary tube diameter is 10% of the effective size and the sand surface is similar to smooth glass. (b) Plot the distribution of vertical effective stress and porewater pressure with depth if the void ratio of the sand is 0.6 and the degree of saturation is 90%.

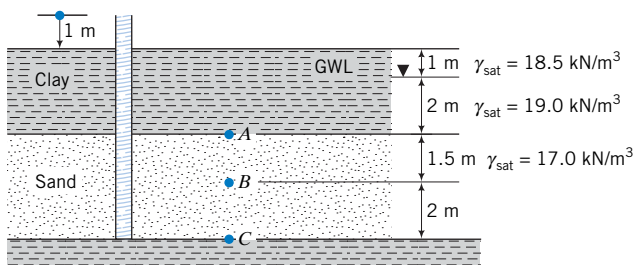


FIGURE P5.15

- 5.15** A soil profile consists of a clay layer underlain by a sand layer as shown in Fig. P5.15. If a tube is inserted into the bottom sand layer (Fig. P5.15) and the water level rises to 1 m above the ground surface, determine the vertical effective stresses and porewater pressures at A , B , and C . If K_o is 0.5, determine the effective and total lateral stresses at A , B , and C . What is the value of the porewater pressure at A to cause the effective stress there to be zero?
- 5.16** A pole is held vertically on a soil surface by three wires tied to the top of the pole. Each wire has a tension of 1 kN and is inclined at 45° to the vertical. Calculate:
- The increase in vertical stress at a depth 1 m below the surface
 - The amount of elastic settlement below the axis of the pole if $E = 40$ MPa and $\nu = 0.35$
- 5.17** A rectangular foundation $4\text{ m} \times 6\text{ m}$ (Fig. P5.17) transmits a stress of 150 kPa on the surface of a soil deposit. Plot the distribution of increases of vertical stresses with depth under points A , B , and C up to a depth of 20 m. At what depth is the increase in vertical stress below A less than 10% of the surface stress?

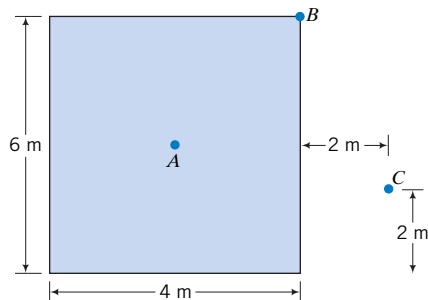


FIGURE P5.17

- 5.18** Determine the increase in vertical stress at a depth of 5 m below the centroid of the foundation shown in Fig. P5.18.

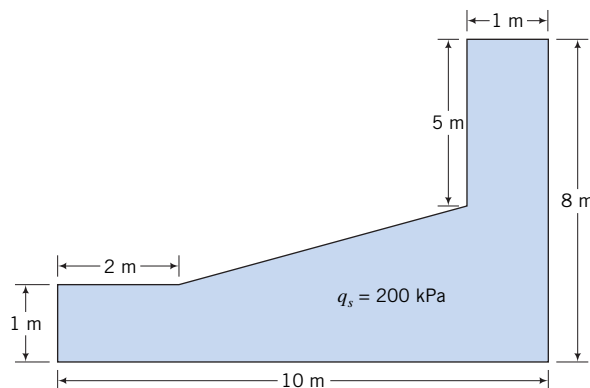


FIGURE P5.18

- 5.19** Three foundations are located next to each other (Fig. P5.19). Determine the stress increases at A , B , and C at a depth of 2 m below the ground surface.

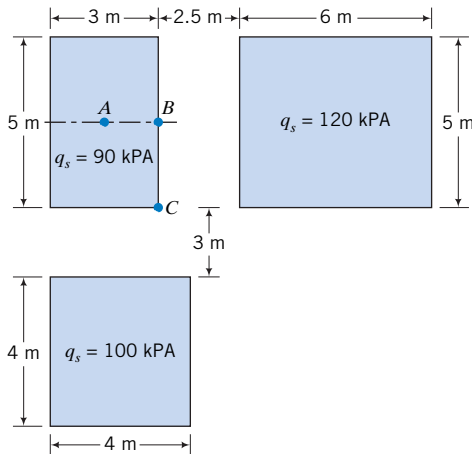


FIGURE P5.19

- 5.20** A cylindrical sample of soil is isotropically compressed under a drained condition with a vertical stress of 100 kPa and a radial stress of 100 kPa. Subsequently, the axial stress was held constant and the radial stress was increased to 300 kPa under an undrained condition. Plot the total and effective stress paths (assume the soil is a linear, isotropic, elastic material). Determine the slopes of the stress paths and the maximum excess porewater pressure.
- 5.21** The initial stresses on a soil element at a certain depth in a soil are $\sigma_1 = 80$ kPa, $\sigma_2 = 40$ kPa, and $\sigma_3 = 40$ kPa. The changes in stresses resulting from a sudden outward movement of a retaining wall are as follows: $\Delta\sigma_1 = 0$ kPa, $\Delta\sigma_2 = -10$ kPa, and $\Delta\sigma_3 = -40$ kPa. Plot the total stress path. Plot the effective stress path assuming that the soil is a linearly elastic material, and determine the maximum excess porewater pressure.

Practical

- 5.22** You are the geotechnical engineer for a proposed office building in a densely clustered city. The office building will be constructed adjacent to an existing office complex. The soil at the site is a deposit of very dense sand with $E' = 45$ MPa and $\nu' = 0.3$. The sand rests on a deep deposit of dense gravel. The existing high rise complex is founded on a concrete slab, $100\text{ m} \times 120\text{ m}$ located at 2 m below ground surface, that transmits a load of 2400 MN to the soil. Your office foundation is $50\text{ m} \times 80\text{ m}$ and transmits a load of 1000 MN. You also intend to locate your foundation at 2 m below ground level. The front of your building is aligned with the existing office complex and the side distance is 0.5 m. The lesser dimension of each building is the frontal dimension. The owners of the existing building are concerned about possible settlement of their building due to your building. You are invited to a meeting with your client, the owners of the existing building, and their technical staff. You are expected to determine what effects your office building would have on the existing building. You only have one hour to make the preliminary calculations and you are expected to present the increase in stresses and the amount of settlement of the existing office complex due to the construction of your office building. Prepare your analysis and presentation.
- 5.23** A house (plan dimension: $10\text{ m} \times 15\text{ m}$) is located on a deep deposit of sand mixed with some clays and silts. The groundwater at the time the house was completed was 0.5 m below the surface. A utility trench, 4 m deep, was later dug on one side along the length of the house. Any water that accumulated in the trench was pumped out so that the trench remained dry. Because of a labor dispute, work on laying the utility in the trench ceased but the open trench was continuously pumped. Sometime during the dispute, the owners noticed cracking of the walls in the house. Assuming $S = 0.9$ for the soil above the groundwater level and a void ratio of 0.7, write a short, preliminary, technical report (not more than a page) to the owner explaining why the cracks developed. The walls of the trench did not move laterally. The hydraulic conductivities of the soil in the vertical and horizontal directions are 0.5×10^{-4} cm/sec and 2.3×10^{-4} cm/sec respectively. The calculations should be in an appendix to the report.

5.24 A farmer requires two steel silos to store wheat. Each silo is 8 m in external diameter and 10 m high. The foundation for each silo is a circular concrete slab thickened at the edge. The total load of each silo filled with wheat is 9552 kN. The soil consists of a 30 m thick deposit of medium clay above a deep deposit of very stiff clay. The farmer desires that the silos be a distance of 2 m apart and hires you to recommend whether this distance is satisfactory. The area is subjected to a gust wind speed of 100 kilometers per hour.

- (a) Plot the distribution of vertical stress increase at the edges and at the center of one of the silos up to a depth of 16 m. Assume the soft clay layer is semi-infinite and the concrete slab is flexible. Use a spreadsheet to tabulate and plot your results.
- (b) Calculate the elastic settlement at the surface of one of the silos at the edges and at the center, assuming $E = 30 \text{ MPa}$ and $\nu = 0.5$.
- (c) Calculate the elastic tilt of the foundation of one of the silos and sketch the deformed shape of the foundation slab.
- (d) Would the tops of the silos touch each other based on the elastic tilt? Show calculations in support of your answer.
- (e) What is the minimum separation distance you would recommend? Make clear sketches to explain your recommendation to the owner.
- (f) Explain how the wind would alter the stress distribution below the silos.

(Hint: Use the charts in Appendix B.)

CHAPTER 6

ONE-DIMENSIONAL CONSOLIDATION SETTLEMENT OF FINE-GRAINED SOILS

	ES	ED
ABET	85	15

6.0 INTRODUCTION

In this chapter, we will consider one-dimensional consolidation settlement of fine-grained soils. We will restrict settlement consideration to the vertical direction. Under load, all soils will settle, causing settlement of structures founded on or within them. If the settlement is not kept to a tolerable limit, the desired use of the structure may be impaired and the design life of the structure may be reduced. Structures may settle uniformly or nonuniformly. The latter is called differential settlement and is often the crucial design consideration.

The total settlement usually consists of three parts—immediate or elastic compression, primary consolidation, and secondary compression. We have considered elastic settlement in Chapter 5 and we will consider some modifications to the elastic analysis for practical applications in Chapter 9. Here we will develop the basic concepts of consolidation and show how these concepts can be applied to calculate the consolidation settlement from applied loads. After that, we will formulate the theory of one-dimensional consolidation and use it to predict the time rate of settlement.

After you have studied this chapter, you should:

- Have a basic understanding of the consolidation of soils under vertical loads.
- Be able to calculate one-dimensional consolidation settlement and time rate of settlement.

You will make use of the following concepts learned from the previous chapters and your courses in mechanics and mathematics.

- Stresses in soils—effective stresses and vertical stress increases from surface loads (Chapter 5)
- Strains in soils—vertical and volumetric strains (Chapter 5)
- Elasticity (Chapter 5)
- Flow of water through soils (Chapter 4, Darcy's law)
- Solutions of partial differential equations

Sample Practical Situation The foundation for an office building is to be constructed on a sand deposit. Below the sand deposit is a layer of soft clay followed by a thick layer of sand and gravel. The office building is asymmetrical and the loads are not uniformly distributed. The allowable total settlement is 50 mm and the differential settlement is limited to 6 mm. You are expected to determine



FIGURE 6.1 The leaning tower of Pisa. (© Photo Disc.)

the total and differential settlement of the building, the rate of settlement, and the time to achieve 90% consolidation settlement.

The leaning tower of Pisa is the classic example of differential settlement (Fig. 6.1). Construction of the tower started in 1173 and by the end of 1178 when two-thirds of the tower was completed it tilted. Since then the tower has been settling differentially. The foundation of the tower is located about 3 m into a bed of silty sand that is underlain by 30 m of clay on a deposit of sand. A sand layer approximately 5 m thick intersects the clay. The structure of the tower is intact but its function is impaired by differential settlement.

6.1 DEFINITIONS OF KEY TERMS

Consolidation is the time-dependent settlement of soils resulting from the expulsion of water from the soil pores.

Primary consolidation is the change in volume of a fine-grained soil caused by the expulsion of water from the voids and the transfer of load from the excess porewater pressure to the soil particles.

Secondary compression is the change in volume of a fine-grained soil caused by the adjustment of the soil fabric (internal structure) after primary consolidation has been completed.

Excess porewater pressure, Δu , is the porewater pressure in excess of the current equilibrium porewater pressure. For example, if the porewater pressure in a soil is u_0 and a load is applied to the soil so that the existing porewater pressure increases to u_1 , then the excess porewater pressure is $\Delta u = u_1 - u_0$.

Drainage path, H_{dr} , is the longest vertical path that a water particle will take to reach the drainage surface.

Preconsolidation stress or past maximum effective stress, σ'_{zc} , is the maximum vertical effective stress that a soil was subjected to in the past.

Normally consolidated soil is one that has never experienced vertical effective stresses greater than its current vertical effective stress ($\sigma'_{z0} = \sigma'_{zc}$).

Overconsolidated soil is one that has experienced vertical effective stresses greater than its existing vertical effective stress ($\sigma'_{z0} < \sigma'_{zc}$).

Overconsolidation ratio, OCR, is the ratio by which the current vertical effective stress in the soil was exceeded in the past ($OCR = \sigma'_{zc}/\sigma'_{z0}$).

Compression index, C_c , is the slope of the normal consolidation line in a plot of the logarithm of vertical effective stress versus void ratio.

Unloading/reloading index or **recompression index, C_r** , is the average slope of the unloading/reloading curves in a plot of the logarithm of vertical effective stress versus void ratio.

Modulus of volume compressibility, m_v , is the slope of the curve between two stress points in a plot of vertical effective stress versus vertical strain.

6.2 QUESTIONS TO GUIDE YOUR READING

1. What is the process of soil consolidation?
2. What is the difference between consolidation and compaction?
3. What is the governing equation in one-dimensional consolidation theory?
4. How is the excess porewater pressure distributed within the soil when a load is applied and after various elapsed times?
5. What factors determine the consolidation settlement of soils?
6. What are the average degree of consolidation, time factor, modulus of volume compressibility, and compression and recompression indices?
7. What is the difference between primary consolidation and secondary compression?
8. What is the drainage path for single drainage and double drainage?
9. Why do we need to carry out consolidation tests, how are they conducted, and what parameters are deduced from the test results?
10. How is time rate of settlement and consolidation settlement calculated?
11. Are there significant differences between the calculated settlements and field settlements?

6.3 BASIC CONCEPTS



Interactive Concept Learning and Self-Assessment

Access Chapter 6, Section 6.3 on the CD to learn about the basic concepts of consolidation through interactive animation. You will conduct a virtual experiment and will be guided step by step to learn the fundamentals of one-dimensional consolidation. You will be able to explore one-dimensional consolidation settlement for different boundary conditions. Take Quiz 6.3 to evaluate your understanding.

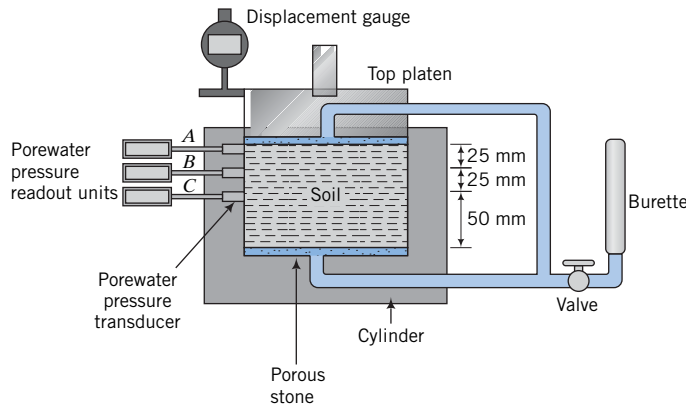


FIGURE 6.2a Experimental setup for illustrating basic concepts on consolidation.

In our development of the various ideas on consolidation settlement, we will assume:

- A homogeneous, saturated soil
- The soil particles and the water to be incompressible
- Vertical flow of water
- The validity of Darcy's law
- Small strains

We will conduct a simple experiment to establish the basic concepts of the one-dimensional consolidation settlement of fine-grained soils. Let us take a thin, soft, saturated sample of clay and place it between porous stones in a rigid, cylindrical container whose inside wall is frictionless (Fig. 6.2a). The porous stones are used to facilitate drainage of the porewater from the top and bottom faces of the soil. The top half of the soil will drain through the top porous stone and the bottom half of the soil will drain through the bottom porous stone. A platen on the top porous stone transmits applied loads to the soil. Expelled water is transported by plastic tubes to a burette. A valve is used to control the flow of the expelled water into the burette. Three porewater pressure transducers are mounted in the side wall of the cylinder to measure the excess porewater pressure near the porous stone at the top (*A*), at a distance of one-quarter the height (*B*), and at midheight of the soil (*C*). Excess porewater pressure is the additional porewater pressure induced in a soil mass by loads. A displacement gauge with its stem on the platen keeps track of the vertical settlement of the soil.

We will assume that the porewater and the soil particles are incompressible, and the initial porewater pressure is zero. The volume of excess porewater that drains from the soil is then a measure of the volume change of the soil resulting from the applied loads. Since the side wall of the container is rigid, no radial displacement can occur. The lateral and circumferential strains are then equal to zero ($\epsilon_r = \epsilon_\theta = 0$), and the volumetric strain ($\epsilon_p = \epsilon_z + \epsilon_\theta + \epsilon_r$) is equal to the vertical strain, $\epsilon_z = \Delta z/H_o$, where Δz is the change in height or thickness and H_o is the initial height or thickness of the soil.

6.3.1 Instantaneous Load

Let us now apply a load P to the soil through the load platen and keep the valve closed. Since no excess porewater can drain from the soil, the change in volume of the soil is zero ($\Delta V = 0$) and no load or stress is transferred to the soil particles ($\Delta\sigma'_z = 0$). The porewater carries the total load. The initial excess porewater pressure in the soil (Δu_o) is then equal to the change in applied vertical stress, $\Delta\sigma_z = P/A$, where A is the cross-sectional area of the soil, or more appropriately the change in mean total stress, $\Delta p = (\Delta\sigma_z + 2\Delta\sigma_r)/3$, where $\Delta\sigma_r$ is the change in radial stress. For our thin soil layer, we

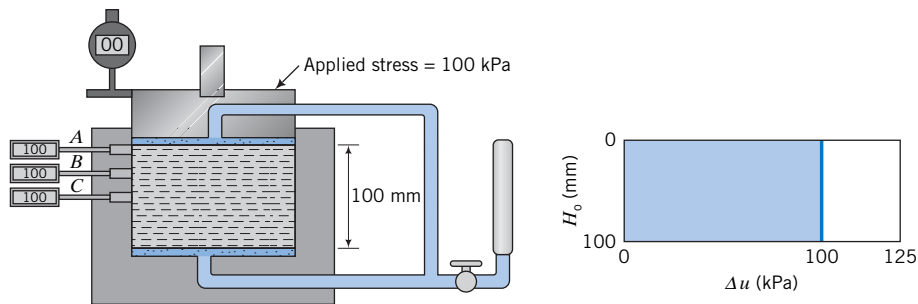


FIGURE 6.2b Instantaneous or initial excess porewater pressure when a vertical load is applied.

will assume that the initial excess porewater pressure will be distributed uniformly with depth so that at every point in the soil layer, the initial excess porewater pressure is equal to the applied vertical stress. For example, if $\Delta\sigma_z = 100$ kPa, then $\Delta u_o = 100$ kPa as shown in Fig. 6.2b.

6.3.2 Consolidation Under a Constant Load—Primary Consolidation

Let us now open the valve and allow the initial excess porewater to drain. The total volume of soil at time t_1 decreases by the amount of excess porewater that drains from it as indicated by the change in volume of water in the burette (Fig. 6.2c). At the top and bottom of the soil sample, the excess porewater pressure is zero because these are the drainage boundaries. The decrease of initial excess porewater pressure at the middle of the soil (position C) is the slowest because a water particle must travel from the middle of the soil to either the top or bottom boundary to exit the system.

You may have noticed that the settlement of the soil (Δz) with time t (Fig. 6.2c) is not linear. Most of the settlement occurs shortly after the valve was opened. The rate of settlement, $\Delta z/t$, is also much faster soon after the valve was opened compared with later times. Before the valve was opened, an initial hydraulic head, $\Delta u_o/\gamma_w$, was created by the applied vertical stress. When the valve was opened, the initial excess porewater was forced out of the soil by this initial hydraulic head. With time, the initial hydraulic head decreases and, consequently, smaller amounts of excess porewater are forced out. An analogy can be drawn with a pipe containing pressurized water that is ruptured. A large

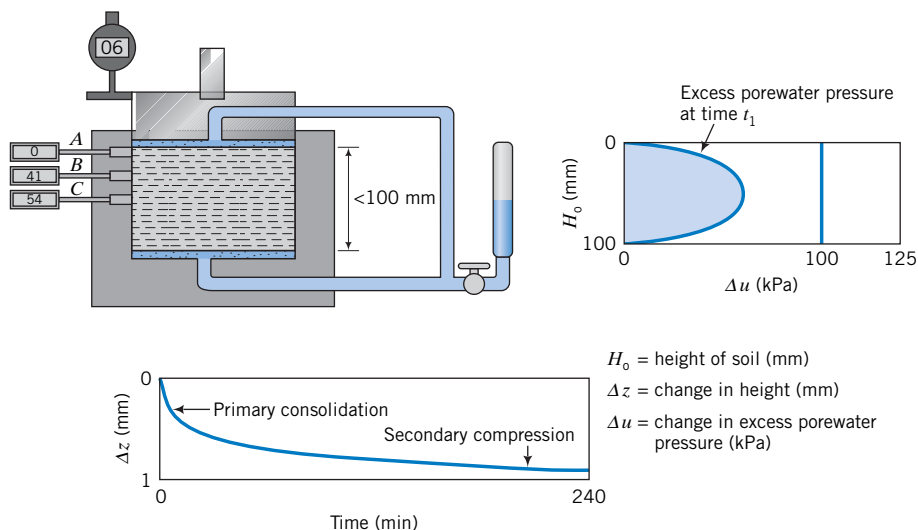


FIGURE 6.2c Excess porewater pressure distribution and settlement during consolidation.

volume of water gushes out as soon as the pipe is ruptured, but soon after the flow becomes substantially reduced. We will call the initial settlement response, soon after the valve was opened, the early time response or primary consolidation. Primary consolidation is the change in volume of the soil caused by the expulsion of water from the voids and the transfer of load from the excess porewater pressure to the soil particles.

6.3.3 Secondary Compression

Primary consolidation ends when $\Delta u_o = 0$. The later time settlement response is called secondary compression or creep. Secondary compression is the change in volume of a fine-grained soil caused by the adjustment of the soil fabric (internal structure) after primary consolidation has been completed. The term consolidation is reserved for the process in which settlement of a soil occurs from changes in effective stresses resulting from decreases in excess porewater pressure. The rate of settlement from secondary compression is very slow compared with primary consolidation.

We have separated primary consolidation and secondary compression. In reality, the distinction is not clear because secondary compression occurs as part of the primary consolidation phase especially in soft clays. The mechanics of consolidation is still not fully understood and to make estimates of settlement, it is convenient to separate primary consolidation and secondary compression.

6.3.4 Drainage Path

The distance of the longest vertical path taken by a particle to exit the soil is called the length of the drainage path. Because we allowed the soil to drain on the top and bottom faces (double drainage), the length of the drainage path, H_{dr} , is

$$H_{dr} = \frac{H_{av}}{2} = \frac{H_o + H_f}{4} \quad (6.1)$$

where H_{av} is the average height and H_o and H_f are the initial and final heights, respectively, under the current loading. If drainage were permitted from only one face of the soil, then $H_{dr} = H_{av}$. Shorter drainage paths will cause the soil to complete its settlement in a shorter time than a longer drainage path. You will see later that, for single drainage, our soil sample will take four times longer to reach a particular settlement than for double drainage.

6.3.5 Rate of Consolidation

The rate of consolidation for a homogeneous soil depends on the soil's permeability, the thickness, and the length of the drainage path. A soil with a hydraulic conductivity lower than that of our current soil will take longer to drain the initial excess porewater and settlement will proceed at a slower rate.

6.3.6 Effective Stress Changes

Since the applied vertical stress (total stress) remains constant, then according to the principle of effective stress ($\Delta\sigma'_z = \Delta\sigma_z - \Delta u$), any reduction of the initial excess porewater pressure must be balanced by a corresponding increase in vertical effective stress. Increases in vertical effective stresses lead to soil settlement caused by changes to the soil fabric. As time increases, the initial excess porewater continues to dissipate and the soil continues to settle (Fig. 6.2c).

After some time, usually within 24 hours for many soils tested in the laboratory, the initial excess porewater pressure in the middle of the soil reduces to approximately zero and the rate of decrease of the volume of the soil becomes very small. Since the initial excess porewater pressure becomes zero, then from the principle of effective stress all of the applied vertical stress is transferred to the soil; that is, the vertical effective stress is equal to the vertical total stress ($\Delta\sigma'_z = \Delta\sigma_z$).

The essential points are:

- 1. When a load is applied to a saturated soil, all of the applied stress is supported initially by the porewater (initial excess porewater pressure); that is, at $t = 0$, $\Delta u_o = \sigma_z$ or $\Delta u_o = \Delta p$. The change in effective stress is zero ($\Delta \sigma'_z = 0$).*
- 2. If drainage of porewater is permitted, the initial excess porewater pressure decreases and soil settlement (Δz) increases with time; that is, $\Delta u(t) < \Delta u_o$ and $\Delta z > 0$. The change in effective stress is $\Delta \sigma'_z = \Delta \sigma_z - \Delta u(t)$.*
- 3. When $t \rightarrow \infty$, the change in volume and the change in excess porewater pressure of the soil approach zero; that is, $\Delta V \rightarrow 0$ and $\Delta u_o \rightarrow 0$. The change in vertical effective stress is $\Delta \sigma'_z = \Delta \sigma_z$.*
- 4. Soil settlement is not linearly related to time except very early in the consolidation process.*
- 5. The change in volume of the soil is equal to the volume of initial excess porewater expelled.*
- 6. The rate of settlement depends on the permeability of the soil.*

6.3.7 Void Ratio and Settlement Changes Under a Constant Load

The initial volume (specific volume) of a soil is $V = 1 + e_o$ (Chapter 3), where e_o is the initial void ratio. Any change in volume of the soil (ΔV) is equal to the change in void ratio (Δe). We can calculate the volumetric strain from the change in void ratio as

$$\varepsilon_p = \frac{\Delta V}{V} = \frac{\Delta e}{1 + e_o} \quad (6.2)$$

Since for one-dimensional consolidation, $\varepsilon_z = \varepsilon_p$, we can write a relationship between settlement and the change in void ratio as

$$\varepsilon_z = \frac{\Delta z}{H_o} = \frac{\Delta e}{1 + e_o} \quad (6.3)$$

where H_o is the initial height of the soil. We can rewrite Eq. (6.3) as

$$\Delta z = H_o \frac{\Delta e}{1 + e_o} \quad (6.4)$$

We are going to use ρ_{pc} to denote primary consolidation settlement rather than Δz , so

$$\rho_{pc} = H_o \frac{\Delta e}{1 + e_o} \quad (6.5)$$

The void ratio at the end of consolidation under load P is

$$e = e_o - \Delta e = e_o - \frac{\Delta z}{H_o} (1 + e_o) \quad (6.6)$$

6.3.8 Effects of Vertical Stresses on Primary Consolidation

We can apply additional loads to the soil and for each load increment we can calculate the final void ratio from Eq. (6.6) and plot the results as shown by segment AB in Fig. 6.3. Three types of graph

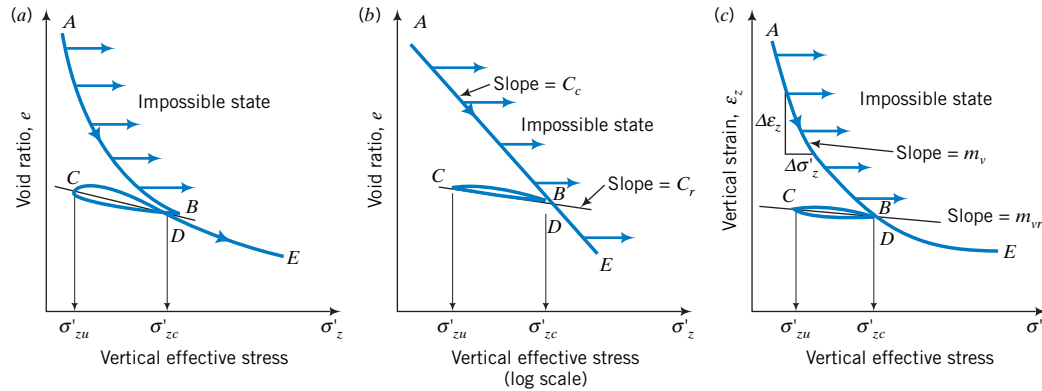


FIGURE 6.3 Three plots of settlement data from soil consolidation.

are shown in Fig. 6.3 to illustrate three different arbitrary ways of plotting the data from our test. Figure 6.3a is an arithmetic plot of the void ratio versus vertical effective stress. Figure 6.3b is a similar plot except the vertical effective stress is plotted on a logarithmic scale. Figure 6.3c is an arithmetic plot of the vertical strain (ϵ_z) versus vertical effective stress. The segment AB in Figs. 6.3a,c is not linear because the settlement that occurs for each increment of loading brings the soil to a denser state from its initial state and the soil's permeability decreases. Therefore, doubling the load from a previous increment, for example, would not cause a twofold increase in settlement. The segment AB (Fig. 6.3a–c) is called the virgin consolidation line or normal consolidation line (NCL). In a plot of σ'_z (log scale) versus e , the NCL is approximately a straight line.

At some value of vertical effective stress, say, σ'_{zc} , let us unload the soil incrementally. Each increment of unloading is carried out only after the soil reaches equilibrium under the previous loading step. When an increment of load is removed, the soil will start to swell by absorbing water from the burette. The void ratio increases but the increase is much less than the decrease in void ratio for the same magnitude of loading that was previously applied.

Let us reload the soil after unloading it to, say, σ'_{zu} . The reloading path CD is convex compared with the concave unloading path, BC . The average slopes of the unloading path and the reloading path are not equal but the difference is generally small for many soils. We will represent the unloading–reloading path by an average slope BC and refer to it as the recompression line or the unloading–reloading line (URL).

A comparison of the soil's response with typical material responses to loads as shown in Figs. 5.5 and 5.6 reveals that soils can be considered to be an elastoplastic material (see Fig. 5.6). The path BC represents the elastic response while the path AB represents the elastoplastic response of the soil. Loads that cause the soil to follow path BC will produce elastic settlement (recoverable settlement of small magnitude). Loads that cause the soil to follow path AB will produce settlements that have both elastic and plastic (permanent) components.

Once the past maximum vertical effective stress, σ'_{zc} , is exceeded the slope of the path followed by the soil, DE , is the same as that of the initial loading path AB . Unloading and reloading the soil at any subsequent vertical effective stress would result in a soil's response similar to paths $BCDE$.

6.3.9 Primary Consolidation Parameters

The primary consolidation settlement of the soil (settlement that occurs along path AB in Fig. 6.3) can be expressed through the slopes of the curves in Fig. 6.3. We are going to define two slopes for primary

consolidation. One is called the coefficient of compression or compression index, C_c , and is obtained from the plot of e versus $\log \sigma'_z$ (Fig. 6.3b) as

$$C_c = -\frac{e_2 - e_1}{\log \frac{(\sigma'_z)_2}{(\sigma'_z)_1}} = -\frac{|\Delta e|}{\log \frac{(\sigma'_z)_2}{(\sigma'_z)_1}} \quad (\text{no units}) \quad (6.7)$$

where the subscripts 1 and 2 denote two arbitrarily selected points on the NCL.

The other is called the modulus of volume compressibility, m_v , and is obtained from the plot of ε_z versus σ'_z (Fig. 6.3c) as

$$m_v = -\frac{(\varepsilon_z)_2 - (\varepsilon_z)_1}{(\sigma'_z)_2 - (\sigma'_z)_1} = -\frac{|\Delta \varepsilon_z|}{(\sigma'_z)_2 - (\sigma'_z)_1} \quad \left(\frac{\text{m}^2}{\text{kN}}\right) \quad (6.8)$$

where the subscripts 1 and 2 denote two arbitrarily selected points on the NCL.

Similarly, we can define the slope BC in Fig. 6.3b as the recompression index, C_r , which we can express as

$$C_r = -\frac{e_2 - e_1}{\log \frac{(\sigma'_z)_2}{(\sigma'_z)_1}} = -\frac{|\Delta e_r|}{\log \frac{(\sigma'_z)_2}{(\sigma'_z)_1}} \quad (6.9)$$

where the subscripts 1 and 2 denote two arbitrarily selected points on the URL.

The slope BC in Fig. 6.3c is called the modulus of volume recompressibility, m_{vr} , and is expressed as

$$m_{vr} = -\frac{(\varepsilon_z)_2 - (\varepsilon_z)_1}{(\sigma'_z)_2 - (\sigma'_z)_1} = -\frac{|\Delta \varepsilon_{zr}|}{\Delta \sigma'_z} \quad \left(\frac{\text{m}^2}{\text{kN}}\right) \quad (6.10)$$

where the subscripts 1 and 2 denote two arbitrarily selected points on the URL.

From Hooke's law, we know that Young's modulus of elasticity is

$$E'_c = \frac{\Delta \sigma'_z}{\Delta \varepsilon_z} \quad (6.11)$$

where the subscript c denotes constrained because we are constraining the soil to settle only in one direction (one-dimensional consolidation). We can rewrite Eq. (6.11) as

$$E'_c = \frac{1}{m_{vr}} \quad (6.12)$$

The slopes C_c , C_r , m_v , and m_{vr} are taken as positive values to satisfy our sign convention of compression or recompression as positive.

6.3.10 Effects of Loading History

In our experiment, we found that during reloading the soil follows the normal consolidation line once the past maximum vertical effective stress is exceeded. The history of loading of a soil is locked in its fabric and the soil maintains a memory of the past maximum effective stress. To understand how the soil will respond to loads, we have to unlock its memory. If a soil were to be consolidated to stresses below its past maximum vertical effective stress, then settlement would be small because the soil fabric was permanently changed by a higher stress in the past. However, if the soil were to be consolidated beyond its past maximum effective stress, settlement would be large for stresses beyond its past

maximum effective stress because the soil fabric would now undergo further change from a current loading that is higher than its past maximum effective stress.

The practical significance of this soil behavior is that if the loading imposed on the soil by a structure is such that the vertical effective stress in the soil does not exceed its past maximum vertical effective stress, the settlement of the structure would be small, otherwise significant permanent settlement would occur. The preconsolidation stress defines the limit of elastic behavior. For stresses that are lower than the preconsolidation stress, the soil will follow the URL and we can reasonably assume that the soil will behave like an elastic material. For stresses greater than the preconsolidation stress the soil would behave like an elastoplastic material.

6.3.11 Overconsolidation Ratio

We will create a demarcation for soils based on their consolidation history. We will label a soil whose current vertical effective stress or overburden effective stress, σ'_{zo} , is less than its past maximum vertical effective stress or preconsolidation stress, σ'_{zc} , as an overconsolidated soil. An overconsolidated soil will follow a void ratio versus vertical effective stress path similar to *CDE* (Fig. 6.3) during loading. The degree of overconsolidation, called overconsolidation ratio, OCR, is defined as

$$\text{OCR} = \frac{\sigma'_{zc}}{\sigma'_{zo}} \quad (6.13)$$

If $\text{OCR} = 1$, the soil is normally consolidated soil. Normally consolidated soils follow paths similar to *ABE* (Fig. 6.3).

6.3.12 Possible and Impossible Consolidation Soil States

The normal consolidation line delineates possible from impossible soil states. Unloading of a soil or reloading it cannot bring it to soil states right of the normal consolidation line, which we will call impossible soil states (Fig. 6.3). Possible soil states only occur on or to the left of the normal consolidation line.

The essential points are:

1. *Path AB (Fig. 6.3), called the normal consolidation line (NCL), describes the response of a normally consolidated soil—a soil that has never experienced a vertical effective stress greater than its current vertical effective stress. The NCL is approximately a straight line in a plot of e versus $\log \sigma'_z$ and is defined by a slope, C_c , called the compression index.*
2. *A normally consolidated soil would behave like an elastoplastic material. That is, part of the settlement under the load is recoverable while the other part is permanent.*
3. *An overconsolidated soil has experienced vertical effective stresses greater than its current vertical effective stress.*
4. *An overconsolidated soil will follow paths such as CDE (Fig. 6.3). For stresses below the preconsolidation stress, an overconsolidated soil would approximately behave like an elastic material and settlement would be small. However, for stresses greater than the preconsolidation stress, an overconsolidated soil will behave like an elastoplastic material, similar to a normally consolidated soil.*

What's next... Next, we will consider how to use the basic concepts to calculate one-dimensional settlement.

6.4 CALCULATION OF PRIMARY CONSOLIDATION SETTLEMENT



Interactive Concept Learning and Self-Assessment

Access Chapter 6, Section 6.4 on the CD to learn how to calculate one-dimensional consolidation settlement. Take Quiz 6.4 to assess your understanding and get problem-solving practice.

6.4.1 Effects of Unloading/Reloading of a Soil Sample Taken from the Field

Let us consider a soil sample that we wish to take from the field at a depth z (Fig. 6.4a). We will assume that the groundwater level is at the surface. The current vertical effective stress or overburden effective stress is

$$\sigma'_{z0} = (\gamma_{\text{sat}} - \gamma_w)z = \gamma'z$$

and the current void ratio can be found from γ_{sat} using Eq. 3.11. On a plot of e versus σ'_z (log scale), the current vertical effective stress can be represented as A as depicted in Fig. 6.4b.

To obtain a sample, we would have to make a borehole and remove the soil above it. The act of removing the soil and extracting the sample reduces the total stress to zero; that is, we have fully unloaded the soil. From the principle of effective stress [Eq. (5.39)], $\sigma'_z = -\Delta u$. Since σ' cannot be negative—that is, soil cannot sustain tension—the porewater pressure must be negative. As the porewater pressure dissipates with time, volume changes (swelling) occur. Using the basic concepts of consolidation described in Section 6.3, the sample will follow an unloading path AB (Fig. 6.4b). The point B does not correspond to zero effective stress because we cannot represent zero on a logarithmic scale. However, the effective stress level at the start of the logarithmic scale is assumed to be small (≈ 0). If we were to reload our soil sample, the reloading path followed depends on the OCR. If $\text{OCR} = 1$ (normally consolidated soil), the path followed during reloading would be BCD (Fig. 6.4b). The average slope of ABC is C_r . Once σ'_{z0} is exceeded, the soil will follow the normal consolidation line, CD , of slope C_c . If the soil were overconsolidated, $\text{OCR} > 1$, the reloading path followed would be BEF because we have to reload the soil beyond its preconsolidation stress before it behaves like a

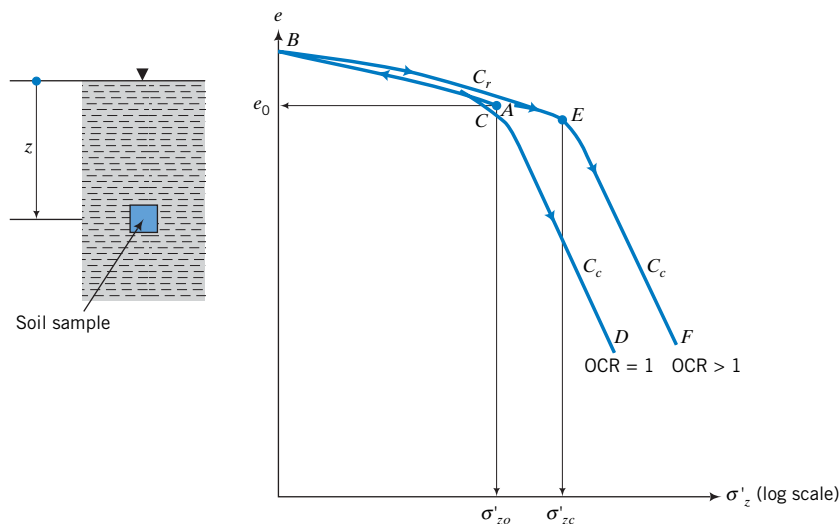


FIGURE 6.4 (a) Soil sample at a depth z below ground surface. (b) Expected one-dimensional response.

normally consolidated soil. The average slope of ABE is C_r and the slope of EF is C_c . The point E marks the preconsolidation stress. Later in this chapter, we will determine the position of E from laboratory tests (Section 6.7).

6.4.2 Primary Consolidation Settlement of Normally Consolidated Fine-Grained Soils

Let us consider a site consisting of a normally consolidated soil on which we wish to construct a building. We will assume that the increase in vertical stress due to the building at depth z , where we took our soil sample, is $\Delta\sigma_z$. (Recall that you can find $\Delta\sigma_z$ using the methods described in Section 5.11.) The final vertical stress is

$$\sigma'_{\text{fin}} = \sigma'_{z0} + \Delta\sigma_z$$

The increase in vertical stress will cause the soil to settle following the NCL and the primary consolidation settlement is

$$\rho_{\text{pc}} = H_o \frac{\Delta e}{1 + e_o} = \frac{H_o}{1 + e_o} C_c \log \frac{\sigma'_{\text{fin}}}{\sigma'_{z0}}; \text{OCR} = 1 \quad (6.14)$$

where $\Delta e = C_c \log(\sigma'_{\text{fin}}/\sigma'_{z0})$.

6.4.3 Primary Consolidation Settlement of Overconsolidated Fine-Grained Soils

If the soil is overconsolidated, we have to consider two cases depending on the magnitude of $\Delta\sigma_z$. We will approximate the curve in the σ'_z (log scale) versus e space as two straight lines, as shown in Fig. 6.5. In Case 1, the increase in $\Delta\sigma_z$ is such that $\sigma'_{\text{fin}} = \sigma'_{z0} + \Delta\sigma_z$ is less than σ'_{zc} (Fig. 6.5a). In this case, consolidation occurs along the URL and

$$\rho_{\text{pc}} = \frac{H_o}{1 + e_o} C_r \log \frac{\sigma'_{\text{fin}}}{\sigma'_{z0}}; \sigma'_{\text{fin}} < \sigma'_{zc} \quad (6.15)$$

In Case 2, the increase in $\Delta\sigma_z$ is such that $\sigma'_{\text{fin}} = \sigma'_{z0} + \Delta\sigma_z$ is greater than σ'_{zc} (Fig. 6.5b). In this case, we have to consider two components of settlement—one along the URL and the other along the

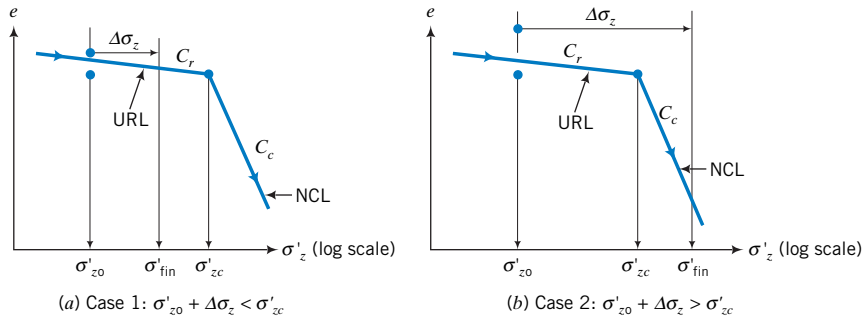


FIGURE 6.5 Two cases to consider for calculating settlement of overconsolidated fine-grained soils.

NCL. The equation to use in Case 2 is

$$\rho_{pc} = \frac{H_o}{1 + e_o} \left(C_r \log \frac{\sigma'_{zc}}{\sigma'_{zo}} + C_c \log \frac{\sigma'_{fin}}{\sigma'_{zc}} \right); \sigma'_{fin} > \sigma'_{zc} \quad (6.16)$$

or

$$\rho_{pc} = \frac{H_o}{1 + e_o} \left[C_r \log(\text{OCR}) + C_c \log \frac{\sigma'_{fin}}{\sigma'_{zc}} \right]; \sigma'_{fin} > \sigma'_{zc} \quad (6.17)$$

6.4.4 Procedure to Calculate Primary Consolidation Settlement

The procedure to calculate primary consolidation settlement is as follows:

1. Calculate the current vertical effective stress (σ'_{zo}) and the current void ratio (e_o) at the center of the soil layer for which settlement is required.
2. Calculate the applied vertical stress increase ($\Delta\sigma_z$) at the center of the soil layer using the appropriate method in Section 5.11.
3. Calculate the final vertical effective stress $\sigma'_{fin} = \sigma'_{zo} + \Delta\sigma_z$.
4. Calculate the primary consolidation settlement.

(a) If the soil is normally consolidated ($\text{OCR} = 1$), the primary consolidation settlement is

$$\rho_{pc} = \frac{H_o}{1 + e_o} C_c \log \frac{\sigma'_{fin}}{\sigma'_{zo}}$$

(b) If the soil is overconsolidated and $\sigma'_{fin} < \sigma'_{zc}$, the primary consolidation settlement is

$$\rho_{pc} = \frac{H_o}{1 + e_o} C_r \log \frac{\sigma'_{fin}}{\sigma'_{zo}}$$

(c) If the soil is overconsolidated and $\sigma'_{fin} > \sigma'_{zc}$, the primary consolidation settlement is

$$\rho_{pc} = \frac{H_o}{1 + e_o} \left(C_r \log(\text{OCR}) + C_c \log \frac{\sigma'_{fin}}{\sigma'_{zc}} \right)$$

where H_o is the thickness of the soil layer.

You can also calculate the primary consolidation settlement using m_v . However, unlike C_c , which is constant, m_v varies with stress levels. You should compute an average value of m_v over the stress range σ'_{zo} to σ'_{fin} . The primary consolidation settlement, using m_v , is

$$\rho_{pc} = H_o m_v \Delta\sigma_z \quad (6.18)$$

The advantage of using Eq. (6.18) is that m_v is readily determined from displacement data in consolidation tests; you do not have to calculate void ratio changes from the test data as required to determine C_c .

6.4.5 Thick Soil Layers

For better accuracy, when dealing with thick layers ($H_o > 2m$), you should divide the soil layer into sublayers (about two to five sublayers) and find the settlement for each sublayer. Add up the settlement of each sublayer to find the total primary consolidation settlement. You must remember that the value of H_o in the primary consolidation equations is the thickness of the sublayer. An alternative method is to use a harmonic mean value of the vertical stress increase for the sublayers in

the equations for primary consolidation settlement. The harmonic mean stress increase is

$$\Delta\sigma_z = \frac{n(\Delta\sigma_z)_1 + (n-1)(\Delta\sigma_z)_2 + (n-2)(\Delta\sigma_z)_3 + \cdots + (\Delta\sigma_z)_n}{n + (n-1) + (n-2) + \cdots + 1} \quad (6.19)$$

where n is the number of sublayers and the subscripts 1, 2, etc., mean the first (top) layer, the second layer, and so on. The advantage of using the harmonic mean is that the settlement is skewed in favor of the upper part of the soil layer. You should recall from Chapter 5 that the increase in vertical stress decreases with depth. Therefore, the primary consolidation settlement of the upper portion of the soil layer can be expected to be more than the lower portion because the upper portion of the soil layer is subjected to higher vertical stress increases.

EXAMPLE 6.1 Consolidation Settlement of a Normally Consolidated Clay

Interactive Problem Solving

Access Chapter 6, click Problem Solver on the sidebar menu and then click on Example 6.1 to interactively solve problems dealing with one-dimensional consolidation settlement.

The soil profile at a site for a proposed office building consists of a layer of fine sand 10.4 m thick above a layer of soft normally consolidated clay 2 m thick. Below the soft clay is a deposit of coarse sand. The groundwater table was observed at 3 m below ground level. The void ratio of the sand is 0.76 and the water content of the clay is 43%. The building will impose a vertical stress increase of 140 kPa at the middle of the clay layer. Estimate the primary consolidation settlement of the clay. Assume the soil above the water table to be saturated, $C_c = 0.3$ and $G_s = 2.7$.

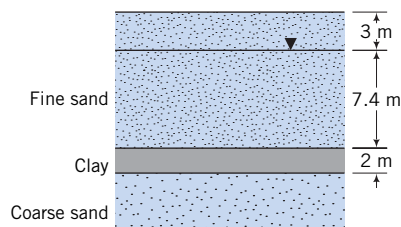


FIGURE E6.1

Strategy You should write down what is given or known and draw a diagram of the soil profile (see Fig. E6.1). In this problem, you are given the stratigraphy, groundwater level, vertical stress increase, and the following soil parameters and soil condition:

$$e_o \text{ (for sand)} = 0.76; w \text{ (for clay)} = 43\%$$

$$H_o = 2 \text{ m}, \Delta\sigma_2 = 140 \text{ kPa}, C_c = 0.3, G_s = 2.7$$

Since you are given a normally consolidated clay, the primary consolidation settlement response of the soil will follow path ABE (Fig. 6.3). The appropriate equation to use is Eq. (6.14).

Solution 6.1

Step 1: Calculate σ'_{z0} and e_o at the center of the clay layer.

$$\text{Sand: } \gamma_{\text{sat}} = \left(\frac{G_s + e}{1 + e} \right) \gamma_w = \left(\frac{2.7 + 0.76}{1 + 0.76} \right) 9.8 = 19.3 \text{ kN/m}^3$$

$$\gamma' = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w = \left(\frac{2.7 - 1}{1 + 0.76} \right) 9.8 = 9.5 \text{ kN/m}^3$$

$$\text{or } \gamma' = \gamma_{\text{sat}} - \gamma_w = 19.3 - 9.8 = 9.5 \text{ kN/m}^3$$

$$\text{Clay: } e_o = wG_s = 2.7 \times 0.43 = 1.16;$$

$$\gamma' = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w = \left(\frac{2.7 - 1}{1 + 1.16} \right) 9.8 = 7.7 \text{ kN/m}^3$$

The vertical effective stress at the mid-depth of the clay layer is

$$\sigma'_{z_o} = (19.3 \times 3) + (9.5 \times 7.4) + (7.7 \times 1) = 135.9 \text{ kPa}$$

Step 2: Calculate the increase of stress at the mid-depth of the clay layer. You do not need to calculate $\Delta\sigma_z$ for this problem. It is given as $\Delta\sigma_z = 140 \text{ kPa}$.

Step 3: Calculate σ'_{fin} .

$$\sigma'_{\text{fin}} = \sigma'_{z_o} + \Delta\sigma_z = 135.9 + 140 = 275.9 \text{ kPa}$$

Step 4: Calculate the primary consolidation settlement.

$$\rho_{\text{pc}} = \frac{H_o}{1 + e_o} C_c \log \frac{\sigma'_{\text{fin}}}{\sigma'_{z_o}} = \frac{2}{1 + 1.16} \times 0.3 \log \frac{275.9}{135.9} = 0.085 \text{ m} = 85 \text{ mm}$$

EXAMPLE 6.2 Consolidation Settlement of an Overconsolidated Clay

Assume the same soil stratigraphy as in Example 6.1. But now the clay is overconsolidated with an $\text{OCR} = 2.5$, $w = 38\%$, and $C_r = 0.05$. All other soil values given in Example 6.1 remain unchanged. Determine the primary consolidation settlement of the clay.

Strategy Since the soil is overconsolidated, you will have to check whether the preconsolidation stress is less than or greater than the sum of the current vertical effective stress and the applied vertical stress at the center of the clay. This check will determine the appropriate equation to use. In this problem, the unit weight of the sand is unchanged but the clay has changed.

Solution 6.2

Step 1: Calculate σ'_{z_o} and e_o at mid-depth of the clay layer.

You should note that this settlement is small compared with the settlement obtained in Example 6.1.

$$\text{Clay: } e_o = wG_s = 0.38 \times 2.7 = 1.03$$

$$\gamma' = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w = \left(\frac{2.7 - 1}{1 + 1.03} \right) 9.8 = 8.2 \text{ kN/m}^3$$

$$\sigma'_{z_o} = (19.3 \times 3) + (9.5 \times 7.4) + (8.2 \times 1) = 136.4 \text{ kPa}$$

(note that the increase in vertical effective stress from the unit weight change in this overconsolidated clay is very small)

Step 2: Calculate the preconsolidation stress.

$$\sigma'_{zc} = 136.4 \times 2.5 = 341 \text{ kPa}$$

Step 3: Calculate σ'_{fin}

$$\sigma'_{\text{fin}} = \sigma'_{z_0} + \Delta\sigma_z = 136.4 + 140 = 276.4 \text{ kPa}$$

Step 4: Check if σ'_{fin} is less than or greater than σ'_{zc} .

$$(\sigma'_{\text{fin}} = 276.4 \text{ kPa}) < (\sigma'_{zc} = 341 \text{ kPa})$$

Therefore, use Eq. (6.15).

Step 5: Calculate the total primary consolidation settlement.

$$\rho_{\text{pc}} = \frac{H_o}{1 + e_o} C_r \log \frac{\sigma'_{\text{fin}}}{\sigma'_{z_0}} = \frac{2}{1 + 1.03} \times 0.05 \log \frac{276.4}{136.4} = 0.015 \text{ m} = 15 \text{ mm}$$

EXAMPLE 6.3 Consolidation Settlement of an Overconsolidated Clay

Assume the same soil stratigraphy and soil parameters as in Example 6.2 except that the clay has an overconsolidation ratio of 1.5. Determine the primary consolidation settlement of the clay.

Strategy Since the soil is overconsolidated, you will have to check whether the preconsolidation stress is less than or greater than the sum of the current vertical effective stress and the applied vertical stress at the center of the clay. This check will determine the appropriate equation to use.

Solution 6.3

Step 1: Calculate σ'_{z_0} and e_o .

From Example 6.2, $\sigma'_{z_0} = 136.4 \text{ kPa}$.

Step 2: Calculate the preconsolidation stress.

$$\sigma'_{zc} = 136.4 \times 1.5 = 204.6 \text{ kPa}$$

Step 3: Calculate σ'_{fin} .

$$\sigma'_{\text{fin}} = \sigma'_{z_0} + \Delta\sigma_z = 136.4 + 140 = 276.4 \text{ kPa}$$

Step 4: Check if σ'_{fin} is less than or greater than σ'_{zc} .

$$(\sigma'_{\text{fin}} = 276.4 \text{ kPa}) > (\sigma'_{zc} = 204.6 \text{ kPa})$$

Therefore, use either Eq. (6.16) or (6.17).

Step 5: Calculate the total primary consolidation settlement.

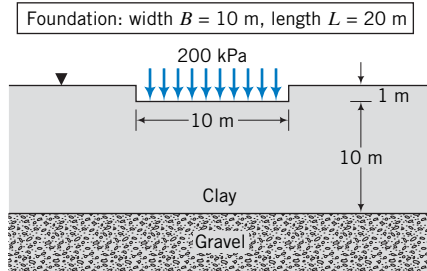
$$\begin{aligned} \rho_{\text{pc}} &= \frac{H_o}{1 + e_o} \left\{ C_r \log \frac{\sigma'_{zc}}{\sigma'_{z_0}} + C_c \log \frac{\sigma'_{\text{fin}}}{\sigma'_{zc}} \right\} = \frac{2}{1 + 1.03} \\ &\times \left(0.05 \log \frac{204.6}{136.4} + 0.3 \log \frac{276.4}{204.6} \right) = 0.047 \text{ m} = 47 \text{ mm} \end{aligned}$$

or

$$\begin{aligned} \rho_{\text{pc}} &= \frac{H_o}{1 + e_o} \left\{ C_r \log(\text{OCR}) + C_c \log \frac{\sigma'_{\text{fin}}}{\sigma'_{zc}} \right\} = \frac{1}{1 + 1.03} \\ &\times \left(0.05 \log 1.5 + 0.3 \log \frac{276.4}{204.6} \right) = 0.047 \text{ m} = 47 \text{ mm} \end{aligned}$$

EXAMPLE 6.4 Consolidation Settlement Using m_v

A vertical section through a building foundation at a site is shown in Fig. E6.4. The average modulus of volume compressibility of the clay is $m_v = 5 \times 10^{-5} \text{ m}^2/\text{kN}$. Determine the primary consolidation settlement.

**FIGURE E6.4**

Strategy To find the primary consolidation settlement, you need to know the vertical stress increase in the clay layer from the building load. Since the clay layer is finite, we will have to use the vertical stress influence values in Appendix C. If we assume a rough base, we can use the influence values specified by Milovic and Tournier (1971) or if we assume a smooth base we can use the values specified by Sovinc (1961). The clay layer is 10 m thick, so it is best to subdivide the clay layer into sublayers $\leq 2 \text{ m}$ thick.

Solution 6.4

Step 1: Find the vertical stress increase at the center of the clay layer below the foundation. Divide the clay layer into five sublayers, each of thickness 2 m—that is, $H_o = 2 \text{ m}$. Find the vertical stress increase at the middle of each sublayer under the center of the rectangular foundation. Assume a rough base and use Table C1 (Appendix C).

$$B = 10 \text{ m}, \quad L = 20 \text{ m}, \quad \frac{L}{B} = 2, \quad q_s = 200 \text{ kPa}$$

Layer	$z \text{ (m)}$	$\frac{z}{B}$	I_{zp}	$\Delta\sigma_z = I_{zp}q_s$ (kPa)
1	1	0.1	0.992	198.4
2	3	0.3	0.951	190.2
3	5	0.5	0.876	175.2
4	7	0.7	0.781	156.2
5	9	0.9	0.686	137.2

Step 2: Calculate the primary consolidation settlement. Use Eq. (6.18).

$$\begin{aligned} \rho_{pc} &= \sum_{i=1}^n (H_o m_v \Delta\sigma_z)_i = 2 \times 5 \times 10^{-5} \times (198.4 + 190.2 + 175.2 \\ &\quad + 156.2 + 137.2) = 0.086 \text{ m} = 86 \text{ mm} \end{aligned}$$

Alternatively: Use the harmonic mean value of $\Delta\sigma_z$ with $n = 5$; that is, Eq. (6.19)

$$\Delta\sigma_z = \frac{5(198.4) + 4(190.2) + 3(175.2) + 2(156.2) + 1(137.2)}{5 + 4 + 3 + 2 + 1} = 181.9 \text{ kPa}$$

$$\rho_{pc} = 10 \times 5 \times 10^{-5} \times 181.9 = 0.091 \text{ m} = 91 \text{ mm}$$

The greater settlement in this method results from the bias toward the top layer. ■

EXAMPLE 6.5 Estimating OCR Variation with Depth

A laboratory test on a saturated clay taken at a depth of 10 m below the ground surface gave the following results: $C_c = 0.3$, $C_r = 0.08$, OCR = 5, $w = 23\%$, and $G_s = 2.7$. The groundwater level is at the surface. Determine and plot the variation of water content and overconsolidation ratio with depth up to 50 m.

Strategy The overconsolidation state lies on the unloading/reloading line (Fig. 6.3), so you need to find an equation for this line using the data given. Identify what given data is relevant to finding the equation for the unloading/reloading line. Here you are given the slope, C_r , so you need to use the other data to find the coordinate of one point on the unloading/reloading line from the water content and the depth as shown in Step 1.

Solution 6.5

Step 1: Determine e_o and σ'_{zo} .

$$\gamma' = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w = \left(\frac{2.7 - 1}{1 + 0.621} \right) 9.8 = 10.3 \text{ kN/m}^3$$

$$e_o = G_s w = 2.7 \times 0.23 = 0.621$$

$$\sigma'_{zo} = \gamma' z = 10.3 \times 10 = 103 \text{ kPa}$$

Step 2: Determine the preconsolidation stress.

$$\sigma'_{zc} = \sigma'_{zo} \times \text{OCR} = 103 \times 5 = 515 \text{ kPa}$$

Step 3: Find the equation for the URL (slope BC in Fig. E6.5a).

$$e_B = e_o - C_r \log \frac{\sigma'_{zc}}{\sigma'_{zo}}$$

Therefore,

$$e_B = 0.621 - 0.08 \log(5) = 0.565$$

Hence, the equation for the unloading/reloading line is

$$e = 0.565 + 0.08 \log(\text{OCR}) \quad (1)$$

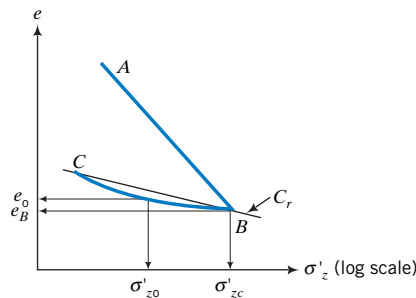


FIGURE E6.5a

Substituting $e = wG_s$ ($G_s = 2.7$) and $\text{OCR} = 515/\gamma'z$ ($\gamma' = 10.3 \text{ kN/m}^3$, z is depth) in Eq. (1) gives

$$w = 0.209 + 0.03 \log\left(\frac{50}{z}\right)$$

You can now substitute values of z from 1 to 50 and find w and substitute $e = wG_s$ in Eq. (1) to find the OCR. The table below shows the calculated values and the results, which are plotted in Fig. E6.5b.

Depth	w (%)	OCR
1	26.0	51.4
5	23.9	10
10	23.0	5
20	22.1	2.5
30	21.6	1.7
40	21.2	1.2
50	20.9	1.0

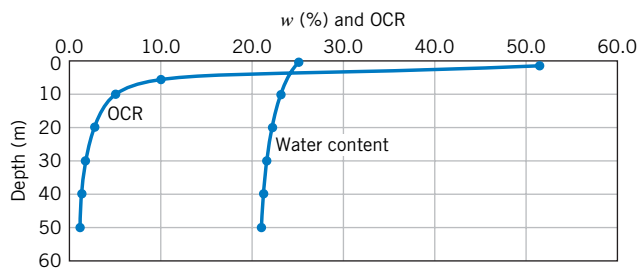


FIGURE E6.5b

You should note that the soil becomes normally consolidated as the depth increases. This is a characteristic of real soils. ■

What's next... So far, we have only considered how to determine the final primary consolidation settlement. This settlement might take months or years to occur, depending essentially on the permeability of the soil, the soil thickness, drainage conditions, and the magnitude of the applied stress. Geotechnical engineers have to know the magnitude of the final primary consolidation settlement and also the rate of settlement so that the settlement at any given time can be evaluated.

The next section deals with a theory to determine the settlement at any time. Several assumptions are made in developing this theory. However, you will see that many of the observations we made in Section 6.3 are well described by this theory.

6.5 ONE-DIMENSIONAL CONSOLIDATION THEORY



Interactive Concept Learning and Self-Assessment

Access Chapter 6, Section 6.5 on the CD to learn the derivation of the dimensional consolidation equation step by step. Take Quiz 6.5 to test your understanding on the interpretation of this equation.

6.5.1 Derivation of Governing Equation

We now return to our experiment described in Section 6.3 to derive the theory for time rate of settlement using an element of the soil sample of thickness dz and cross-sectional area $dA = dx dy$ (Fig. 6.6). We will assume the following:

1. The soil is saturated, isotropic, and homogeneous.
2. Darcy's law is valid.
3. Flow only occurs vertically.
4. The strains are small.

We will use the following observations made in Section 6.3:

1. The change in volume of the soil (ΔV) is equal to the change in volume of porewater expelled (ΔV_w), which is equal to the change in the volume of the voids (ΔV_v). Since the area of the soil is constant (the soil is laterally constrained), the change in volume is directly proportional to the change in height.
2. At any depth, the change in vertical effective stress is equal to the change in excess porewater pressure at that depth. That is, $\partial \sigma'_z = \partial u$.

For our soil element in Fig. 6.6, the inflow of water is $q_v dA$ and the outflow over the elemental thickness dz is $q_v + (\partial q_v / \partial z) dz dA$. The change in flow is then $(\partial q_v / \partial z) dz dA$. The rate of change in volume of water expelled, which is equal to the rate of change of volume of the soil, must equal the change in flow. That is,

$$\frac{\partial V}{\partial t} = \frac{\partial q_v}{\partial z} dz dA \quad (6.20)$$

Recall [Eq. (6.2)] that the volumetric strain $\epsilon_p = \partial V / V = \partial e / (1 + e_o)$, and therefore

$$\partial V = \frac{\partial e}{1 + e_o} dz dA = m_v \partial \sigma'_z dz dA = m_v \partial u dz dA \quad (6.21)$$

Substituting Eq. (6.21) into Eq. (6.20) and simplifying, we obtain

$$\frac{\partial q_v}{\partial z} = \frac{\partial u}{\partial t} m_v \quad (6.22)$$

The one-dimensional flow of water from Darcy's law is

$$q_v = Ak_z i = Ak_z \frac{\partial h}{\partial z} \quad (6.23)$$

where k_z is the hydraulic conductivity in the vertical direction.

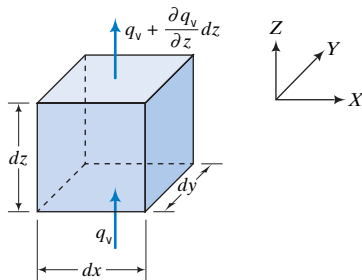


FIGURE 6.6 One-dimensional flow through a two-dimensional soil element.

Partial differentiation of Eq. (6.23) with respect to z gives

$$\frac{\partial q_v}{\partial z} = k_z \frac{\partial^2 h}{\partial z^2} \quad (6.24)$$

The porewater pressure at any time from our experiment in Section 6.3 is

$$u = h\gamma_w \quad (6.25)$$

where h is the height of water in the burette.

Partial differentiation of Eq. (6.25) with respect to z gives

$$\frac{\partial^2 h}{\partial z^2} = \frac{1}{\gamma_w} \frac{\partial^2 u}{\partial z^2} \quad (6.26)$$

By substitution of Eq. (6.26) into Eq. (6.24), we get

$$\frac{\partial q_v}{\partial z} = \frac{k_z}{\gamma_w} \frac{\partial^2 u}{\partial z^2} \quad (6.27)$$

Equating Eq. (6.22) and Eq. (6.27), we obtain

$$\frac{\partial u}{\partial t} = \frac{k_z}{m_v \gamma_w} \frac{\partial^2 u}{\partial z^2} \quad (6.28)$$

We can replace

$$\frac{k_z}{m_v \gamma_w} \text{ by a coefficient } C_v \text{ called the coefficient of consolidation}$$

The units for C_v are length²/time, for example, cm²/min. Rewriting Eq. (6.28) by substituting C_v , we get the general equation for one-dimensional consolidation as

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} \quad (6.29)$$

This equation describes the spatial variation of excess porewater pressure (Δu) with time (t) and depth (z). It is a common equation in many branches of engineering. For example, the heat diffusion equation commonly used in mechanical engineering is similar to Eq. (6.29) except that temperature, T , replaces u and heat factor, K , replaces C_v . Equation (6.29) is sometimes called the Terzaghi one-dimensional consolidation equation because Terzaghi (1925) developed it.

In the derivation of Eq. (6.29), we tacitly assumed that k_z and m_v are constants. This is usually not the case because as the soil consolidates the void spaces are reduced and k_z decreases. Also, m_v is not linearly related to σ'_z (Fig. 6.3c). The consequence of k_z and m_v not being constants is that C_v is not a constant. In practice, C_v is assumed to be a constant and this assumption is reasonable only if the stress changes are small enough such that k_z and m_v do not change significantly.

The essential point is:

1. *The one-dimensional consolidation equation allows us to predict the changes in excess porewater pressure at various depths within the soil with time. We need to know the excess porewater pressure at a desired time because we have to determine the vertical effective stress to calculate the primary consolidation settlement.*

What's next... In the next section, the solution to the one-dimensional consolidation equation is found for the case where the soil can drain from the top and bottom boundaries using two methods. One method is based on the Fourier series and the other method is based on the finite difference numerical scheme. The latter is simpler in programming and in spreadsheet applications for any boundary condition.

6.5.2 Solution of Governing Consolidation Equation Using Fourier Series

The solution of any differential equation requires a knowledge of the boundary conditions. By specification of the initial distribution of excess porewater pressures at the boundaries, we can obtain solutions for the spatial variation of excess porewater pressure with time and depth. Various distributions of porewater pressures within a soil layer are possible. Two of these are shown in Fig. 6.7. One of these is a uniform distribution of initial excess porewater pressure with depth (Fig. 6.7a). This may occur in a thin layer of fine-grained soils. The other (Fig. 6.7b) is a triangular distribution. This may occur in a thick layer of fine-grained soils.

The boundary conditions for a uniform distribution of initial excess porewater pressure in which double drainage occurs are

When $t = 0$, $\Delta u = \Delta u_o = \Delta \sigma_z$.

At the top boundary, $z = 0$, $\Delta u = 0$.

At the bottom boundary, $z = 2H_{dr}$, $\Delta u = 0$, where H_{dr} is the length of the drainage path.

A solution for the governing consolidation equation (6.29), which satisfies these boundary conditions, is obtained using the Fourier series,

$$\Delta u(z, t) = \sum_{m=0}^{\infty} \frac{2\Delta u_o}{M} \sin\left(\frac{Mz}{H_{dr}}\right) \exp(-M^2 T_v) \quad (6.30)$$

where $M = (\pi/2)(2m + 1)$ and m is a positive integer with values from 0 to ∞ and

$$T_v = \frac{C_v t}{H_{dr}^2} \quad (6.31)$$

where T_v is known as the time factor; it is a dimensionless term.

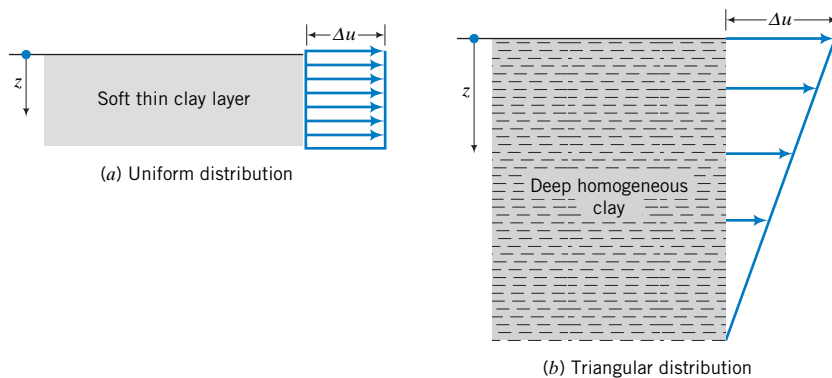


FIGURE 6.7 Two types of excess porewater pressure distribution with depth: (a) uniform distribution with depth in a thin layer and (b) triangular distribution with depth in a thick layer.

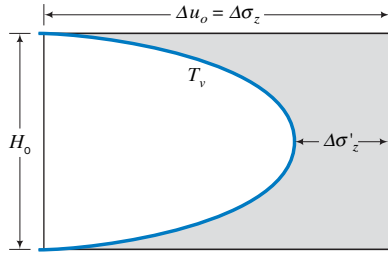


FIGURE 6.8 An isochrone illustrating the theoretical excess porewater pressure distribution with depth.

A plot of Eq. (6.30) gives the variation of excess porewater pressure with depth at different times. Let us examine Eq. (6.30) for an arbitrarily selected isochrone at any time t or time factor T_v as shown in Fig. 6.8. At time $t = 0$ ($T_v = 0$), the initial excess porewater pressure, Δu_o , is equal to the applied vertical stress throughout the soil layer. As soon as drainage occurs, the initial excess porewater pressure will immediately fall to zero at the permeable boundaries. The maximum excess porewater pressure occurs at the center of the soil layer because the drainage path there is the longest, as obtained earlier in our experiment in Section 6.3.

At time $t > 0$, the total applied vertical stress increment $\Delta \sigma_z$ at a depth z is equal to the sum of the vertical effective stress increment $\Delta \sigma'_z$ and the excess porewater pressure Δu_z . After considerable time ($t \rightarrow \infty$), the excess porewater pressure decreases to zero and the vertical effective stress increment becomes equal to the vertical total stress increment.

We now define a parameter, U_z , called the degree of consolidation or consolidation ratio, which gives us the amount of consolidation completed at a particular time and depth. This parameter can be expressed mathematically as

$$U_z = 1 - \frac{\Delta u_z}{\Delta u_o} = 1 - \sum_{m=0}^{\infty} \frac{2}{M} \sin\left(\frac{Mz}{H_{dr}}\right) \exp(-M^2 T_v) \quad (6.32)$$

The consolidation ratio is equal to zero everywhere at the beginning of the consolidation ($\Delta u_z = \Delta u_o$) but increases to unity as the initial excess porewater pressure dissipates.

A geotechnical engineer is often concerned with the average degree of consolidation, U , of a whole layer at a particular time rather than the consolidation at a particular depth. The shaded area in Fig. 6.8 represents the amount of consolidation of a soil layer at any given time. The average degree of consolidation can be expressed mathematically from the solution of the one-dimensional consolidation equation as

$$U = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp(-M^2 T_v) \quad (6.33)$$

Figure 6.9 shows the variation of the average degree of consolidation with time factor T_v for a uniform and a triangular distribution of excess porewater pressure.

A convenient set of equations for double drainage, found by curve fitting Fig. 6.9, is

$$T_v = \frac{\pi}{4} \left(\frac{U}{100} \right)^2 \quad \text{for } U < 60\% \quad (6.34)$$

and

$$T_v = 1.781 - 0.933 \log(100 - U) \quad \text{for } U \geq 60\% \quad (6.35)$$

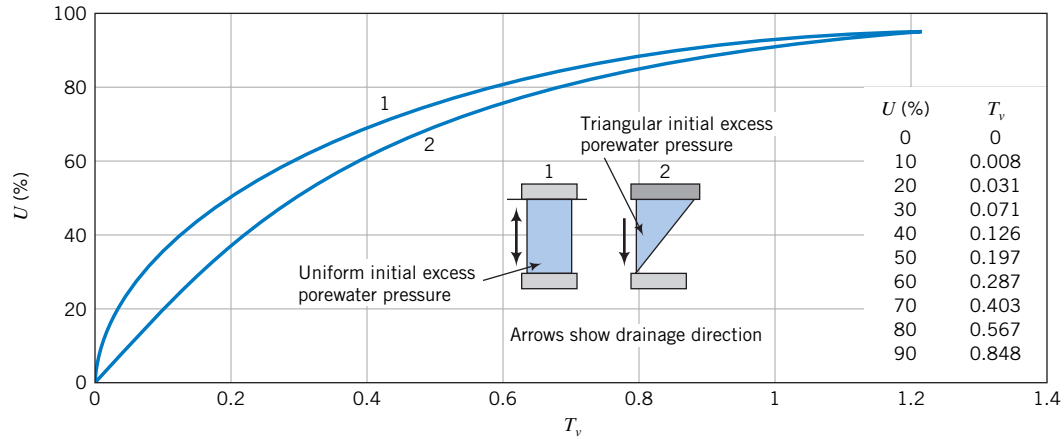


FIGURE 6.9 Relationship between time factor and average degree of consolidation for a uniform distribution and a triangular distribution of initial excess porewater pressure.

The time factor corresponding to every 10% of average degree of consolidation for double drainage conditions is shown in the inset table in Fig. 6.9. The time factors corresponding to 50% and 90% consolidation are often used in interpreting consolidation test results. You should remember that $T_v = 0.848$ for 90% consolidation, and $T_v = 0.197$ for 50% consolidation.

6.5.3 Finite Difference Solution of the Governing Consolidation Equation



Computer Program Utility

Access Chapter 6 on the CD, and click on Spreadsheet on the sidebar menu to view and use a spreadsheet solution of the finite difference equation of the one-dimensional consolidation equation. You can modify the copy of this spreadsheet to solve other problems. Read this section before accessing the spreadsheet.

Numerical methods (finite difference, finite element, and boundary element) provide approximate solutions to differential and integral equations for boundary conditions in which closed form solutions (analytical solutions) are not possible. We will use the finite difference method here to find a solution to the consolidation equation because it involves only the expansion of the differential equation using Taylor's theorem and can easily be adopted for spreadsheet applications.

Using Taylor's theorem,

$$\frac{\partial u}{\partial t} = \frac{1}{\Delta t} (u_{i,j+1} - u_{i,j}) \quad (6.36)$$

and

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{(\Delta z)^2} (u_{i-1,j} - 2u_{i,j} + u_{i+1,j}) \quad (6.37)$$

where (i, j) denotes a nodal position at the intersection of row i and column j . Columns represent time divisions and rows represent soil depth divisions. The assumption implicit in Eq. (6.36) is that the

excess porewater pressure between two adjacent nodes changes linearly with time. This assumption is reasonable if the distance between the two nodes is small. Substituting Eqs. (6.36) and (6.37) in the governing consolidation equation (6.29) and rearranging, we get

$$u_{i,j+1} = u_{i,j} + \frac{C_v \Delta t}{(\Delta z)^2} (u_{i-1,j} - 2u_{i,j} + u_{i+1,j}) \quad (6.38)$$

Equation (6.38) is valid for nodes that are not boundary nodes. There are special conditions that apply to boundary nodes. For example, at an impermeable boundary, no flow across it can occur and, consequently, $\partial u / \partial z = 0$ for which the finite difference equation is

$$\frac{\partial u}{\partial z} = 0 = \frac{1}{2\Delta z} (u_{i-1,j} - u_{i+1,j}) = 0 \quad (6.39)$$

and the governing consolidation equation becomes

$$u_{i,j+1} = u_{i,j} + \frac{C_v \Delta t}{(\Delta z)^2} (2u_{i-1,j} - 2u_{i,j}) \quad (6.40)$$

To determine how the porewater pressure is distributed within a soil at a given time, we have to establish the initial excess porewater pressure at the boundaries. Once we do this, we have to estimate the variation of the initial excess porewater pressure within the soil. We may, for example, assume a linear distribution of initial excess porewater pressure with depth if the soil layer is thin or a triangular distribution for a thick soil layer. If you cannot estimate the initial excess porewater pressure, then you can guess reasonable values for the interior of the soil or use linear interpolation. Then you successively apply Eq. (6.38) to each interior nodal point and replace the old value by the newly calculated value until the old value and the new value differ by a small tolerance. At impermeable boundaries, you have to apply Eq. (6.40).

The procedure to apply the finite difference form of the governing consolidation equation to determine the variation of excess porewater pressure with time and depth is as follows:

1. Divide the soil layer into a depth–time grid (Fig. 6.10). Rows represent subdivisions of the depth, columns represent subdivisions of time. Let's say we divide the depth into m rows and the time into n columns; then $\Delta z = H_o/m$ and $\Delta t = t/n$, where H_o is the thickness of the soil layer and t is the total time. A nodal point represents the i th depth position and the j th elapsed time. To avoid convergence problems, researchers have found that $\alpha = C_v \Delta t / (\Delta z)^2$ must be less than $\frac{1}{2}$. This places a limit on the number of subdivisions in the grid. Often, the depth is subdivided arbitrarily and the time step Δt is selected so that $\alpha < \frac{1}{2}$. In many practical situations, $\alpha = 0.25$ usually ensures convergence.
2. Identify the boundary conditions. For example, if the top boundary is a drainage boundary then the excess porewater pressure there is zero. If, however, the top boundary is an impermeable boundary, then no flow can occur across it and Eq. (6.40) applies.
3. Estimate the distribution of initial excess porewater pressure and determine the nodal initial excess porewater pressures.
4. Calculate the excess porewater pressure at interior nodes using Eq. (6.38) and at impermeable boundary nodes using Eq. (6.40). If the boundary is permeable, then the excess porewater pressure is zero at all nodes on this boundary.

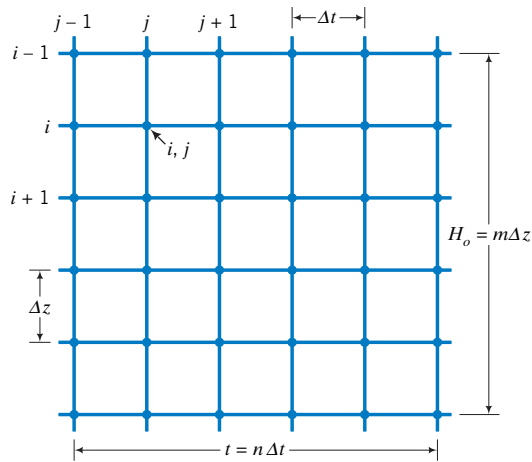


FIGURE 6.10 Division of a soil layer into a depth (row)–time (column) grid.

The essential points are:

1. *An ideal soil (isotropic, homogeneous, saturated) is assumed in developing the governing one-dimensional consolidation equation.*
2. *Strains are assumed to be small.*
3. *Excess porewater pressure dissipation depends on the time, soil thickness, drainage conditions, and permeability of the soil.*
4. *The decrease in initial excess porewater pressure causes an equivalent increase in vertical effective stress and settlement increases.*
5. *The average degree of consolidation is conventionally used to find the time rate of settlement.*

EXAMPLE 6.6 *Change in Vertical Effective Stress at a Given Degree of Consolidation*

A soft clay layer 1.5 m thick is sandwiched between layers of sand. The initial vertical total stress at the center of the clay layer is 200 kPa and the porewater pressure is 100 kPa. The increase in vertical stress at the center of the clay layer from a building foundation is 100 kPa. What is the vertical effective stress and excess porewater pressure at the center of the clay layer when 60% consolidation occurs?

Strategy You are given the increment in applied stress and the degree of consolidation. You know that the initial change in excess porewater pressure is equal to the change in applied vertical stress. From the data given decide on the appropriate equation, which in this case is Eq. (6.32).

Solution 6.6

Step 1: Calculate the initial excess porewater pressure.

$$\Delta u_o = \Delta \sigma_z = 100 \text{ kPa}$$

Step 2: Calculate the current excess porewater pressure at 60% consolidation. From Eq. (6.32),

$$\Delta u_z = \Delta u_o(1 - U_z) = 100(1 - 0.6) = 40 \text{ kPa}$$

Step 3: Calculate the vertical total stress and total excess porewater pressure.

$$\text{Vertical total stress: } \Delta\sigma_z = 200 + 100 = 300 \text{ kPa}$$

$$\text{Total pore water pressure: } 100 + 40 = 140 \text{ kPa}$$

Step 4: Calculate the current vertical effective stress.

$$\sigma'_z = \sigma_z - \Delta u_z = 300 - 140 = 160 \text{ kPa}$$

Alternatively:

Step 1: Calculate the initial vertical effective stress.

$$\text{Initial vertical effective stress} = 200 - 100 = 100 \text{ kPa}$$

Step 2: Same as Step 2 above.

Step 3: Calculate the increase in vertical effective stress at 60% consolidation.

$$\Delta\sigma'_z = 100 - 40 = 60 \text{ kPa}$$

Step 4: Calculate the current vertical effective stress.

$$\sigma'_z = 100 + 60 = 160 \text{ kPa}$$

EXAMPLE 6.7 *Application of Finite Difference Method to Calculate Porewater Pressure Distribution*

Computer Program Utility

Access Chapter 6 on the CD, and click on Spreadsheet on the sidebar menu to view and use a spreadsheet on the finite difference solution of the one-dimensional consolidation equation. You can modify the copy of this spreadsheet to solve other problems.

A layer of soft clay, 5 m thick, is drained at the top surface only. The initial excess porewater pressure from an applied load at time $t = 0$ is distributed according to $\Delta u_o = 80 - 2z^2$, where z is the depth measured from the top boundary. Determine the distribution of excess porewater pressure with depth after 6 months using the finite difference method. The coefficient of consolidation, C_v , is $8 \times 10^{-4} \text{ cm}^2/\text{s}$.

Strategy Divide the clay layer into, say, five equal layers of 1 m thickness and find the value of the initial excess porewater pressure at each node at time $t = 0$ using $\Delta u_o = 80 - 2z^2$. Then find a time step Δt that will lead to $\alpha < \frac{1}{2}$. The top boundary is a drainage boundary. Therefore, the excess porewater pressure is zero for all times at all nodes along this boundary. Use a spreadsheet or write a short program to do the iterations to solve the governing consolidation equation. You must note that the bottom boundary is not a drainage boundary and the relevant equation to use for the nodes at this boundary is Eq. (6.40).

Solution 6.7

Step 1: Divide the soil layer into a grid.
Divide the depth into five layers:

$$\Delta z = \frac{5}{5} = 1 \text{ m}$$

$$C_v = 8 \times 10^{-4} \text{ cm}^2/\text{s} = 8 \times 10^{-8} \text{ m}^2/\text{s} = 2.52 \text{ m}^2/\text{yr}$$

Assume $\Delta t = 0.1$ yr.

$$\alpha = \frac{C_v \Delta t}{\Delta z^2} = \frac{2.52 \times 0.1}{1^2} = 0.252 < 0.5$$

Step 2: Identify boundary conditions.

The bottom boundary is impermeable, therefore Eq. (6.40) applies to the nodes along this boundary. The top boundary is pervious, therefore the excess porewater pressure is zero at all times greater than zero.

Step 3: Determine the distribution of initial excess porewater pressure.

You are given the distribution of initial excess porewater pressure as $\Delta u_o = 80 - 2z^2$. At time $t = 0$ (column 1), insert the nodal values of initial excess for water pressure. For example, at row 2, column 1 (node 2), $\Delta u_o = 80 - 2 \times 1^2 = 78$ kPa. The initial excess porewater pressures are listed in column 1; see the table below.

Step 4: Calculate the excess porewater pressure at each node of the grid. The governing equation, except at the impermeable boundary, is

$$u_{i,j+1} = u_{i,j} + 0.252(u_{i-1,j} - 2u_{i,j} + u_{i+1,j})$$

Let us calculate the excess porewater pressure after 0.1 yr at the node located at row 2, column 2.

$$u_{2,1+1} = 78 + 0.252(0 - 2 \times 78 + 72) = 57 \text{ kPa}$$

At the bottom impermeable boundary (row 6, column 2), we have to apply Eq. (6.40), which is

$$u_{i,j+1} = u_{i,j} + 0.252(2u_{i-1,j} - 2u_{i,j})$$

Therefore,

$$u_{2,1+1} = 30 + 0.252(2 \times 48 - 2 \times 30) = 39 \text{ kPa}$$

The complete results, after 6 months, using a spreadsheet program are shown in the table below. The results are plotted in Fig. E6.7.

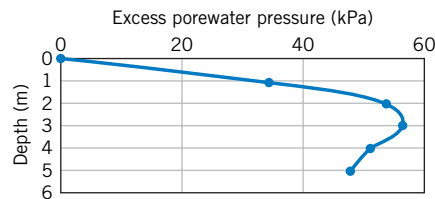


FIGURE E6.7

Column	1	2	3	4	5	6
Depth (m)	Time (yr)					
	0.00	0.10	0.20	0.30	0.4	0.5
0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	78.0	57	46.3	39.4	34.5	30.7
2.0	72.0	71	65.0	59.1	54.0	49.7
3.0	62.0	61	60.0	58.4	56.5	54.5
4.0	48.0	47	48.5	50.0	51.0	51.6
5.0	30.0	39	43.0	45.8	47.9	49.5

What's next... We have only described primary consolidation settlement. The other part of the total consolidation settlement is secondary compression, which will be discussed next.

6.6 SECONDARY COMPRESSION SETTLEMENT



Concept Learning and Self-Assessment

Access Chapter 6, Section 6.6 on the CD to learn about secondary compression and take Quiz 6.6 to assess your understanding of secondary compression.

You will recall from our experiment in Section 6.3 that consolidation settlement consisted of two parts. The first part is primary consolidation, which occurs at early times. The second part is secondary compression, or creep, which takes place under a constant vertical effective stress. The physical reasons for secondary compression in soils are not fully understood. One plausible explanation is the expulsion of water from micropores; another is viscous deformation of the soil structure.

We can make a plot of void ratio versus the logarithm of time from our experimental data in Section 6.3, as shown in Fig. 6.11. Primary consolidation is assumed to end at the intersection of the projection of the two straight parts of the curve.

The secondary compression index is

$$C_{\alpha} = -\frac{(e_t - e_p)}{\log(t/t_p)} = \frac{|\Delta e|}{\log(t/t_p)}; t > t_p \quad (6.41)$$

where (t_p, e_p) is the coordinate at the intersection of the tangents to the primary consolidation and secondary compression parts of the void ratio versus logarithm of time curve and (t, e_t) is the coordinate of any point on the secondary compression curve as shown in Fig. 6.11. The secondary consolidation settlement is

$$\rho_{sc} = \frac{H_o}{(1 + e_p)} C_{\alpha} \log\left(\frac{t}{t_p}\right) \quad (6.42)$$

Overconsolidated soils do not creep significantly but creep settlements in normally consolidated soils can be very significant.

What's next... You will recall that there are several unknown parameters in the theoretical solution of the one-dimensional consolidation equation. We need to know these parameters to calculate consolidation settlement

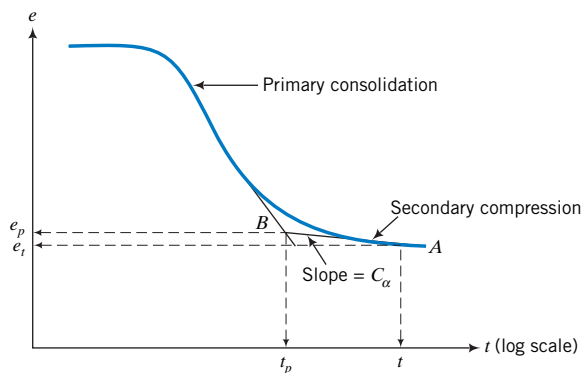


FIGURE 6.11 Secondary compression.

and rate of settlement for fine-grained soils. The one-dimensional consolidation test is used to find these parameters. This test is usually one of the tests that you will perform in the laboratory component of your course. In the next section, the test procedures will be briefly discussed followed by the methods used to determine the various soil consolidation parameters.

6.7 ONE-DIMENSIONAL CONSOLIDATION LABORATORY TEST



Virtual Laboratory and Self-Assessment

Access Chapter 6, Virtual Lab on the CD to conduct a virtual consolidation test. You will have your own 3-D apparatus and your own soil sample. You should complete the pretest and view a digital video describing how your soil sample was taken in the field. Follow the instructions to prepare your soil sample, set up the apparatus including the cell to run the test, interpret the results, and apply them to a practical situation. Answer Quiz 6.7 on the CD to test your understanding.

6.7.1 Oedometer Test

The one-dimensional consolidation test, called the oedometer test, is used to find C_c , C_r , C_α , C_v , m_v , and σ'_{zc} . The coefficient of permeability, k_z , can also be calculated from the test data. The experimental arrangement we used in Section 6.3 is similar to an oedometer test setup. The details of the test apparatus and the testing procedures are described in ASTM D2435. A disk of soil is enclosed in a stiff metal ring and placed between two porous stones in a cylindrical container filled with water, as shown in Fig. 6.12. A metal load platen mounted on top of the upper porous stone transmits the applied vertical stress (vertical total stress) to the soil sample. Both the metal platen and the upper porous stone can move vertically inside the ring as the soil settles under the applied vertical stress. The ring containing the soil sample can be fixed to the container by a collar (fixed ring cell, Fig. 6.12b) or is unrestrained (floating ring cell, Fig. 6.12c).

Incremental loads, including unloading sequences, are applied to the platen and the settlement of the soil at various fixed times, under each load increment, is measured by a displacement gauge. Each loading increment is allowed to remain on the soil until the change in settlement is negligible and the excess porewater pressure developed under the current load increment has dissipated. For many soils, this usually occurs within 24 hours, but longer monitoring times may be required for exceptional soil types, for example, montmorillonite. Each loading increment is doubled. The ratio of the load increment to the previous load is called the load increment ratio (LIR); conventionally, $LIR = 1$. To determine C_r , the soil sample is unloaded using a load decrement ratio—load decrement divided by current load—of 2.

At the end of the oedometer test, the apparatus is dismantled and the water content of the sample is determined. It is best to unload the soil sample to a small pressure before dismantling the apparatus, because if you remove the final consolidation load completely, a negative excess porewater pressure that equals the final consolidation pressure would develop. This negative excess porewater pressure can cause water to flow into the soil and increase the soil's water content. Consequently, the final void ratio calculated from the final water content would be erroneous.

The data obtained from the one-dimensional consolidation test are as follows:

1. Initial height of the soil, H_o , which is fixed by the height of the ring.
2. Current height of the soil at various time intervals under each load (time–settlement data).
3. Water content at the beginning and at the end of the test, and the dry weight of the soil at the end of the test.

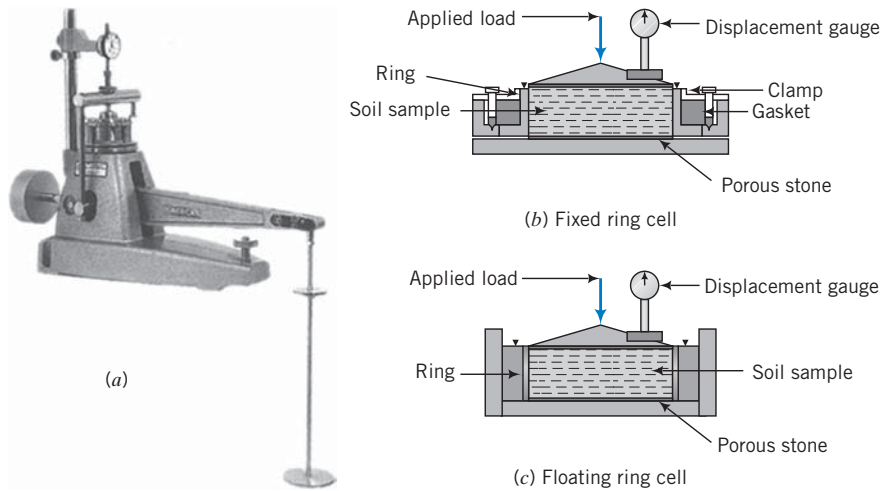


FIGURE 6.12 (a) A typical consolidation apparatus (photo courtesy of Geotest), (b) a fixed ring cell, and (c) a floating ring cell.

You now have to use these data to determine C_c , C_r , C_α , C_v , m_v , and σ'_{zc} . We will start with finding C_v .

6.7.2 Determination of the Coefficient of Consolidation

There are two popular methods that can be used to calculate C_v . Taylor (1942) proposed one method called the root time method. Casagrande and Fadum (1940) proposed the other method called the log time method. The root time method utilizes the early time response, which theoretically should appear as a straight line in a plot of square root of time versus displacement gauge reading.

6.7.2.1 Root Time Method (Square Root Time Method) Let us arbitrarily choose a point, C , on the displacement versus square root of time factor gauge reading as shown in Fig. 6.13. We will assume that this point corresponds to 90% consolidation ($U = 90\%$) for which $T_v = 0.848$ (Fig. 6.9). If point C were to lie on a straight line, the theoretical relationship between U and T_v would be $U = 0.98\sqrt{T_v}$; that is, if you substitute $T_v = 0.848$, you get $U = 90\%$.

At early times, the theoretical relationship between U and T_v is given by Eq. (6.34); that is,

$$U = \sqrt{\frac{4}{\pi}} T_v = 1.13\sqrt{T_v}; \quad U < 0.6$$

The laboratory early time response is represented by the straight line OA in Fig. 6.13. You should note that O is below the initial displacement gauge reading because there is an initial compression of the soil before consolidation begins. The ratio of the gradient of OA and the gradient of the theoretical early time response, line OCB , is

$$\frac{1.13\sqrt{T_v}}{0.98\sqrt{T_v}} = 1.15$$

We can use this ratio to establish the time when 90% consolidation is achieved in the one-dimensional consolidation test.

The procedure, with reference to Fig. 6.14, is as follows:

1. Plot the displacement gauge readings versus square root of times.

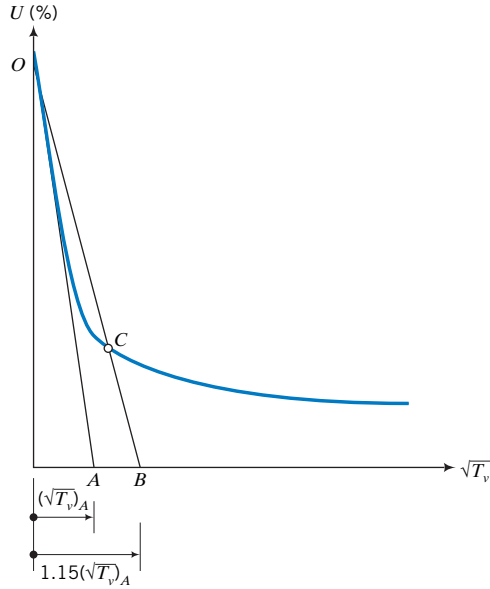


FIGURE 6.13 Correction of laboratory early time response to determine C_v .

2. Draw the best straight line through the initial part of the curve intersecting the ordinate (displacement reading) at O and the abscissa ($\sqrt{\text{time}}$) at A .
3. Note the time at point A ; let us say it is $\sqrt{t_A}$.
4. Locate a point B , $1.15\sqrt{t_A}$, on the abscissa.
5. Join OB .
6. The intersection of the line OB with the curve, point C , gives the displacement gauge reading and the time for 90% consolidation (t_{90}). You should note that the value read off the abscissa is $\sqrt{t_{90}}$. Now when $U = 90\%$, $T_v = 0.848$ (Fig. 6.9) and from Eq. (6.31) we obtain

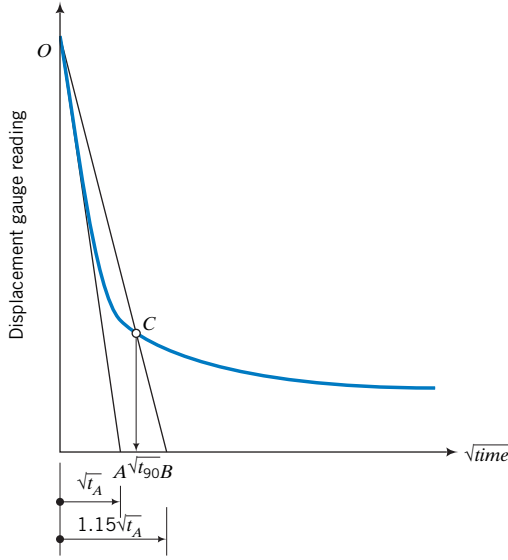
$$C_v = \frac{0.848 H_{dr}^2}{t_{90}} \quad (6.43)$$

where H_{dr} is the length of the drainage path.

6.7.2.2 Log Time Method In the log time method, the displacement gauge readings are plotted against the times (log scale). The logarithm of times is arbitrary and is only used for convenience. A typical curve obtained is shown in Fig. 6.15. The theoretical early time settlement response in a plot of logarithm of times versus displacement gauge readings is a parabola (Section 6.3). The experimental early time curve is not normally a parabola and a correction is often required.

The procedure, with reference to Fig. 6.15, is as follows:

1. Project the straight portions of the primary consolidation and secondary compression to intersect at A . The ordinate of A , d_{100} , is the displacement gauge reading for 100% primary consolidation.
2. Correct the initial portion of the curve to make it a parabola. Select a time t_1 , point B , near the head of the initial portion of the curve ($U < 60\%$) and then another time t_2 , point C , such that $t_2 = 4t_1$.
3. Calculate the difference in displacement reading, $\Delta d = d_2 - d_1$, between t_2 and t_1 . Plot a point D at a vertical distance Δd from B . The ordinate of point D is the corrected initial displacement gauge reading, d_0 , at the beginning of primary consolidation.

FIGURE 6.14 Root time method to determine C_v .

4. Calculate the ordinate for 50% consolidation as $d_{50} = (d_{100} + d_0)/2$. Draw a horizontal line through this point to intersect the curve at E . The abscissa of point E is the time for 50% consolidation, t_{50} .
5. You will recall (Fig. 6.9) that the time factor for 50% consolidation is 0.197 and from Eq. (6.31) we obtain

$$C_v = \frac{0.197 H_{dr}^2}{t_{50}} \quad (6.44)$$

The log time method makes use of the early (primary consolidation) and later time responses (secondary compression) while the root time method only utilizes the early time response, which is expected to be a straight line. In theory, the root time method should give good results except when nonlinearities arising from secondary compression cause substantial deviations from the expected straight line. These deviations are most pronounced in fine-grained soils with organic materials.

6.7.3 Determination of Void Ratio at the End of a Loading Step

To determine σ'_{zc} , C_c , C_r , and C_{α} , we need to know the void ratio for each loading step. Recall that at the end of the consolidation test, we determined the water content (w) of the soil sample. Using this data, initial height (H_0), and the specific gravity (G_s) of the soil sample, you can calculate the void ratio for each loading step as follows:

1. Calculate the final void ratio, $e_{fin} = wG_s$.
2. Calculate the total consolidation settlement of the soil sample during the test, $(\Delta z)_{fin} = d_{fin} - d_i$, where d_{fin} is the final displacement gauge reading and d_i is the displacement gauge reading at the start of the test.
3. Back-calculate the initial void ratio, $e_0 = e_{fin} + \Delta e$, where Δe is found from Eq. (6.4) as

$$\Delta e = \frac{(\Delta z)_{fin}}{H_0} (1 + e_0)$$

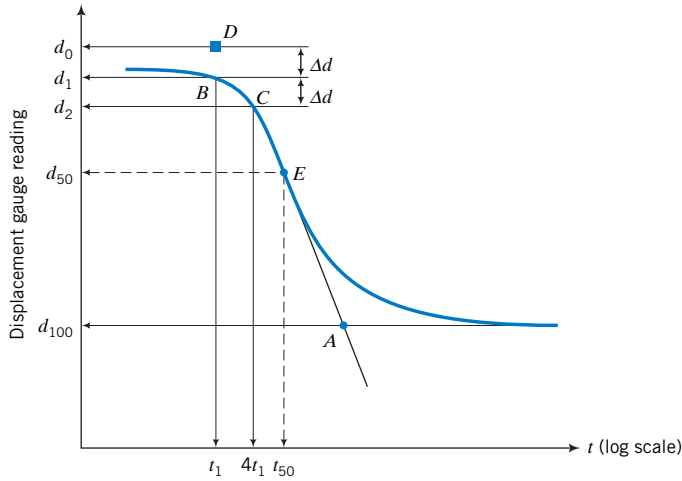


FIGURE 6.15 Log time method to determine C_v .

Therefore,

$$e_o = \frac{e_{\text{fin}} + \frac{(\Delta z)_{\text{fin}}}{H_o}}{1 - \frac{(\Delta z)_{\text{fin}}}{H_o}}$$

4. Calculate e for each loading step using Eq. (6.6).

6.7.4 Determination of the Preconsolidation Effective Stress

Now that we have calculated e for each loading step, we can plot a graph of the void ratio versus the vertical effective stress (log scale) as shown in Fig. 6.16. We will call Fig. 6.16 the e versus σ'_z (log scale) curve. You will now determine the preconsolidation effective stress using a method proposed by Casagrande (1936).

The procedure, with reference to Fig. 6.16, is as follows:

1. Identify the point of maximum curvature, point D , on the initial part of the curve.
2. Draw a horizontal line through D .

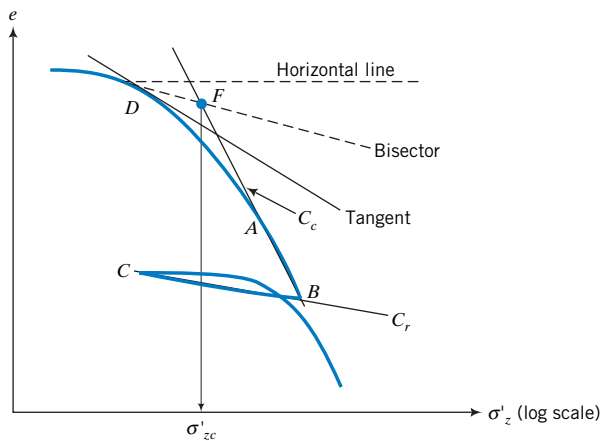


FIGURE 6.16 Determination of the preconsolidation stress using Casagrande's method.

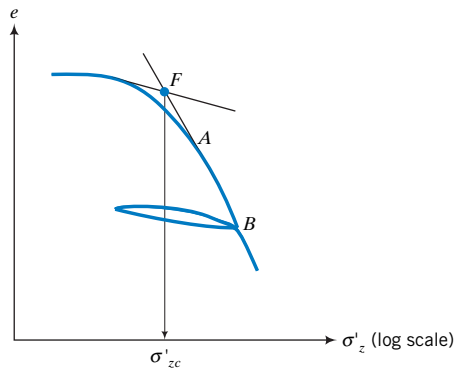


FIGURE 6.17 A simplified method of determining the preconsolidation effective stress.

3. Draw a tangent to the curve at D .
4. Bisect the angle formed by the tangent and the horizontal line at D .
5. Extend backward the straight portion of the curve (the normal consolidation line), BA , to intersect the bisector line at F .
6. The abscissa of F is the preconsolidation effective stress, σ'_{zc} .

A simpler method that is also used in practice is to project the straight portion of the initial recompression curve to intersect the backward projection of the normal consolidation line at F as shown in Fig. 6.17. The abscissa of F is σ'_{zc} .

Both of these methods are based on individual judgment. The actual value of σ'_{zc} for real soils is more difficult to ascertain than described above. Degradation of the soil from its intact condition caused by sampling, transportation, handling, and sample preparation usually does not produce the ideal curve shown in Fig. 6.16.

6.7.5 Determination of Compression and Recompression Indices

The slope of the normal consolidation line, BA , gives the compression index, C_c . To determine the recompression index, C_r , draw a line (BC) approximately mid-way between the unloading and reloading curves (Fig. 6.16). The slope of this line is the recompression index (C_r).

Field observations indicate that, in many instances, the predictions of the total settlement and the rate of settlement using oedometer test results do not match recorded settlement from actual structures. Disturbances from sampling and sample preparation tend to decrease C_c and C_v . Schmertmann (1953) suggested a correction to the laboratory curve to obtain a more representative in situ value of C_c . His method is as follows. Locate a point A at coordinate (σ'_{z0}, e_0) and a point B at ordinate $0.42e_0$ on the laboratory e versus σ'_z (log scale) curve, as shown in Fig. 6.18. The slope of the line AB is the corrected value for C_c .

6.7.6 Determination of the Modulus of Volume Change

The modulus of volume compressibility, m_v , is found from plotting a curve similar to Fig. 6.3c and determining the slope as shown in this figure. You do not need to calculate void ratio to determine m_v . You need the final change in height at the end of each loading (Δz) and then you calculate the vertical strain, $\epsilon_z = \Delta z/H_0$, where H_0 is the initial height. The modulus of volume compressibility is not constant but depends on the range of vertical effective stress that is used in the calculation. A representative value for m_v can be obtained by finding the slope between the current vertical effective

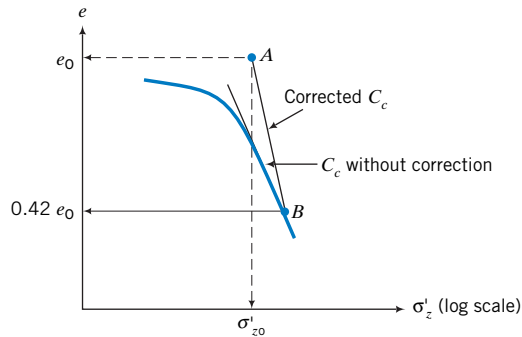


FIGURE 6.18 Schmertmann's method to correct C_c for soil disturbances.

stress and the final vertical effective stress ($\sigma'_{z0} + \Delta\sigma_z$) at the center of the soil layer in the field or 100 kPa, whichever is less.

6.7.7 Determination of the Secondary Compression Index

The secondary compression index, C_α , can be found by making a plot similar to Fig. 6.11. You should note that Fig. 6.11 is for a single load. The value of C_α usually varies with the magnitude of the applied loads and other factors such as the LIR.

What's next... Three examples and their solutions are presented next to show you how to find various consolidation soil parameters as discussed above. The first two examples are intended to illustrate the determination of the compression indices and how to use them to make predictions. The third example illustrates how to find C_v using the root time method.

EXAMPLE 6.8 Calculating C_c From Test Data

At a vertical stress of 200 kPa, the void ratio of a saturated soil sample tested in an oedometer is 1.52 and lies on the normal consolidation line. An increment of vertical stress of 150 kPa compresses the sample to a void ratio of 1.43.

- (1) Determine the compression index C_c of the soil.
- (2) The sample was unloaded to a vertical stress of 200 kPa and the void ratio increased to 1.45. Determine the slope of the recompression index, C_r .
- (3) What is the overconsolidation ratio of the soil at stage (2)?
- (4) If the soil were reloaded to a vertical stress of 500 kPa, what void ratio would be attained?

Strategy Draw a sketch of the soil response on an e versus log scale curve. Use this sketch to answer the various questions.

Solution 6.8

Step 1: Determine C_c .

C_c is the slope AB shown in Fig. E6.8.

$$C_c = \frac{-(1.43 - 1.52)}{\log(350/200)} = 0.37$$

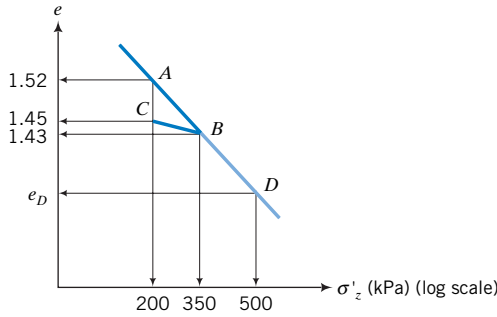


FIGURE E6.8

Step 2: Determine C_r .

C_r is the slope of BC in Fig. E6.8.

$$C_r = \frac{-(1.43 - 1.45)}{\log(350/200)} = 0.08$$

Step 3: Determine the overconsolidation ratio.

Preconsolidation effective stress: $\sigma'_{zc} = 350$ kPa

Current vertical effective stress: $\sigma'_z = 200$ kPa

$$\text{OCR} = \frac{\sigma'_{zc}}{\sigma'_z} = \frac{350}{200} = 1.75$$

Step 4: Calculate the void ratio at 500 kPa.

The void ratio at 500 kPa is the void ratio at D on the normal consolidation line (Fig. E6.8).

$$e_D = e_B - C_c \log\left(\frac{500}{350}\right) = 1.43 - 0.37 \log 1.43 = 1.37$$

EXAMPLE 6.9 Determination of Elastic Parameters, m_{vr} and E'_c

During a one-dimensional consolidation test, the height of a normally consolidated soil sample at a vertical effective stress of 200 kPa was 18 mm. The vertical effective stress was reduced to 50 kPa, causing the soil to swell by 0.5 mm. Determine m_{vr} and E'_c .

Strategy From the data, you can find the vertical strain. You know the increase in vertical effective stress, so the appropriate equations to use to calculate m_{vr} and E'_c are Eqs. (6.10) and (6.11).

Solution 6.9

Step 1: Calculate the vertical strain.

$$\Delta\epsilon_z = \frac{\Delta z}{H_o} = \frac{0.5}{18} = 0.028$$

Step 2: Calculate the modulus of volume recompressibility.

$$m_{vr} = \frac{\Delta\epsilon_{zr}}{\Delta\sigma'_z} = \frac{0.028}{150} = 1.9 \times 10^{-4} \text{ m}^2/\text{kN}$$

Step 3: Calculate the constrained elastic modulus.

$$E'_c = \frac{1}{m_{vr}} = \frac{1}{1.9 \times 10^{-4}} = 5623 \text{ kPa}$$

EXAMPLE 6.10 *Determination of C_v /Using Root Time Method*

The following readings were taken for an increment of vertical stress of 20 kPa in an oedometer test on a saturated clay sample, 75 mm in diameter and 20 mm thick. Drainage was permitted from the top and bottom boundaries.

Time (min)	0.25	1	2.25	4	9	16	25	36	24 hours
ΔH (mm)	0.12	0.23	0.33	0.43	0.59	0.68	0.74	0.76	0.89

Determine the coefficient of consolidation using the root time method.

Strategy Plot the data in a graph of displacement reading versus $\sqrt{\text{time}}$ and follow the procedures in Section 6.7.2.1.

Solution 6.10

Step 1: Make a plot of settlement (decrease in thickness) versus $\sqrt{\text{time}}$ as shown in Fig. E6.10.

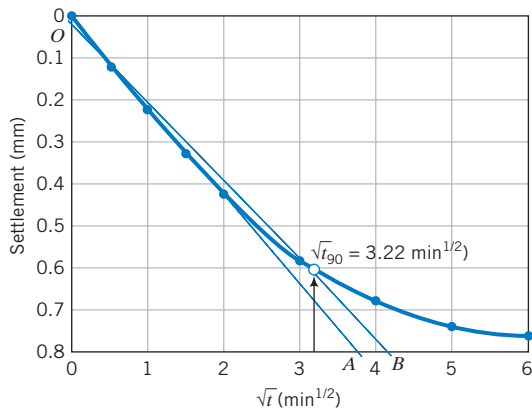


FIGURE E6.10

Step 2: Follow the procedures outlined in Section 6.7.2.1 to find t_{90} .
From Fig. E6.10,

$$\sqrt{t_{90}} = 3.22 \text{ min}^{1/2}; \quad t_{90} = 10.4 \text{ min}$$

Step 3: Calculate C_v from Eq. (6.43).

$$C_v = \frac{0.848 H_{dr}^2}{t_{90}}$$

where H_{dr} is the length of the drainage path. The current height is $20 - 0.89 = 19.1$ mm.
From Eq. (6.1),

$$H_{dr} = \frac{H_o + H_f}{4} = \frac{20 + 19.1}{4} = 9.8 \text{ mm}$$

$$\therefore C_v = \frac{0.848 \times 9.8^2}{10.4} = 7.8 \text{ mm}^2/\text{min}$$

What's next... We have described the consolidation test of a small sample of soil and the soil consolidation parameters that can be obtained. What is the relationship between this small test sample and the soil in the field? Can you readily calculate the settlement of the soil in the field based on the results of your consolidation test? The next section provides the relationship between the small test sample and the soil in the field.

6.8 RELATIONSHIP BETWEEN LABORATORY AND FIELD CONSOLIDATION



Concept Learning and Self-Assessment

Access Chapter 6, Section 6.8 and take Quiz 6.8 to get problem-solving practice to estimate the amount of settlement in the field from lab test results.

The time factor (T_v) provides a useful expression to estimate the settlement in the field from the results of a laboratory consolidation test. If two layers of the same clay have the same degree of consolidation, then their time factors and coefficients of consolidation are the same. Hence,

$$T_v = \frac{(C_v t)_{\text{lab}}}{(H_{\text{dr}}^2)_{\text{lab}}} = \frac{(C_v t)_{\text{field}}}{(H_{\text{dr}}^2)_{\text{field}}} \quad (6.45)$$

and, by simplification,

$$\frac{t_{\text{field}}}{t_{\text{lab}}} = \frac{(H_{\text{dr}}^2)_{\text{field}}}{(H_{\text{dr}}^2)_{\text{lab}}} \quad (6.46)$$

What's next... The following example shows you how to find the expected field settlement from consolidation test results for a particular degree of consolidation. ■

EXAMPLE 6.11 Time-Settlement Calculations

A sample, 75 mm in diameter and 20 mm high, taken from a clay layer 10 m thick, was tested in an oedometer with drainage at the upper and lower boundaries. It took the laboratory sample 15 minutes to reach 50% consolidation.

- (1) If the clay layer in the field has the same drainage condition as the laboratory sample, calculate how long it will take the 10 m clay layer to achieve 50% and 90% consolidation.
- (2) How much more time would it take the 10 m clay layer to achieve 50% consolidation if drainage existed only on one boundary?

Strategy You are given all the data to directly use Eq. (6.46). For part (1) there is double drainage in the field and the lab, so the drainage path is one-half the soil thickness. For part (2), there is single drainage in the field, so the drainage path is equal to the soil thickness.

Solution 6.11

- (1) We proceed as follows:

Step 1: Calculate the drainage path.

$$(H_{\text{dr}})_{\text{lab}} = \frac{20}{2} = 10 \text{ mm} = 0.01 \text{ m}; \quad (H_{\text{dr}})_{\text{field}} = \frac{10}{2} = 5 \text{ m}$$

Step 2: Calculate the field time using Eq. (6.46).

$$t_{\text{field}} = \frac{t_{\text{lab}}(H_{\text{dr}}^2)_{\text{field}}}{(H_{\text{dr}}^2)_{\text{lab}}} = \frac{15 \times 5^2}{0.01^2} = 375 \times 10^4 \text{ min} = 7.13 \text{ years}$$

(2) We proceed as follows:

Step 1: Calculate the drainage path.

$$(H_{\text{dr}})_{\text{lab}} = \frac{20}{2} = 10 \text{ mm} = 0.01 \text{ m}; \quad (H_{\text{dr}})_{\text{field}} = 10 \text{ m}$$

Step 2: Calculate field time using Eq. (6.46).

$$t_{\text{field}} = \frac{t_{\text{lab}}(H_{\text{dr}}^2)_{\text{field}}}{(H_{\text{dr}}^2)_{\text{lab}}} = \frac{15 \times 10^2}{0.01} = 15 \times 10^6 \text{ min} = 28.54 \text{ years}$$

You should take note that if drainage exists only on one boundary rather than both boundaries of the clay layer, the time taken for a given percent consolidation in the field is four times longer. ■

What's next... Several empirical equations are available linking consolidation parameters to simple, less time consuming soil tests such as the Atterberg limits and water content. In the next section, some of these relationships are presented.

6.9 TYPICAL VALUES OF CONSOLIDATION SETTLEMENT PARAMETERS AND EMPIRICAL RELATIONSHIPS



Self-Assessment

Access Chapter 7, click on Quiz and take Quiz 7.9 to use these correlations.

Some relationships between simple soil tests and consolidation settlement parameters are given below. You should be cautious in using these relationships because they may not be applicable to your soil type.

Typical Values

$$C_c = 0.1 \text{ to } 0.8$$

$$C_r = C_c/5 \text{ to } C_c/10$$

$$C_{\alpha}/C_c = 0.01 \text{ to } 0.07$$

Empirical Relationships

$$C_c = 0.009(LL - 10)$$

(Skempton, 1944)

$$C_c = 0.40(e_o - 0.25)$$

(Azzouz et al., 1976)

$$C_c = 0.01(w - 5)$$

(Azzouz et al., 1976)

$$C_c = 0.37(e_o + 0.003 LL - 0.34)$$

(Azzouz et al., 1976)

$$C_c = 0.00234 LL G_s$$

(Nagaraj and Murthy, 1986)

$$C_r = 0.15(e_o + 0.007)$$

(Azzouz et al., 1976)

$$C_r = 0.003(w + 7)$$

(Azzouz et al., 1976)

$$C_r = 0.126(e_o + 0.003 LL - 0.06)$$

(Azzouz et al., 1976)

$$C_r = 0.000463 LL G_s$$

(Nagaraj and Murthy, 1985)

w is the natural water content (%), LL is the liquid limit (%), e_o is the initial void ratio

What's next... Sometimes, we may have to build structures on a site for which the calculated settlement of the soil is intolerable. One popular method to reduce the consolidation settlement to tolerable limits is to preload the soil and use wick drains to speed up the drainage of the excess porewater pressure. Next, we will discuss wick drains.

6.10 PRECONSOLIDATION OF SOILS USING WICK DRAINS



Concept Learning, Computer Program Utility, and Self-Assessment

Access Chapter 6, Section 6.10 to learn about wick drains. Take Quiz 6.10 to test your understanding. Access Spreadsheet on the sidebar to solve for wick drain spacing.

The purpose of wick drains is to accelerate the consolidation settlement of soft saturated clays by reducing the drainage path. A wick drain is a prefabricated drainage strip that consists of a plastic core surrounded by a nonwoven polypropylene geotextile jacket (Fig. 6.19). The geotextile (a filter fabric) allows passage of water into the core that is then pumped out. A wick drain is installed by enclosing the drain in a tubular steel mandrel and supporting it at the base by an anchor plate. The mandrel is then driven into the soil by a vibratory rig or pushed by a hydraulic rig. At the desired depth, the mandrel is removed, the drain and anchor plate remain in place, and the drain is cut off with a tail about 300 mm long. The effect of drains on the soil consolidation is illustrated in Fig. 6.20. A wick drain in which one end is on an impervious boundary is called a half-closed drain (Fig. 6.21). Sometimes, the drain may penetrate into a pervious layer below an impervious layer, allowing the porewater to be expelled from the top and bottom of the drain. Such a drain provides two-way drainage and would accelerate the consolidation of the soil. Recall that for one dimensional consolidation two-way drainage reduces the time for a given degree of consolidation by four times compared with single drainage.

The governing equation for axisymmetric radial drainage is

$$\frac{\partial u}{\partial t} = C_h \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \quad (6.47)$$

where r is the radial distance from the center of the drain and C_h is the coefficient of consolidation in the horizontal or radial direction. The boundary conditions to solve Eq. (6.47) are:

$$\text{At } r = r_d : \quad u = 0 \quad \text{when } t > 0$$

$$\text{At } r = R : \quad \frac{\partial u}{\partial r} = 0$$

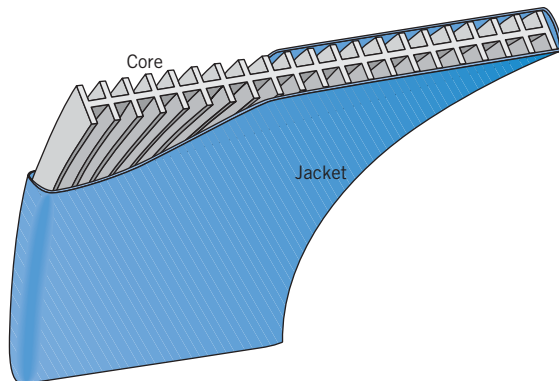


FIGURE 6.19 Wick drain.

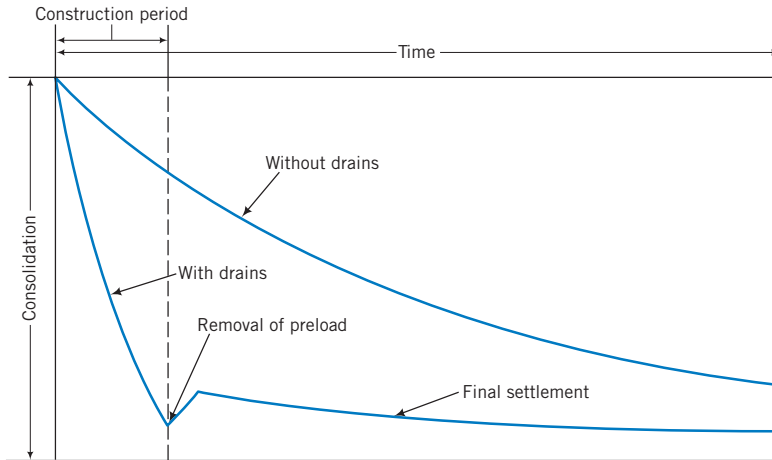


FIGURE 6.20 Effects of wick drains on time to achieve a given degree of consolidation.

and the initial condition is $t = 0$, $u = u_o$, where r_d is the radius of the drains, t is time, and R is the radius of the cylindrical influence zone (Fig. 6.21). Richart (1959) reported solutions to Eq. (6.47) for two cases—free strain and equal strain. Free strain occurs when the surface load is uniformly distributed (flexible foundation, Fig. 6.21) and the resulting surface settlement is uneven. Equal strain

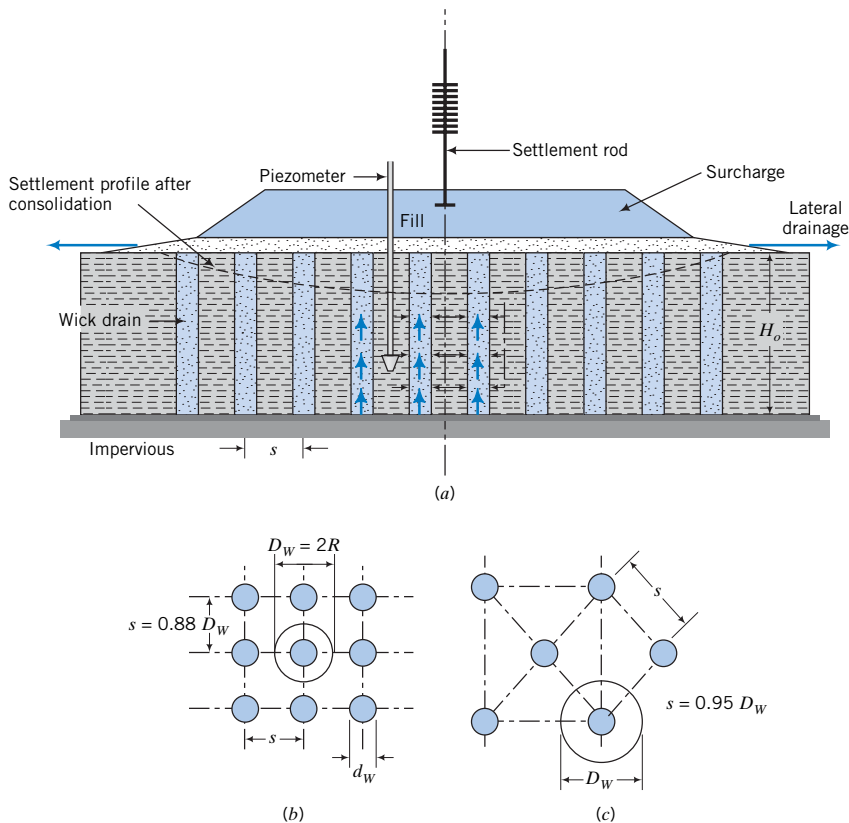


FIGURE 6.21 (a) Vertical section of a half-closed wick drain. (b) Plan of a square grid wick drain. (c) Plan of a triangular grid wick drain.

occurs when the surface settlement is forced to be uniform (rigid foundation) and the resulting surface load is not uniformly distributed. Richart (1959) showed that the differences in the two cases are small and the solution for equal strain is often used in practice.

The time factor for consolidation in the vertical direction is given by Eq. (6.31) while the time factor for consolidation in the radial direction (T_r) is

$$T_r = \frac{C_h t}{4R^2} \quad (6.48)$$

The permeability of the soil in the horizontal or radial direction is sometimes much greater (2 to 10 times for many soils) than in the vertical direction (Chapter 4) and, consequently, C_h is greater than C_v , usually $C_h/C_v \simeq 1.2$ to 2. During drilling of the borehole and installation of the drain, a thin layer of soil at the interface of the drain is often remolded. This thin layer of remolded soil is called a smear zone. The values of C_v and C_h are often much lower in the smear zone than in the natural soil. It is customary to use reduced values of C_v and C_h to account for the smear zone.

The average degree of consolidation for vertical and radial dissipation of porewater pressure (U_{vr}) is

$$U_{vr} = 1 - (1 - U)(1 - U_r) \quad (6.49)$$

where U is the average degree of consolidation for vertical drainage [Eq. (6.33)] and U_r is the average degree of consolidation for radial drainage.

The flow into wick drains is predominantly radial. Assuming no soil disturbances during installation, the time for soil consolidation for a finite vertical discharge capacity is

$$t = \frac{D_w^2}{8C_h} \left[\frac{1}{1 - \left(\frac{d_w}{D_w}\right)^2} \ln \frac{D_w}{d_w} - 0.75 + 0.25 \left(\frac{d_w}{D_w}\right)^2 + z\pi(2L - z) \frac{k_s}{q_w} \right] \ln \frac{1}{1 - U} \quad (6.50)$$

where t is time(s) required to achieve the desired consolidation, D_w is diameter (m) of the zone of influence, C_h is the coefficient of consolidation for horizontal flow (m^2/s), d_w is the equivalent drain diameter (m) $= 2(h + b)/\pi$, where b is the width (m) and h is the thickness (m) of the drain, z is the distance to flow point (m), L is the effective drain length (total drain length when drainage occurs at one end only, half length when drainage occurs at both ends, m), k_s is the hydraulic conductivity of the soils (m/s), q_w is discharge capacity of the wick drain at a gradient of 1 (m^3/s), and U is average degree of consolidation. For a square arrangement of drains, the spacing, s , is about $0.88D_w$ while for a triangular arrangement, $s = 0.95D_w$.

The spacing of the wick drains is the primary design parameter for a desired amount of settlement in a desired time period. The other design parameter is the surcharge height. In general, by varying the spacing and surcharge height, the designer can obtain the most economical combination of wick spacing, consolidation time, and surcharge height for a project.

EXAMPLE 6.12 Spacing of Wick Drains

A foundation for a structure is to be constructed on a soft deposit of clay, 20 m thick. Below the soft clay is a stiff overconsolidated clay. The calculated settlement cannot be tolerated and it was decided that the soft soil should be preconsolidated by an embankment equivalent to the building load to achieve 90% consolidation in 12 months. Wick drains are required to speed up the time for soil consolidation. The wick drains are 100 mm wide and 3 mm thick with a discharge of $0.1 \times 10^{-6} \text{ m}^3/\text{s}$.

The properties of the soils are $k_s = 0.01 \times 10^{-8}$ m/s and $C_h = 1 \times 10^{-8}$ m²/s. The distance to the flow point is 10 m and the system is half-closed. Determine the spacing of the wick drains arranged in a square grid.

Strategy You have to assume a spacing such that the calculated time using Eq. (6.50) matches the desired time of 12 months. It is best to set up a spreadsheet to do this calculation and use the Goal Seek function. Such a spreadsheet is available on the CD in the sidebar of Chapter 6.

Solution 6.12

The spreadsheet solution follows.

	A	B	C	D	E
1					
2	Spacing	s	0.97	m	
3	Drain configuration		s	$s = \text{square}, t = \text{triangular}$	
4	Drain width	h	100	mm	
5	Thickness	b	3	mm	
6	Degree of consolidation	U	90	%	
7	Coef. of horz. consolidation	C_h	1	$\times 10^{-8}$ m ² /s	
8	Desired time	t	365	days	
9	Soil thickness	L	20	m	
10	Distance to flow point	z	10	m	
11	Discharge capacity	q_w	0.1	$\times 10^{-6}$ m ³ /s	
12	Soil permeability	k_s	0.01	$\times 10^{-8}$ m/s	
13	Diameter of influence	D_w	1.107	m	
14	Equivalent diameter	d_w	65.57	mm	
15	k_s/q_w ratio		0.1	$\times 10^{-2}$	
16	Calculated time		365	days	
17					

Cell C16 is Eq. (6.50). Using the Goal Seek function in Excel with the following input

Set cell	C16
To value	365
By changing cell	C2

This gives spacing at 0.97 m. Use 1 m spacing. ■

6.11 SUMMARY

Consolidation settlement of a soil is a time-dependent process that depends on the soil's hydraulic conductivity, thickness, and the drainage conditions. When an increment of vertical stress is applied to a soil, the instantaneous (initial) excess porewater pressure is equal to the vertical stress increment. With time, the initial excess porewater pressure decreases, the vertical effective stress increases by the amount of decrease of the initial excess porewater pressure, and settlement increases. The consolidation settlement is made up of two parts—the early time response called primary consolidation and a later time response called secondary compression.

Soils retain a memory of the past maximum effective stress, which may be erased by loading to a higher stress level. If the current vertical effective stress on a soil was never exceeded in the past (a normally consolidated soil), it would behave elastoplastically when stressed. If the current vertical

effective stress on a soil was exceeded in the past (an overconsolidated soil), it would behave elastically (approximately) for stresses less than its past maximum effective stress.

Practical Examples



Interactive Problem Solving

Access Chapter 6, click on Problem Solver on the sidebar and then solve one-dimensional consolidation problems interactively.

EXAMPLE 6.13 Lateral Stress During Soil Consolidation in the Lab

A soil was consolidated in an oedometer to a vertical stress of 100 kPa and then unloaded incrementally to 50 kPa. The excess porewater pressure is zero. If the frictional soil constant ϕ'_{CS} is 25° , determine the lateral stress.

Strategy The soil in this case becomes overconsolidated—the past maximum effective stress is 100 kPa and the current effective stress is 50 kPa. You need to find K_o^{nc} and then K_o^{oc} using the OCR of your soil. (See Section 5.10.)

Step 1: Calculate K_o^{nc} .

$$\text{Eq. (5.51): } K_o^{nc} = 1 - \sin \phi'_{cs} = 1 - \sin 25 = 0.58$$

Step 2: Calculate OCR.

$$\text{OCR} = \frac{\sigma'_{zc}}{\sigma'_{zo}} = \frac{100}{50} = 2$$

Step 3: Calculate K_o^{oc} .

$$\text{Eq. (5.52): } K_o^{oc} = K_o^{nc}(\text{OCR})^{1/2} = 0.58(2)^{1/2} = 0.82$$

Step 4: Calculate the lateral effective stress.

$$\sigma'_x = K_o^{oc} \sigma'_z = 0.82 \times 50 = 41 \text{ kPa}$$

Step 5: Calculate the lateral total stress.

$$\sigma_x = \sigma'_x + \Delta u = 41 + 0 = 41 \text{ kPa}$$

EXAMPLE 6.14 Consolidation Settlement Due to a Foundation

A foundation for an oil tank is proposed for a site with a soil profile as shown in Fig. E6.14a. A specimen of the fine-grained soil, 75 mm in diameter and 20 mm thick, was tested in an oedometer in a laboratory. The initial water content was 62% and $G_s = 2.7$. The vertical stresses were applied incrementally—each increment remaining on the specimen until the porewater pressure change was negligible. The cumulative settlement values at the end of each loading step are as follows:

Vertical stress (kPa)	15	30	60	120	240	480
Settlement (mm)	0.10	0.11	0.21	1.13	2.17	3.15

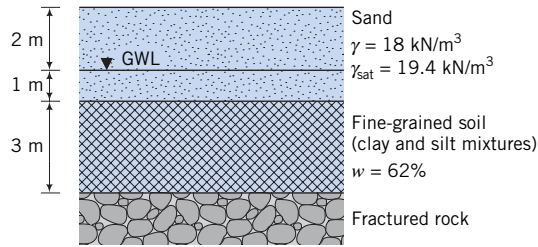


FIGURE E6.14a

The time–settlement data when the vertical stress was 200 kPa are:

Time (min)	0	0.25	1	4	9	16	36	64	100
Settlement (mm)	0	0.22	0.42	0.6	0.71	0.79	0.86	0.91	0.93

The tank, when full, will impose vertical stresses of 90 kPa and 75 kPa at the top and bottom of the fine-grained soil layer, respectively. You may assume that the vertical stress is linearly distributed in this layer.

- Determine the primary consolidation settlement of the fine-grained soil layer when the tank is full.
- Calculate and plot the settlement–time curve.

Strategy To calculate the primary consolidation settlement you need to know C_c and C_r or m_v , and σ'_{zo} , $\Delta\sigma$, and σ'_{zc} . Use the data given to find the values of these parameters. To find time for a given degree of consolidation, you need to find C_v from the data.

Solution 6.14

Step 1: Find C_v using the root time method.

Use the data from the 240 kPa load step to plot a settlement versus $\sqrt{\text{time}}$ curve as depicted in Fig. E6.14b. Follow the procedures set out in Section 6.7 to find C_v . From the curve, $t_{90} = 1.2$ min.

$$\text{Height of sample at beginning of loading} = 20 - 1.2 = 18.8 \text{ mm}$$

$$\text{Height of sample at end of loading} = 20 - 2.17 = 17.83 \text{ mm}$$

$$\text{Equation (6.1): } H_{dr} = \frac{H_o + H_f}{4} = \frac{18.8 + 17.83}{4} = 9.16 \text{ mm}$$

$$\begin{aligned} \text{Equation (6.31): } C_v &= \frac{T_v H_{dr}^2}{t_{90}} = \frac{0.848 \times (9.16)^2}{1.2} = 59.3 \text{ mm}^2/\text{min} \\ &= 59.3 \times 10^{-6} \text{ m}^2/\text{min} \end{aligned}$$

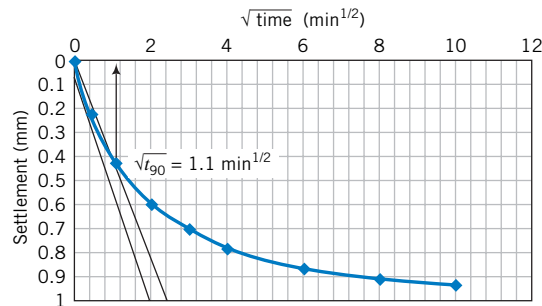


FIGURE E6.14b

Step 2: Determine the void ratio at the end of each load step.

$$\text{Initial void ratio: } e_o = wG_s = 0.62 \times 2.7 = 1.67$$

$$\begin{aligned} \text{Equation (6.6): } e &= e_o - \frac{\Delta z}{H_o} (1 + e_o) = 1.67 - \frac{\Delta z}{20} (1 + 1.67) \\ &= 1.67 - 13.35 \times 10^{-2} \Delta z \end{aligned}$$

The void ratio for each load step is shown in the table below.

σ'_z (kPa)	15	30	60	120	240	480
Void ratio	1.66	1.65	1.64	1.52	1.38	1.25

A plot of $e - \log \sigma'_z$ versus e is shown in Fig. E6.14c.

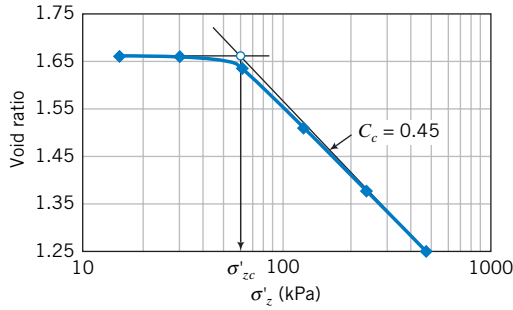


FIGURE E6.14c

Step 3: Determine σ'_{zc} and C_c .

Follow the procedures in Section 6.7 to find σ'_{zc} .

$$\begin{aligned} \sigma'_{zc} &= 60 \text{ kPa} \\ C_c &= \frac{1.52 - 1.25}{\log\left(\frac{480}{120}\right)} = 0.45 \end{aligned}$$

Step 4: Calculate σ'_{zo} .

$$\begin{aligned} \text{Clay: } \gamma_{\text{sat}} &= \frac{G_s + e_o}{1 + e_o} \gamma_w = \left(\frac{2.7 + 1.67}{1 + 1.67} \right) 9.8 = 16 \text{ kN/m}^3 \\ \sigma'_{zo} &= (18 \times 2) + (19.4 - 9.8)1 + (16 - 9.8)1.5 = 54.9 \text{ kPa} \end{aligned}$$

Step 5: Calculate settlement.

$$\text{OCR} = \frac{\sigma'_{zc}}{\sigma'_{zo}} = \frac{60}{54.9} = 1.1$$

For practical purpose, the OCR is very close to 1; that is, $\sigma'_z \approx \sigma'_{zc}$. Therefore, the soil is normally consolidated. Also, inspection of the e versus $\log \sigma'_z$ curve shows that C_r is approximately zero, which lends further support to the assumption that the soil is normally consolidated.

$$\rho_{\text{pc}} = \frac{H_o}{1 + 1.67} 0.45 \left(\frac{\sigma'_{zo} + \Delta \sigma_z}{\sigma'_{zo}} \right) = 0.17 H_o \log \left(\frac{\sigma'_{zo} + \Delta \sigma_z}{\sigma'_{zo}} \right)$$

Divide the clay layer into three sublayers of 1.0 m thick and compute the settlement for each sublayer. The primary consolidation settlement is the sum of the settlement of each sublayer. The vertical stress increase in the fine-grained soil layer is

$$90 - \left(\frac{90 - 75}{3} \right) z = 90 - 5z$$

where z is the depth below the top of the layer. Calculate the vertical stress increase at the center of each sublayer and then the settlement from the above equation. The table below summarizes the computation.

Layer	z (m)	σ'_{z0} at center of sublayer (kPa)	$\Delta\sigma_z$	$\sigma'_{z0} + \Delta\sigma_z$ (kPa)	ρ_{pc} (mm)
1	0.5	48.7	87.5	136.2	75.9
2	1.5	54.9	82.5	137.4	67.7
3	2.5	61.1	77.5	138.6	60.5
Total					204.1

Alternatively, by considering the whole fine-grained soil layer and taking the average vertical stress increment, we obtain

$$\rho_{pc} = \frac{3000}{1 + 1.67} 0.45 \log \left(\frac{137.4}{54.9} \right) = 201.4 \text{ mm}$$

In general, the former approach is more accurate for thick layers.

Step 6: Calculate settlement–time values.

$$C_v = 59.3 \times 10^{-6} \times 60 \times 24 = 85392 \times 10^{-6} \text{ m}^2/\text{day}$$

$$t = \frac{T_v H_{dr}^2}{C_v} = \frac{T_v \times \left(\frac{3}{2}\right)^2}{85392 \times 10^{-6}} = 26.3 T_v \text{ days}$$

The calculation of settlement at discrete times is shown in the table below and the data are plotted in Fig. E6.14d.

U (%)	T_v	Settlement (mm)	
		$\rho_{pc} \times \frac{U}{100}$	$t = 26.3 T_v$ (days)
10	0.008	20.4	0.2
20	0.031	20.8	0.8
30	0.071	61.3	1.9
40	0.126	81.6	3.3
50	0.197	102.1	5.2
60	0.287	122.5	7.6
70	0.403	142.9	10.6
80	0.567	163.3	14.9
90	0.848	183.7	22.3

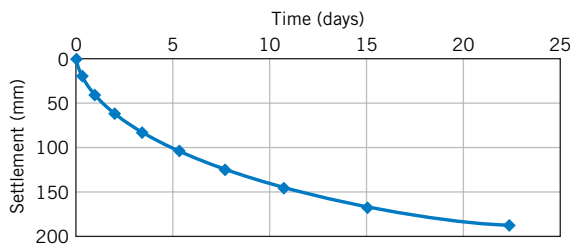


FIGURE E6.14d.

EXAMPLE 6.15 *Estimating Consolidation Settlement Due to Unexpected Field Condition*

A geotechnical engineer made a preliminary settlement analysis for a foundation of an office building, which is to be constructed at a location where the soil strata contain a compressible clay layer. She calculated 50 mm of primary consolidation settlement. The building will impose an average vertical stress of 150 kPa in the clay layer. As often happens in design practice, design changes are required. In this case, the actual thickness of the clay is 30% more than the original soil profile indicated and, during construction, the groundwater table has to be lowered by 2 m. Estimate the new primary consolidation settlement.

Strategy From Section 6.3, the primary consolidation settlement is proportional to the thickness of the soil layer and also to the increase in vertical stress [see Eq. (6.18)]. Use proportionality to find the new primary consolidation settlement.

Solution 6.15

Step 1: Estimate the new primary consolidation settlement due to the increase in thickness.

$$\frac{(\rho_{pc})_n}{(\rho_{pc})_o} = \frac{H_n}{H_o}$$

where subscripts o and n denote original and new, respectively.

$$(\rho_{pc})_n = 50 \times \frac{1.3 H_o}{H_o} = 65 \text{ mm}$$

Step 2: Estimate primary consolidation settlement from vertical stress increase due to lowering of the groundwater level.

Increase in vertical effective stress due to lowering of water table = $2 \times 9.8 = 19.6 \text{ kPa}$

Primary consolidation settlement is also proportional to the vertical effective stress:

$$\begin{aligned} \frac{(\rho_{pc})_n}{(\rho_{pc})_o} &= \frac{(\sigma'_z)_n}{(\sigma'_z)_o} \\ \therefore (\rho_{pc})_n &= 65 \times \frac{(150 + 19.6)}{150} = 73.5 \text{ mm} \end{aligned}$$

EXAMPLE 6.16 *Consolidation Differential Settlement in Nonuniform Soil*

The foundations supporting two columns of a building are shown in Fig. E6.16. An extensive soil investigation was not carried out and it was assumed in the design of the footing that the clay layer has a uniform thickness of 1.2 m. Two years after construction, the building settled with a differential settlement of 10 mm. Walls of the building began to crack. The doors have not jammed but by measuring the out-of-vertical distance of the doors, it is estimated that they would become jammed if the differential settlement exceeded 24 mm. A subsequent soil investigation showed that the thickness of the clay layer was not uniform but varies as shown in Fig. E6.16. The owners would like to get an estimate of the expected total differential settlement and how long it would take before the doors become jammed.

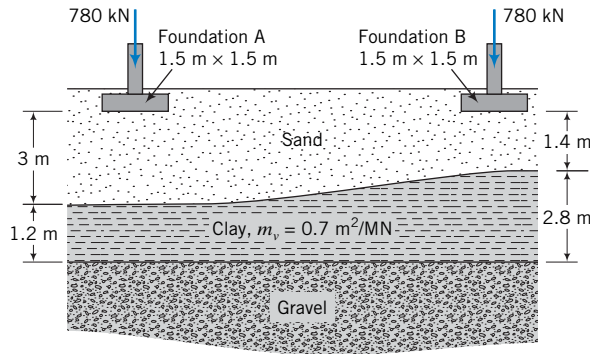


FIGURE E6.16

Strategy Determine the settlement under each foundation and then calculate the differential settlement. Since you know the differential settlement that occurs over a two year period after construction, you can find the degree of consolidation and then use this information to calculate the expected time for the doors to become jammed.

Solution 6.16

Step 1: Calculate the vertical stress increase at the center of the clay layer under each foundation. You can use Eq. (5.81) or Eq. (5.85) or, for a quick estimate, Eq. (5.95) to determine the stress increase under the foundations. Let's use the approximate method, Eq. (5.95).

$$\Delta\sigma_z = \frac{P}{(B+z)(L+z)}$$

$$(\Delta\sigma_z)_A = \frac{780}{(1.5+3.6)(1.5+3.6)} = 30 \text{ kPa}$$

$$(\Delta\sigma_z)_B = \frac{780}{(1.5+2.8)(1.5+2.8)} = 42.2 \text{ kPa}$$

Note: For a more accurate value of $\Delta\sigma_z$ you should use the vertical stress increase due to surface loads on multilayered soils (Poulos and Davis, 1974).

Step 2: Calculate the primary consolidation settlement.

Use Eq. (6.18), $\rho_{pc} = H_o m_v \Delta\sigma$, to calculate the primary consolidation settlement.

$$(\rho_{pc})_A = 1.2 \times 0.7 \times 10^{-3} \times 30 = 25.2 \times 10^{-3} \text{ m} = 25.2 \text{ mm}$$

$$(\rho_{pc})_B = 2.8 \times 0.7 \times 10^{-3} \times 42.2 = 82.7 \times 10^{-3} \text{ m} = 82.7 \text{ mm}$$

Step 3: Calculate the differential settlement.

$$\text{Differential settlement: } \delta = 82.7 - 25.2 = 57.5 \text{ mm}$$

Step 4: Calculate the time for 24 mm differential settlement to occur.

Current differential settlement: $\delta_c = 10 \text{ mm}$

Degree of consolidation: $U = \frac{\delta_c}{\delta} = \frac{10}{57.5} = 0.17$

From Eq. (6.34): $T_v = \frac{4}{\pi} U^2 = \frac{4}{\pi} \times 0.17^2 = 0.037$

From Eq. (6.31): $C_v = \frac{T_v H_{dr}^2}{t} = \frac{0.037 \times (2.8/2)^2}{2} = 0.036 \text{ m}^2/\text{yr}$

For 24 mm differential settlement: $U = \frac{24}{57.5} = 0.42$, $T_v = \frac{4}{\pi} \times 0.42^2 = 0.225$

From Eq. (6.31): $t = \frac{T_v H_{dr}^2}{C_v} = \frac{0.225 \times (2.8/2)^2}{0.036} = 12.25 \text{ years}$

Therefore, in the next 12.25 years, the total differential settlement would be 24 mm. ■

EXERCISES

For all problems, assume $G_s = 2.7$ unless otherwise stated.

Theory

- 6.1** A clay soil of thickness H is allowed to drain on the top boundary through a thin sand layer. A vertical stress of σ was applied to the clay. The excess porewater pressure distribution was linear in the soil layer with a value of u_t at the top boundary and u_b ($u_b > u_t$) at the bottom boundary. The excess porewater pressure at the top boundary was not zero because the sand layer was partially blocked. Derive an equation for the excess porewater pressure distribution with soil thickness and time.
- 6.2** A soil layer of thickness H_o has only single drainage through the top boundary. The excess porewater pressure distribution when a vertical stress, σ , is applied varies parabolically with a value of zero at the top boundary and u_b at the bottom boundary. Show that

$$C_v = \frac{H_o^2}{2u_b} \frac{d\sigma'}{dt} \quad \text{and} \quad k_z = \frac{\gamma_w H_o}{2u_b} \frac{dH_o}{dt}$$

- 6.3** Show that, for a linear elastic soil,

$$m_v = \frac{(1 + \nu')(1 - 2\nu')}{E'(1 - \nu')}$$

- 6.4** Show that, if an overconsolidated soil behaves like a linear elastic material,

$$K_o^{\text{oc}} = (\text{OCR}) K_o^{\text{nc}} - \frac{\nu'}{1 - \nu'} (\text{OCR} - 1)$$

- 6.5** The excess porewater pressure distribution in a 10 m thick clay varies linearly from 100 kPa at the top to 10 kPa at the bottom of the layer when a vertical stress was applied. Assuming drainage only at the top of the clay layer, determine the excess porewater pressure in 1 year's time using the finite difference method if $C_v = 1.5 \text{ m}^2/\text{yr}$.
- 6.6** At a depth of 4 m in a clay deposit, the overconsolidation ratio is 3.0. Plot the variation of overconsolidation ratio and water content with depth for this deposit up to a depth of 15 m. The recompression index is $C_r = 0.05$ and the water content at 4 m is 32%.

- 6.7** The overconsolidation ratio of a saturated clay at a depth of 5 m is 6.0 and its water content is 38%. It is believed that the clay has become overconsolidated as a result of erosion. Calculate the thickness of the soil layer that was eroded. Assume that the groundwater level is at the ground surface for both the past and present conditions.

Problem Solving



Interactive Problem Solving

Access Chapter 6 and click on Problem Solver on the sidebar to interactively solve problems similar to some of the problems below.

- 6.8** An oedometer test on a saturated clay soil gave the following results: $C_c = 0.4$, $C_r = 0.08$, $\text{OCR} = 4.5$. The existing vertical effective stress in the field is 130 kPa. A building foundation will increase the vertical stress at the center of the clay by 150 kPa. The thickness of the clay layer is 2 m and its water content is 28%. Calculate the primary consolidation settlement. What would be the difference in settlement if the OCR were 1.5 instead of 4.5?
- 6.9** A building is expected to increase the vertical stress at the center of a 2 m thick clay layer by 100 kPa. If m_v is $4 \times 10^{-4} \text{ m}^2/\text{kN}$, calculate the primary consolidation settlement.
- 6.10** Two adjacent bridge piers rest on clay layers of different thickness but with the same properties. Pier #1 imposes a stress increment of 100 kPa to a 3 m thick layer while Pier #2 imposes a stress increment of 150 kPa to a 5 m thick layer. What is the differential settlement between the two piers if $m_v = 3 \times 10^{-4} \text{ m}^2/\text{kN}$?
- 6.11** The table below shows data recorded during an oedometer test on a soil sample for an increment of vertical stress of 200 kPa. At the start of the loading, the sample height was 19.17 mm.

Time (min)	0	0.25	1	4	9	16	36	64	100
Settlement (mm)	0	0.30	0.35	0.49	0.61	0.73	0.90	0.95	0.97

After 24 hours the settlement was negligible and the void ratio was 1.20, corresponding to a sample height of 18.2 mm. Determine C_v using the root time and the log time methods.

- 6.12** A sample of saturated clay of height 20 mm and water content of 30% was tested in an oedometer. Loading and unloading of the sample were carried out. The thickness H_f of the sample at the end of each stress increment/decrement is shown in the table below.

σ'_z (kPa)	100	200	400	200	100
H_f (mm)	20	19.31	18.62	18.68	18.75

- (a) Plot the results as void ratio versus $\log \sigma'_z$.
- (b) Determine C_c and C_r .
- (c) Determine m_v between $\sigma'_z = 200 \text{ kPa}$ and $\sigma'_z = 300 \text{ kPa}$.
- 6.13** A sample of saturated clay, taken from a depth of 5 m, was tested in a conventional oedometer. The table below gives the vertical stress and the corresponding thickness recorded during the test.

σ'_z (kPa)	100	200	400	800	1600	800	400	100
h (mm)	19.2	19.0	17.0	14.8	12.6	13.1	14.3	15.9

The water content at the end of the test was 40% and the initial height was 20 mm.

- (a) Plot a graph of void ratio versus σ'_z (log scale).
- (b) Determine C_c and C_r .
- (c) Determine m_v between $\sigma'_z = 400$ kPa and $\sigma'_z = 500$ kPa.
- (d) Determine the relationship between e (void ratio) and h (thickness).
- (e) Determine σ'_{zc} using Casagrande's method.

6.14 The following observations were recorded in an oedometer test on a clay sample 100 mm in diameter and 30 mm high.

Load (N)	0	50	100	200	400	800	0
Displacement gauge reading (mm)	0	0.48	0.67	0.98	1.24	1.62	1.4

At the end of the test, the wet mass of the sample was 507.3 grams and, after oven drying, its dry mass was 412.5 grams. The specific gravity was 2.65.

- (a) Calculate the void ratio at the end of the test.
 - (b) Calculate the void ratio at the end of each loading step.
 - (c) Calculate the initial thickness of the soil sample from the initial void ratio and compare this with the initial thickness.
 - (d) Determine m_v between $\sigma'_z = 50$ kPa and $\sigma'_z = 150$ kPa.
- 6.15** A laboratory consolidation test on a 20 mm thick sample of soil shows that 90% consolidation occurs in 30 minutes. Plot a settlement (degree of consolidation)–time curve for a 10 m layer of this clay in the field for (a) single drainage and (b) double drainage.
- 6.16** A clay layer below a building foundation settles 15 mm in 200 days after the building was completed. According to the oedometer results, this settlement corresponds to an average degree of consolidation of 25%. Plot the settlement–time curve for a 10 year period, assuming double drainage.
- 6.17** An oil tank is to be sited on a soft alluvial deposit of clay. Below the soft clay is a thick layer of stiff clay. It was decided that a circular embankment, 10 m diameter, with wick drains inserted into the clay would be constructed to preconsolidate the soil. The height of the embankment is 6 m and the saturated unit weight of the soil comprising the embankment is 18 kN/m³. The following data are available: thickness of soft clay = 7 m, $k_s = 1 \times 10^{-10}$ m/s and $C_h = 0.6$ m²/yr. The desired degree of consolidation is 90% in 12 months. Determine the spacing of a square grid of the wick drains. Assume wick drain of size 100 mm $\times$ 3 mm and $q_w = 0.1 \times 10^{-6}$ m³/s. The flow point distance is 4 m. Assume a half-closed system.

Practical

- 6.18** Fig. P6.18 shows the soil profile at a site for a proposed office building. It is expected that the vertical stress at the top of the clay will increase by 150 kPa and at the bottom by 90 kPa. Assuming a linear stress distribution within the clay, calculate the consolidation settlement. [Hint: You should divide the clay into five equal layers, compute the settlement for each layer, and then find the total settlement.] An oedometer test on a sample of the clay revealed that the time for 90% consolidation on a 20 mm thick sample occurred in 40 minutes. The sample was drained on the upper and lower boundaries. How long will it take for 50% consolidation to occur in the field?

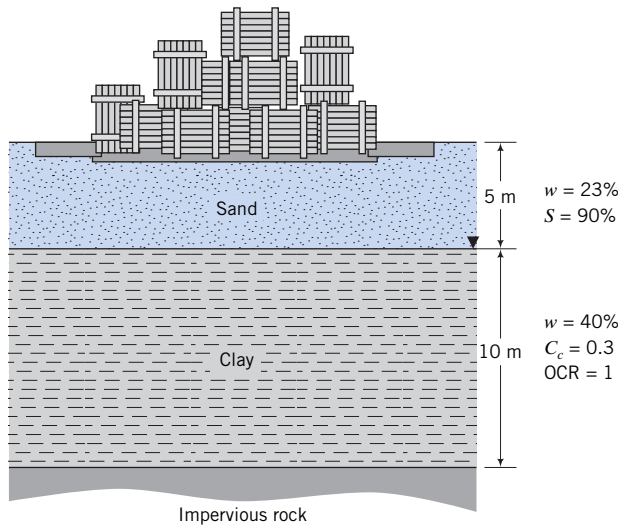


FIGURE P6.18

6.19 A borehole at a site for a proposed building reveals the following soil profile:

0–5 m	Dense sand, $\gamma = 18 \text{ kN/m}^3$, $\gamma_{\text{sat}} = 19 \text{ kN/m}^3$
At 4 m	Groundwater level
5–10 m	Soft normally consolidated clay, $\gamma_{\text{sat}} = 17.5 \text{ kN/m}^3$
Below 10 m	Impervious rock

A building is to be constructed on this site with its foundation at 2 m below ground level. The building load is 30 MN and the foundation is rectangular with a width of 10 m and length of 15 m. A sample of the clay was tested in an oedometer and the following results were obtained:

Vertical stress (kPa)	50	100	200	400	800
Void ratio	0.945	0.895	0.815	0.750	0.705

Calculate the primary consolidation settlement. Assuming that the primary consolidation took 5 years to achieve in the field, calculate the secondary compression for a period of 10 years beyond primary consolidation. The secondary compression index is $C_c/6$. [Hint: Determine e_p for your σ'_{fin} from a plot of e versus σ'_z (log scale).]

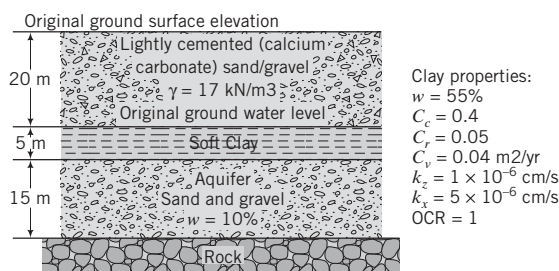


FIGURE P6.20

- 6.20 Water is pumped from an aquifer as shown in Fig. P6.20 for domestic use. The groundwater level drops by 10 m and the surface subsides. Assuming that the subsidence is due to the settlement of the soft clay, estimate the settlement of the ground surface. A decision was made to recharge the aquifer by pumping water from a canal. If the groundwater level were to return to its original level, would the ground surface return to its original elevation? If so, why? If not, why not? Show calculations to support your answer. How much recovery (expansion of soil) is possible using the one-dimensional consolidation theory?

CHAPTER

7

SHEAR STRENGTH OF SOILS

ABET	ES	ED
	85	15

7.0 INTRODUCTION

The safety of any geotechnical structure is dependent on the strength of the soil. If the soil fails, a structure founded on it can collapse, endangering lives and causing economic damage. The strength of soils is therefore of paramount importance to geotechnical engineers. The word strength is used loosely to mean shear strength, which is the internal frictional resistance of a soil to shearing forces. Shear strength is required to make estimates of the load bearing capacity of soils, the stability of geotechnical structures, and in analyzing the stress–strain characteristics of soils.

In this chapter we will define, describe, and determine the shear strength of soils. When you complete this chapter, you should be able to:

- Determine the shear strength of soils.
- Understand the differences between drained and undrained shear strength.
- Determine the type of shear test that best simulates field conditions.
- Interpret laboratory and field test results to obtain shear strength parameters.

You will use the following principles learned from previous chapters and other courses.

- Stresses, strains, Mohr's circle of stresses and strains, and stress paths (Chapter 5)
- Friction (statics and/or physics)

Sample Practical Situation You are the geotechnical engineer in charge of a soil exploration program for a dam and housing project. You are expected to specify laboratory and field tests to determine the shear strength of the soil and to recommend soil strength parameters for the design of the dam.

In Fig. 7.1 a house is shown in a precarious position because the shear strength of the soil within the slope near the house was exceeded. Would you like this to be your house? The content of this chapter will help you to understand the shear behavior of soils so that you can prevent catastrophes like that shown in Fig. 7.1.

7.1 DEFINITIONS OF KEY TERMS

Shear strength of a soil (τ_f) is the maximum internal resistance to applied shearing forces.

Effective friction angle (ϕ') is a measure of the shear strength of soils due to friction.

Cohesion (c_o) is a measure of the forces that cement particles of soil.

Undrained shear strength (s_u) is the shear strength of a soil when sheared at constant volume.



FIGURE 7.1 Shear failure of soil under a house. (© Leverett Bradley/Tony Stone Images/New York, Inc.)

Critical state is a stress state reached in a soil when continuous shearing occurs at constant shear stress to normal effective stress ratio and constant volume.

Dilation is a measure of the change in volume of a soil when the soil is distorted by shearing.

7.2 QUESTIONS TO GUIDE YOUR READING

1. What is meant by the shear strength of soils?
2. What factors affect the shear strength?
3. How is shear strength determined?
4. What are the assumptions in the Mohr–Coloumb failure criterion?
5. Do soils fail on a plane?
6. What are the differences between peak, critical, and residual effective friction angles?
7. What are peak shear strength, critical shear strength, and residual shear strength?
8. Are there differences between the shear strengths of dense and loose sands or normally and overconsolidated clays?
9. What are the differences between drained and undrained shear strengths?
10. Under what conditions should the drained shear strength or the undrained shear strength parameters be used?
11. What laboratory and field tests are used to determine shear strength?
12. What are the differences between the results of various laboratory and field tests?
13. How do I know what laboratory test to specify for a project?

7.3 TYPICAL RESPONSE OF SOILS TO SHEARING FORCES



Interactive Concept Learning and Self-Assessment

Access Chapter 7, Section 7.3 on the CD to learn the basic concept of shear strength of soils through an interactive animation. You will perform a virtual experiment and will be guided to interpret the results and learn about the responses of soils to shearing. Take Quiz 7.3 to assess your understanding.

We are going to describe the behavior of two groups of soils when they are subjected to shearing forces. One group, called uncemented soils, has very weak interparticle bonds and comprises most soils. The other group, called cemented soils, has strong interparticle bonds through ion exchange or substitution. The particles of cemented soils are chemically bonded or cemented together. An example of a cemented soil is caliche, which is a mixture of clay, sand, and gravel cemented by calcium carbonate.

Let us incrementally deform two samples of soil by applying simple shear deformation (Fig. 7.2) to each of them. One sample, which we call Type I, represents mostly loose sands and normally consolidated and lightly overconsolidated clays ($\text{OCR} \leq 2$). The other, which we call Type II, represents mostly dense sands and overconsolidated clays ($\text{OCR} > 2$). In classical mechanics, simple shear deformation refers to shearing under constant volume. In soil mechanics, we relax this restriction (constant volume) because we wish to know the volume change characteristics of soils under simple shear (Fig. 7.2). The implication is that the mathematical interpretation of simple shear tests becomes complicated because we now have to account for the influence of volumetric strains on soil behavior. Since in simple shear $\epsilon_x = \epsilon_y = 0$, the volumetric strain is equal to the vertical strain, $\epsilon_z = \Delta z/H_o$, where Δz is the vertical displacement (positive for compression) and H_o is the initial sample height. The shear strain is the small angular distortion expressed as $\gamma_{zx} = \Delta x/H_o$, where Δx is the horizontal displacement.

We are going to summarize the important features of the responses of these two groups of soils—Type I and Type II—when subjected to a constant vertical (normal) effective stress and increasing shear strain. We will consider the shear stress versus the shear strain, the volumetric strain versus the shear strain, and the void ratio versus the shear strain responses, as illustrated in Fig. 7.3.

Type I soils—loose sands, normally consolidated and lightly overconsolidated clays ($\text{OCR} \leq 2$)—are observed to:

- Show gradual increase in shear stresses as the shear strain increases (strain hardens) until an approximately constant shear stress, which we will call the critical state shear stress, τ_{cs} , is attained (Fig. 7.3a).

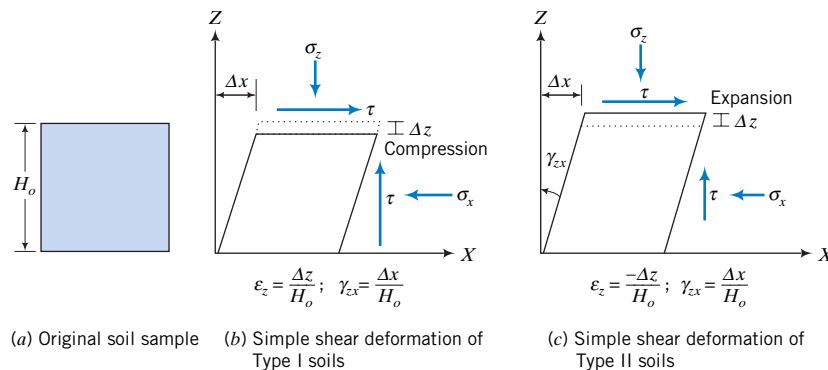


FIGURE 7.2 Simple shear deformation of Type I and Type II soils.

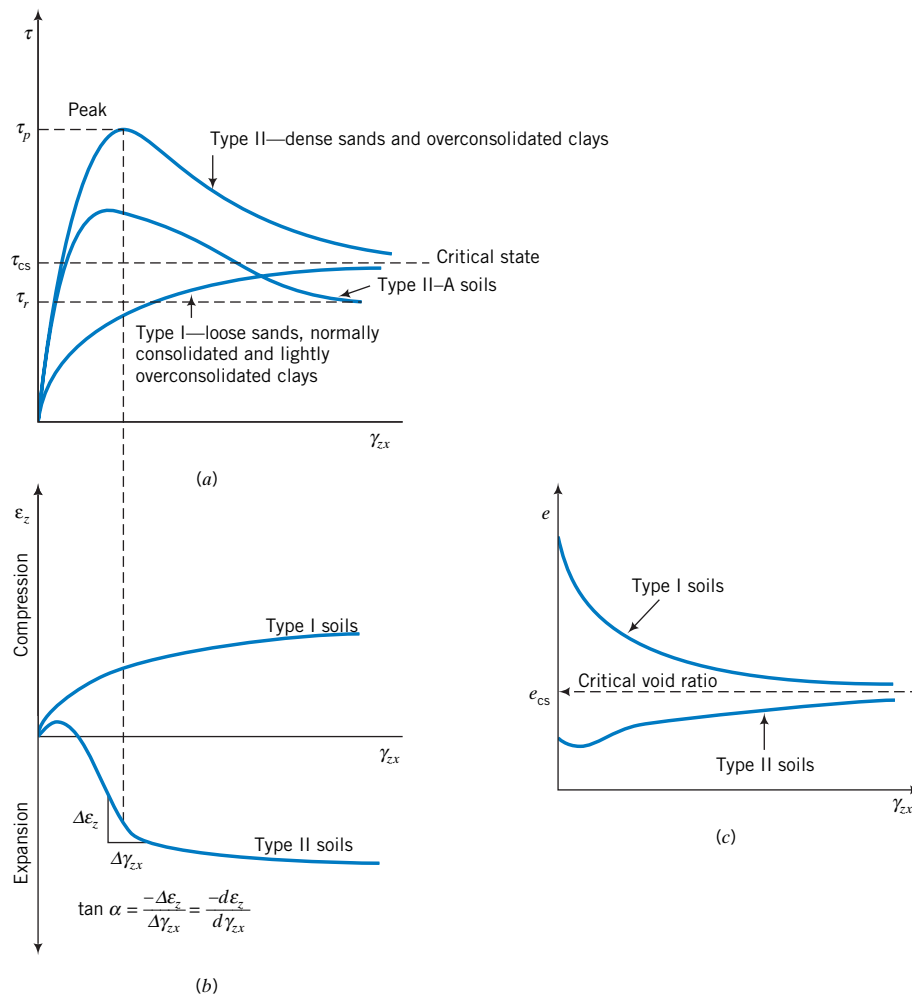


FIGURE 7.3 Response of soils to shearing.

- Compress, that is, they become denser (Fig. 7.2b and Figs. 7.3b,c) until a constant void ratio, which we will call the critical ratio, e_{cs} , is reached (Fig. 7.3c).

Type II soils—dense sands and heavily overconsolidated clays ($OCR \leq 2$)—are observed to:

- Show a rapid increase in shear stress reaching a peak value, τ_p , at low shear strains (compared to Type I soils) and then show a decrease in shear stress with increasing shear strain (strain softens) ultimately attaining a critical state shear stress (Fig. 7.3a). The strain softening response generally results from localized failure zones called shear bands (Fig. 7.4). These shear bands are soil pockets that have loosened and reached the critical state shear stress. Between the shear bands are denser soils that gradually loosen as shearing continues. The initial thickness of shear bands was observed from laboratory tests to be about 10 to 15 grain diameters. The soil within a shear band undergoes intense shearing while the soil masses above and below it behave as rigid bodies. The development of shear bands depends on the boundary conditions imposed on the soil, the homogeneity of the soil, the grain size, uniformity of loads, and initial density.

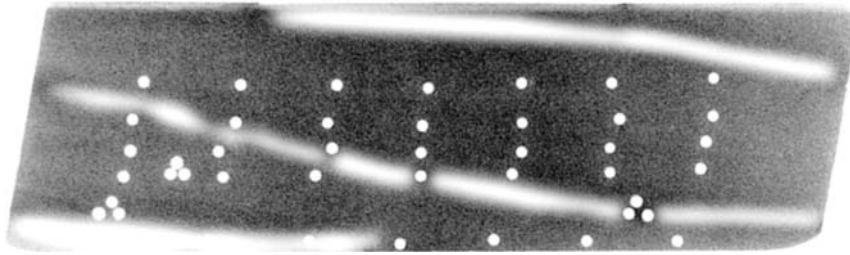


FIGURE 7.4 Radiographs of shear bands in a dense fine sand (the white circles are lead shot used to trace internal displacements; white lines are shear bands). (After Budhu, 1979)

When a shear band(s) develops in some types of overconsolidated clays, the particles become oriented parallel to the direction of the shear band, causing the final shear stress of these clays to decrease below the critical state shear stress. We will call this type of soil, Type II-A, and the final shear stress attained the residual shear stress, τ_r . Type I soils at very low normal effective stress can also exhibit a peak shear stress during shearing.

- Compress initially (attributed to particle adjustment) and then expand, that is, they become looser (Fig. 7.2c and Figs. 7.3b,c) until a critical void ratio (the same void ratio as Type I soils) is attained.

The critical state shear stress is reached for all soils when no further volume change occurs under continued shearing. We will use the term critical state to define the stress state reached by a soil when no further change in shear stress and volume occurs under continuous shearing at a constant normal effective stress.

7.3.1 Effects of Increasing the Normal Effective Stress

So far, we have only used a single normal effective stress in our presentation of the responses of Type I and Type II soils. What is the effect of increasing the normal effective stress? For Type I soils, the amount of compression and the magnitude of the critical state shear stress will increase (Figs. 7.5a,b). For Type II soils, the peak shear stress tends to disappear, the critical shear stress increases and the change in volume expansion decreases (Figs. 7.5a,b).

If we were to plot the peak shear stress and the critical state shear stress for each constant normal effective stress for Type I and II soils, we would get:

1. An approximate straight line (OA, Fig. 7.5c) that links all the critical state shear stress values of Type I and Type II soils. We will call the angle between OA and the σ'_n axis, the critical state friction angle, ϕ'_{cs} . The line OA will be called the failure envelope because any shear stress that lies on it is a critical state shear stress.
2. A curve (OBCA, Fig. 7.5c) that links all peak shear stress values for Type II soils. We will call OBC (the curved part of OBCA), the peak shear stress envelope because any shear stress that lies on it is a peak shear stress.

At large normal effective stresses, the peak shear stress for Type II soils is suppressed and only a critical state shear stress is observed and appears as a point (point 9) located on OA (Fig. 7.5c). For Type II-A soils, the residual shear stresses would lie on a line OD below OA. We will call the angle between OD and the σ'_n axis, the residual friction angle, ϕ'_r . As the normal effective stress increases, the critical void ratio decreases (Fig. 7.5d). Thus, the critical void ratio is dependent on the magnitude of the normal effective stress.

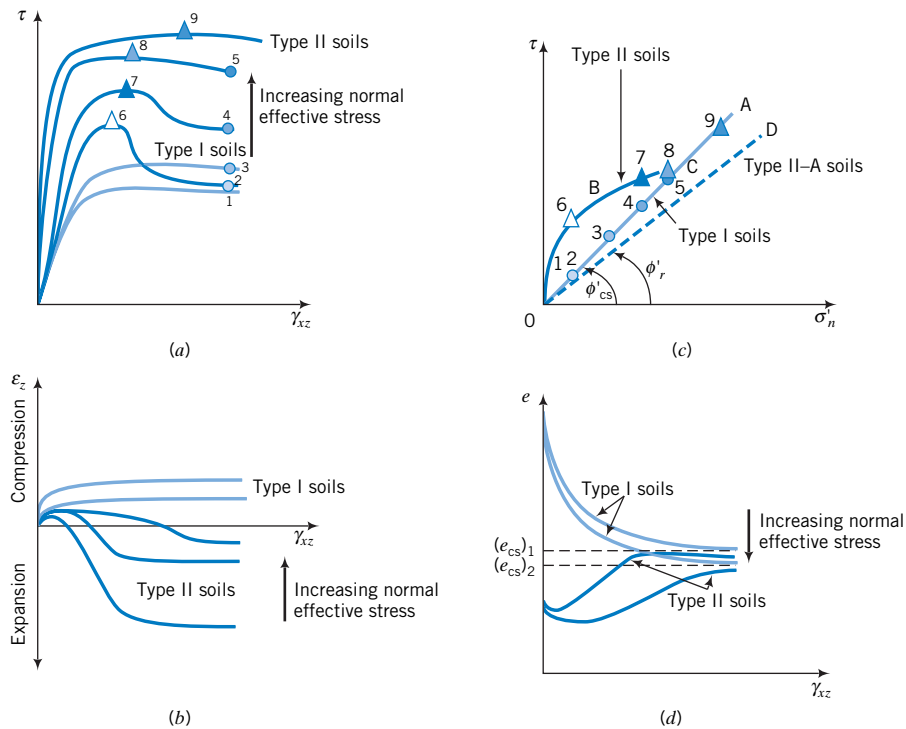


FIGURE 7.5 Effects of increasing normal effective stresses on the response of soils.

7.3.2 Effects of Overconsolidation Ratio

The initial state of the soil dictates the response of the soil to shearing forces. For example, two overconsolidated soils with different overconsolidation ratios but the same mineralogical composition would exhibit different peak shear stresses and volume expansion as shown in Fig. 7.6. The higher overconsolidated soil gives a higher peak shear strength and greater volume expansion.

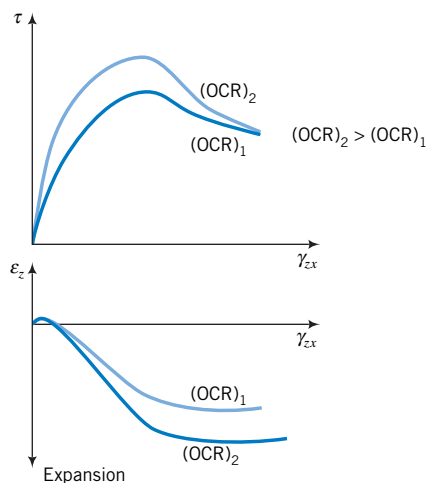


FIGURE 7.6 Effects of OCR on peak strength and volume expansion.

The essential points are:

1. *Type I soils—loose sands and normally consolidated and lightly overconsolidated clays—strain harden to a critical state shear stress and compress toward a critical void ratio.*
2. *Type II soils—dense sands and overconsolidated clays—reach a peak shear stress, strain soften to a critical state shear stress and expand toward a critical void ratio after an initial compression at low shear strains.*
3. *The peak shear stress of Type II soils is suppressed and the volume expansion decreases when the normal effective stress is large.*
4. *Just before peak shear stress is attained in Type II soils, shear bands develop. Shear bands are loose pockets or bands of soil mass that have reached the critical state shear stress. Denser soil masses adjacent to the shear bands gradually become looser as shearing continues.*
5. *All soils reach a critical state, irrespective of their initial state, at which continuous shearing occurs without changes in shear stress and volume for a given normal effective stress.*
6. *The critical state shear stress and the critical void ratio depend on the normal effective stress. Higher normal effective stresses result in higher critical state shear stresses and lower critical void ratios.*
7. *At large strains, the particles of some overconsolidated clays become oriented parallel to the direction of shear bands and the final shear stress attained is lower than the critical state shear stress.*
8. *Higher overconsolidation ratios result in higher peak shear stresses and greater volume expansion.*

7.3.3 Cemented Soils

The particles of some soils are cemented (chemically bonded). These soils possess shear strength even when the normal effective stress is zero. Cemented soils behave much like Type II soils except they have an initial shear strength under zero normal effective stress. In a plot of peak shear stress versus normal effective stress, an intercept shear stress, c_o , called cohesion is observed (Fig. 7.7). You should not rely on c_o in design because at large shear strains any shear strength due to cementation in the soil is destroyed.

7.3.4 Unsaturated Soils

Unsaturated soils generally behave like Type II soils because negative excess porewater pressure increases the effective stress. Recall that effective stress is equal to total stress minus porewater pressure. Thus, if the porewater pressure is negative, the effective stress increases.

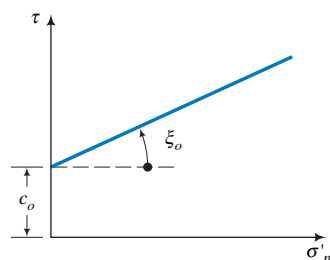


FIGURE 7.7 Peak shear stress envelope for cemented soils.

What's next... You should now have a general idea on the responses of soils to shearing forces. Can we interpret these responses using a simple model? In the next section, we will develop a simple model to gain an insight into the behavior of soils that will later help us to interpret the shear strength of soils.

7.4 TWO SIMPLE MODELS FOR THE SHEAR STRENGTH OF SOILS



Interactive Concept Learning and Self-Assessment

Access Chapter 7, Section 7.4 on the CD to learn about Coulomb's model through interactive visualizations. Take Quiz 7.4 to test your knowledge.

7.4.1 Coulomb's Model

You may recall Coulomb's frictional law from your courses in statics or physics. If a wooden block is pushed horizontally across a table (Fig. 7.8a), the horizontal force (H) required to initiate movement is

$$H = \mu W \quad (7.1)$$

where μ is the coefficient of static sliding friction between the block and the table and W is the weight of the block. The angle between the resultant force and the normal force is called the friction angle, $\phi' = \tan^{-1} \mu$.

In terms of stresses, Coulomb's law is expressed as

$$\tau_f = (\sigma'_n)_f \tan \phi' \quad (7.2)$$

where $\tau_f (=T/A$, where T is the shear force at impending slip and A is the area of the plane parallel to T) is the shear stress when slip is initiated, $(\sigma'_n)_f$ is the normal effective stress on the plane on which slip is initiated. The subscript f denotes failure, which according to Coulomb's law occurs when rigid body movement of one body relative to another is initiated. Failure does not necessarily mean collapse but the initiation of movement of one rigid body relative to another.

Coulomb's law requires the existence or the development of a critical sliding plane, also called slip plane. In the case of the wooden block on the table, the slip plane is the horizontal plane at the interface between the wooden block and the table. Unlike the wooden block, we do not know where the sliding plane is located for soils.

If you plot Coulomb's equation (7.2) on a graph of shear stress, τ_f , versus normal effective stress, $(\sigma'_n)_f$, you get a straight line similar to OA (Fig. 7.5) if $\phi' = \phi'_{cs}$. Thus, Coulomb's law may be used to model soil behavior at or near the critical state. But what about modeling the peak behavior that is characteristic of Type II soils?

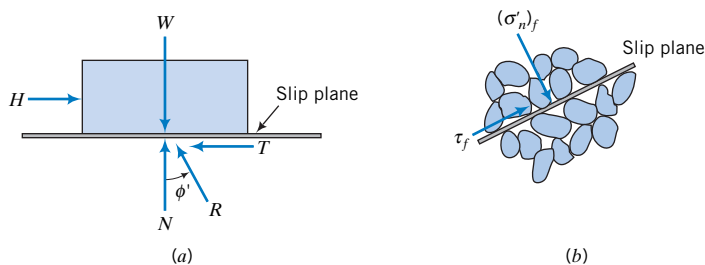


FIGURE 7.8 (a) Slip of a wooden block. (b) A slip plane in a soil mass.

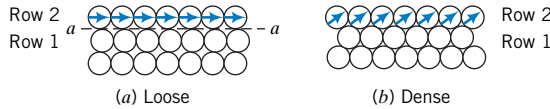


FIGURE 7.9 Packing of disks representing loose and dense sand.

You should recall from Chapter 3 that soils can have different unit weights depending on the arrangement of the particles. Let us simulate two extreme arrangements of soil particles for coarse-grained soils—one loose, the other dense. We will assume that the soil particles are spheres. In two dimensions, arrays of spheres become arrays of disks. The loose array is obtained by stacking the spheres one on top of another while the dense packing is obtained by staggering the rows as illustrated in Fig. 7.9.

For simplicity, let us consider the first two rows. If we push (shear) row 2 relative to row 1 in the loose assembly, sliding would be initiated on the horizontal plane, $a-a$, consistent with Coulomb's frictional law [Eq. (7.2)]. Once motion is initiated the particles in the loose assembly would tend to move into the void spaces. The direction of movement would have a downward component, that is, compression.

In the dense packing, relative sliding of row 2 with respect to row 1 is restrained by the interlocking of the disks. Sliding, for the dense assembly, would be initiated on an inclined plane rather than on a horizontal plane. For the dense assembly, the particles must ride up over each other or be pushed aside or both. The direction of movement of the particles would have an upward component, that is, expansion.

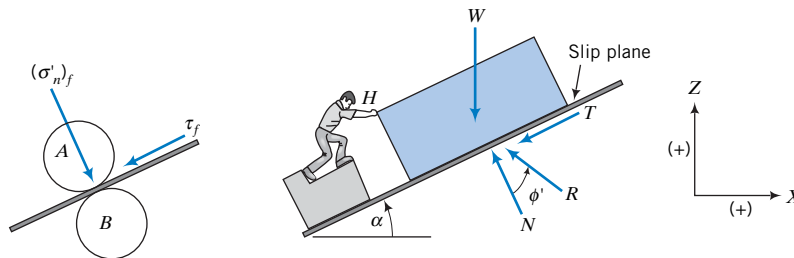
We are going to use our knowledge of statics to investigate impending sliding of particles up or down a plane to assist us in interpreting the shearing behavior of soils using Coulomb's frictional law. The shearing of the loose array can be idealized by analogy with the sliding of our wooden block on the horizontal plane. At failure (impending motion),

$$\frac{\tau_f}{(\sigma'_n)_f} = \frac{H}{W} = \tan \phi' \quad (7.3)$$

Consider two particles A and B in the dense assembly and draw the free-body diagram of the stresses at the sliding contact between A and B as depicted in Fig. 7.10. We now appeal to our wooden block for an analogy to describe the shearing behavior of the dense array. For the dense array, the wooden block is placed on a plane oriented at an angle α to the horizontal (Fig. 7.10b). Our goal is to find the horizontal force to initiate movement of the block up the incline. You may have solved this problem in statics. Anyway, we are going to solve it again. At impending motion, $T = \mu N$ where N is the normal force. Using the force equilibrium equations in the X and Z directions, we get

$$\Sigma F_x = 0: \quad H - N \sin \alpha - \mu N \cos \alpha = 0 \quad (7.4)$$

$$\Sigma F_z = 0: \quad N \cos \alpha - \mu N \sin \alpha - W = 0 \quad (7.5)$$



(a) Stresses on failure plane

(b) Simulated shearing of a dense array of particles

FIGURE 7.10 Simulation of failure in dense sand.

Solving for H and W , we obtain

$$H = N(\sin \alpha + \mu \cos \alpha) \quad (7.6)$$

$$W = N(\cos \alpha - \mu \sin \alpha) \quad (7.7)$$

Dividing Eq. (7.6) by Eq. (7.7) and simplifying, we obtain

$$\frac{H}{W} = \frac{\mu + \tan \alpha}{1 - \mu \tan \alpha} = \frac{\tan \phi' + \tan \alpha}{1 - \tan \phi' \tan \alpha}$$

By analogy with the loose assembly, we can replace H by τ_f and W by $(\sigma'_n)_f$, resulting in

$$\tau_f = (\sigma'_n)_f \frac{\tan \phi' + \tan \alpha}{1 - \tan \phi' \tan \alpha} = (\sigma'_n)_f \tan(\phi' + \alpha) \quad (7.8)$$

Let us investigate the implications of Eq. (7.8). If $\alpha = 0$, Eq. (7.8) reduces to Coulomb's frictional equation (7.2). If α increases, the shear strength, τ_f , gets larger. For instance, assume $\phi' = 30^\circ$ and $(\sigma'_n)_f$ is constant; then for $\alpha = 0$ we get $\tau_f = 0.58(\sigma'_n)_f$, but if $\alpha = 10^\circ$ we get $\tau_f = 0.84(\sigma'_n)_f$, that is, an increase of 45% in shear strength for a 10% increase in α .

If the normal effective stress increases on our dense disk assembly, the amount of “riding up” of the disks will decrease. In fact, we can impose a sufficiently high normal effective stress to suppress the “riding up” tendencies of the dense disk assembly. Therefore, the ability of the dense disk assembly to expand depends on the magnitude of the normal effective stress. The lower the normal effective stress, the greater the value of α . The net effect of α due to normal effective stress increases is that the failure envelope becomes curved as illustrated by OBC in Fig. 7.11, which is similar to the expected peak shear stress response of Type II soils (Fig. 7.5c).

The geometry of soil grains and the structural arrangement of soil fabrics are much more complex than our loose and dense assembly of disks. However, the model using disks is applicable to soils if we wish to interpret their (soils) shear strength using Coulomb's frictional law. In real soils, the particles are randomly distributed and often irregular. Shearing of a given volume of soil would cause impending slip of some particles to occur up the plane while others occur down the plane. The general form of Eq. (7.8) is then

$$\tau_f = (\sigma'_n)_f \tan(\phi' \pm \alpha) \quad (7.9)$$

where the positive sign refers to soils in which the net movement of the particles is initiated up the plane and the negative sign refers to net particle movement down the plane.

We will call the angle α the dilation angle. It is a measure of the change in volumetric strain with respect to the change in shear strain. Soils that have positive values of α expand during shearing while

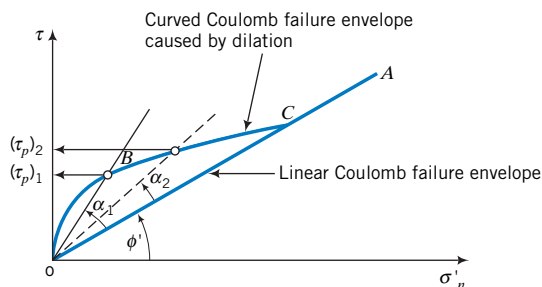


FIGURE 7.11 Effects of dilation on Coulomb's failure envelope.

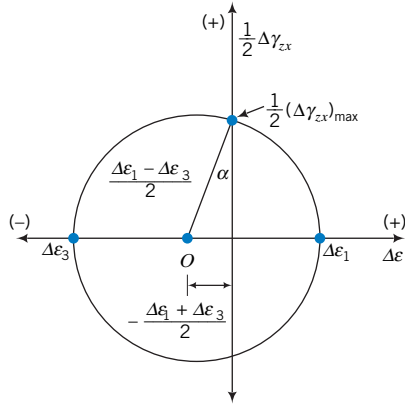


FIGURE 7.12 Mohr's circle of strain and angle of dilation.

soils with negative values of α contract during shearing. In Mohr's circle of strain (Fig. 7.12), the dilation angle is

$$\alpha = \sin^{-1} \left(\frac{\Delta \varepsilon_1 + \Delta \varepsilon_3}{\Delta \varepsilon_1 - \Delta \varepsilon_3} \right) = \sin^{-1} \left(- \frac{\Delta \varepsilon_1 + \Delta \varepsilon_3}{(\Delta \gamma_{zx})_{\max}} \right) \quad (7.10)$$

where Δ denotes change. The negative sign is used because we want α to be positive when the soil is expanding. You should recall that compression is taken as positive in soil mechanics. The angle α is also the tangent to the curve in a plot of volumetric strain versus shear strain as illustrated for simple shear in Fig. 7.3b.

If a soil mass is constrained in the lateral directions, the dilation angle is represented as

$$\alpha = \tan^{-1} \left(\frac{-\Delta z}{\Delta x} \right) \quad (7.11)$$

Dilation is not a peculiarity of soils but occurs in many other materials, for example, rice and wheat. The ancient traders of grains were well aware of the phenomenon of volume expansion of grains. However, it was Osborne Reynolds (1885) who described the phenomenon of dilatancy and brought it to the attention of the scientific community. Dilation can be seen in action at a beach. If you place your foot on a beach just following the receding wave, you will notice that the initially wet saturated sand around your foot momentarily appears to be dry (whitish color). This occurs because the sand mass around your foot dilates, sucking water up into the voids. This water is released, showing up as surface water, when you lift up your foot. View this phenomenon as reproduced in the lab by clicking Section 7.4. on the CD.

For cemented soils, Coulomb's frictional law can be written as

$$\tau_f = c_o + (\sigma'_n)_f \tan(\xi_o) \quad (7.12)$$

where c_o is called cohesion (Fig. 7.7) and ξ_o is the apparent friction angle.

Coulomb's model strictly applies to soil failures that occur along a slip plane such as a joint or the interface of two soils or the interface between a structure and a soil. Stratified soil deposits such as overconsolidated varved clays (regular layered soils that depict seasonal variations in deposition) and fissured clays are likely candidates for failure following Coulomb's model, especially if the direction of shearing is parallel to the direction of the bedding plane.

The essential points are:

1. *Shear failure of soils may be modeled using Coulomb's frictional law, $\tau_f = (\sigma'_n)_f \tan(\phi' \pm \alpha)$ where τ_f is the shear stress when slip is initiated, $(\sigma'_n)_f$ is the normal effective stress on the slip plane, ϕ' is the friction angle, and α is the dilation angle.*
2. *The effect of dilation is to increase the shear strength of the soil and cause the Coulomb's failure envelope to be curved.*
3. *Large normal effective stresses tend to suppress dilation.*
4. *At the critical state, the dilation angle is zero.*
5. *For cemented soils, Coulomb's frictional law is $\tau_f = c_o + (\sigma'_n)_f \tan(\xi_o)$ where c_o is called cohesion and ξ_o is the apparent friction angle.*

7.4.2 Taylor's Model

Taylor (1948) used an energy method to derive a simple soil model. He assumed that the shear strength of soil is due to sliding friction from shearing and the interlocking of soil particles. Consider a rectangular soil element that is sheared by a shear stress τ under a constant vertical effective stress, σ'_z . (Fig. 7.2c) Let us assume that the increment of shear strain is $d\gamma$ and the increment of vertical strain is $d\varepsilon_z$.

The external energy (force $\times$ distance moved in the direction of the force or stress $\times$ compatible strain) is $\tau d\gamma$. The internal energy is the work done by friction, $\mu_f \sigma'_z d\gamma$, where μ_f is the static, sliding friction coefficient and the work done by the movement of the soil against the vertical effective stress, $\pm \sigma'_z d\varepsilon_z$. The negative sign indicates the vertical strain is in the opposite direction (expansion) to the direction of the vertical effective stress. The energy $\pm \sigma'_z d\varepsilon_z$ is the interlocking energy due to the arrangement of the soil particles or soil fabric.

For equilibrium,

$$\tau d\gamma = \mu_f \sigma'_z d\gamma \pm \sigma'_z d\varepsilon_z$$

Dividing by $\sigma'_z d\gamma$ we get

$$\frac{\tau}{\sigma'_z} = \mu_f \mp \frac{d\varepsilon_z}{d\gamma} \quad (7.13)$$

At critical state, $\mu_f = \tan \phi'_{cs}$ and $\alpha = \frac{d\varepsilon_z}{d\gamma} = 0$. Therefore,

$$\left(\frac{\tau}{\sigma'_z} \right)_{cs} = \tan \phi'_{cs} \quad (7.14)$$

At peak shear strength,

$$-\frac{d\varepsilon_z}{d\gamma} = \tan \alpha_p$$

Therefore,

$$\left(\frac{\tau}{\sigma'_z} \right)_p = \tan \phi'_{cs} + \tan \alpha_p \quad (7.15)$$

Unlike Coulomb's model, Taylor's model does not require the assumption of any physical mechanism of failure, such as a plane of sliding. It can be applied at every stage of loading for soils that are homogeneous and deform under plane strain conditions similar to simple shear. This model would not apply to soils that fail along a joint or an interface between two soils. Taylor's model gives a higher peak dilation angle than Coulomb's model.

Equation (7.13) applies to two-dimensional stress systems. An extension of Taylor's model to account for three-dimensional stress is presented in Chapter 8. Neither Taylor's model nor Coulomb's model explicitly considers the rotation of the soil particles during shearing.

The essential points are:

1. *The shear strength of soils is due to friction and to interlocking of soil particles.*
2. *The critical state shear strength is: $\tau_{cs} = \sigma'_n \tan \phi'_{cs}$.*
3. *The peak shear strength is: $\tau_p = \sigma'_n (\tan \phi'_{cs} + \tan \alpha_p)$.*

What's next... In the next section, we will define and describe various parameters to interpret the shear strength of soils. It is an important section, which you should read carefully, because it is an important juncture in our understanding of shear strength of soils for soil stability analyses and design considerations.

7.5 INTERPRETATION OF THE SHEAR STRENGTH OF SOILS



Concept Learning

Access Chapter 7, Section 7.5 to learn about how to interpret soil strength.

The shear strength of a soil is its resistance to shearing stresses. In this book, we will interpret the shear strength of soils based on their capacity to dilate. Dense sands and overconsolidated clays ($OCR > 2$) tend to show peak shear stresses and expand (positive dilation angle), while loose sands and normally consolidated and lightly overconsolidated clays do not show peak shear stresses except at very low normal effective stresses and tend to compress (negative dilation angle). In our interpretation of shear strength, we will describe soils as dilating soils when they exhibit peak shear stresses at $\alpha > 0$ and nondilating soils when they exhibit no peak shear stress and attain a maximum shear stress at $\alpha = 0$. However, a nondilating soil does not mean that it does not change volume (expand or contract) during shearing. The terms dilating and nondilating only refer to particular stress states (peak and critical) during soil deformation.

The peak shear strength of a soil is provided by a combination of the shearing resistance due to sliding (Coulomb's sliding), dilatancy effects, crushing, and rearrangement of particles (Fig. 7.13). At low normal effective stresses, rearrangement of soil particles and dilatancy are more readily facilitated than at high normal effective stresses. At high normal effective stresses, particle crushing significantly influences the shearing resistance. The contribution of crushing and rearrangement of particles is difficult to quantify. In this book, we will take a simple approach and lump the strength not due to sliding friction as strength due to dilatancy.

We will refer to key soil shear strength parameters using the following notation. The peak shear strength, τ_p , is the peak shear stress attained by a dilating soil (Fig. 7.3). The dilation angle at peak shear stress will be denoted as α_p . The shear stress attained by all soils at large shear strains ($\gamma_{zx} > 10\%$), when the dilation angle is zero, is the critical state shear strength denoted by τ_{cs} . The void ratio corresponding to the critical state shear strength is the critical void ratio denoted by e_{cs} . The effective friction angle corresponding to the critical state shear strength and critical void ratio is ϕ'_{cs} .

The peak effective friction angle for a dilating soil according to Coulomb's model is

$$\phi'_p = \phi'_{cs} + \alpha_p \quad (7.16)$$

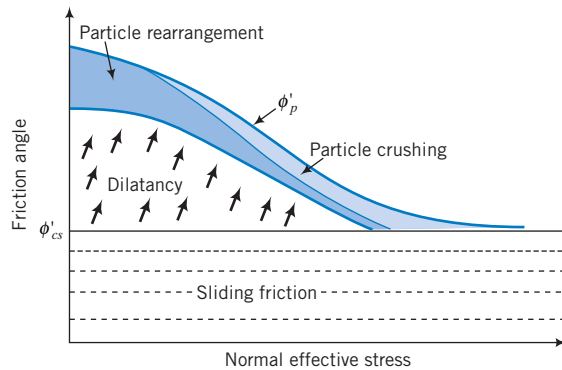


FIGURE 7.13 Contribution of sliding friction, dilatancy, crushing, and rearrangement of particles on the peak shear strength of soils.

Test results (Bolton, 1986) show that for plane strain tests

$$\phi'_p = \phi'_{cs} + 0.8\alpha_p \quad (7.17)$$

We will continue to use Eq. (7.16) but in practice you can make the adjustment [Eq. (7.17)] suggested by Bolton (1986).

Typical values of ϕ'_{cs} , ϕ'_p and ϕ'_r for soils are shown in Table 7.1. The peak dilatancy angle, α_p , generally has values ranging from 0 to 15°.

We will drop the term effective in describing friction angle and accept it by default such that effective critical state friction angle becomes critical state friction angle, ϕ'_{cs} , and effective peak friction angle becomes peak friction angle, ϕ'_p .

All soils, critical state shear strength: $\tau_{cs} = (\sigma'_n)_f \tan \phi'_{cs}$ (7.18)

Dilating soils, peak shear strength:

Coulomb: $\tau_p = (\sigma'_n)_f \tan(\phi'_{cs} + \alpha_p) = (\sigma'_n)_f \tan \phi'_p$ (7.19)

TABLE 7.1 Ranges of Friction Angles for Soils (degrees)

Soil type	ϕ'_{cs}	ϕ'_p	ϕ'_r
Gravel	30–35	35–50	
Mixtures of gravel and sand with fine-grained soils	28–33	30–40	
Sand	27–37*	32–50	
Silt or silty sand	24–32	27–35	
Clays	15–30	20–30	5–15

* Higher values (32°–37°) in the range are for sands with significant amount of feldspar (Bolton, 1986). Lower values (27–32°) in the range are for quartz sands.

Taylor:

$$\tau_p = (\sigma'_n)_f (\tan \phi'_{cs} + \tan \alpha_p) \quad (7.20)$$

The Coulomb equation for soils that exhibit residual shear strength is

$$\tau_r = (\sigma'_n)_f \tan \phi'_r \quad (7.21)$$

where ϕ'_r is the residual friction angle. The residual shear strength is very important in the analysis and design of slopes in overconsolidated clays and previously failed slopes.

Coulomb's model gives no information on the shear strains required to initiate slip. Strains (shear and volumetric) are important in the evaluation of shear strength and deformation of soils for design of safe foundations, slopes, and other geotechnical systems. In Chapter 8, we will develop a simple model in which we will consider strains at which soil failure occurs.

If the shear stress (τ) induced in a soil is less than the peak or critical shear strength, then the soil has reserved shear strength and we can characterize this reserved shear strength by a factor of safety (FS).

For peak condition in dilating soils:

$$FS = \frac{\tau_p}{\tau} \quad (7.22)$$

For critical state condition in all soils:

$$FS = \frac{\tau_{cs}}{\tau} \quad (7.23)$$

The essential points are:

1. The friction angle at the critical state, ϕ'_{cs} , is a fundamental soil parameter.
2. The friction angle at peak shear stress for dilating soils, ϕ'_p , is not a fundamental soil parameter but depends on the capacity of the soil to dilate.
3. Coulomb equation only gives information of the soil shear strength when slip is initiated. It does not give any information on the strains at which soil failure occurs.
4. The friction angle interpreted using Coulomb's law relates to sliding friction.

EXAMPLE 7.1 Interpreting Direct Shear Data

A dry sand was tested in a shear device. The shear force–shear displacement for a normal force of 100 N is shown in Fig. E7.1a.

- (1) Is the soil a dense or loose sand?
- (2) Identify and determine the peak shear force and critical state shear force.
- (3) Calculate the peak and critical state friction angles, and the peak dilation angle using Coulomb's model.
- (4) Determine the peak dilation angle using Taylor's model.
- (5) If the sand were loose, determine the critical state shear stress for a normal effective stress of 200 kPa using Coulomb's model.

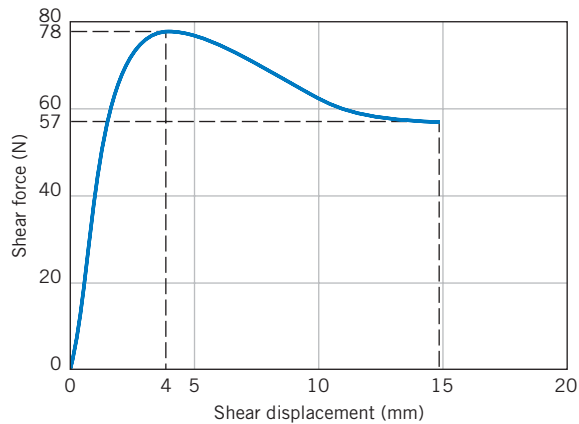


FIGURE E7.1a

Strategy Identify the peak and critical state shear force. The friction angle is the arctangent of the ratio of the shear force to the normal force. The constant shear force at large displacement gives the critical state shear force. From this force, you can calculate the critical state friction angle. Similarly, you can calculate ϕ'_p from the peak shear force.

Solution 7.1

Step 1: Determine whether soil is dense or loose. Since the curve shows a peak, the soil is likely to be dense.

Step 2: Identify peak and critical state shear forces.

With reference to Fig. E7.1b,

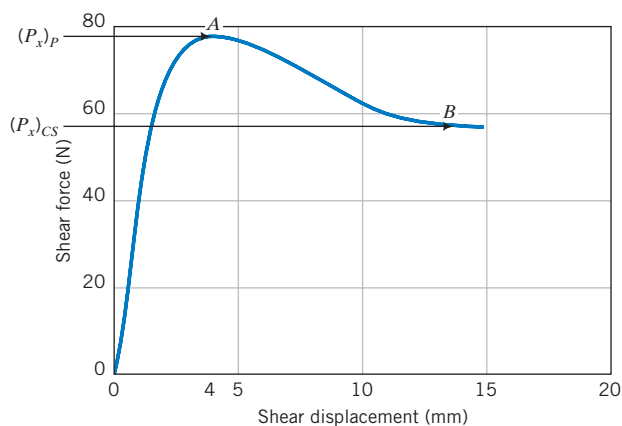


FIGURE E7.1b

Point A is the peak shear force.

Point B is the critical state shear force.

Step 3: Read values of peak and critical state shear forces from the graph in Fig. E7.1b.

$$\begin{aligned}(P_x)_P &= 78 \text{ N} \\ (P_x)_{CS} &= 57 \text{ N} \\ P_z &= 100 \text{ N}\end{aligned}$$

Step 4: Determine friction angles.

$$\begin{aligned}\phi'_P &= \tan^{-1} \left(\frac{(P_x)_P}{P_z} \right) = \tan^{-1} \left(\frac{78}{100} \right) = 38^\circ \\ \phi'_{CS} &= \tan^{-1} \left(\frac{(P_x)_{CS}}{P_z} \right) = \tan^{-1} \left(\frac{57}{100} \right) = 29.7^\circ\end{aligned}$$

Step 5: Determine peak dilation angle using Coulomb's model.

$$\alpha_P = \phi'_P - \phi'_{CS} = 38 - 29.7 = 8.3^\circ$$

Step 6: Calculate the peak dilation angle using Taylor's method.

$$\begin{aligned}\tan \alpha_P &= \frac{(P_x)_P}{P_z} - \tan \phi'_{CS} \\ \alpha_P &= \tan^{-1} \left(\frac{78}{100} - 0.57 \right) = 11.9^\circ\end{aligned}$$

Step 7: Calculate critical state shear stress.

$$\tau_f = \sigma'_n \tan \phi'_{CS} = 200 \tan (29.7^\circ) = 200 \times 0.57 = 114 \text{ kPa}$$

What's next... Coulomb's frictional law for finding the shear strength of soils requires that we know the friction angle and the normal effective stress on the slip plane. Both of these are not readily known for soils because soils are usually subjected to a variety of stresses. You should recall from Chapter 5 that Mohr's circle can be used to determine the stress within a soil mass. By combining Mohr's circle for finding stress states with Coulomb's frictional law we can develop a generalized failure criterion.

7.6 MOHR-COULOMB FAILURE CRITERION



Interactive Concept Learning and Self-Assessment

Access Chapter 7, Section 7.6 on the CD to learn about the Mohr-Coulomb failure criterion through interactive animation. Take Quiz 7.6 to assess your understanding.

Let us draw a Coulomb frictional failure line as illustrated by AB in Fig. 7.14 and subject a cylindrical sample of soil to principal stresses so that Mohr's circle touches the Coulomb failure line.

Of course, several circles can share AB as the common tangent but we will show only one for simplicity. The point of tangency is at $B[\tau_f, (\sigma'_n)_f]$ and the center of the circle is at O . We are going to discuss the top half of the circle; the bottom half is a reflection of the top half. The major and minor principal effective stresses at failure are $(\sigma'_1)_f$ and $(\sigma'_3)_f$. Our objective is to find a relationship between the principal effective stresses and ϕ' at failure. We will discuss the appropriate ϕ' later in this section.

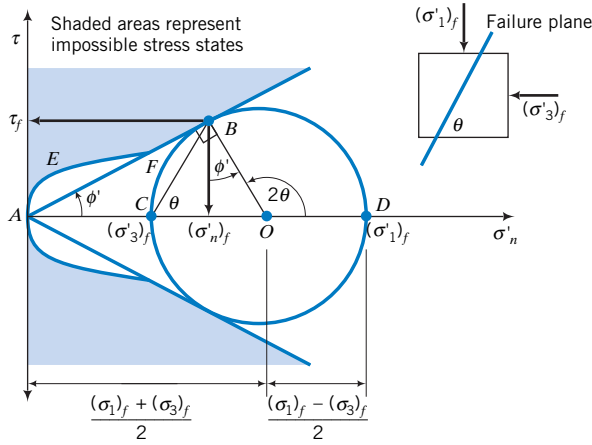


FIGURE 7.14 The Mohr–Coulomb failure envelope.

From the geometry of Mohr's circle,

$$\sin \phi' = \frac{OB}{OA} = \frac{\frac{(\phi'_1)_f - (\sigma'_3)_f}{2}}{\frac{(\sigma'_1)_f + (\sigma'_3)_f}{2}}$$

which reduces to

$$\sin \phi' = \frac{(\sigma'_1)_f - (\sigma'_3)_f}{(\sigma'_1)' + (\sigma'_3)_f} \quad (7.24)$$

Rearranging Eq. (7.24) gives

$$\left| \frac{(\sigma'_1)_f}{(\sigma'_3)_f} = \frac{1 + \sin \phi'}{1 - \sin \phi'} = \tan^2 \left(45 + \frac{\phi'}{2} \right) = K_p \right. \quad (7.25)$$

or

$$\frac{(\sigma'_3)_f}{(\sigma'_1)_f} = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \tan^2 \left(45 - \frac{\phi'}{2} \right) = K_a \quad (7.26)$$

where K_p and K_a are called the passive and active earth pressure coefficients. In Chapter 12, we will discuss K_p and K_a and use them in connection with the analysis of earth retaining walls. The angle $BCO = \theta$ represents the inclination of the failure plane or slip plane to the plane on which the major principal effective stress acts in Mohr's circle. Let us find a relationship between θ and ϕ' . From the geometry of Mohr's circle (Fig. 7.14),

$$\angle BOC = 90 - \phi' \quad \text{and} \quad \angle BOD = 2\theta = 90^\circ + \phi'$$

$$\therefore \theta = 45^\circ + \frac{\phi'}{2} = \frac{\pi}{4} + \frac{\phi'}{2} \quad (7.27)$$

The failure stresses are

$$(\sigma'_n)_f = \frac{\sigma'_1 + \sigma'_3}{2} - \frac{\sigma'_1 - \sigma'_3}{2} \sin \phi' \quad (7.28)$$

$$\tau_f = \frac{\sigma'_1 - \sigma'_3}{2} \cos \phi' \quad (7.29)$$

The Mohr–Coulomb failure criterion requires that stresses on the soil mass cannot lie within the shaded region shown in Fig. 7.14. That is, the soil cannot have stress states greater than the failure stress state. The shaded areas are called the regions of impossible stress states. The bounding curve for possible stress states is the failure envelope, $A E F B$, for dilating soils. For nondilating soils, the bounding curve is the linear line $A F B$. The Mohr–Coulomb failure criterion derived here is independent of the intermediate principal effective stress σ'_2 and does not consider the strains at which failure occurs.

Traditionally, failure criteria are defined in terms of stresses. Strains are considered at working stresses (stresses below the failure stresses) using stress–strain relationships (also called constitutive relationships) such as Hooke’s law. Strains are important because although the stress or load imposed on a soil may not cause it to fail, the resulting strains or displacements may be intolerable. We will describe a simple model in Chapter 8 that considers the effects of the intermediate principal effective stress and strains on soil behavior.

If we normalize (make the quantity a number, i.e., no units) Eq. (7.24) by dividing the nominator and denominator by σ'_{3f} , we get

$$\sin \phi' = \frac{\frac{(\sigma'_1)_f}{(\sigma'_{3f})} - 1}{\frac{(\sigma'_1)_f}{(\sigma'_{3f})} + 1}$$

The implication of this equation is that the Mohr–Coulomb failure criterion defines failure when the maximum principal effective stress ratio, called maximum effective stress obliquity, $\frac{(\sigma'_1)_f}{(\sigma'_{3f})}$, is achieved and not when the maximum shear stress, $[(\sigma'_1 - \sigma'_3)/2]_{\max}$, is achieved. The failure shear stress is then less than the maximum shear stress.

You should use the appropriate value of ϕ' in the Mohr–Coulomb equation [Eq. (7.24)]. For nondilating soils, $\phi' = \phi'_{cs}$; while for dilating soils, $\phi' = \phi'_p$.

The essential points are:

1. *Coupling Mohr’s circle with Coulomb’s frictional law allows us to define shear failure based on the stress state of the soil.*
2. *The Mohr–Coulomb failure criterion is*

$$\sin \phi' = \frac{(\sigma'_1)_f - (\sigma'_{3f})}{(\sigma'_1)_f + (\sigma'_{3f})}$$

or

$$\frac{(\sigma'_1)_f}{(\sigma'_{3f})} = \frac{1 + \sin \phi'}{1 - \sin \phi'} = \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

or

$$\frac{(\sigma'_{3f})}{(\sigma'_1)_f} = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \tan^2 \left(45 - \frac{\phi'}{2} \right)$$

3. *Failure occurs, according to the Mohr–Coulomb failure criterion, when the soil reaches the maximum effective stress obliquity, that is, $\left\{ \frac{(\sigma'_1)_f}{(\sigma'_{3f})} \right\}_{\max}$.*

4. The failure plane or slip plane is inclined at an angle $\theta = \pi/4 + \phi'/2$ to the plane on which the major principal effective stress acts.

5. The maximum shear stress, $\tau_{\max} = [(\sigma'_1 - \sigma'_3)/2]_{\max}$, is not the failure shear stress.

6. The failure stresses are

$$\begin{aligned}(\sigma'_n)_f &= \frac{\sigma'_1 + \sigma'_3}{2} - \frac{\sigma'_1 - \sigma'_3}{2} \sin \phi' \\ \tau_f &= \frac{\sigma'_1 - \sigma'_3}{2} \cos \phi'\end{aligned}$$

EXAMPLE 7.2 Friction Angle and Inclination of Slip Plane

A cylindrical soil sample was subjected to axial principal stresses (σ'_1) and radial principal stresses (σ'_3). The soil could not support additional stresses when $\sigma'_1 = 300$ kPa and $\sigma'_3 = 100$ kPa. Determine the friction angle and the inclination of the slip plane to the horizontal. Assume no significant dilational effects.

Strategy This example is a straightforward application of Eqs. (7.24) and (7.27). Since dilation is neglected $\phi' = \phi'_{\text{cs}}$.

Solution 7.2

Step 1: Find ϕ'_{cs} .

From Eq. (7.24),

$$\begin{aligned}\sin'_{\text{cs}} &= \frac{(\sigma'_1)_f - (\sigma'_3)_f}{(\sigma'_1)_f + (\sigma'_3)_f} = \frac{300 - 100}{300 + 100} = \frac{2}{4} = \frac{1}{2} \\ \therefore \phi'_{\text{cs}} &= 30^\circ\end{aligned}$$

Step 2: Find θ .

From Eq. (7.27),

$$\theta = 45^\circ + \frac{\phi'_{\text{cs}}}{2} = 45^\circ + \frac{30^\circ}{2} = 60^\circ$$

EXAMPLE 7.3 Failure Stress Due to a Foundation

Figure E7.3 shows the soil profile at a site for a proposed building. Determine the increase in vertical effective stress at which a soil element at a depth of 3 m, under the center of the building, will fail if the increase in lateral effective stress is 40% of the increase in vertical effective stress. The coefficient of lateral earth pressure at rest, K_o , is 0.5.

Strategy You are given a uniform deposit of sand and its properties. Use the data given to find the initial stresses and then use the Mohr–Coulomb equation to solve the problem. Since the soil element is under the center of the building, axisymmetric conditions prevail. Also, you are given that $\Delta\sigma'_3 = 0.4\Delta\sigma'_1$. Therefore, all you need to do is to find $\Delta\sigma'_1$.

Solution 7.3**Step 1:** Find the initial effective stresses.

Assume the top 1 m of soil to be saturated.

$$\sigma'_{zo} = (\sigma'_1)_o = (18 \times 3) - 9.8 \times 2 = 34.4 \text{ kPa}$$

The subscript o denotes original or initial.

The lateral earth pressure is

$$(\sigma'_x)_o = (\sigma'_3)_o = K_o(\sigma'_z)_o = 0.5 \times 34.4 = 17.2 \text{ kPa}$$

Step 2: Find $\Delta\sigma'_1$.

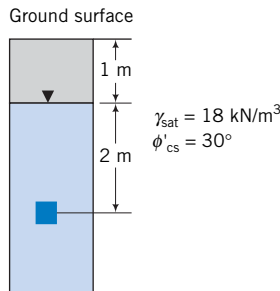
$$\text{At failure : } \frac{(\sigma'_1)_f}{(\sigma'_3)_f} = \frac{1 + \sin \phi'_{cs}}{1 - \sin \phi'_{cs}} = \frac{1 + \sin 30^\circ}{1 - \sin 30^\circ} = 3$$

But

$$(\sigma'_1)_f = (\sigma'_1)_o + \Delta\sigma'_1 \quad \text{and} \quad (\sigma'_3)_f = (\sigma'_3)_o + 0.4\Delta\sigma'_1$$

where $\Delta\sigma'_1$ is the additional vertical effective stress to bring the soil to failure.

$$\therefore \frac{(\sigma'_1)_o + \Delta\sigma'_1}{(\sigma'_3)_o + 0.4\Delta\sigma'_1} = \frac{34.4 + \Delta\sigma'_1}{17.2 + 0.4\Delta\sigma'_1} = 3$$

The solution is $\Delta\sigma'_1 = 86 \text{ kPa}$.**FIGURE E7.3**

What's next... In the next section, we will discuss the implications of Coulomb and Mohr-Coulomb failure criteria for soils. You need to be aware of some important ramifications of these criteria when you use them to interpret soil properties for practical applications.

7.7 PRACTICAL IMPLICATIONS OF COULOMB AND MOHR-COULOMB FAILURE CRITERIA

When we interpret soil failure using Coulomb and Mohr-Coulomb criteria, we are using a particular mechanical model—a sliding block model. For this model, we assume that:

1. There is a slip plane upon which one part of the soil mass slides relative to the other. Each part of the soil above and below the slip plane is a rigid mass. However, soils generally do not fail on a slip plane. Rather, in dense soils, there are pockets or bands of soil that reached critical state while other pockets are still dense. As the soil approaches peak shear stress and beyond, more dense pockets become loose as the soil strain softens. At the critical state, the whole soil mass becomes loose and behaves like a viscous fluid. Loose soils do not normally show slip planes or shear bands, and strain hardens to the critical state.
2. No deformation of the soil mass occurs prior to failure. In reality, significant soil deformation (shear strains $\sim 2\%$) is required to mobilize the peak shear stress and much more (shear strains $> 10\%$) for the critical state shear stress.
3. Failure occurs according to Coulomb by impending sliding along a plane and according to Mohr–Coulomb when the maximum stress obliquity on a plane is mobilized.

We are going to define three regions of soil states as illustrated in Fig. 7.15 and consider practical implications of soils in these regions.

Region I. Impossible soil states. A soil cannot have soil states above the boundary AEFB.

Region II. Impending instability (risky design). Soil states within the region AEFA (Fig. 7.15a) or 1-2-3 (Fig. 7.15b) are characteristic of dilating soils that show peak shear strength and are associated with the formation of shear bands. The shear bands consist of soils that have reached the critical state and are embedded within soil zones with high interlocking stresses due to particle rearrangement. These shear bands grow as the peak shear strength is mobilized and as the soil strain softens subsequently to the critical state.

Soil stress states within AEFA (Fig. 7.15a) or 1-2-3 (Fig. 7.15b) are analogous to brittle material type behavior. Brittle material (e.g., cast iron) will fail suddenly. Because of the shear bands that are formed within Region II, the soil permeability increases and if water is available it will migrate to and flow through them. This could lead to catastrophic failures in soil structures such as slopes since the flow of water through the shear bands could trigger flow slides. Design that allows the soil to mobilize stress states that lie within Region II can be classified as a risky design. Soil stress states that lie on the curve AEF can lead to sudden failure (collapse). We will call this curve the peak strength envelope (a curve linking the loci of peak shear strengths).

Region III. Stable soil states (safe design). One of your aims as a geotechnical engineer is to design geotechnical systems on the basis that if the failure state were to occur, the soil would not collapse suddenly but will continuously deform under constant load. This is called ductility. Soil states that are below the failure line or failure envelop AB (Fig. 7.15a) or 0-1-3 (Fig. 7.15b) would lead to safe design. Soil states on AB are failure states.

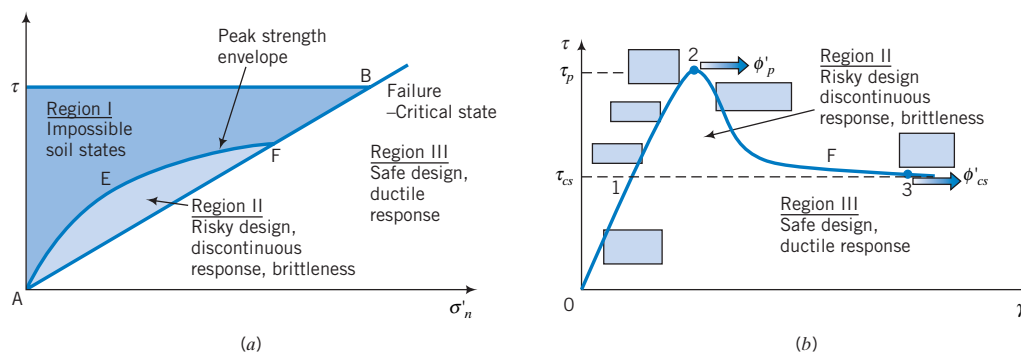


FIGURE 7.15 Interpretation of soil states.

The essential points are:

1. *Soil states above the peak shear strength boundary are impossible.*
2. *Soil states within the peak shear strength boundary and the failure line (critical state) are associated with brittle, discontinuous soil responses and risky design.*
3. *Soil states below the failure line lead to ductile responses and are safe.*
4. *You should not rely on ϕ'_p in geotechnical design because the amount of dilation one measures in laboratory or field tests may not be mobilized by the soil under the construction loads. You should use ϕ'_{cs} unless experience dictates otherwise. A higher factor of safety is warranted if ϕ'_p rather than ϕ'_{cs} is used in design.*

What's next. . . In the next section, we will consider two rather extreme conditions—drained and undrained conditions—under which soil is loaded and the effects these loading conditions have on the shear strength. Drained and undrained conditions are the bounds to evaluate soil stability.

7.8 DRAINED AND UNDRAINED SHEAR STRENGTH



Concept Learning and Self-Assessment

Access Chapter 7, Section 7.8 to learn about undrained shear strength. Take Quiz 7.8 to test your understanding.

You were introduced to drained and undrained conditions when we discussed stress paths in Chapter 5. Drained condition occurs when the excess porewater pressure developed during loading of a soil dissipates, i.e., $\Delta u = 0$. Undrained condition occurs when the excess porewater pressure cannot drain, at least quickly, from the soil; that is, $\Delta u \neq 0$. The existence of either condition—drained or undrained—depends on the soil type, geological formation (fissures, sand layers in clays, etc.), and the rate of loading.

The rate of loading under the undrained condition is often much faster than the rate of dissipation of the excess porewater pressure and the volume change tendency of the soil is suppressed. The result of this suppression is a change in excess porewater pressure during shearing. A soil with a tendency to compress during drained loading will exhibit an increase in excess porewater pressure (positive excess porewater pressure, Fig. 7.16) under undrained condition resulting in a decrease in effective stress. A soil that expands during drained loading will exhibit a decrease in excess porewater pressure (negative excess porewater pressure, Fig. 7.16) under undrained condition resulting in an increase in effective stress. These changes in excess porewater pressure occur because the void ratio does not change during undrained loading; that is, the volume of the soil remains constant.

During the life of a geotechnical structure, called the long-term condition, the excess porewater pressure developed by a loading dissipates and drained condition applies. Clays usually take many years to dissipate the excess porewater pressures. During construction and shortly after, called the short-term condition, soils with low permeability (fine-grained soils) do not have sufficient time for the excess porewater pressure to dissipate and undrained condition applies. The permeability of coarse-grained soils is sufficiently large that under static loading conditions the excess porewater pressure dissipates quickly. Consequently, undrained condition does not apply to clean coarse-grained soils under static loading but only to fine-grained soils and to mixtures of coarse and fine-grained soils. Dynamic loading, such as during an earthquake, is imposed so quickly that even coarse-grained soils do not have sufficient time to dissipate the excess porewater pressure and undrained condition applies.

The shear strength of a fine-grained soil under undrained condition is called the undrained shear strength, s_u . We use the Tresca's failure criterion—shear stress at failure is one half the principal stress

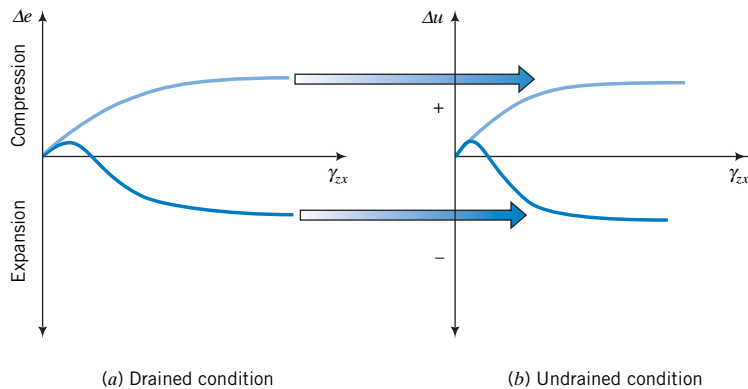


FIGURE 7.16 Effects of drained and undrained conditions on volume changes.

difference—to interpret the undrained shear strength. The undrained shear strength s_u , is the radius of the Mohr total stress circle; that is,

$$s_u = \frac{(\sigma_1)_f - (\sigma_3)_f}{2} = \frac{(\sigma'_1)_f - (\sigma'_3)_f}{2} \quad (7.30)$$

as shown in Fig. 7.17a. The shear strength under undrained loading depends only on the initial void ratio or the initial water content. An increase in initial normal stress, sometimes called confining pressure, causes a decrease in initial void ratio and a larger change in excess porewater pressure when a soil is sheared under undrained condition. The result is that the Mohr's circle of total stress expands and the undrained shear strength increases (Fig. 7.17b). Thus, s_u is not a fundamental soil shear strength property. The value of s_u depends on the magnitude of the initial confining pressure or initial void ratio. Analyses of soil strength and soil stability problems using s_u are called total stress analyses (TSA).

When designing geotechnical systems, geotechnical engineers must consider both drained and undrained conditions to determine which of these conditions is critical. The decision on what shear strength parameters to use depends on whether you are considering the short-term (undrained) or the long-term (drained) conditions. In the case of analyses for drained condition, called effective stress analyses (ESA), the shear strength parameters are ϕ'_p and ϕ'_{cs} . The value of ϕ'_{cs} is constant for a soil regardless of its initial condition and the magnitude of the normal effective stress. But the value of ϕ'_p depends on the normal effective stress. In the case of fine-grained soils, the shear strength parameter for short-term loading is s_u .

A summary of the essential differences between drained and undrained conditions is shown in Table 7.2.

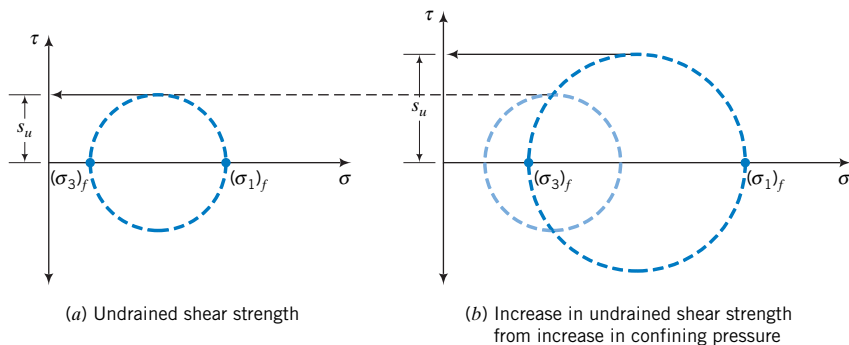


FIGURE 7.17 Mohr's circles for undrained conditions.

TABLE 7.2 Differences Between Drained and Undrained Loading

Condition	Drained	Undrained
Excess porewater pressure	~ 0	Not zero; could be positive or negative
Volume change	Compression Expansion	Positive excess porewater pressure Negative excess porewater pressure
Consolidation	Yes, fine-grained soils	No
Compression	Yes	Yes, but lateral expansion must occur so that the volume change is zero
Analysis	Effective stress	Total stress
Design parameters	ϕ'_{cs} (or ϕ'_p)	s_u

The essential points are:

- 1 Volume changes that occur under drained condition are suppressed under undrained condition. The result of this suppression is that a soil with a compression tendency under drained condition will respond with positive excess porewater pressures during undrained condition, and a soil with an expansion tendency during drained condition will respond with negative excess porewater pressures during undrained condition.*
- 2 For an effective stress analysis, the shear strength parameters are ϕ'_{cs} and ϕ'_p .*
- 3 For a total stress analysis, which applies to fine-grained soils, the shear strength parameter is the undrained shear strength, s_u .*
- 4. The undrained shear strength depends on the initial void ratio. It is not a fundamental soil shear strength parameter.*

What's next... We have identified the shear strength parameters (ϕ'_{cs} , ϕ'_p and s_u) that are important for analyses and design of geotechnical systems. A variety of laboratory tests and field tests are used to determine these parameters. We will describe many of these tests and the interpretation of the results. You may have to perform some of these tests in the laboratory section of your course.

7.9 LABORATORY TESTS TO DETERMINE SHEAR STRENGTH PARAMETERS



Virtual Laboratory and Self-Assessment

Access Chapter 7, click Virtual Lab on the left sidebar to conduct a virtual direct shear test and a virtual triaxial test. Click on Section 7.9 to learn about lab tests to determine the shear strength of soils. Test your understanding by taking Quiz 7.9.

7.9.1 A Simple Test to Determine Friction Angle of Clean Coarse-Grained Soils

The critical state friction angle, ϕ'_{cs} , for a clean coarse-grained soil, can be found by pouring the soil into a loose heap on a horizontal surface and measuring the slope angle of the heap relative to the horizontal. This angle is sometimes called the angle of repose, but it closely approximates ϕ'_{cs} . Access Chapter 7, Section 7.9 on the CD to view a video on how to determine ϕ'_{cs} for coarse-grained soil.

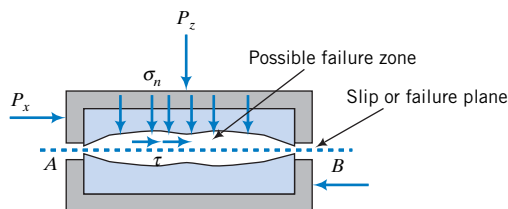


FIGURE 7.18 Shear box.

7.9.2 Shear Box or Direct Shear Test

A popular apparatus to determine the shear strength parameters is the shear box. This test is useful when a soil mass is likely to fail along a thin zone under plane strain conditions. The shear box (Fig. 7.18) consists of a horizontally split, open metal box. Soil is placed in the box, and one-half of the box is moved relative to the other half. Failure is thereby constrained along a thin zone of soil on the horizontal plane (AB). Serrated or grooved metal plates are placed at the top and bottom faces of the soil to generate the shearing force.

Vertical forces are applied through a metal platen resting on the top serrated plate. Horizontal forces are applied through a motor for displacement control or by weights through a pulley system for load control. Most shear box tests are conducted using displacement control because we can get both the peak shear force and the critical shear force. In load control tests, you cannot get data beyond the maximum or peak shear force.

The horizontal displacement, Δx , the vertical displacement, Δz , the vertical loads, P_z , and the horizontal loads, P_x , are measured. Usually, three or more tests are carried out on a soil sample using three different constant vertical forces. Failure is determined when the soil cannot resist any further increment of horizontal force. The stresses and strains in the shear box test are difficult to calculate from the forces and displacements measured. The stresses in the thin (dimension unknown) constrained failure zone (Fig. 7.18) are not uniformly distributed and strains cannot be determined.

The shear box apparatus cannot prevent drainage, but one can get an estimate of the undrained shear strength of clays by running the shear box test at a fast rate of loading so that the test is completed quickly. However, the shear box test should not be used for an accurate determination of the undrained shear strength of soils.

In summary, drained tests are generally conducted in a shear box test. Three or more tests are performed on a soil. The soil sample in each test is sheared under a constant vertical force, which is different in each test. The data recorded for each test are the horizontal displacements, the horizontal forces, the vertical displacements, and the constant vertical force under which the test is conducted. From the recorded data, you can find the following parameters: τ_p , τ_{cs} , ϕ'_p , ϕ'_{cs} , α (and s_u , if fine-grained soils are tested quickly). These parameters are generally determined from plotting the data, as illustrated in Fig. 7.19 for sand.

Only the results of one test at a constant value of P_z are shown in Figs. 7.19a,b. The results of $(P_x)_p$ and $(P_x)_{cs}$ plotted against P_z for all tests are shown in Fig. 7.19c. If the soil is dilatant, it would exhibit a peak shear force (Fig. 7.19a, dense sand) and expand (Fig. 7.19b, dense sand), and the failure envelope would be curved (Fig. 7.19c, dense sand). The peak shear stress is the peak shear force divided by the cross-sectional area (A) of the test sample; that is,

$$\tau_p = \frac{(P_x)_p}{A} \quad (7.31)$$

The critical shear stress is

$$\tau_{cs} = \frac{(P_x)_{cs}}{A} \quad (7.32)$$

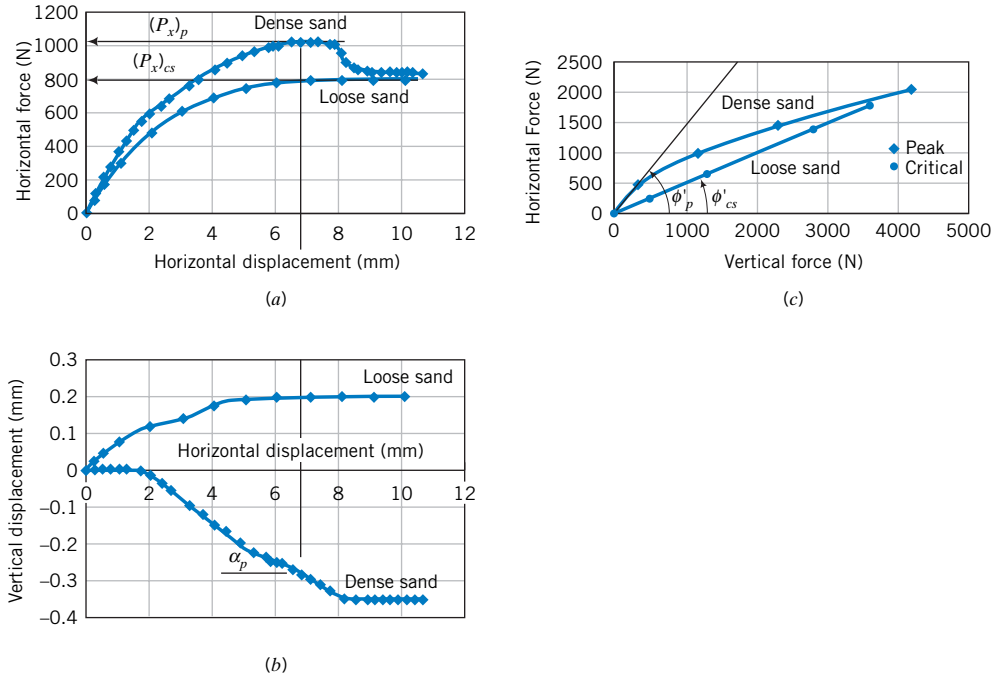


FIGURE 7.19 Results from a shear box test on a dense and a loose sand.

In a plot of vertical forces versus horizontal forces (Fig. 7.19c), the points representing the critical horizontal forces should ideally lie along a straight line through the origin. Experimental results usually show small deviations from this straight line and a “best-fit” straight line is conventionally drawn. The angle subtended by this straight line and the horizontal axis is ϕ'_{cs} . Alternatively,

$$\phi'_{cs} = \tan^{-1} \frac{(P_x)_{cs}}{P_z} \quad (7.33)$$

For dilatant soils, each point representing the peak horizontal force in Fig. 7.19c that does not lie on the “best-fit” straight line represents a value of ϕ'_p at the corresponding vertical force. Recall from Section 7.5 that ϕ'_p is not constant but varies with the magnitude of the normal effective stress (P_z/A). Usually, the normal effective stress at which ϕ'_p is determined should correspond to the maximum anticipated normal effective stress in the field. The value of ϕ'_p is largest at the lowest value of the applied normal effective stress, as illustrated in Fig. 7.19c. You would determine ϕ'_p by drawing a line from the origin to the point representing the peak horizontal force at the desired normal force, and measuring the angle subtended by this line and the horizontal axis. Alternatively,

$$\phi'_p = \tan^{-1} \frac{(P_x)_p}{P_z} \quad (7.34)$$

You can also determine the peak dilation angle directly for each test from a plot of horizontal displacement versus vertical displacement, as illustrated in Fig. 7.19b. The peak dilation angle is

$$\alpha_p = \tan^{-1} \left(\frac{-\Delta z}{\Delta x} \right) \quad (7.35)$$

We can find α_p from

$$\alpha_p = \phi'_p - \phi'_{cs} \quad (7.36)$$

EXAMPLE 7.4 *Interpretation of Shear Box Test*

The shear box test results of two samples of the same soil but with different initial unit weights are shown in the table below. Sample A did not show any peak values but Sample B did.

Soil	Test number	Vertical force (N)	Horizontal force (N)
A	Test 1	250	150
	Test 2	500	269
	Test 3	750	433
B	Test 1	100	98
	Test 2	200	175
	Test 3	300	210
	Test 4	400	248

Determining the following:

- (a) ϕ'_{cs}
- (b) ϕ'_p at vertical forces of 200 N and 400 N for sample B
- (c) The dilation angle at vertical forces of 200 N and 400 N for sample B

Strategy To obtain the desired values, it is best to plot a graph of vertical force versus horizontal force.

Solution 7.4

Step 1: Plot a graph of the vertical forces versus failure horizontal forces for each sample. See Fig. E7.4.

Step 2: Extract ϕ'_{cs} .

All the plotted points for sample A fall on a straight line through the origin. Sample A is a nondilatant soil, possibly a loose sand or a normally consolidated clay. The effective friction angle is $\phi'_{cs} = 30^\circ$.

Step 3: Determine ϕ'_p .

The horizontal forces corresponding to vertical forces at 200 N and 400 N for sample B do not lie on the straight line corresponding to ϕ'_{cs} . Therefore, each of these forces has a ϕ'_p associated with it.

$$(\phi'_p)_{200\text{ N}} = \tan^{-1}\left(\frac{175}{200}\right) = 41.2^\circ$$

$$(\phi'_p)_{400\text{ N}} = \tan^{-1}\left(\frac{248}{400}\right) = 31.8^\circ$$

Step 4: Determine α_p .

$$\alpha_p = \phi'_p - \phi'_{cs}$$

$$(\alpha_p)_{200\text{ N}} = 41.2 - 30 = 11.2^\circ$$

$$(\alpha_p)_{400\text{ N}} = 31.8 - 30 = 1.8^\circ$$

Note that as the normal force increases α_p decreases.

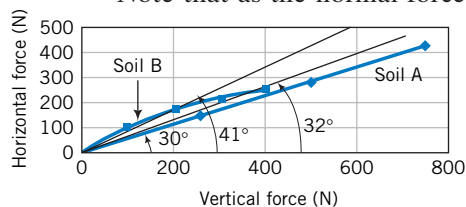


FIGURE E7.4

EXAMPLE 7.5 *Using Coulomb's Law*

The critical state friction angle of a soil is 28° . Determine the critical state shear stress if the normal effective stress is 200 kPa.

Strategy This is a straightforward application of the Coulomb's frictional equation.

Solution 7.5

Step 1: Determine the failure shear stress.

$$\begin{aligned}\tau_f &= \tau_{cs} = (\sigma'_n)_f \tan \phi'_{cs} \\ \tau_{cs} &= 200 \tan 28^\circ = 106.3 \text{ kPa}\end{aligned}$$

EXAMPLE 7.6 *Interpreting Shear Box Test Data*

The data recorded during a shear box test on a sand sample, $10 \text{ cm} \times 10 \text{ cm} \times 3 \text{ cm}$, at a constant vertical force of 1200 N are shown in the table below. A negative sign denotes vertical expansion.

- Plot graphs of (1) horizontal forces versus horizontal displacements and (2) vertical displacements versus horizontal displacements.
- Would you characterize the behavior of this sand as that of a dense or a loose sand? Explain your answer.
- Determine (1) the maximum or peak shear stress, (2) the critical state shear stress, (3) the peak dilation angle, (4) ϕ'_p , and (5) ϕ'_{cs} .

Horizontal displacement (mm)	Horizontal force (N)	Vertical displacement (mm)	Horizontal displacement (mm)	Horizontal force (N)	Vertical displacement (mm)
0.00	0.00	0.00	6.10	988.29	−0.40
0.25	82.40	0.00	6.22	988.29	−0.41
0.51	157.67	0.00	6.48	993.68	−0.45
0.76	249.94	0.00	6.60	998.86	−0.46
1.02	354.31	0.00	6.86	991.52	−0.49
1.27	425.72	0.01	7.11	999.76	−0.51
1.52	488.90	0.00	7.37	1005.26	−0.53
1.78	538.33	0.00	7.75	1002.51	−0.57
2.03	571.29	−0.01	7.87	994.27	−0.57
2.41	631.62	−0.03	8.13	944.83	−0.58
2.67	663.54	−0.05	8.26	878.91	−0.58
3.30	759.29	−0.09	8.51	807.50	−0.58
3.68	807.17	−0.12	8.64	791.02	−0.59
4.06	844.47	−0.16	8.89	774.54	−0.59
4.45	884.41	−0.21	9.14	766.30	−0.60
4.97	928.35	−0.28	9.40	760.81	−0.59
5.25	939.34	−0.31	9.65	760.81	−0.59
5.58	950.32	−0.34	9.91	758.06	−0.60
5.72	977.72	−0.37	10.16	758.06	−0.59
5.84	982.91	−0.37	10.41	758.06	−0.59
5.97	988.29	−0.40	10.67	755.32	−0.59

Strategy After you plot the graphs, you can get an idea as to whether you have a loose or dense sand. A dense sand may show a peak horizontal force in the plot of horizontal force versus horizontal displacement and would expand.

Solution 7.6

Step 1: Plot graphs.

See Fig. E7.6.

Step 2: Determine whether the sand is dense or loose.

The sand appears to be dense—it shows a peak horizontal force and dilated.

Step 3: Extract the required values.

Cross-sectional area of sample: $A = 10 \times 10 = 100 \text{ cm}^2 = 10^{-2} \text{ m}^2$

$$(c1) \quad \tau_p = \frac{(P_x)_p}{A} = \frac{1005 \text{ N}}{10^{-2}} \times 10^{-3} = 100.5 \text{ kPa}$$

$$(c2) \quad \tau_{cs} = \frac{(P_x)_{cs}}{A} = \frac{758 \text{ N}}{10^{-2}} \times 10^{-3} = 75.8 \text{ kPa}$$

$$(c3) \quad \alpha_p = \tan^{-1} \left(\frac{-\Delta z}{\Delta x} \right) = \tan^{-1} \left(\frac{0.1}{0.8} \right) = 7.1^\circ$$

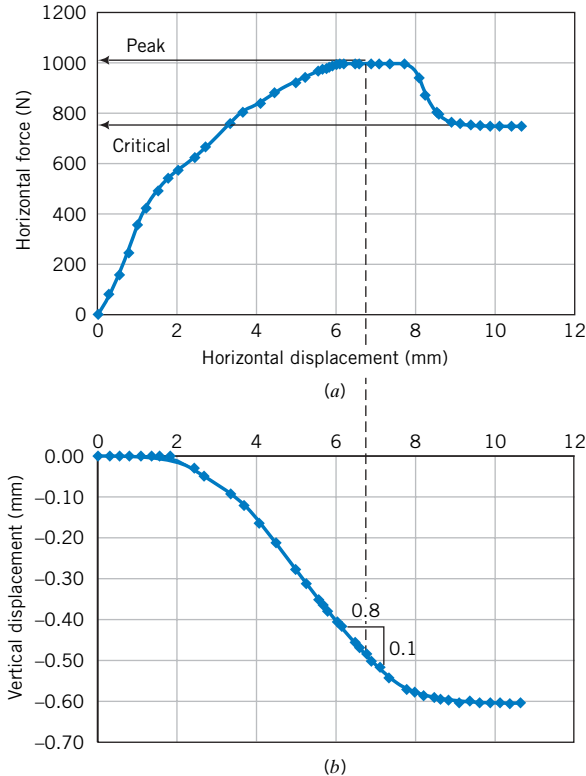


FIGURE E7.6

$$(c4) \quad \text{Normal stress : } \sigma'_n = \left(\frac{1200 \text{ N}}{10^{-2}} \right) \times 10^{-3} = 120 \text{ kPa}$$

$$\phi'_p = \tan^{-1} \left(\frac{\tau_p}{\sigma'_n} \right) = \tan^{-1} \left(\frac{100.5}{120} \right) = 39.9^\circ$$

$$(c5) \quad \phi'_{cs} = \tan^{-1} \left(\frac{\tau_{cs}}{\sigma'_n} \right) = \tan^{-1} \left(\frac{75.8}{120} \right) = 32.3^\circ$$

Also,

$$\alpha_p = \sigma'_p - \sigma'_{cs} = 39.9 - 32.3 = 7.6^\circ$$

7.9.3 Conventional Triaxial Apparatus

A widely used apparatus to determine the shear strength parameters and the stress–strain behavior of soils is the triaxial apparatus. The name is a misnomer since two, not three, stresses can be controlled. In the triaxial test, a cylindrical sample of soil, usually with a length to diameter ratio of 2, is subjected to either controlled increases in axial stresses or axial displacements and radial stresses. The sample is laterally confined by a membrane and radial stresses are applied by pressuring water in a chamber (Fig. 7.20). The axial stresses are applied by loading a plunger. If the axial stress is greater than the radial stress, the soil is compressed vertically and the test is called triaxial compression. If the radial

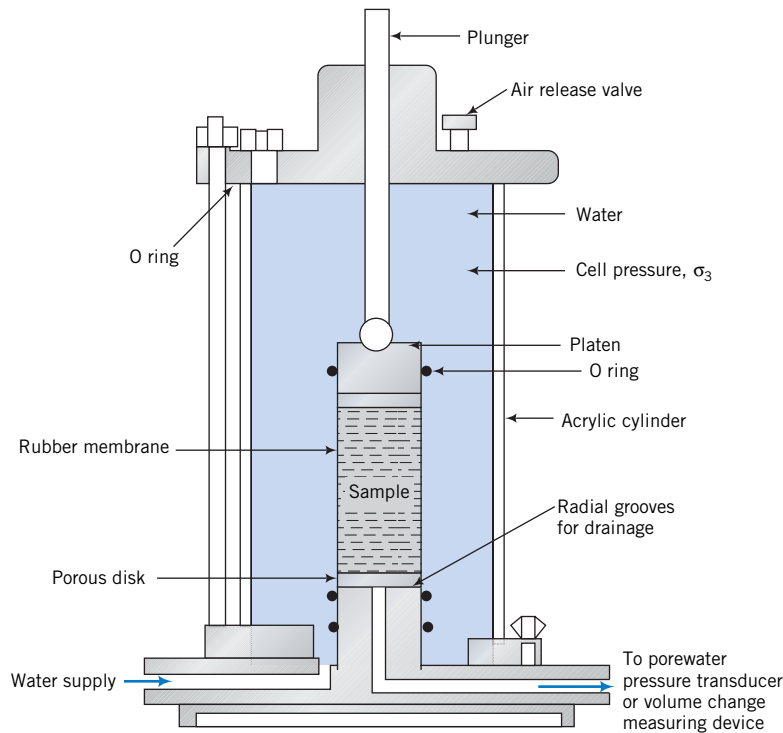


FIGURE 7.20 Schematic of a triaxial cell.

stress is greater than the axial stress, the soil is compressed laterally and the test is called triaxial extension.

The applied stresses (axial and radial) are principal stresses and the loading condition is axisymmetric. For compression tests, we will denote the radial stresses σ_r as σ_3 and the axial stresses σ_z as σ_1 . For extension tests, we will denote the radial stresses σ_r as σ_1 and the axial stresses σ_z as σ_3 .

The average stresses and strains on a soil sample in the triaxial apparatus for compression tests are as follows:

$$\text{Axial total stress: } \sigma_1 = \frac{P_z}{A} + \sigma_3 \quad (7.37)$$

$$\text{Deviatoric stress: } \sigma_1 - \sigma_3 = \frac{P_z}{A} \quad (7.38)$$

$$\text{Axial strain: } \varepsilon_3 = \frac{\Delta z}{H_o} \quad (7.39)$$

$$\text{Radial strain: } \varepsilon_3 = \frac{\Delta r}{r_o} \quad (7.40)$$

$$\text{Volumetric strain: } \varepsilon_p = \frac{\Delta V}{V_o} = \varepsilon_1 + 2\varepsilon_3 \quad (7.41)$$

$$\text{Deviatoric strain: } \varepsilon_p = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \quad (7.42)$$

where P_z is the load on the plunger, A is the cross-sectional area of the soil sample, r_o is the initial radius of the soil sample, Δr is the change in radius, V_o is the initial volume, ΔV is the change in volume, H_o is the initial height, and Δz is the change in height. We will call the plunger load, the deviatoric load, and the corresponding stress, the deviatoric stress, $q = (\sigma_1 - \sigma_3)$. The shear stress is $\tau = q/2$.

The area of the sample changes during loading and at any given instance the area is

$$A = \frac{V}{H} = \frac{V_o - \Delta V}{H_o - \Delta z} = \frac{V_o \left(1 - \frac{\Delta V}{V_o}\right)}{H_o \left(1 - \frac{\Delta z}{H_o}\right)} = \frac{A_o(1 - \varepsilon_p)}{1 - \varepsilon_1} \quad (7.43)$$

where $A_o (= \pi r_o^2)$ is the initial cross-sectional area and H is the current height of the sample. The dilation angle for a triaxial test is given by Eq. (7.10).

The triaxial apparatus is versatile because we can (1) independently control the applied axial and radial stresses, (2) conduct tests under drained and undrained conditions, and (3) control the applied displacements or stresses.

A variety of stress paths can be applied to soil samples in the triaxial apparatus. However, only a few stress paths are used in practice to mimic typical geotechnical problems. We will discuss the tests most often used, why they are used, and typical results obtained.

7.9.4 Unconfined Compression (UC) Test

The purpose of this test is to determine the undrained shear strength of saturated clays quickly. In the UC test, no radial stress is applied to the sample ($\sigma_3 = 0$). The plunger load, P_z , is increased rapidly until the soil sample fails, that is, cannot support any additional load. The loading is applied quickly so that the porewater cannot drain from the soil; the sample is sheared at constant volume.

The stresses applied on the soil sample and the total stress path followed are shown in Figs. 7.21a,b. The effective stress path is unknown since porewater pressure changes are not normally measured. Mohr's circle using total stresses is depicted in Fig. 7.21c. If the excess porewater pressures were to be measured, they would be negative. The theoretical reason for negative excess porewater pressures is as follows. Since $\sigma_3 = 0$, then from the principle of effective stresses, $\sigma'_3 = \sigma_3 - \Delta u = 0 - \Delta u = -\Delta u$. The effective radial stress, σ'_3 , cannot be negative because soils cannot sustain tension. Therefore, the excess porewater pressure must be negative so that σ'_3 is positive. Mohr's circle of effective stresses would be to the right of the total stress circle as shown in Fig. 7.21c.

The undrained shear strength is

$$s_u = \frac{P_z}{2A} = \frac{1}{2}\sigma_1 \quad (7.44)$$

where, from Eq. (7.43), $A = A_o/(1 - \varepsilon_1)$ (no volume change, i.e., $\varepsilon_p = 0$). The undrained elastic modulus, E_u , is determined from a plot of ε_1 versus σ_1 .

The results from UC tests are used to:

- Estimate the short-term bearing capacity of fine-grained soils for foundations.
- Estimate the short-term stability of slopes.
- Compare the shear strengths of soils from a site to establish soil strength variability quickly and cost-effectively (the UC test is cheaper to perform than other triaxial tests).
- Determine the stress-strain characteristics under fast (undrained) loading conditions.

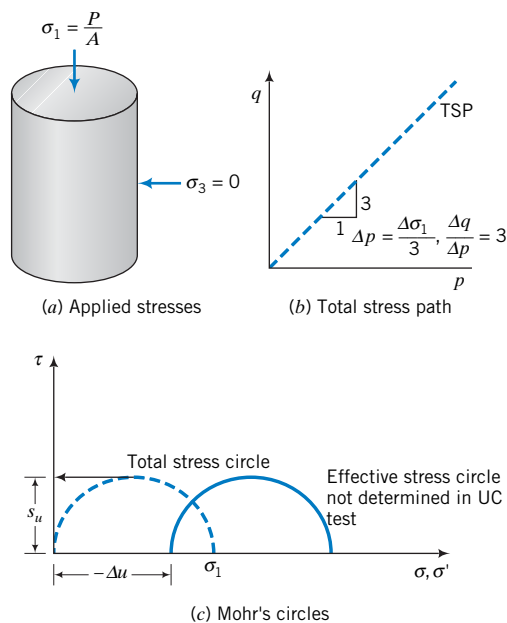


FIGURE 7.21 Stresses, stress paths, and Mohr's circle for UC test.

EXAMPLE 7.7 Undrained Shear Strength from a UC Test

An unconfined compression test was carried out on a saturated clay sample. The maximum load the clay sustained was 127 N and the vertical displacement was 0.8 mm. The size of the sample was 38 mm diameter $\times$ 76 mm long. Determine the undrained shear strength. Draw Mohr's circle of stress for the test and locate s_u .

Strategy Since the test is a UC test, $\sigma_3 = 0$ and $(\sigma_1)_f$ is the failure axial stress. You can find s_u by calculating one-half the failure axial stress.

Solution 7.7

Step 1: Determine the sample area at failure.

Diameter $D_o = 38$ mm, length, $H_o = 76$ mm.

$$A_o = \frac{\pi \times D_o^2}{4} = \frac{\pi \times 0.038^2}{4} = 11.3 \times 10^{-4} \text{ m}^2, \quad \epsilon_1 = \frac{\Delta z}{H_o} = \frac{0.8}{76} = 0.01$$

$$A = \frac{A_o}{1 - \epsilon_1} = \frac{11.3 \times 10^{-4}}{1 - 0.01} = 11.4 \times 10^{-4} \text{ m}^2$$

Step 2: Determine the major principal stress at failure.

$$(\sigma_1)_f = \frac{P_z}{A} = \frac{127 \times 10^{-3}}{11.4 \times 10^{-4}} = 111.4 \text{ kPa}$$

Step 3: Calculate s_u .

$$s_u = \frac{(\sigma_1)_f - (\sigma_3)_f}{2} = \frac{111.4 - 0}{2} = 55.7 \text{ kPa}$$

Step 4: Draw Mohr's circle.

See Fig. E7.7. The values extracted from the graphs are

$$(\sigma_3)_f = 0, \quad (\sigma_1)_f = 111 \text{ kPa}, \quad s_u = 56 \text{ kPa}$$

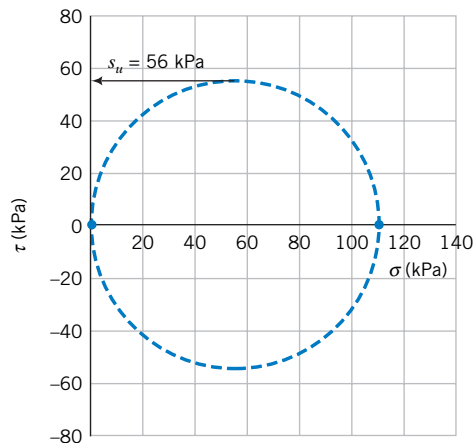


FIGURE E7.7

7.9.5 Consolidated Drained (CD) Compression Test

The purpose of a CD test is to determine the drained shear strength parameters, ϕ'_{cs} and ϕ'_p , to analyze long-term loading of a soil mass. The effective elastic moduli for drained condition E' and E'_s are also obtained from this test. A consolidated drained compression test is performed in two stages. The first stage is consolidating the soil to a desired effective stress level by pressurizing the water in the cell and allowing the soil sample to drain until the excess porewater pressure dissipates. In the second stage, the pressure in the cell (cell pressure or confining pressure) is kept constant and additional axial loads or displacements are added very slowly until the soil sample fails. The displacement rate (or strain rate) used must be slow enough to allow the excess porewater pressure to dissipate. Because the permeability of fine-grained soils is much lower than coarse-grained soils, the displacement rate for testing fine-grained soils is much lower than for coarse-grained soils. Drainage of the excess porewater is permitted and the amount of water expelled is measured. It is customary to perform a minimum of three tests at different cell pressures. The stresses on the soil sample for the two stages of a CD test are as follows:

Stage 1: Isotropic consolidation phase

$$\Delta\sigma_1 = \Delta\sigma_3 = \Delta\sigma'_1 = \Delta\sigma'_3; \Delta\sigma_1 > 0, \Delta u = 0$$

When the load is applied, the initial excess porewater pressure for a saturated soil is equal to the cell pressure (Chapter 6); that is, $\Delta u = \Delta\sigma_3$. At the end of the consolidation phase, the excess porewater pressure dissipates; that is, $\Delta u = 0$.

$$\Delta p' = \Delta p = \frac{\Delta\sigma_1 + 2\Delta\sigma_3}{3} = \Delta\sigma_1; \Delta q = \Delta\sigma_1 - \Delta\sigma_3 = 0, \text{ and } \frac{\Delta q}{\Delta p'} = \frac{\Delta q}{\Delta p} = 0$$

Stage 2: Shearing phase

$$\Delta\sigma_1 = \Delta\sigma'_1 > 0; \Delta\sigma_3 = \Delta\sigma'_3 = 0; \Delta u = 0$$

Therefore,

$$\Delta p' = \Delta p = \frac{\Delta\sigma'_1}{3}; \Delta q = \sigma_1; \frac{\Delta q}{\Delta p'} = \frac{\Delta q}{\Delta p} = 3$$

The stresses and stress path applied are illustrated in Fig. 7.22. The change in volume of the soil is measured by continuously recording the volume of water expelled. The volumetric strain is

$$\varepsilon_p = \frac{\Delta V}{V_o} = \varepsilon_1 + 2\varepsilon_3 \quad (7.45)$$

where ΔV is the change in volume and V_o is the original volume of the soil. Also, the axial displacements are recorded and the axial strain is calculated as $\varepsilon_1 = \Delta z/H_o$. The radial strains are calculated by rearranging Eq. (7.45) to yield

$$\varepsilon_3 = \frac{1}{2}(\varepsilon_p - \varepsilon_1) \quad (7.46)$$

The maximum shear strain is

$$(\gamma_{zx})_{\max} = (\varepsilon_1 - \varepsilon_3)_{\max}$$

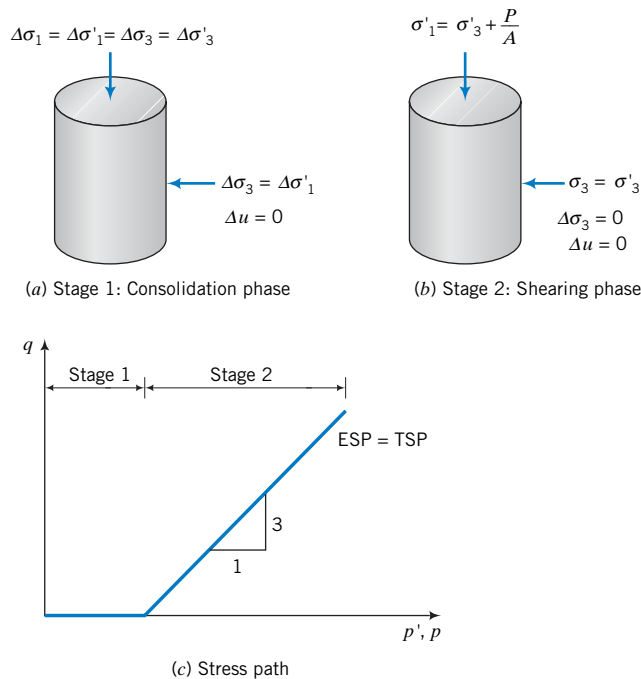


FIGURE 7.22 Stresses and stress paths during a CD test.

which, by substitution of Eq. (7.45), gives

$$(\gamma_{zx})_{\max} = \frac{1}{2}(3\varepsilon_1 - \varepsilon_p) \quad (7.47)$$

Since the CD test is a drained test, a single test can take several days if the permeability of the soil is low (e.g., fine-grained soils). Typical results of consolidated drained tests on a sand are shown in Fig. 7.23.

The elastic moduli for drained conditions, E' and E'_s , are obtained from the CD test from the plot of deviatoric stress, $(\sigma'_1 - \sigma'_3)$, as ordinate and ε_1 as abscissa as shown in Fig. 7.23a. The results of CD tests are used to determine the long-term stability of slopes, foundations, retaining walls, excavations and other earthworks.

EXAMPLE 7.8 Interpreting CD Triaxial Test Data

The results of three CD tests on a soil at failure are as follows:

Test number	σ'_3 (kPa)	Deviatoric stress (kPa)
1	100	247.8 (peak)
2	180	362.0 (peak)
3	300	564 (no peak observed)

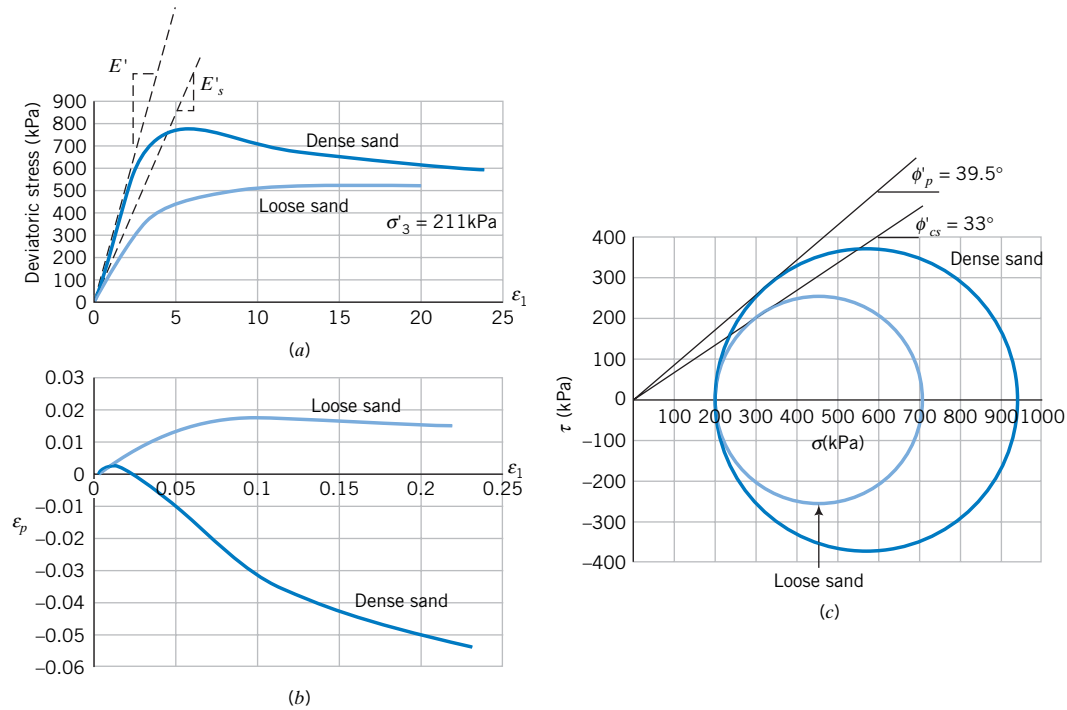


FIGURE 7.23 Results from CD tests on dense and loose sand. (From “The Measurement of Soil Properties in the Triaxial Test,” by Bishop and Henkel, Edward Arnold, 1962.)

The detailed results for Test 1 are as follows. The negative sign indicates expansion.

Δz (mm)	ΔV (cm ³)	Plunger load— P_z (N)
0	0.00	0.0
0.152	0.02	61.1
0.228	0.03	94.3
0.38	−0.09	124.0
0.76	−0.50	201.5
1.52	−1.29	257.5
2.28	−1.98	292.9
2.66	−2.24	298.9
3.04	−2.41	298.0
3.8	−2.55	279.2
4.56	−2.59	268.4
5.32	−2.67	252.5
6.08	−2.62	238.0
6.84	−2.64	229.5
7.6	−2.66	223.2
8.36	−2.63	224.3

The initial size of the sample is 38 mm in diameter and 76 mm in length.

- Determine the friction angle for each test.
- Determine τ_p , τ_{cs} , E' , and E'_s at peak shear stress for Test 1.

- (c) Determine ϕ'_{cs} .
 (d) Determine α_p for Test 1.

Strategy From a plot of deviatoric stress versus axial strain for Test 1, you will get τ_p, τ_{cs} , E' and E'_s . The friction angles can be calculated or found from Mohr's circle.

Solution 7.8

Step 1: Determine the friction angles. Use a table to do the calculations.

Test No.	σ'_3 (kPa)	$\sigma'_1 - \sigma'_3$ (kPa)	σ'_1 (kPa)	$\sigma'_1 + \sigma'_3$ (kPa)	$\phi'_p = \sin^{-1} \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3}$
Test 1	100	247.8	347.8	447.8	33.6°
Test 2	180	362	542	722	30.1°
Test 3	300	564	864	1164	29°

Alternatively, plot Mohr's circles and determine the friction angles as shown for Test 1 and Test 2 in Fig. E7.8a.

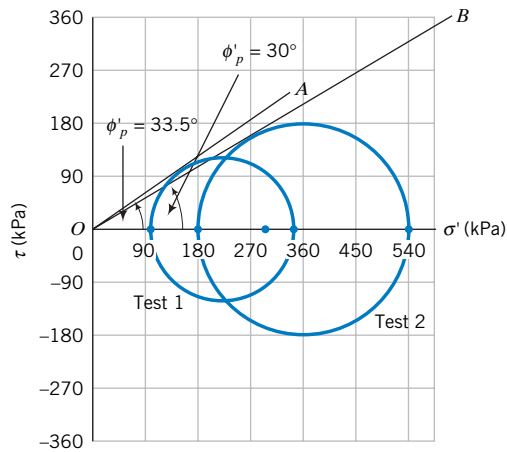


FIGURE E7.8a

Step 2: Determine τ_p and τ_{cs} from a plot of deviatoric stress versus axial strain response for Test 1.

$$\begin{aligned} \text{The initial area is: } A_o &= \frac{\pi D_o^2}{4} = \frac{\pi \times 38^2}{4} = 1134 \text{ mm}^2 \\ V_o &= \frac{\pi D_o H_o}{4} = \frac{\pi \times 38^2 \times 76}{4} = 86193 \text{ mm}^3 \\ A &= \frac{A_o(1 - \epsilon_p)}{1 - \epsilon_1} \end{aligned}$$

See the table for calculations and Fig. E7.8b for a plot of the results.
 Extract τ_p and τ_{cs} .

$$\tau_p = \frac{(\sigma'_1 - \sigma'_3)_p}{2} = \frac{247.8}{2} = 124 \text{ kPa}, \quad \tau_{cs} = \frac{(\sigma'_1 - \sigma'_3)_{cs}}{2} = \frac{170.7}{2} = 85.4 \text{ kPa}$$

$A_o = 1134 \text{ mm}^2$					
$\Delta z \text{ (mm)}$	$\varepsilon_1 \frac{\Delta z}{H_0}$	$\Delta V \text{ (cm}^3\text{)}$	$\varepsilon_p \frac{\Delta V}{V_0}$	$A \text{ (mm}^2\text{)}$	$q = P_z/A \text{ (kPa)}$
0.00	0.00	0.00	0.00	1134	0
0.15	0.20	0.02	0.02	1136	53.8
0.23	0.30	0.03	0.03	1137	82.9
0.38	0.50	-0.09	-0.10	1141	108.7
0.76	1.00	-0.50	-0.58	1152	174.9
1.52	2.00	-1.29	-1.50	1175	219.2
2.28	3.00	-1.98	-2.30	1196	244.9
2.66	3.50	-2.24	-2.60	1206	247.8
3.04	4.00	-2.41	-2.80	1215	245.3
3.80	5.00	-2.55	-2.97	1229	227.1
4.56	6.00	-2.59	-3.01	1243	215.9
5.32	7.00	-2.67	-3.10	1257	200.8
6.08	8.00	-2.62	-3.05	1270	187.3
6.84	9.00	-2.64	-3.07	1285	178.7
7.60	10.00	-2.66	-3.09	1299	171.8
8.36	11.00	-2.63	-3.06	1313	170.7

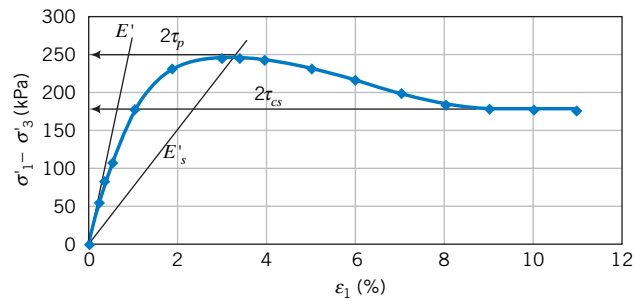


FIGURE E7.8b

Step 3: Determine E' and E'_s .

The initial slope of Fig. E7.8b gives E' and the slope of the line from the origin to $2\tau_p$ gives E'_s .

$$E' = \frac{54}{0.002} = 27,000 \text{ kPa}$$

$$E'_s = \frac{247.8}{0.035} = 7081 \text{ kPa}$$

Step 4: Determine ϕ'_{cs} .

The deviatoric stress and the volumetric change appear to be constant from about $\varepsilon_1 \approx 10\%$. We can use the result at $\varepsilon_1 = 11\%$ to determine ϕ'_{cs} (σ'_3)_{cs} = 100 kPa, (σ'_1)_{cs} = 170.7 + 100 = 270.7 kPa.

$$\phi'_{cs} = \sin^{-1} \left(\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} \right)_{cs} = \sin^{-1} \frac{170.7}{275.8 + 100} = 27.4^\circ$$

Step 5: Determine α_p

$$\alpha_p = \phi'_p - \phi'_{cs} = 33.6 - 27.4 = 6.2^\circ$$

7.9.6 Consolidated Undrained (CU) Compression Test

The purpose of a CU test is to determine the undrained and drained shear strength parameters (s_u , ϕ'_{cs} , ϕ'_p). The CU test is conducted in a similar manner to the CD test except that after isotropic consolidation, the axial load is increased under undrained condition and the excess porewater pressure is measured. The applied stresses are as follows:

Stage 1: Isotropic consolidation phase

$$\Delta\sigma_1 = \Delta\sigma_3 = \Delta\sigma'_1 = \Delta\sigma'_3; \quad \Delta\sigma_1 > 0, \quad \Delta u = 0$$

Therefore,

$$\Delta p' = \Delta p = \Delta\sigma_1; \quad \Delta q = 0, \quad \frac{\Delta q}{\Delta p'} = \frac{\Delta q}{\Delta p} = 0$$

Stage 2: Shearing phase

$$\Delta\sigma_1 > 0, \quad \Delta\sigma_3 = 0; \quad \Delta\sigma'_1 = \Delta\sigma_1 - \Delta u, \quad \Delta\sigma'_3 = -\Delta u$$

$$\Delta p = \frac{\Delta\sigma_1}{3}; \quad \Delta q = \Delta\sigma_1, \quad \frac{\Delta q}{\Delta p} = 3$$

$$\Delta p' = \Delta p - \Delta u = \frac{\Delta\sigma_1}{3} - \Delta u$$

$$\Delta q = \Delta\sigma_1, \quad \frac{\Delta q}{\Delta p'} = \frac{\Delta\sigma_1}{\frac{\Delta\sigma_1}{3} - \Delta u} = \frac{3}{1 - \frac{3\Delta u}{\Delta\sigma_1}}$$

While the total stress path is determinate, the effective stress path can be determined only if we measure the changes in excess porewater pressures. The stresses on the soil samples and the stress paths in a CU test are illustrated in Fig. 7.24. The effective stress path is nonlinear because when the soil yields, the excess porewater pressures increase nonlinearly causing the ESP to bend.

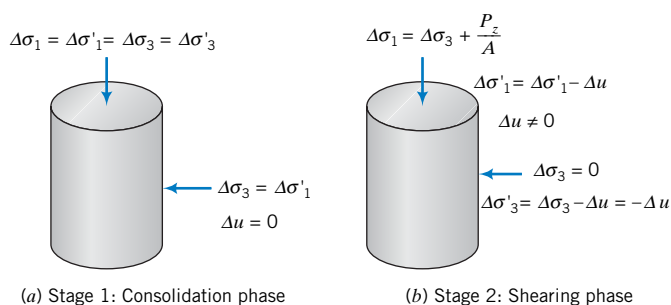


FIGURE 7.24 Stresses and stress paths for triaxial CU test.

In a CU test, the volume of the soil is constant during the shear phase. Therefore,

$$\varepsilon_p = \varepsilon_1 + 2\varepsilon_3 = 0$$

which leads to

$$\varepsilon_3 = -\frac{\varepsilon_1}{2} \quad (7.48)$$

The axial displacement is measured and $\varepsilon_1 = \Delta z/H_0$ is calculated. The maximum shear strain is

$$(\gamma_{zx})_{\max} = \varepsilon_1 - \varepsilon_3 = \varepsilon_1 - \left(-\frac{\varepsilon_1}{2}\right) = 1.5\varepsilon_1 \quad (7.49)$$

The elastic moduli E_u and $(E_u)_s$ are determined from a plot of $(\sigma_1 - \sigma_3)$ as ordinate and ε_1 as abscissa.

Typical results from a CU test are shown in Fig. 7.25. Two sets of Mohr's circles at failure can be drawn, as depicted in Fig. 7.25. One represents total stress condition and the other, effective stress

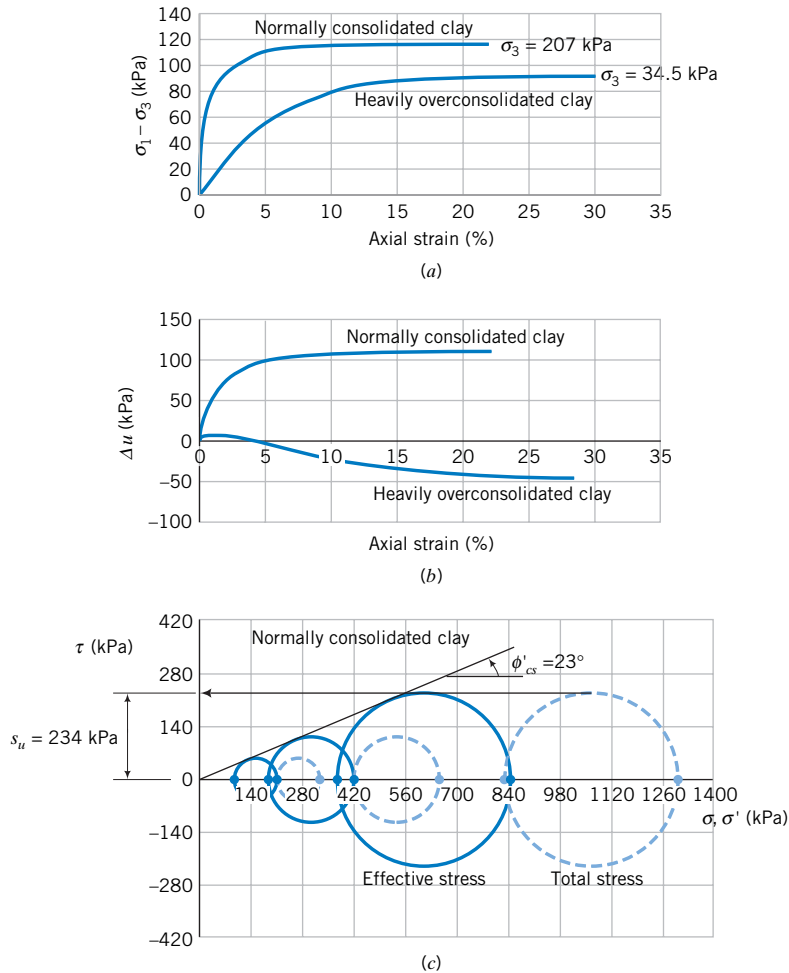


FIGURE 7.25 Triaxial CU tests on clays. (From "The Measurement of Soil Properties in the Triaxial Test," by Bishop and Henkel, Edward Arnold, 1962.)

condition. For each test, Mohr's circle representing the total stresses has the same size as Mohr's circle representing the effective stresses but they are separated horizontally by the excess porewater pressure. Mohr's circle of effective stresses is shifted to the right if the excess porewater pressure at failure is negative and to the left if the excess porewater pressure is positive.

Each Mohr's circle of total stress is associated with a particular value of s_u because each test has a different initial water content resulting from the different confining pressure, or applied consolidating stresses. The value of s_u is obtained by drawing a horizontal line from the top of the desired Mohr's circle of total stress to intersect the vertical shear axis. The intercept is s_u . The value of s_u at a cell pressure of about 830 kPa is 234 kPa, as shown in Fig. 7.25. Alternatively, you can calculate s_u from

$$s_u = \frac{(\sigma_1 - \sigma_3)_f}{2} = \frac{(P_z)_{\max}}{2A}$$

The drained shear strength parameters (ϕ'_{cs} and ϕ'_p) are found as described in the previous section dealing with the CD test. You would normally determine s_u at the maximum anticipated stress level in the field.

The CU test is the most popular triaxial test because you can obtain not only s_u but ϕ'_{cs} and ϕ'_p , and most tests can be completed within a few minutes after consolidation compared with more than a day for a CD test. Fine-grained soils with low k values must be sheared slowly to allow the excess porewater pressure to equilibrate throughout the test sample. The results from CU tests are used to analyze the stability of slopes, foundations, retaining walls, excavations, and other earthworks.

EXAMPLE 7.9 Interpreting CU Triaxial Test Data

A CU test was conducted on a saturated clay soil by isotropically consolidating the soil using a cell pressure of 150 kPa and then incrementally applying loads on the plunger while keeping the cell pressure constant. Failure was observed when the stress exerted by the plunger was 160 kPa and the porewater pressure recorded was 54 kPa. Determine (a) s_u and (b) ϕ'_{cs} . Illustrate your answer by plotting Mohr's circle for total and effective stresses.

Strategy You can calculate the effective strength parameters by using the Mohr–Coulomb failure criterion [Eq. (7.24)] or you can determine them from plotting Mohr's circle. Remember that the stress imposed by the plunger is not the major principal stress σ_1 but $(\sigma_1 - \sigma_3) = (\sigma'_1 - \sigma'_3)$.

Solution 7.9

Step 1: Calculate the stresses at failure.

$$\begin{aligned}\frac{P_z}{A} &= (\sigma_1)_f - (\sigma_3)_f = 160 \text{ kPa} \\ (\sigma_1)_f &= \frac{P_z}{A} + \sigma_3 = 160 + 150 = 310 \text{ kPa} \\ (\sigma'_1)_f &= (\sigma_1)_f - \Delta u_f = 310 - 54 = 256 \text{ kPa} \\ (\sigma_3)_f &= 150 \text{ kPa}, \quad (\sigma'_3)_f = (\sigma_3)_f - \Delta u_f = 150 - 54 = 96 \text{ kPa}\end{aligned}$$

Step 2: Determine the undrained shear strength.

$$s_u = \frac{(\sigma_1)_f - (\sigma_3)_f}{2} = \frac{160}{2} = 80 \text{ kPa}$$

Step 3: Determine ϕ'_{cs} .

$$\begin{aligned}\sin \phi'_{cs} &= \frac{(\sigma_1)_f - (\sigma_3)_f}{(\sigma_1)_f + (\sigma_3)_f} = \frac{160}{256 + 96} = 0.45 \\ \phi'_{cs} &= 26.7^\circ\end{aligned}$$

Step 4: Draw Mohr's circle.

See Fig. E7.9.

$$\phi'_{cs} = 27^\circ$$

$$s_u = 80 \text{ kPa}$$

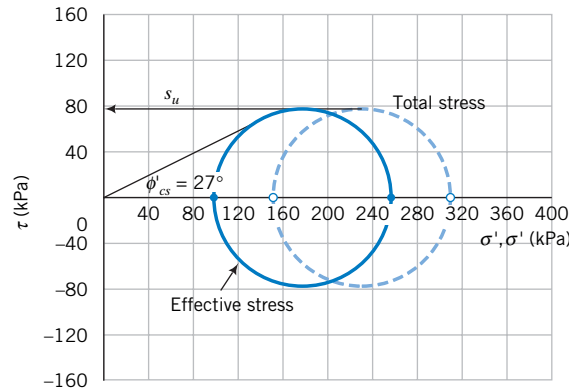


FIGURE E7.9

7.9.7 Unconsolidated Undrained (UU) Test

The purpose of a UU test is to determine the undrained shear strength of a saturated soil. The UU test consists of applying a cell pressure to the soil sample without drainage of porewater followed by increments of axial stress. The cell pressure is kept constant and the test is completed very quickly because in neither of the two stages—consolidation and shearing—is the excess porewater pressure allowed to drain. The stresses applied are:

Stage 1: Isotropic compression (not consolidation) phase

$$\Delta\sigma_1 = \Delta\sigma_3, \quad \Delta u \neq 0$$

$$\Delta p = \Delta\sigma_1, \quad \Delta q = 0, \quad \frac{\Delta q}{\Delta p} = 0$$

Stage 2: Shearing phase

$$\Delta\sigma_1 > 0, \quad \Delta\sigma_3 = 0$$

$$\Delta p = \frac{\Delta\sigma_1}{3}, \quad \Delta q = \Delta\sigma_1, \quad \frac{\Delta q}{\Delta p} = 3$$

Two or more samples of the same soil and the same initial void ratio are normally tested at different cell pressures. Each Mohr's circle is the same size but the circles are translated horizontally by the difference in the magnitude of the cell pressures. Mohr's circles, stresses, and stress paths for a UU test are shown in Fig. 7.26. Only the total stress path is known since the porewater pressures are not measured to enable the calculation of the effective stresses.

The undrained shear strength, s_u , and the undrained elastic moduli, E_u and $(E_u)_s$, are obtained from a UU test. The UU tests, like the UC tests, are quick and inexpensive compared with CD and CU tests. The advantage that the UU test has over the UC test is that the soil sample is stressed in the lateral direction to simulate the field condition. Both the UU and UC tests are useful in preliminary analyses for the design of slopes, foundations, retaining walls, excavations, and other earthworks.

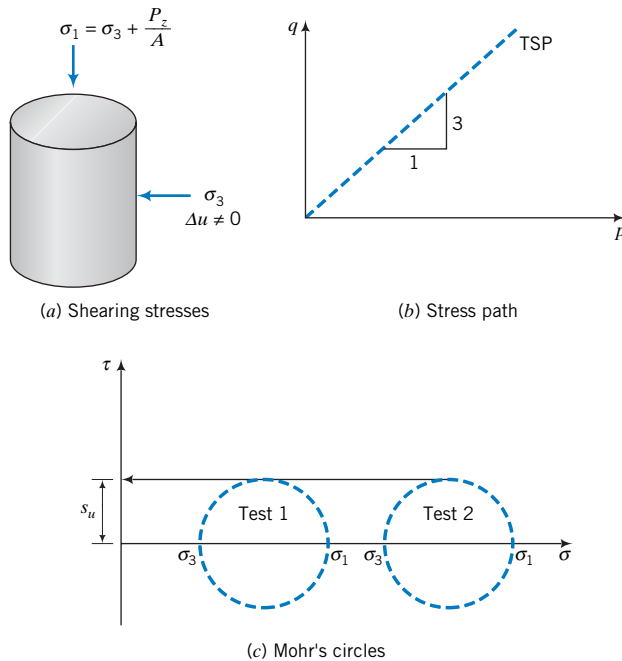


FIGURE 7.26 Stresses, stress path, and Mohr's circles for UU tests.

EXAMPLE 7.10 *Undrained Shear Strength from a UU Triaxial Test*

A UU test was conducted on saturated clay. The cell pressure was 200 kPa and failure occurred under a deviatoric stress of 220 kPa. Determine the undrained shear strength.

Strategy You can calculate the radius of the total stress circle to give s_u but a plot of Mohr's circle is instructive.

Solution 7.10

Step 1: Draw Mohr's circle.

See Fig. E7.10

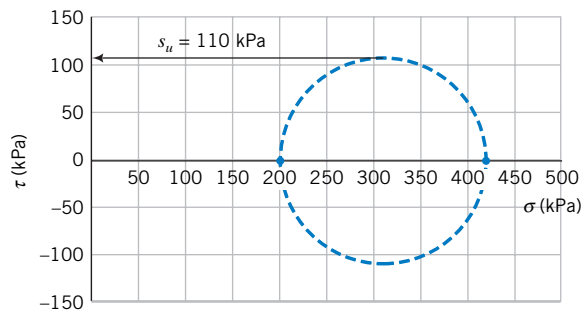


FIGURE E7.10

Step 2: Determine the undrained shear strength.

Draw a horizontal line to the top of Mohr's circle. The intersection of the horizontal line with the ordinate is the undrained shear strength.

$$s_u = 110 \text{ kPa}$$

$$\text{By calculation : } s_u = \frac{(\sigma_1)_f - (\sigma_3)_f}{2} = \frac{220}{2} = 110 \text{ kPa}$$

What's next... In the UU test and sometimes in the CU test, the excess porewater pressures are not measured. However, we need to know the magnitude of the excess porewater pressures to calculate effective stresses. Next, we will present a method to predict the excess porewater pressure from axisymmetric tests.

7.10 POREWATER PRESSURE UNDER AXISYMMETRIC UNDRAINED LOADING



Concept Learning and Self-Assessment

Access Chapter 7, Section 7.10 to learn about porewater pressures under axisymmetric loading. Take Quiz 7.10 to test your understanding.

The porewater pressure changes in soils are due to the changes in mean total and deviatoric stresses. Skempton (1954) proposed the following equation to determine the porewater pressure under axisymmetric conditions:

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] \quad (7.50)$$

where $\Delta\sigma_3$ is the increase in lateral principal stress, $\Delta\sigma_1 - \Delta\sigma_3$ is the deviatoric stress increase, B is a coefficient indicating the level of saturation, and A is an excess porewater pressure coefficient. The coefficient B is 1 for saturated soils and 0 for dry soils. However, B is not directly correlated with saturation except at high values of saturation ($S > 90\%$). At failure,

$$A = A_f - \left(\frac{\Delta u_d}{\Delta\sigma_1 - \Delta\sigma_3} \right)_f \quad (7.51)$$

where Δu_d is the change in excess porewater pressure resulting from changes in deviatoric (shear) stresses. Experimental results of A_f presented by Skempton (1954) are shown in Table 7.3. The coefficient A was found to be dependent on the overconsolidation ratio (OCR). A typical variation of A_f with OCR is shown in Fig. 7.27.

Equation (7.50) is very useful in determining whether a soil is saturated in an axisymmetric test. Let us manipulate Eq. (7.50) by dividing both sides by $\Delta\sigma_3$, resulting in

$$\frac{\Delta u}{\Delta\sigma_3} = B \left[1 + A \left(\frac{\Delta\sigma_1}{\Delta\sigma_3} - 1 \right) \right] \quad (7.52)$$

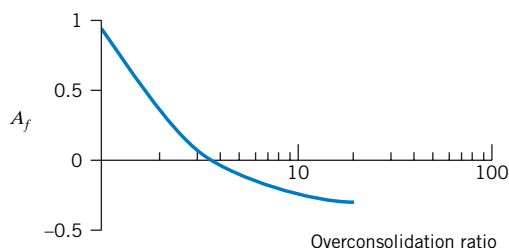
During isotropic consolidation $\Delta\sigma_3 = \Delta\sigma_1$ and Eq. (7.52) becomes

$$\frac{\Delta u}{\Delta\sigma_3} = B \quad (7.53)$$

TABLE 7.3 A_f Values

Type of clay	A_f
Highly sensitive	0.75 to 1.0
Normally consolidated	0.5 to 1
Compacted sandy clay	0.25 to 0.75
Lightly overconsolidated clays	0 to 0.5
Compacted clay-gravel	−0.25 to 0.25
Heavily overconsolidated clays	−0.5 to 0

SOURCE: From "The Measurement of Soil Properties in the Triaxial Test," by Bishop and Henkel, Edward Arnold, 1962.

**FIGURE 7.27** Variation of OCR with A_f .

If a soil is saturated, then $B = 1$ and $\Delta u = \Delta \sigma_3$. That is, if we were to increase the consolidation stress or confining pressure by $\Delta \sigma_3$, the instantaneous excess porewater pressure change must be equal to the increase of confining pressure. Equation (7.53) then provides a basis to evaluate the level of saturation of a soil sample in an axisymmetric test. The coefficients A and B are referred to as Skempton's porewater pressure coefficients.

The essential points are:

1. Under an axisymmetric loading condition, the porewater pressure can be predicted using Skempton's porewater pressure coefficients, A and B .
2. For a saturated soil, $B = 1$.

What's next... In the next section, we will briefly describe a few other laboratory apparatuses to determine the shear strength of soils. These apparatuses are more complex and were developed to more closely represent the field stresses on the laboratory soil sample than the triaxial and direct shear apparatuses.

7.11 OTHER LABORATORY DEVICES TO MEASURE SHEAR STRENGTH



Concept Learning and Self-Assessment

Access Chapter 7, Section 7.11 to learn about more advanced shear apparatus. Take Quiz 7.11 to test your understanding.

There are several other types of apparatus that are used to determine the shear strength of soils in the laboratory. These apparatuses are, in general, more sophisticated than the shear box and the triaxial apparatus.

7.11.1 Simple Shear Apparatuses

The purpose of a simple shear test is to determine shear strength parameters and the stress–strain behavior of soils under loading conditions that closely simulate plane strain and allow for the principal axes of stresses and strains to rotate. Principal stress rotations also occur in the direct shear test but are indeterminate. The stress states in soils for many geotechnical structures are akin to simple shear.

There are two types of commercially available simple shear devices. One deforms an initial cuboidal sample under plane strain conditions into a parallelepiped (Fig. 7.28a). The sample is contained in a box made by stacking square hollow plates between two platens. The top platen can be maintained at a fixed height for constant-volume tests or allowed to move vertically to permit volume change to occur. By displacing the bottom of the box relative to the top, the soil is transformed from a cube to a parallelepiped.

A load cell mounted on the top platen measures the excess porewater pressures. The lateral stresses are deduced from one of the hollow plates outfitted with strain gauges. The stresses and strains deduced from measurements in the cuboidal simple shear apparatus are shown in Fig. 7.28b. If the excess porewater pressures are measured in undrained (constant-volume) tests, then the effective stresses can be determined.

The other apparatus tests a cylindrical sample whose vertical side is enclosed by a wire reinforced rubber membrane (Fig. 7.28c). Rigid rough metal plates are placed at the top and bottom of the sample. Displacing the top of the sample relative to the bottom deforms the sample. The vertical and horizontal loads (usually on the top boundary) as well as displacements on the boundaries are measured and thus the average normal and shear stresses and boundary strains can be deduced. In the cylindrical apparatus, the stresses measured are σ_z and τ_{zx} , and the test is referred to as direct simple shear.

Simple shear apparatuses do not subject the sample as a whole to uniform stresses and strains. However, the stresses and strains in the central region of the sample are uniform. In simple shear, the strains are $\epsilon_x = \epsilon_y = 0$, $\epsilon_z = \Delta z/H_o$, and $\gamma_{zx} = \Delta x/H_o$. A plot of shear stress τ_{zx} versus γ_{zx} is used to determine G .

The shear displacement Δx must be applied in small increments to comply with the above definition. The principal strains from Eqs. (5.35) and (5.36) are

$$\epsilon_1 = \frac{1}{2}(\epsilon_z + \sqrt{\epsilon_z^2 + \gamma_{zx}^2}) \quad (7.54)$$

$$\epsilon_3 = \frac{1}{2}(\epsilon_z - \sqrt{\epsilon_z^2 + \gamma_{zx}^2}) \quad (7.55)$$

and

$$(\gamma_{zx})_{\max} = \epsilon_1 - \epsilon_3 = \sqrt{\epsilon_z^2 + \gamma_{zx}^2} \quad (7.56)$$

The dilation angle is determined from Eq. (7.10).

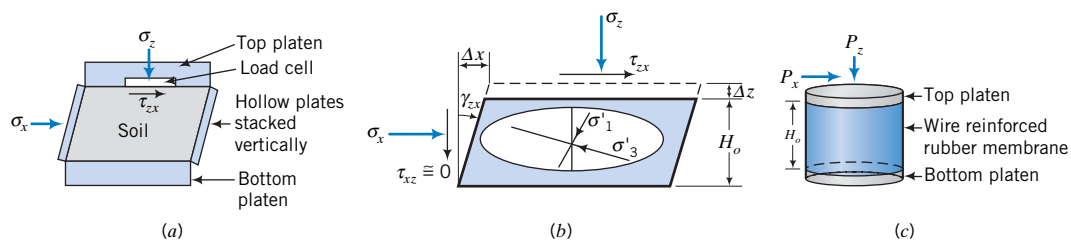


FIGURE 7.28 Cuboidal simple shear apparatus: (a) simple shear box, (b) stresses imposed on samples, and (c) direct simple shear.

EXAMPLE 7.11 *Interpreting Simple Shear Test Data*

A cuboidal soil sample, with 50 mm sides, fails in a simple shear constant-volume test under a vertical load (P_z) of 500 N, a horizontal load (P_x) of 375 N, and a shear load (T) of 150 N. The excess porewater pressure developed is 60 kPa.

- Plot Mohr's circles for total and effective stresses.
- Determine the friction angle and the undrained shear strength, assuming the soil is nondilating.
- Determine the failure stresses.
- Find the magnitudes of the principal effective stresses and the inclination of the major principal axis of stress to the horizontal.
- Determine the shear and normal stresses on a plane oriented at 20° clockwise from the horizontal.

Strategy Draw a diagram of the forces on the soil sample, calculate the stresses, and plot Mohr's circle. You can find all the required values from Mohr's circle or you can calculate them. You must use effective stresses to calculate the friction angle.

Solution 7.11

Step 1: Determine the total and effective stresses.

$$\sigma_z = \frac{P_z}{A} = \frac{500 \times 10^{-3}}{(0.05)^2} = 200 \text{ kPa}$$

$$\sigma_x = \frac{P_x}{A} = \frac{375 \times 10^{-3}}{(0.05)^2} = 150 \text{ kPa}$$

$$\tau_{zx} = \frac{T}{A} = \frac{150 \times 10^{-3}}{(0.05)^2} = 60 \text{ kPa}$$

$$\sigma'_z = \sigma_z - \Delta u = 200 - 60 = 140 \text{ kPa}$$

$$\sigma'_x = \sigma_x - \Delta u = 150 - 60 = 90 \text{ kPa}$$

Step 2: Draw Mohr's circle of total and effective stresses.

See Fig. E7.11.

Step 3: Determine ϕ'_{cs} and s_u .

Draw a tangent to Mohr's circle of effective stress from the origin of the axes.

$$\text{From Mohr's circle : } \phi'_{cs} = 34.5^\circ$$

The undrained shear strength is found by drawing a horizontal line from the top of Mohr's circle of total stresses to intersect the ordinate.

$$s_u = 65 \text{ kPa}$$

Step 4: Determine the failure stresses.

At the point of tangency of the failure envelope and Mohr's circle of effective stress, we get

$$\tau_f = 54 \text{ kPa}, \quad (\sigma'_n)_f = 79 \text{ kPa}$$

Step 5: Determine σ'_1 , σ'_3 and ψ .

From Mohr's circle of effective stresses, we get

$$\sigma'_1 = 180 \text{ kPa} \quad \text{and} \quad \sigma'_3 = 50 \text{ kPa}; \quad 2\psi = 66.5^\circ \quad \text{and} \quad \psi = 33.3^\circ$$

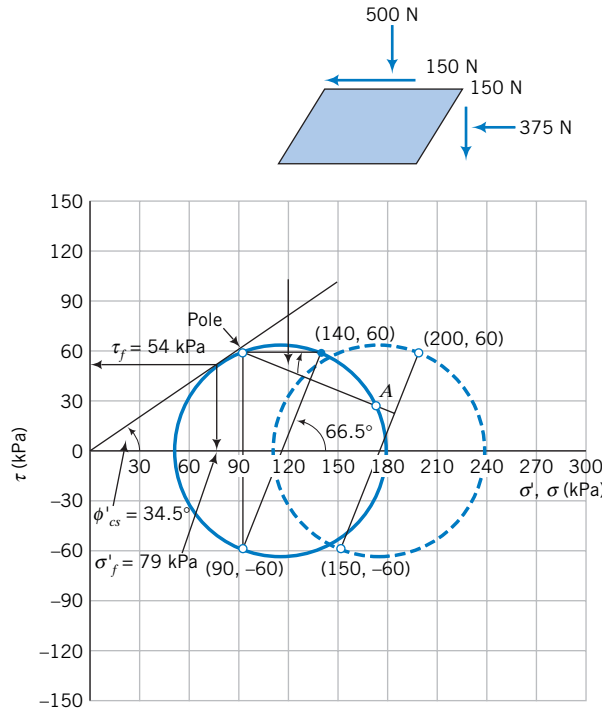


FIGURE E7.11

Step 6: Determine the stresses on a plane oriented at 20° .

Identify the pole as shown in Fig. E7.11. Draw a line inclined at 20° to the horizontal from the pole as shown in Fig. E7.11. Remember that we are using counterclockwise shear as positive. Point A gives the stresses on a plane oriented at 20° from the horizontal.

$$\tau_{20^\circ} = 30 \text{ kPa}; \quad \sigma'_{20^\circ} = 173 \text{ kPa}$$

Alternatively, by calculation,

$$\text{Equation (5.27): } \sigma'_1 = \frac{140 + 90}{2} + \sqrt{\left(\frac{140 - 90}{2}\right)^2 + 60^2} = 180 \text{ kPa}$$

$$\text{Equation (5.28): } \sigma'_3 = \frac{140 + 90}{2} - \sqrt{\left(\frac{140 - 90}{2}\right)^2 + 60^2} = 50 \text{ kPa}$$

$$s_u = \frac{(\sigma'_1 - \sigma'_3)_f}{2} = \frac{180 - 50}{2} = 65 \text{ kPa}$$

$$\phi'_{cs} = \sin^{-1} \left(\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} \right) = \sin^{-1} \left(\frac{180 - 50}{180 + 50} \right) = 34.4^\circ$$

$$\text{Equation (5.29): } \tan \psi = \frac{\tau_{zx}}{\sigma'_1 - \sigma'_x} = \frac{60}{180 - 90} = 0.67; \quad \psi = 33.7^\circ$$

$$\begin{aligned} \text{Equation (5.32): } \sigma'_{20^\circ} &= \frac{140 + 90}{2} + \frac{140 - 90}{2} \cos 40^\circ + 60 \sin 40^\circ \\ &= 172.7 \text{ kPa} \end{aligned}$$

$$\text{Equation (5.33): } \tau_{20^\circ} = 60 \cos 40^\circ - \frac{140 - 90}{2} \sin 40^\circ = 29.9 \text{ kPa}$$

EXAMPLE 7.12 *Interpreting Strains in Simple Shear*

A cuboidal soil sample, with 50 mm sides, was tested under drained conditions in a simple shear apparatus and the maximum shear stress occurred when the shear displacement was 1 mm and the vertical movement was -0.05 mm.

- (a) Plot Mohr's circle of strain.
- (b) Determine the principal strains.
- (c) Determine the maximum shear strain.
- (d) Determine the dilation angle, α .

Strategy Calculate the vertical and shear strains and then plot Mohr's circle of strain. You can determine all the required values from Mohr's circle or you can calculate them.

Solution 7.12

Step 1: Determine the strains.

$$\varepsilon_z = \frac{\Delta z}{H_o} = -\frac{0.05}{50} = -0.001$$

(negative sign because the sample expands; compression is positive)

$$\varepsilon_x = \varepsilon_y = 0$$

$$\gamma_{zx} = \frac{\Delta x}{H_o} = \frac{1}{50} = 0.02$$

Step 2: Plot Mohr's circle of strain and determine ε_1 , ε_3 , $(\gamma_{zx})_{\max}$, and α .

See Mohr's circle of strain, Fig. E7.12.

$$\varepsilon_1 = 9.5 \times 10^{-3}, \quad \varepsilon_3 = -10.5 \times 10^{-3}$$

$$\alpha = 3^\circ$$

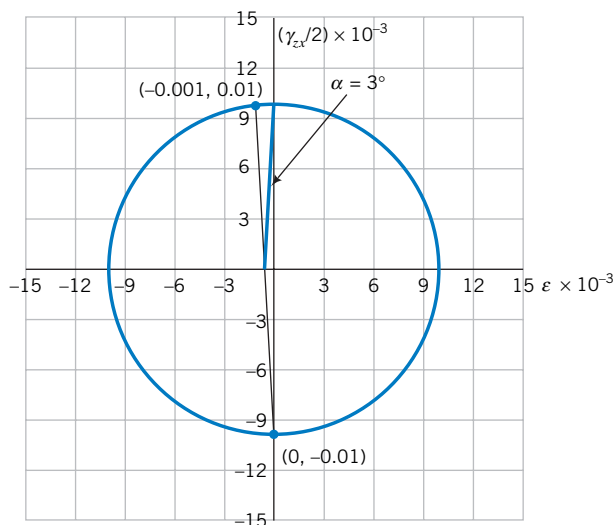


FIGURE E7.12

By calculation,

$$\text{Equation (5.35): } \varepsilon_1 = \frac{1}{2}(-0.001 + \sqrt{(-0.001)^2 + (0.02)^2}) = 9.5 \times 10^{-3}$$

$$\text{Equation (5.36): } \varepsilon_3 = \frac{1}{2}(-0.001 - \sqrt{(-0.001)^2 + (0.02)^2}) = -10.5 \times 10^{-3}$$

7.11.2 True Triaxial Apparatus

The purpose of a true triaxial test is to determine soil behavior and properties by loading the soil in three dimensions. In a true triaxial test, a cuboidal sample is subjected to independent displacements or stresses on three Cartesian axes. Displacements are applied through a system of rigid metal plates moving perpendicularly and tangentially to each face, as shown by the arrows in Fig. 7.29a. Pressure transducers are fixed to the inside of the faces to measure the three principal stresses. Like the conventional triaxial apparatus, the directions of principal stresses are prescribed and can only be changed instantaneously through an angle of 90° . The stresses and strains that can be measured in a true triaxial test are shown in Fig. 7.29b.

7.11.3 Hollow Cylinder Apparatus

The purpose of a hollow cylinder test is to determine soil properties from a variety of plane strain stress paths. In the hollow cylinder apparatus (Fig. 7.30a), a hollow thin-walled cylindrical sample is enclosed in a pressure chamber and can be subjected to vertical loads or displacements, radial stresses on the inner and outer cylindrical surfaces, and a torque as shown in Fig. 7.30b.

The vertical stress acting on a typical element of soil in this device is

$$\sigma_z = \frac{P_z}{A}$$

If the internal and external radial pressures are equal, then

$$\sigma_r = \sigma_\theta$$

The shearing stress applied is

$$\tau_{z\theta} = \frac{3M}{2\pi(r_2^3 - r_1^3)} \quad (7.57)$$

where M is the applied torque and r_1 and r_3 are the inner and outer radii. We can obtain ϕ'_p , ϕ'_{cs} , s_u , and G from the hollow cylinder test.

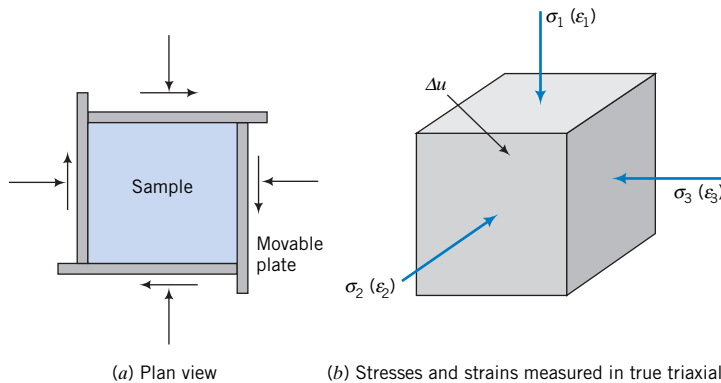


FIGURE 7.29 Schematic of true triaxial cell and stresses imposed on a sample of soil.

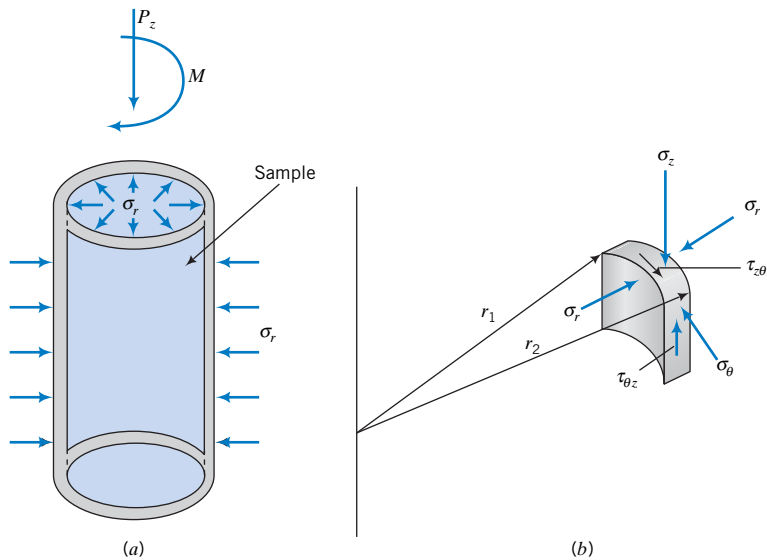


FIGURE 7.30 (a) Hollow cylinder cell and (b) stresses on an element of soil.

What's next... Sampling disturbances and sample preparation for laboratory tests may significantly impair the shear strength parameters. Consequently, a variety of field tests (Chapter 2) have been developed to obtain more reliable soil shear strength parameters by testing soils in situ. Field test results are often related to laboratory test results using empirical factors. In the next section, the shear strength parameters from popular in situ tests are presented.

7.12 FIELD TESTS



Concept Learning and Self-Assessment

Access Chapter 7, Section 7.12 to learn more about in situ tests. Take Quiz 7.12 to test your understanding.

7.12.1 Vane Shear Test (VST)

The undrained shear strength from a vane shear test (Chapter 2, Section 2.6.7a) is calculated from

$$s_u = \frac{2T}{\pi d^3 (h/d + \frac{1}{3})} \quad (7.58)$$

where T is the maximum torque, h is the height, and d is the diameter of the vane.

7.12.2 The Standard Penetration Test (SPT)

Results from SPT (Chapter 2) have been correlated to several soil parameters. Most of these correlations are weak. Typical correlation among N values, relative density, and ϕ' are given in

TABLE 7.4 Correlation of N , N_{60} , γ , D_r , and ϕ' for Coarse-Grained Soils

N	N_{60}	Compactness	$\gamma(\text{kN/m}^3)$	$D_r(\%)$	ϕ' (degrees)
0–4	0–3	Very loose	11–13	0–20	26–28
4–10	3–9	Loose	14–16	20–40	29–34
10–30	9–25	Medium	17–19	40–70	35–40 ^a
30–50	25–45	Dense	20–21	70–85	38–45 ^a
>50	>45	Very dense	>21	>85	>45 ^a

^aThese values correspond to ϕ'_p .

TABLE 7.5 Correlation of N_{60} and s_u for Saturated Fine-Grained Soils

N_{60}	Description	$s_u(\text{kPa})$
0–2	Very soft	<10
3–5	Soft	10–25
6–9	Medium	25–50
10–15	Stiff	50–100
15–30	Very stiff	100–200
>30	Extremely stiff	>200

Tables 7.4 and 7.5. You should be cautious in using the correlation in Tables 7.4. SPTs are not recommended for fine-grained soils, so the correlation shown in Table 7.5 should be used only to provide an assessment of the relative shear strength of fine-grained soils.

7.12.3 Cone Penetrometer Test (CPT)

The cone resistance q_c is normally correlated with the undrained shear strength. Several adjustments are made to q_c . One correlation equation is

$$s_u = \frac{q_c - \sigma_z}{N_k} \quad (7.59)$$

where N_k is a cone factor that depends on the geometry of the cone and the rate of penetration. Average values of N_k as a function of plasticity index can be estimated from

$$N_k = 19 - \frac{\text{PI} - 10}{5}; \text{PI} > 10 \quad (7.60)$$

Results of cone penetrometer tests have been correlated with the peak friction angle. A number of correlations exist. Based on published data for sand (Robertson and Campanella, 1983), you can estimate ϕ'_p using

$$\phi'_p = 35^\circ + 11.5 \log \left(\frac{q_c}{30\sigma'_{z0}} \right); \quad 25^\circ < \phi'_p < 50^\circ \quad (7.61)$$

The essential points are:

1. Various field tests are used to determine soil strength parameters.
2. You should be cautious in using these correlations of field test results, especially SPT, with soil strength parameters in design.

What's next... Several empirical relationships have been proposed to obtain soil strength parameters from laboratory tests, for example, the Atterberg limits, or from statistical analyses of field and laboratory test results. Some of these relationships are presented in the next section.

7.13 EMPIRICAL RELATIONSHIPS FOR SHEAR STRENGTH PARAMETERS

Some suggested empirical relationships for the shear strength of soils are shown in Table 7.6. These relationships should only be used as a guide and in preliminary design calculations.

TABLE 7.6 Empirical Soil Strength Relationships

Soil type	Equation	Reference
Normally consolidated clays	$\left(\frac{s_u}{\sigma'_z}\right)_{nc} = 0.11 + 0.0037 \text{ PI}$	Skempton (1957)
Overconsolidated clays	$\frac{(s_u/\sigma'_z)_{oc}}{(s_u/\sigma'_z)_{nc}} = (\text{OCR})^{0.8}$ $\frac{s_u}{\sigma'_z} = (0.23 \pm 0.04) \text{OCR}^{0.8}$	Ladd et al. (1977) Jamiolkowski et al. (1985)
All clays	$\frac{s_u}{\sigma'_{zc}} = 0.22$	Mesri (1975)
Clean quartz sand	$\phi'_p = \phi'_{cs} + 3D_r(10 - \ln p'_f) - 3$, where p'_f is the mean effective stress at failure (in kPa) and D_r is relative density. This equation should only be used if $12 > (\phi'_p - \phi'_{cs}) > 0$.	Bolton (1986)

7.14 SUMMARY

The strength of soils is interpreted using Coulomb's frictional law. All soils regardless of their initial state of stress will reach a critical state characterized by continuous shearing at constant shear stress and constant volume. The initial void ratio of a soil and the normal effective stresses determine whether the soil will dilate or not. Dilating soils often exhibit (1) a peak shear stress and then strain soften to a constant shear stress, and (2) initial contraction followed by expansion toward a critical void ratio. Nondilating soils (1) show a gradual increase of shear stress, ultimately reaching a constant shear stress, and (2) contract toward a critical void ratio. The shear strength parameters are the friction angles (ϕ'_p and ϕ'_{cs}) for drained conditions and s_u for undrained conditions. Only ϕ'_{cs} is a fundamental soil strength parameter.

A number of laboratory and field tests are available to determine the shear strength parameters. All these tests have shortcomings and you should use careful judgment in deciding on the test to be used for a particular project and in the interpretation of the results.

Practical Examples

EXAMPLE 7.13 Failure Calculations for a Foundation

A rectangular foundation $4 \text{ m} \times 5 \text{ m}$ transmits a total load of 5 MN to a deep, uniform deposit of stiff overconsolidated clay with an $\text{OCR} = 4$ and $\gamma_{\text{sat}} = 18 \text{ kN/m}^3$ (Fig. E7.13). Groundwater level is at 1 m below the ground surface. A CU test was conducted on a soil sample taken at a depth 5 m below the center of the foundation. The results obtained are $s_u = 40 \text{ kPa}$, $\phi'_p = 27^\circ$, and $\phi'_{\text{cs}} = 24^\circ$. Determine if the soil will reach the failure state for short-term and long-term loading conditions. If the soil does not reach the failure state, what are the factors of safety? Assume the soil above the groundwater level is saturated.

Strategy The key is to find the stresses imposed on the soil at a depth of 5 m and check whether the imposed shear stress exceeds the critical shear stress. For short-term condition, the critical shear strength is s_u while for long-term conditions the critical shear strength is $(\sigma'_n)_f \tan \phi'_{\text{cs}}$.

Solution 7.13

Step 1: Determine the initial stresses.

$$\begin{aligned}\sigma'_z &= \sigma'_1 = \gamma_{\text{sat}} z_1 + (\gamma_{\text{sat}} - \gamma_w) z_2 = (18 \times 1) + (18 - 9.8)4 = 50.8 \text{ kPa} \\ K_o^{\text{oc}} &= K_o^{\text{nc}} (\text{OCR})^{0.5} = (1 - \sin \phi'_{\text{cs}}) (\text{OCR})^{0.5} = (1 - \sin 18^\circ)(4)^{0.5} = 1.4 \\ \sigma'_3 &= K_o^{\text{oc}} \sigma'_z = 1.4 \times 50.8 = 71.1 \text{ kPa} \\ \sigma_1 &= \sigma'_z + z_2 \gamma_w = 50.8 + 4 \times 9.8 = 90 \text{ kPa} \\ \sigma_3 &= \sigma'_x + z_2 \gamma_w = 71.1 + 4 \times 9.8 = 110.3 \text{ kPa}\end{aligned}$$

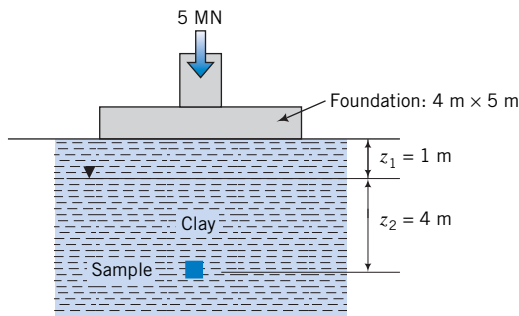


FIGURE E7.13

Step 2: Determine the vertical stress increases at $z = 5 \text{ m}$ under the center of the rectangular foundation.

Use Eq. (5.85) or the computer program on the CD.

$$\Delta \sigma_z = 71.1 \text{ kPa}, \quad \Delta \sigma_x = 5.1 \text{ kPa}$$

Neglect the effect of shear stresses ($\Delta \tau_{zx}$).

Step 3: Determine imposed shear stress for short-term loading.

$$\text{Current vertical total stress: } (\sigma_1)_T = \sigma_1 + \Delta \sigma_z = 90 + 71.1 = 161.1 \text{ kPa}$$

$$\text{Current horizontal total stress: } (\sigma_3)_T = \sigma_3 + \Delta \sigma_x = 110.3 + 5.1 = 115.4 \text{ kPa}$$

$$\text{Current shear stress: } \tau_u = \frac{(\sigma_1)_T - (\sigma_3)_T}{2} = \frac{161.1 - 110.4}{2} = 25.4 \text{ kPa} < 40 \text{ kPa}$$

The soil will not reach the failure state.

$$\text{Factor of safety: } FS = \frac{s_u}{\tau_u} = \frac{40}{25.4} = 1.6$$

Step 4: Determine failure shear stress for long-term loading.

For long-term loading, we will assume that the excess porewater pressure dissipated.

$$\text{Final effective stresses: } (\sigma'_1)_F = \sigma'_1 + \Delta\sigma_z = 50.8 + 71.1 = 121.9 \text{ kPa}$$

$$(\sigma'_3)_F = \sigma'_3 + \Delta\sigma_x = 71.1 + 5.1 = 76.2 \text{ kPa}$$

$$\text{Angle of friction mobilized: } \phi' = \sin^{-1} \left(\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} \right) = \sin^{-1} \left(\frac{121.9 - 76.2}{121.9 + 76.2} \right) = 13.3^\circ$$

The critical state angle of friction is 24° , which is greater than ϕ' . Therefore, failure would not occur.

$$\text{Factor of safety: } FS = \frac{\tan 24^\circ}{\tan 13.3^\circ} = 1.9$$

EXAMPLE 7.14 Specifying Lab Tests for a Dam

An earth dam is proposed for a site consisting of a homogeneous stiff clay, as shown in Fig. E7.14a. You, the geotechnical engineer, are required to specify soil strength tests to determine the stability of the dam. One of the critical situations is the possible failure of the dam by a rotational slip plane in the stiff clay. What laboratory strength tests would you specify and why?

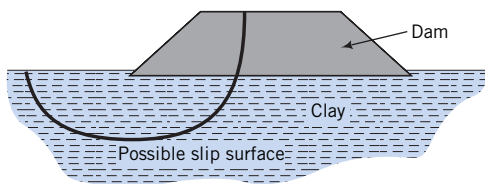


FIGURE E7.14a

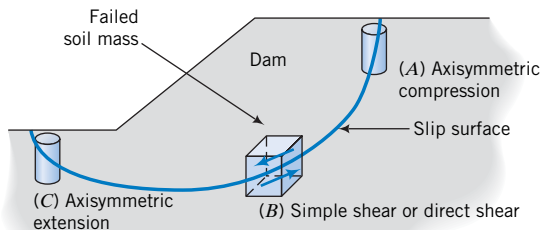


FIGURE E7.14b

Strategy The key is to determine the stress states for short-term and long-term conditions along the failure surface.

Solution 7.14

Step 1: Determine stresses along the failure surface.

Let us select three points—*A*, *B*, and *C*—on the possible failure surface. The rotational slip surface will introduce compression on element *A*, shear on element *B*, and extension on element *C* (Fig. E7.14b). The stresses on element *A* are analogous to a triaxial compression test. Element *B* will deform in a manner compatible with simple shear, while element *C* will suffer from an upward thrust that can be simulated by a triaxial extension test.

Step 2: Make recommendations.

The following strength tests are recommended.

(a) Triaxial CU compression tests with porewater pressure measurements. Parameters required are ϕ'_p , ϕ'_{cs} and s_u .

(b) Simple shear constant-volume tests.

Parameters required are ϕ'_p , ϕ'_{cs} , and s_u . If a simple shear apparatus is unavailable, then direct shear (shear box) tests should be substituted.

Undrained tests should be carried out at the maximum anticipated stress on the soil. You can determine the stress increases from the dam using the methods described in Chapter 5. ■

EXAMPLE 7.15 Estimation of s_u

You have contracted a laboratory to conduct soil tests for a site, which consists of a layer of sand, 6 m thick, with $\gamma_{\text{sat}} = 18 \text{ kN/m}^3$. Below the sand is a deep, soft, bluish clay with $\gamma_{\text{sat}} = 20 \text{ kN/m}^3$ (Fig. E7.15). The site is in a remote area. Groundwater level was located at 2.5 m below the surface. You specified a consolidation test and a triaxial consolidated undrained test for samples of the soil taken at 10 m below ground surface. The consolidation test shows that the clay is lightly over-consolidated with an $\text{OCR} = 1.3$. The undrained shear strength at a cell pressure approximately equal to the initial vertical stress is 72 kPa. Do you think the undrained shear strength value is reasonable, assuming the OCR of the soil is accurate? Show calculations to support your thinking.

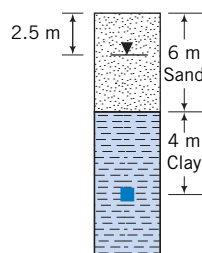


FIGURE E7.15

Strategy Because the site is in a remote area, it is likely that you may not find existing soil results from neighboring constructions. In such a case, you can use empirical relationships as guides but you are warned that soils are notorious for having variable strengths.

Solution 7.15

Step 1: Determine the initial effective stresses. Assume that the sand above the groundwater level is saturated.

$$\sigma'_{z0} = (18 \times 2.5) + (18 - 9.8) \times 3.5 + (20 - 9.8) \times 4 = 114.5 \text{ kPa}$$

$$\sigma'_{zc} = \sigma'_{z0} \times \text{OCR} = 114.5 \times 1.3 = 148.9 \text{ kPa}$$

Step 2: Determine s_u/σ'_{z0} and s_u/σ'_{zc} .

$$\frac{s_u}{\sigma'_{z0}} = \frac{72}{114.5} = 0.63$$

$$\frac{s_u}{\sigma'_{zc}} = \frac{72}{148.9} = 0.48$$

Step 3: Use empirical equations. See Table 7.6

$$\text{Jamolkowski et al. (1985) : } \frac{s_u}{\sigma'_{z0}} = (0.23 \pm 0.04) \text{OCR}^{0.8}$$

$$\begin{aligned} \text{Range of } \frac{s_u}{(\sigma'_z)} &: 0.27(\text{OCR})^{0.8} \text{ to } 0.19(\text{OCR})^{0.8} \\ &= 0.27(1.3)^{0.8} \text{ to } 0.19(1.3)^{0.8} \\ &= 0.33 \text{ to } 0.23 < 0.63 \end{aligned}$$

$$\text{Mesri (1975) : } \frac{s_u}{\sigma'_{zc}} = 0.22 < 0.48$$

The differences between the reported results and the empirical relationships are substantial. The undrained shear strength is therefore suspicious. You should request a repeat of the test. ■

EXERCISES**Interactive Problem Solving**

Access Chapter 7 on the CD, and click on Problem Solver to interactively solve problems similar to some of the problems in this exercise.

Theory

- 7.1** A CD triaxial test was conducted on a loose sand. The axial stress was held constant and the radial stress was increased until failure occurred. At failure the radial stress was greater than the axial stress. Show, using Mohr's circle and geometry, that the slip plane is inclined at $\pi/4 - \phi'_{cs}/2$ to the horizontal plane.
- 7.2** The initial stresses on a soil are $\sigma'_1 = \sigma'_z$ and $\sigma'_3 = K_o \sigma'_z$, where K_o is the lateral earth pressure coefficient at rest (see Chapter 5). The soil was then brought to failure by reducing σ'_3 while keeping σ'_1 constant. At failure, $\sigma'_3 = K \sigma'_1$, where K is a lateral earth pressure coefficient. Show that

$$\frac{\tau_{cs}}{\tau_{zx}} = \frac{1 - K}{1 - K_o} \cos \phi'_{cs}$$

where τ_{zx} is the maximum shear stress under the initial stresses and τ_{cs} is the failure shear stress.

- 7.3** Sand is placed on a clay slope as shown in Fig. P7.3. (a) Show that sand will be unstable (i.e., fail by sliding) if $\theta > \phi'$. (b) Does the thickness of the clay layer influence impending failure? (c) If $\phi' = 25^\circ$ and $\theta = 23^\circ$, determine the factor of safety.

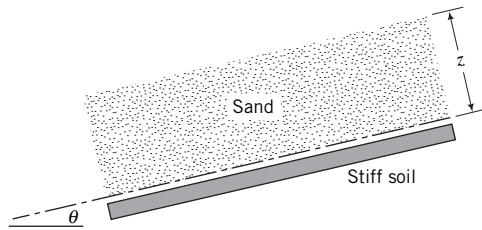


FIGURE P7.3

Problem Solving

- 7.4** A structure will impose a normal effective stress of 100 kPa and a shear stress of 30 kPa on a plane inclined at 58° to the horizontal. The friction angle of the soil is 25° . Will the soil fail? If not, what is the factor of safety?
- 7.5** Figure P7.5 shows the stress–strain behavior of a soil. Determine the peak and critical shear stresses. Estimate the peak and critical state friction angle and the peak dilation angle using (i) Coulomb's model, (ii) Taylor's model. The normal effective stress is 160 kPa.

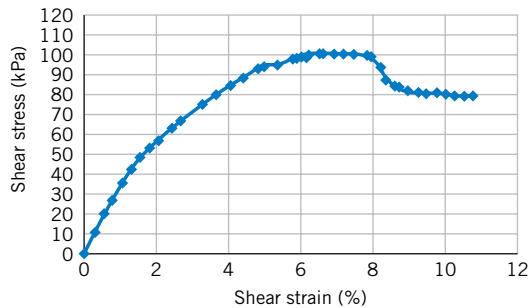


FIGURE P7.5

- 7.6** The following results were obtained from three shear box tests on a sample of sandy clay. The cross section of the shear box is $6 \text{ cm} \times 6 \text{ cm}$.

Normal force (N)	1250	1000	500	250
Shearing force (N)	506	405	325	255

- (a) Determine the critical state friction angle.
- (b) If the soil is dilatant, determine ϕ'_p at a normal force of 250 N.

- 7.7 The results of a direct shear test on a dense sand are shown in the table below. Determine ϕ'_p , ϕ'_{cs} and α_p . The vertical force is 200 N. The sample area is 100 mm $\times$ 100 mm and the sample height is 20 mm.

Horizontal displacement (mm)	0.00	0.25	0.38	0.64	0.76	1.02	1.52	1.78	2.67	2.92
Vertical displacement (mm)	3.18	3.68	4.06	4.57	5.08	5.59	6.10	6.48	6.60	6.86
Horizontal force (N)	7.11	7.62	8.13	8.64	9.14	9.65	10.16			
Vertical force (N)	0.00	0.00	−0.02	−0.03	−0.04	−0.04	−0.03	−0.01	0.04	0.06
	0.09	0.14	0.15	0.19	0.22	0.24	0.26	0.27	0.28	0.28
	0.28	0.28	0.28	0.28	0.28	0.28	0.28			
	0.0	17.7	19.3	33.4	40.3	54.9	83.3	94.9	127.2	133.8
	137.4	148.8	155.9	161.7	169.6	174.4	176.8	177.7	177.7	176.4
	172.9	163.5	161.2	160.8	159.8	158.9	158.8			

- 7.8 Repeat Problem 7.7 but use Taylor's model instead of Coulomb's model and assume that the data are from a simple shear test.
- 7.9 A cylindrical sample of soil 50 mm in diameter $\times$ 100 mm long was subjected to axial and radial effective stresses. When the vertical displacement was 2 mm, the soil failed. The stresses at failure are $(\sigma'_1)_f = 280$ kPa and $(\sigma'_3)_f = 100$ kPa. The change in soil volume at failure was 800 mm³. Determine (a) the axial strain at failure, (b) the volumetric strain at failure, (c) the friction angle, and (d) the inclination of the slip plane to the horizontal.
- 7.10 An unconfined compression test was conducted on a compacted soil sample, 50 mm diameter by 100 mm long. The maximum axial force recorded was 230 N and the sample shortened by 2 mm. Determine the undrained shear strength.
- 7.11 A CD test was conducted on a sample of dense sand, 38 mm in diameter $\times$ 76 mm long. The cell pressure was 200 kPa. The results are shown in the table below.

- (a) Determine the ϕ'_p , ϕ'_{cs} , α_p , τ_p , E' , and E'_s .
- (b) If $\nu' = 0.3$, calculate G .

Axial displacement (mm)	Axial load (N)	Change in volume (cm ³)	Axial displacement (mm)	Axial load (N)	Change in volume (cm ³)
0	0	0	3.2	604.3	−2.7
0.3	128.1	0.1	5.2	593.0	−3.2
0.6	225.9	0.1	6.4	576.0	−3.4
1.0	338.9	0.1	7.1	564.8	−3.5
1.3	451.8	−0.25	9.7	525.2	−3.6
1.6	508.3	−0.7	12.9	497.0	−3.7
1.9	564.8	−1.9	15.5	480.0	−3.7

- 7.12 CU tests were carried out on two samples of a clay. Each sample was isotropically consolidated before the axial stress was increased. The following results were obtained.

Sample No.	$(\sigma_3)_f$ (kPa)	$(\sigma_1 - \sigma_3)_f$ (kPa)	Δu_f (kPa)
I	420	320	205
II	690	365	350

- (a) Draw Mohr's circles (total and effective stresses) for each test on the same graph.
- (b) Why are the total stress circles not the same size?

- (c) Determine the friction angle for each test.
- (d) Determine the undrained shear strength at a cell pressure, $(\sigma_3)_f$ of 690 kPa.
- (e) Determine the shear stress on the failure plane for each sample.

7.13 CU tests were conducted on a compacted clay. Each sample was brought to a saturated state before shearing. The results, when no further change in excess porewater pressure or deviatoric stress occurred, are shown in the table below. Determine (a) ϕ'_{cs} and (b) s_u at a cell pressure of 420 kPa.

σ_3 (kPa)	$(\sigma_1 - \sigma_3)$ (kPa)	Δu (kPa)
140	636	-71
280	1008	-51.2
420	1323	-19.4

7.14 Three samples of a loose sand were tested under CU conditions. The failure stresses and excess porewater pressures for each sample are given below.

Sample No.	$(\sigma_3)_f$ (kPa)	$(\sigma_1 - \sigma_3)_f$ (kPa)	Δu_f (kPa)
1	210	123	112
2	360	252	162
3	685	448	323

- (a) Plot Mohr's circle of effective stress from these data.
- (b) Determine the friction angle for each test.
- (c) If the sand were to be subjected to a vertical effective stress of 300 kPa, what magnitude of horizontal effective stress will cause failure?
- (d) Determine the orientations of (1) the failure plane and (2) the plane of maximum shear stress to the horizontal for Test 2.
- (e) Determine the failure stresses for Test 1.
- (f) Is the failure shear stress the maximum shear stress? Give reasons.

7.15 A CU triaxial test was carried out on a silty clay that was isotropically consolidated with a cell pressure of 125 kPa. The following data were obtained:

Stress imposed by plunger (kPa)	Axial strain, ε_1 (%)	Δu (kPa)	Stress imposed by plunger (kPa)	Axial strain, ε_1 (%)	Δu (kPa)
0	0	0	35.0	0.56	34.8
5.5	0.05	4.0	50.5	1.08	41.0
11.0	0.12	8.6	85.0	2.43	49.7
24.5	0.29	19.1	105.0	4.02	55.8
28.5	0.38	29.3	120.8	9.15	59.0

- (a) Plot the deviatoric stress against axial strain and excess porewater pressure against axial strain.
- (b) Determine the undrained shear strength and the friction angle.
- (c) Determine E' and E'_s .
- (d) Determine Skempton's A_f .

- 7.16** Three CU tests with porewater pressure measurements were made on a clay soil. The results at failure are shown below.

σ_3 (kPa)	$\sigma_1 - \sigma_3$ (kPa)	Δu (kPa)
150	87.5	80
275	207.5	115
500	345	230

- (a) Determine the friction angle for each test.
 (b) Determine the undrained shear strength at a cell pressure of 275 kPa.
 (c) Determine the maximum shear stress and the failure shear stress for each test.
- 7.17** The following data at failure were obtained from two CU tests on a soil:

Test	σ_3 (kPa)	$\sigma_1 - \sigma_3$ (kPa)	Δu (kPa)
1	100	240	−60
2	150	270	−90

- (a) Determine the friction angle for each test.
 (b) Determine the change in dilation angle between the two tests, if a CD test were carried out.
 (c) Determine the undrained shear strength for each test.
- 7.18** A CU test on a stiff, overconsolidated clay at a cell pressure at 150 kPa fails at a deviatoric stress of 448 kPa. The excess porewater pressure at failure is −60 kPa. Determine the friction angle.
- 7.19** The results of UU and CU tests on samples of soil are shown below. Determine (a) the friction angle and (b) s_u at a cell pressure of 210 kPa in the CU tests. Predict the excess porewater pressure at failure for each of the UU test samples.

Type of test	σ_3 (kPa)	$(\sigma_1 - \sigma_3)$ (kPa)	Δu (kPa)
UU	57	105	To be determined
	210	246	To be determined
CU	57	74.2	11.9
	133	137.9	49.0
	210	203	86.1

- 7.20** The failure stresses in a simple shear constant-volume test are shown in the table below.

Total normal stress on the horizontal plane	300 kPa
Total normal stress on the vertical plane	200 kPa
Total shear stress on the horizontal and vertical planes	100 kPa
porewater pressure	50 kPa

- (a) Draw Mohr's circles of total and effective stresses and determine the magnitude of the principal effective stresses and direction of the major principal effective stresses.
 (b) Determine the friction angle assuming the soil is nondilational.
 (c) Determine the undrained shear strength.

Assume the shear stress on the horizontal plane is counterclockwise.

Practical

- 7.21** You are in charge of designing a retaining wall. What laboratory tests would you specify for the backfill soil? Give reasons.
- 7.22** The following results were obtained from CU tests on a clay soil that is the foundation material for an embankment:

σ_3 (kPa)	$\sigma_1 - \sigma_3$ (kPa)	Δu (kPa)
300	331	111
400	420	160
600	487	322

Recommend the shear strength parameters to be used for short-term and long-term analyses. The maximum confining pressure (cell pressure) at the depth of interest is 300 kPa.

- 7.23** An embankment is being constructed to preload a foundation for an oil tank using a soil of unit weight $\gamma_{\text{sat}} = 18 \text{ kN/m}^3$. The soil below the embankment has a friction angle $\phi'_{\text{cs}} = 25^\circ$. The Skempton's pore pressure parameters found from triaxial tests are $A_f = 0.5$ and $B = 0.9$. It is proposed to raise the embankment from 3 m to 6 m. Assume (i) the increases in stresses are principal stresses, (ii) the dissipation of excess pore pressure during this stage of construction is negligible, and (iii) the increase in the lateral total pressure at any point is 0.5 times the increase in vertical total pressure. Determine the following:
- Increase in the vertical total stress ($\Delta\sigma_z$) at the base of the embankment.
 - Increase in the lateral total stress ($\Delta\sigma_x$) at 0.5 m below the base of the embankment. Assume the vertical stress at the base is fully transferred to the soil at 0.5 m depth.
 - Increase in porewater pressure (Δu) at 0.5 m below the base of the embankment.
 - Vertical effective stress (σ'_z) increase at 0.5 m below the base of the embankment when it achieved a height of 6 m.
- 7.24** A trench is proposed for utilities along a very long slope that consists of two soils. The top soil is a fissured saturated clay ($\gamma_{\text{sat}} = 20 \text{ kN/m}^3$, $\phi'_{\text{cs}} = 25^\circ$, $s_u = 20 \text{ kPa}$) and the soil below it is a deep heavily over-consolidated, stiff saturated clay with $s_u = 80 \text{ kPa}$ (Fig. P7.24). The groundwater level is 8 m below the ground surface. One of the potential failure mechanisms consists of the top clay sliding as a rigid block at the interface. Consider 1 m length of slope and calculate the factor of safety (failure shear stress/shear stress imposed) of the slope for (a) short-term and (b) long-term conditions. Assume that the friction angle at the interface of the two soils is 25° .

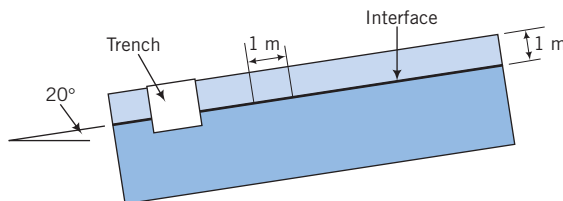


FIGURE P7.24

CHAPTER

8

A CRITICAL STATE MODEL TO INTERPRET SOIL BEHAVIOR

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8.0 INTRODUCTION

So far, we have painted individual pictures of soil behavior. We looked at the physical characteristics of soil in Chapter 3, effective stresses and stress paths in Chapter 5, one-dimensional consolidation in Chapter 6, and shear strength in Chapter 7. You know that if you consolidate a soil to a higher stress state than its current one, the shear strength of the soil will increase. But the amount of increase depends on the soil type, the loading conditions (drained or undrained condition), and the stress paths. Therefore, the individual pictures should all be linked together: But how?

In this chapter, we are going to take the individual pictures and build a mosaic that will provide a base for us to interpret and anticipate soil behavior. Our mosaic is mainly intended to unite consolidation and shear strength. Real soils, of course, require a complex mosaic not only because soils are natural, complex materials but also because the loads and loading paths cannot be anticipated accurately.

We are going to build a mosaic to provide a simple framework to describe, interpret, and anticipate soil responses to various loadings. The framework is essentially a theoretical model based on critical state soil mechanics—critical state model (Schofield and Wroth, 1968). Laboratory and field data, especially results from soft normally consolidated clays, lend support to the underlying concepts embodied in the development of the critical state model. The emphasis in this chapter will be on using the critical state model to provide a generalized understanding of soil behavior rather than on the mathematical formulation.

The critical state model (CSM) we are going to study is a simplification and an idealization of soil behavior. However, the CSM captures the behavior of soils that are of greatest importance to geotechnical engineers. The central idea in the CSM is that all soils will fail on a unique failure surface in (q, p', e) space. Thus, the CSM incorporates volume changes in its failure criterion unlike the Mohr–Coulomb failure criterion, which defines failure only as the attainment of the maximum stress obliquity. According to the CSM, the failure stress state is insufficient to guarantee failure; the soil structure must also be loose enough.

The CSM is a tool to make estimates of soil responses when you cannot conduct sufficient soil tests to completely characterize a soil at a site or when you have to predict the soil's response from changes in loading during and after construction. Although there is a debate on the application of the CSM to real soils, the ideas behind the CSM are simple. It is a very powerful tool to get insights into soil behavior, especially in the case of the “what-if” situation. There is also a plethora of soil models in the literature that have critical state as their core. By studying the CSM, albeit a simplified version in this chapter, you will be able to better understand these other soil models.

When you have studied this chapter, you should be able to:

- Estimate failure stresses for soil.
- Estimate strains at failure.
- Predict stress–strain characteristics of soils from a few parameters obtained from simple soil tests.

- Evaluate possible soil stress states and failure if the loading on a geotechnical system were to change.

You will make use of all the materials you studied in Chapters 3 to 7 but particularly:

- Index properties (Chapter 3)
- Effective stresses, stress invariants, and stress paths (Chapter 5)
- Primary consolidation (Chapter 6)
- Shear strength (Chapter 7)

Sample Practical Situation An oil tank is to be constructed on a soft alluvial clay. It was decided that the clay would be preloaded with a circular embankment imposing a stress equal to, at least, the total applied stress of the tank when filled. Sand drains are to be used to accelerate the consolidation process. The foundation for the tank is a circular slab of concrete and the purpose of the preloading is to reduce the total settlement of the foundation. You are required to advise the owners on how the tank should be filled during preloading to prevent premature failure. After preloading, the owners decided to increase the height of the tank. You are requested to determine whether the soil has enough shear strength to support an additional increase in tank height, and if so the amount of settlement that can be expected. The owners do not want to finance any further preloading and soil testing.

8.1 DEFINITIONS OF KEY TERMS

Overconsolidation ratio (R_o) is the ratio by which the current mean effective stress in the soil was exceeded in the past ($R_o = p'_c/p'_o$ where p'_c is the past maximum mean effective stress and p'_o is the current mean effective stress).

Compression index (λ) is the slope of the normal consolidation line in a plot of void ratio versus the natural logarithm of mean effective stress.

Unloading/reloading index or recompression index (κ) is the average slope of the unloading/reloading curves in a plot of void ratio versus the natural logarithm of mean effective stress.

Critical state line (CSL) is a line that represents the failure state of soils. In (q, p') space the critical state line has a slope M , which is related to the friction angle of the soil at the critical state. In $(e, \ln p')$ space, the critical state line has a slope λ , which is parallel to the normal consolidation line. In three-dimensional (q, p', e) space, the critical state line becomes a critical state surface.

8.2 QUESTIONS TO GUIDE YOUR READING

1. What is soil yielding?
2. What is the difference between yielding and failure in soils?
3. What parameters affect the yielding and failure of soils?
4. Does the failure stress depend on the consolidation pressure?
5. What are the critical state parameters and how can you determine them from soil tests?
6. Are strains important in soil failure?
7. What are the differences in the stress–strain responses of soils due to different stress paths?

8.3 BASIC CONCEPTS



Interactive Concept Learning and Self-Assessment

Access Chapter 8, Section 8.3 on the CD to learn the basic concepts of the critical state model through simulation. In addition, you will get an understanding of how to (1) calculate failure stresses and strains, (2) calculate yield stresses and strains, and (3) predict stress–strain responses for both drained and undrained conditions. Take Quiz 8.3 to test your understanding.

8.3.1 Parameter Mapping

In our development of the basic concepts on critical state, we are going to map certain plots we have studied in Chapters 6 and 7 using stress and strain invariants and concentrate on a saturated soil under axisymmetric loading. However, the concepts and method hold for any loading condition. Rather than plotting τ versus σ'_n or σ'_z , we will plot the data as q versus p' (Fig. 8.1a). This means that you must know the principal stresses acting on the element. For axisymmetric (triaxial) condition, you only need to know two principal stresses.

The Mohr–Coulomb failure line in (τ, σ'_z) space of slope $\phi'_{cs} = \tan^{-1}[\tau_{cs}/(\sigma'_z)_f]$ is now mapped in (q, p') space as a line of slope $M = q_f/p'_f$, where the subscript f denotes failure. Instead of a plot of e versus σ'_z , we will plot the data as e versus p' (Fig. 8.1b) and instead of e versus σ'_z (log scale), we will plot e versus p' (ln scale) (Fig. 8.1c). We will denote the slope of the normal consolidation line in the plot of e versus p' (ln scale) as λ and the unloading/reloading line as κ . There are now relationships between ϕ'_{cs} and M , C_c and λ , and C_r and κ . The relationships for the slopes of the normal consolidation line (NCL), λ , and the unloading/reloading line (URL), κ , are

$$\lambda = \frac{C_c}{\ln(10)} = \frac{C_c}{2.3} = 0.434 C_c \quad (8.1)$$

$$\kappa = \frac{C_r}{\ln(10)} = \frac{C_r}{2.3} = 0.434 C_r \quad (8.2)$$

Both λ and κ are positive for compression. For many soils, κ/λ has values within the range $\frac{1}{10}$ to $\frac{1}{5}$. We will formulate the relationship between ϕ'_{cs} and M later. The overconsolidation ratio using stress invariants is

$$R_o = \frac{p'_c}{p'_o} \quad (8.3)$$

where p'_o is the initial mean effective stress or overburden mean pressure and p'_c is the preconsolidated mean effective stress. The overconsolidation ratio, R_o , defined by Eq. (8.3) is not equal to OCR [Eq. (6.13)]:

$$R_o = \frac{1 + 2K_o^{nc}}{1 + 2K_o^{oc}} \text{OCR} \quad (8.4)$$

(You will be required to prove this equation in Exercise 8.1.) We will refer to e versus p' (ln scale) as $(e, \ln p')$.

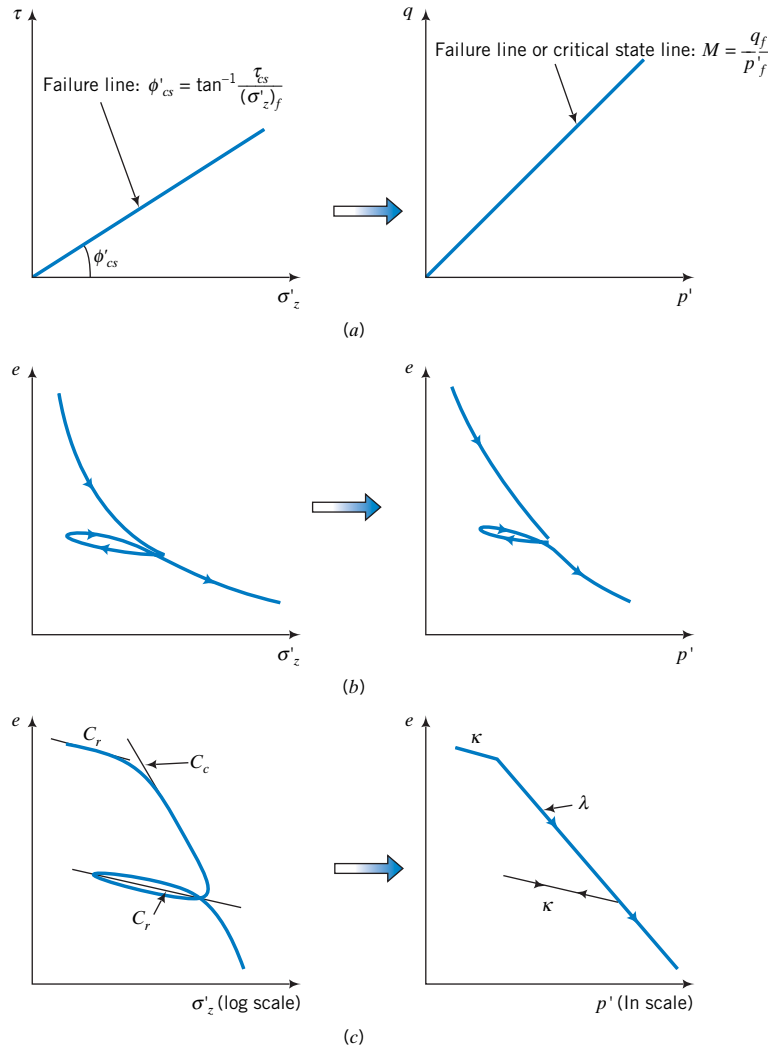


FIGURE 8.1 Mapping of strength and consolidation parameters.

8.3.2 Failure Surface

The fundamental concept in CSM is that a unique failure surface exists in (q, p', e) space, which defines failure of a soil irrespective of the history of loading or the stress paths followed. Failure and critical state are synonymous. We will refer to the failure line as the critical state line (CSL) in this chapter. You should recall that critical state is a constant stress state characterized by continuous shear deformation at constant volume. In stress space (q, p') the CSL is a straight line of slope $M = M_c$, for compression, and $M = M_e$, for extension (Fig. 8.2a). Extension does not mean tension but refers to the case where the lateral stress is greater than the vertical stress. There is a corresponding CSL in (p', e) space (Fig. 8.2b) or $(e, \ln p')$ space (Fig. 8.2c) that is parallel to the normal consolidation line.

We can represent the CSL in a single three-dimensional plot with axes q, p', e , but we will use the projections of the failure surface in the (q, p') space and the (e, p') space for simplicity.

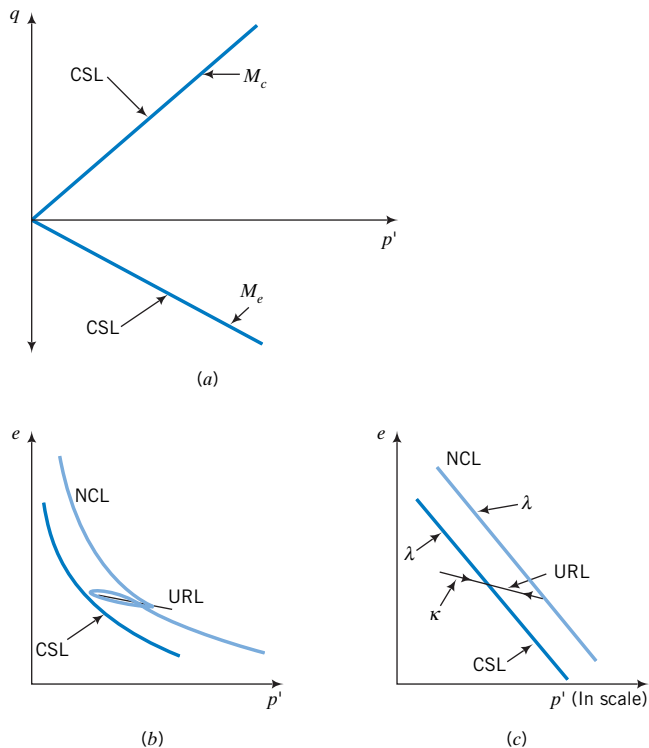


FIGURE 8.2 Critical state lines, normal compression, and unloading/reloading lines.

8.3.3 Soil Yielding

You should recall from Chapter 5 (Fig. 5.8) that there is a yield surface in stress space that separates stress states that produce elastic responses from stress states that produce plastic responses. We are going to use the yield surface in (q, p') space (Fig. 8.3) rather than (σ'_1, σ'_3) space so that our interpretation of soil responses is independent of the axis system.

The yield surface is assumed to be an ellipse and its initial size or major axis is determined by the preconsolidation stress, p'_c . Experimental evidence (Wong and Mitchell, 1975) indicates that an elliptical yield surface is a reasonable approximation for soils. The higher the preconsolidation stress, the larger the initial ellipse. We will consider the yield surface for compression but the ideas are the same for extension except that the minor axis of the elliptical yield surface in extension is smaller than

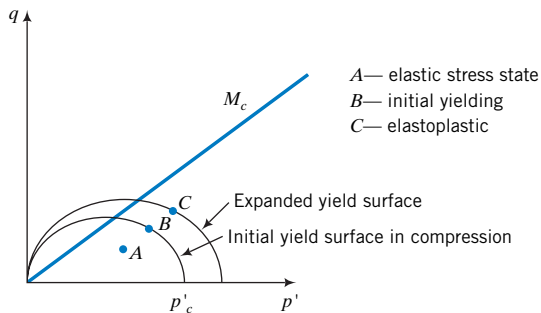


FIGURE 8.3 Expansion of the yield surface.

in compression. All combinations of q and p' that lie within the yield surface, for example, point A in Fig. 8.3, will cause the soil to respond elastically. If a combination of q and p' lies on the initial yield surface (point B, Fig. 8.3), the soil yields similar to the yielding of a steel bar. Any tendency of a stress combination to move outside the current yield surface is accompanied by an expansion of the current yield surface such that during plastic loading the stress point (q, p') lies on the expanded yield surface and not outside, as depicted by C. Effective stress paths such as BC (Fig. 8.3) cause the soil to behave elastoplastically. If the soil is unloaded from any stress state below failure, the soil will respond like an elastic material. As the yield surface expands, the elastic region gets larger.

8.3.4 Prediction of the Behavior of Normally Consolidated and Lightly Overconsolidated Soils Under Drained Conditions

Let us consider a hypothetical situation to illustrate the ideas presented so far. We are going to try to predict how a sample of soil of initial void ratio e_o will respond when tested under drained condition in a triaxial apparatus, that is, a CD test. You should recall that the soil sample in a CD test is isotropically consolidated and then axial loads or displacements are applied, keeping the cell pressure constant. We are going to consolidate our soil sample up to a maximum mean effective stress p'_c , and then unload it to a mean effective stress p'_o such that $R_o = p'_c/p'_o < 2$. We can sketch a curve of e versus p' (AC, Fig. 8.4b) during the consolidation phase. You should recall from Fig. 8.1

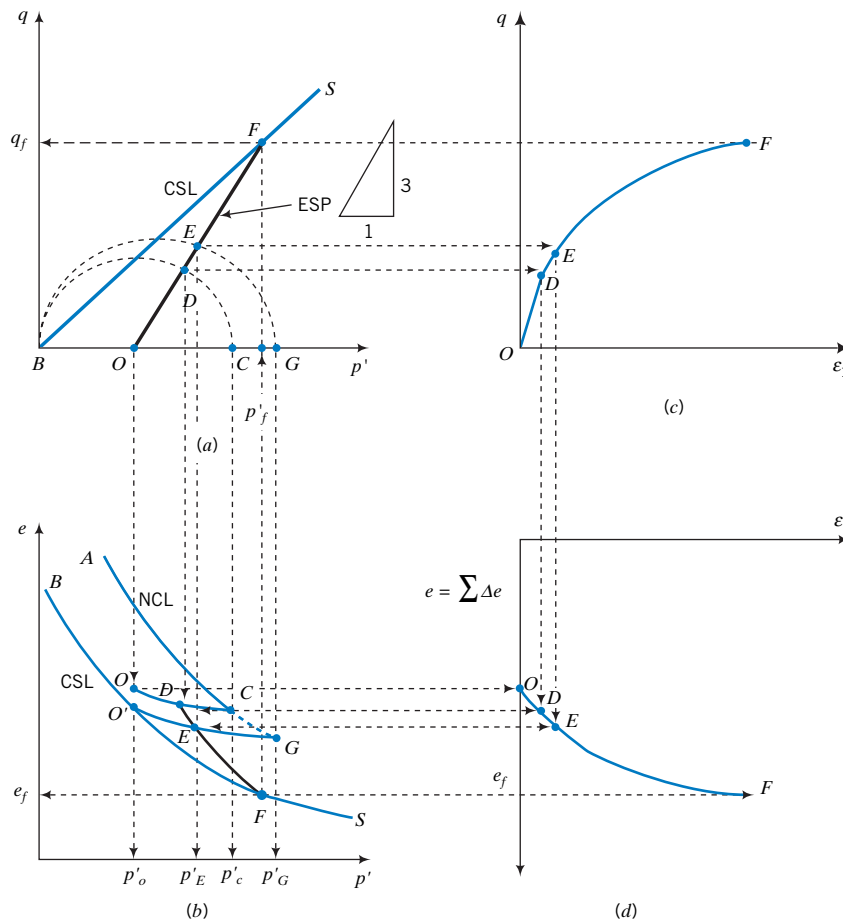


FIGURE 8.4 Illustrative predicted results from a CD test ($R_o \leq 2$) using CSM.

that the line AC is the normal consolidation line of slope λ . Because we are applying isotropic loading, the line AC (Fig. 8.4c) is called the isotropic consolidation line. We can also sketch a curve (CO , Fig. 8.4b) to represent the unloading of the soil. The line CO is the unloading/reloading line of slope κ .

The preconsolidated mean effective stress, p'_c , determines the size of the initial yield surface. A semi-ellipse is sketched in Fig. 8.4a to illustrate the initial yield surface for compression. We can draw a line, BS , from the origin to represent the critical-state line in (q, p') space as shown in Fig. 8.4a and a similar line in (e, p') space as shown in Fig. 8.4b. Of course, we do not know, as yet, the slope M , or the equation to draw the initial yield surface. We have simply selected arbitrary values. Later, we are going to develop equations to define the slope M , the shape of the yield surface, and the critical state line in (e, p') space or $(e, \ln p')$ space.

Let us now shear the soil sample at its current mean effective stress, p'_o , by increasing the axial stress, keeping the cell pressure, σ_3 , constant and allowing the sample to drain. You should recall from Chapter 7 that the effective stress path for a CD test has a slope $q/p' = 3$. The effective stress path is shown by OF in Fig. 8.4a. The effective stress path intersects the initial yield surface at D . All stress states from O to D lie within the initial yield surface and, therefore, from O to D on the ESP the soil behaves elastically. Assuming linear elastic response of the soil, we can draw a line OD in (q, ϵ_1) space (Fig. 8.4c) to represent the elastic stress-strain response. At this stage, we do not know the slope of OD but later you will learn how to get this slope. Since the line CO in (e, p') space represents the unloading/reloading line (URL), the elastic response must lie along this line. The change in void ratio is $\Delta e = e_o - e_D$ (Fig. 8.4b) and we can plot the e versus ϵ_1 response as shown by OD in Fig. 8.4d.

Further loading from D along the stress path CF causes the soil to yield. The initial yield surface expands (Fig. 8.4a) and the stress-strain response is a curved path (Fig. 8.4c) because the soil behaves elastoplastically (Chapter 5). At some arbitrarily chosen loading point, E , along the ESP, the size (major axis) of the yield surface is p'_G corresponding to point G in (e, p') space.

The total change in void ratio as you load the sample from D to E is DE (Fig. 8.4b). Since E lies on the yield surface corresponding to a mean effective stress p'_E , then E must be on the unloading line, GO' , as illustrated in Fig. 8.4b. If you unload the soil sample from E back to O , the soil will follow an unloading path, EO' , parallel to OC as shown in Fig. 8.4b.

We can continue to add increments of loading along the ESP until the soil fails. For each load increment, we can sketch the stress-strain curve and the path followed in (e, p') space. Failure occurs when the ESP intersects the critical state line as indicated by F in Fig. 8.4a. The failure stresses are p'_f and q_f (Fig. 8.4a) and the failure void ratio is e_f (Fig. 8.4b). For each increment of loading, we can determine Δe and plot ϵ_1 versus $\Sigma \Delta e$ [or $\epsilon_p = (\Sigma \Delta e)/(1 + e_o)$] as shown in Fig. 8.4d.

Each point on one of the figures has a corresponding point on another figure in each of the quadrants shown in Fig. 8.4. Thus, each point on any figure can be obtained by projection as illustrated in Fig. 8.4. Of course, the scale of the axis on one figure must match the scale of the corresponding axis on the other figure.

8.3.5 Prediction of the Behavior of Normally Consolidated and Lightly Overconsolidated Soils Under Undrained Conditions

Instead of a CD test we could have conducted a CU test after consolidating the sample. Let us examine what would have occurred according to our CSM. We know (Chapter 7) that for undrained condition the soil volume remains constant, that is, $\Delta e = 0$, and the ESP for stresses that produce an elastic response is vertical, that is, the change in mean effective stress, $\Delta p'$, is zero for linearly elastic soils. Because the change in volume is zero, the mean effective stress at failure can be represented by drawing a horizontal line from the initial void ratio to intersect the critical state line in (e, p') space as

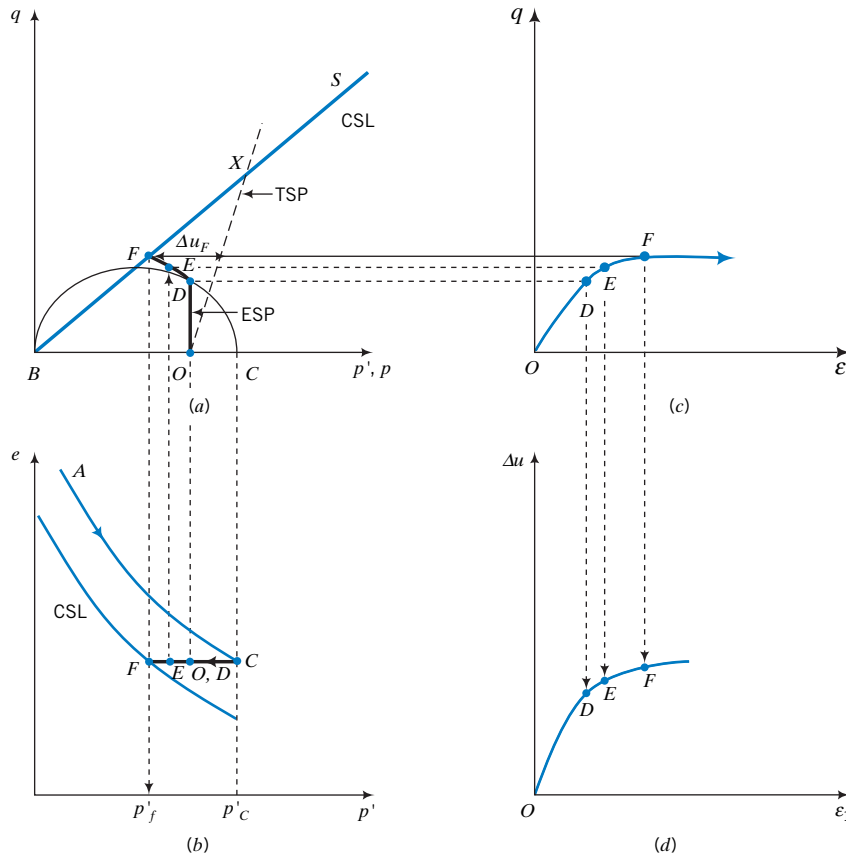


FIGURE 8.5 Illustrative predicted results from a CU test using the CSM ($R_o \leq 2$).

illustrated by OF in Fig. 8.5b. Projecting a vertical line from the mean effective stress at failure in (e, p') space to intersect the critical state line in (q, p') space gives the deviatoric stress at failure (Fig. 8.5a). Since the ESP is vertical within the initial yield surface (OD , Fig. 8.5a), the yield stress can readily be found from the intersection of the ESP and the initial yield surface. Points O and D are coincident in the (e, p') plot as illustrated in Fig. 8.5b because $\Delta p' = 0$. For normally consolidated and lightly overconsolidated soils, the effective stress path after initial yielding (point D , Fig. 8.5a) curves toward the critical state line as the excess porewater pressure increases significantly after yielding occurs.

The TSP has a slope of 3 (Chapter 7) as illustrated by CX in Fig. 8.5a. The difference in mean stress between the total stress path and the effective path gives the change in excess porewater pressure. The intersection of the TSP with the critical state line at X is not the failure point because failure and deformation in a soil mass depend on effective not total stress. By projection, we can sketch the stress–strain response and the excess porewater pressure versus strain as illustrated in Figs. 8.5c,d.

8.3.6 Prediction of the Behavior of Heavily Overconsolidated Soils

So far we have considered a lightly overconsolidated soil ($R_o < 2$). What is the situation regarding heavily overconsolidated soils, that is, $R_o > 2$? We can model a heavily overconsolidated soil by unloading it so that $p'_c/p'_o > 2$ as shown by point O in Figs. 8.6a,b. Heavily overconsolidated soils have initial stress states that lie to the left of the critical state line in the e versus p' plot. The ESP for a CD

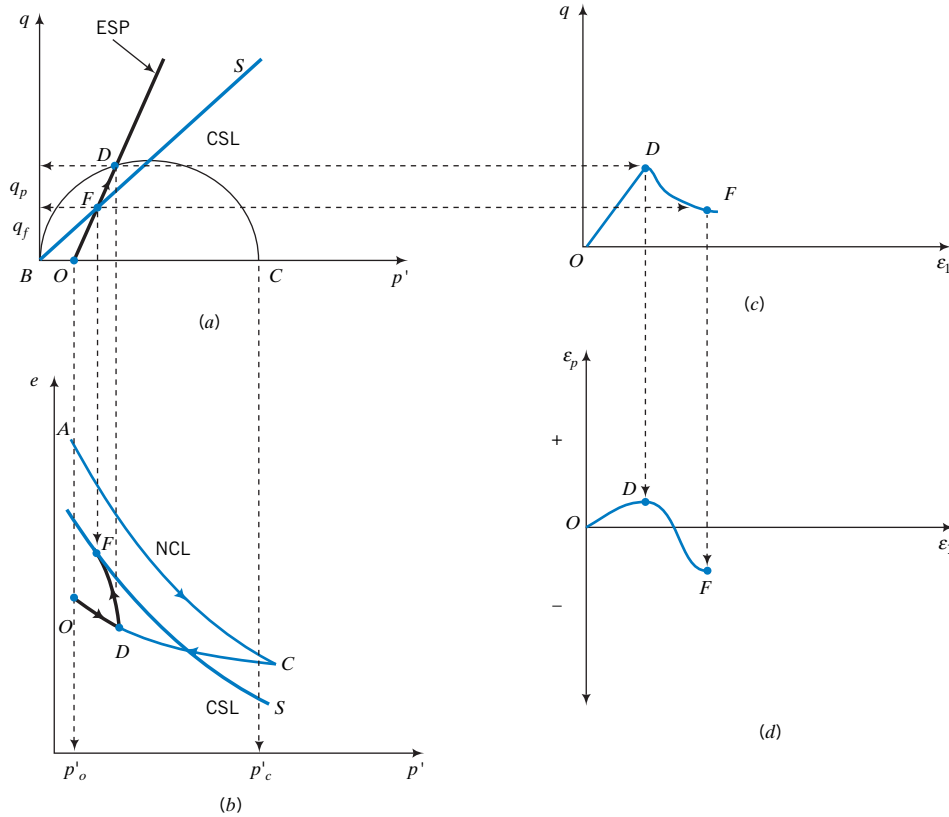


FIGURE 8.6 Illustrative predicted results from a CD test ($R_o > 2$) using the CSM.

test has a slope of 3 and intersects the initial yield surface at D . Therefore, from O to D the soil behaves elastically as shown by OD in Figs. 8.6b,c. The intersection of the ESP with the critical state line is at F (Fig. 8.6a), so that the yield surface must contract as the soil is loaded to failure. The initial yield shear stress is analogous to the peak shear stress for dilating soils. From D , the soil expands (Figs. 8.6b,d) and strain softens (Fig. 8.6c) to failure at F .

The CSM simulates the mechanical behavior of heavily overconsolidated soils as elastic materials up to the peak shear stress and thereafter elastoplastically as the imposed loading causes the soil to strain soften toward the critical state line. In reality, heavily overconsolidated soils may behave elastoplastically before the peak shear stress is achieved but this behavior is not captured by the simple CSM described here.

In the case of a CU test on heavily overconsolidated soils, the path to failure in (e, p') space is OF as shown in Fig. 8.7b. Initial yielding is attained at D and failure at F . The excess porewater pressures at initial yield, Δu_y , and at failure, Δu_f , are shown in the inset of Fig. 8.7a. The excess porewater pressure at failure is negative ($p'_f > p_f$).

8.3.7 Critical State Boundary

The CSL serves as a boundary separating normally consolidated and lightly overconsolidated soils from heavily overconsolidated soils. Stress states that lie to the right of the CSL will result in compression and strain hardening of the soil; stress states that lie to the left of the CSL will result in expansion and strain softening of the soil.

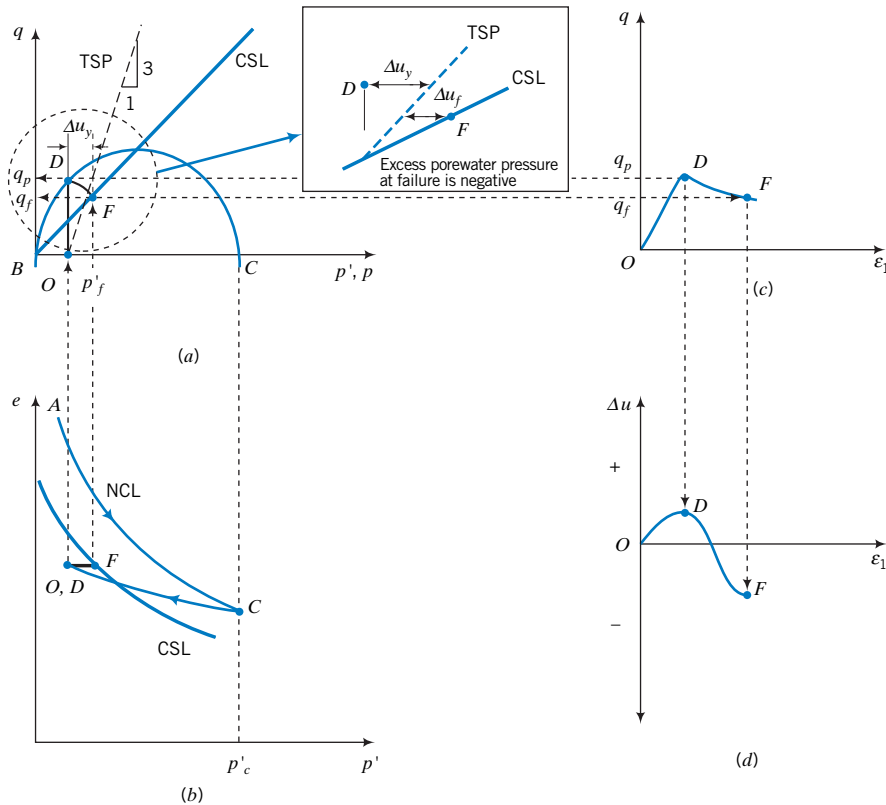


FIGURE 8.7 Illustrative predicted results from a CU test ($R_o \geq 2$) using the CSM.

8.3.8 Volume Changes and Excess Porewater Pressures

If you compare the responses of soils in drained and undrained tests as predicted by the CSM, you will notice that compression in drained tests translates as positive excess porewater pressures in undrained tests, and expansion in drained tests translates as negative excess porewater pressures in undrained tests. The CSM also predicts that normally consolidated and lightly overconsolidated soils strain harden to failure, while heavily overconsolidated soils strain soften to failure. The predicted responses from the CSM then qualitatively match observed soil responses (Chapter 7).

8.3.9 Effects of Effective Stress Paths

The response of a soil depends on the ESP. Effective stress paths with slopes less than the CSL (OA, Fig. 8.8) will not produce shear failure in the soil because the ESP will never intersect the

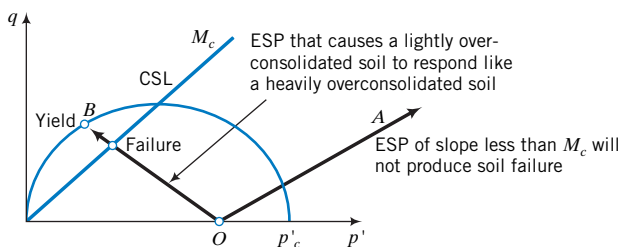


FIGURE 8.8 Effects of effective stress paths on soil response.

critical state line. You can load a normally consolidated or a lightly overconsolidated soil with an ESP that causes it to respond like an overconsolidated soil as shown by *OB* in Fig. 8.8. Effective stress paths similar to *OB* are possible in soil excavation. Remember that a soil must yield before it fails.

The essential points are:

1. *There is a unique critical state line in (q, p') space and a corresponding critical state line in (e, p') space for soils.*
2. *There is an initial yield surface for soils. The size of the initial yield surface depends on the preconsolidation mean effective stress.*
3. *The yield surface expands for $R_o \leq 2$ and contracts for $R_o \geq 2$ when the applied effective stresses exceed the initial yield stress.*
4. *The soil will behave elastically for stresses that are within the yield surface and elastoplastically for stresses outside the yield surface.*
5. *Every stress state must lie on an expanded or contracted yield surface and on a corresponding URL.*
6. *Failure occurs when the ESP intersects the CSL and the change in volume is zero.*
7. *The critical state model qualitatively captures the essential features of soil responses under drained and undrained loading.*

What's next... You were given an illustration using projection geometry of the essential ingredients of the critical state model. There were many unknowns. For example, you did not know the slope of the critical state line and the equation of the yield surface. In the next section we will develop equations to find these unknowns. Remember that our intention is to build a simple mosaic coupling the essential features of consolidation and shear strength.

8.4 ELEMENTS OF THE CRITICAL STATE MODEL

8.4.1 Yield Surface

The equation for the yield surface is an ellipse given by

$$(p')^2 - p'p'_c + \frac{q^2}{M^2} = 0 \quad (8.5)$$

The theoretical basis for the yield surface is presented by Schofield and Wroth (1968) and Roscoe and Burland (1968). You can draw the initial yield surface from the initial stresses on the soil if you know the values of M and p'_c .

8.4.2 Critical State Parameters

8.4.2.1 Failure Line in (p', q) Space The failure line in (p', q) space is

$$q_f = Mp'_f \quad (8.6)$$

where q_f is the deviatoric stress at failure, M is a frictional constant, and p'_f is the mean effective stress at failure. For compression, $M = M_c$ and for extension $M = M_e$. The critical state line intersects the yield surface at $p'_c/2$.

We can build a convenient relationship between M and ϕ'_{cs} for axisymmetric compression and extension as follows.

Axisymmetric Compression

$$M_c = \frac{q_f}{p'_f} = \frac{(\sigma'_1 - \sigma'_3)_f}{\left(\frac{\sigma'_1 + 2\sigma'_3}{3}\right)_f} = \frac{3\left(\frac{\sigma'_1}{\sigma'_3} - 1\right)_f}{\left(\frac{\sigma'_1}{\sigma'_3} + 2\right)_f}$$

We know from Chapter 7 that

$$\left(\frac{\sigma'_1}{\sigma'_3}\right)_f = \frac{1 + \sin \phi'_{cs}}{1 - \sin \phi'_{cs}}$$

Therefore,

$$M_c = \frac{6 \sin \phi'_{cs}}{3 - \sin \phi'_{cs}} \quad (8.7)$$

or

$$\sin \phi'_{cs} = \frac{3M_c}{6 + M_c} \quad (8.8)$$

Axisymmetric Extension In an axisymmetric extension test, the radial stress is the major principal stress. Since in axial symmetry the radial stress is equal to the circumferential stress, we get

$$p'_f = \left(\frac{2\sigma'_1 + \sigma'_3}{3}\right)_f$$

$$q_f = (\sigma'_1 - \sigma'_3)_f$$

and

$$M_e = \frac{q_f}{p'_f} = \frac{\left(2\frac{\sigma'_1}{\sigma'_3} + 1\right)_f}{\left(\frac{\sigma'_1}{\sigma'_3} - 1\right)_f} = \frac{6 \sin \phi'_{cs}}{3 + \sin \phi'_{cs}} \quad (8.9)$$

or

$$\sin \phi'_{cs} = \frac{3M_e}{6 - M_e} \quad (8.10)$$

An important point to note is that while the friction angle, ϕ'_{cs} , is the same for compression and extension, the slope of the critical state line in (q, p') space is not the same. Therefore, the failure deviatoric stresses in compression and extension are different. Since $M_e < M_c$, the failure deviatoric stress of a soil in extension is lower than that for the same soil in compression.

8.4.2.2 Failure Line in (e, p') Space Let us now find the equation for the critical state line in (e, p') space. We will use the $(e, \ln p')$ plot as shown in Fig. 8.9c. The CSL is parallel to the normal consolidation line and is represented by

$$e_f = e_\Gamma - \lambda \ln p'_f \quad (8.11)$$

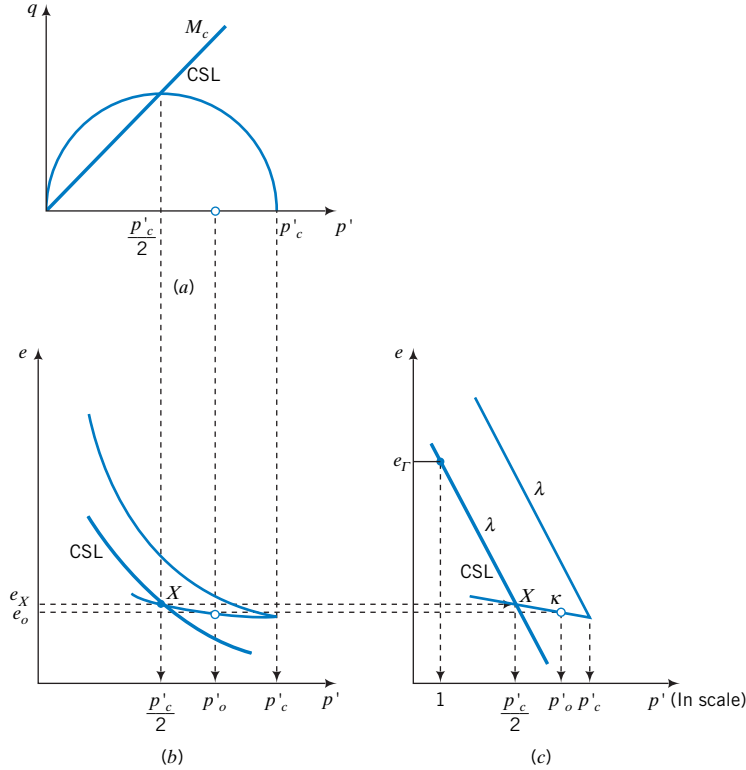


FIGURE 8.9 Void ratio, e_Γ , to anchor critical state line.

where e_Γ is the void ratio on the critical state line when $p' = 1$. The value of e_Γ depends on the units chosen for the p' scale. In this book, we will use kPa for the units of p' .

We will now determine e_Γ from the initial state of the soil. Let us isotropically consolidate a soil to a mean effective stress p'_c and then isotropically unload it to a mean effective stress p'_o (Figs. 8.9a,b). Let X be the intersection of the unloading/reloading line with the critical state line. The mean effective stress at X is $p'_c/2$ and from the unloading/reloading line

$$e_X = e_o + \kappa \ln \frac{p'_o}{p'_c/2} \quad (8.12)$$

where e_o is the initial void ratio. From the critical state line,

$$e_X = e_\Gamma - \lambda \ln \frac{p'_c}{2} \quad (8.13)$$

Therefore, equating Eqs. (8.12) and (8.13) we get

$$e_\Gamma = e_o + (\lambda - \kappa) \ln \frac{p'_c}{2} + \kappa \ln p'_o \quad (8.14)$$

The essential critical state parameters are:

λ —**Compression index, which is obtained from an isotropic or a one-dimensional consolidation test.**

κ —**Unloading/reloading index or recompression index, which is obtained from an isotropic or a one-dimensional consolidation test.**

M —**Critical state frictional constant.**

To use the critical state model, you must also know the initial stresses, for example, p'_o and p'_c , and the initial void ratio, e_o .

EXAMPLE 8.1 Calculation of Failure Stresses

A CD test at a constant cell pressure, $\sigma_3 = \sigma'_3 = 120$ kPa, was conducted on a sample of normally consolidated clay. At failure, $q = \sigma'_1 - \sigma'_3 = 140$ kPa. What is the value of M_c ? If an extension test were to be carried out, determine the mean effective and deviatoric stresses at failure.

Strategy You are given the final stresses, so you have to use these to compute ϕ'_{cs} and then use Eq. (8.7) to calculate M_c and Eq. (8.9) to calculate M_e . You can then calculate q_f for the extension test by proportionality.

Solution 8.1

Step 1: Find the major principal stress at failure.

$$(\sigma'_1)_f = 140 + 120 = 260 \text{ kPa}$$

Step 2: Find ϕ'_{cs} .

$$\sin \phi'_{cs} = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} = \frac{140}{260 + 120} = 0.37$$

$$\phi'_{cs} = 21.6^\circ$$

Step 3: Find M_c and M_e .

$$M_c = \frac{6 \sin \phi'_{cs}}{3 - \sin \phi'_{cs}} = \frac{6 \times 0.37}{3 - 0.37} = 0.84$$

$$M_e = \frac{6 \sin \phi'_{cs}}{3 + \sin \phi'_{cs}} = \frac{6 \times 0.37}{3 + 0.37} = 0.66$$

Step 4: Find q_f for extension.

$$q_f = \frac{0.66}{0.84} \times 140 = 110 \text{ kPa}; \quad p'_f = \frac{q_f}{M_e} = \frac{110}{0.66} = 166.7 \text{ kPa}$$

EXAMPLE 8.2 Determination of λ , κ , and e_Γ

A saturated soil sample was isotropically consolidated in a triaxial apparatus and a selected set of data is shown in the table. Determine λ , κ , and e_Γ .

Condition	Cell pressure (kPa)	Final void ratio
Loading	200	1.72
	1000	1.20
Unloading	500	1.25

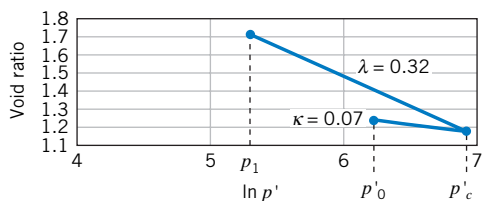


FIGURE E8.2

Strategy Make a sketch of the results in $(e, \ln p')$ space to provide a visual aid for solving this problem.

Solution 8.2

Step 1: Make a plot of $\ln p'$ versus e .

See Fig. E8.2.

Step 2: Calculate λ .

From Fig. E8.2,

$$\lambda = \frac{|\Delta e|}{\ln(p'_c/p'_1)} = \frac{|1.20 - 1.72|}{\ln\left(\frac{1000}{200}\right)} = 0.32$$

Step 3: Calculate κ .

From Fig. E8.2,

$$\kappa = \frac{|\Delta e|}{\ln(p'_c/p'_o)} = \frac{|1.20 - 1.25|}{\ln\left(\frac{1000}{500}\right)} = 0.07$$

Step 4: Calculate e_Γ .

$$e_\Gamma = e_o + (\lambda - \kappa) \ln \frac{p'_c}{2} + \kappa \ln p'_o = 1.25 + (0.32 - 0.07) \ln \frac{1000}{2} + 0.07 \ln 500 = 3.24$$

What's next... We now know the key parameters to use in the CSM. Next, we will use the CSM to predict the shear strength of soils.

8.5 FAILURE STRESSES FROM THE CRITICAL STATE MODEL



Concept Learning and Self-Assessment

Access Chapter 8, Section 8.5 on the CD to learn about failure states. Take Quiz 8.5 to test your understanding.

8.5.1 Drained Triaxial Test

Let us consider a CD test in which we isotropically consolidate a soil to a mean effective stress p'_c and unload it isotropically to a mean effective stress of p'_o (Figs. 8.10a,b) such that $R_o \leq 2$. The slope of the ESP = TSP is 3:1 as shown by OF in Fig. 8.10a. The ESP will intersect the critical state line at F . We need to find the stresses at F . The equation for the ESP is

$$q_f = 3(p'_f - p'_o) \quad (8.15)$$

The equation for the critical state line, using a generic M , which for compression is M_c and for extension is M_e , is

$$q_f = Mp'_f \quad (8.16)$$

The intersection of these two lines is found by equating Eqs. (8.15) and (8.16), which leads to

$$p'_f = \frac{3p'_o}{3 - M} \quad (8.17)$$

and

$$q_f = Mp'_f = \frac{3Mp'_o}{3 - M} \quad (8.18)$$

Let us examine Eqs. (8.17) and (8.18). If $M = M_c = 3$, then $p'_f \rightarrow \infty$ and $q_f \rightarrow \infty$. Therefore, M_c cannot have a value of 3 because soils cannot have infinite strength. If $M_c > 3$, then p'_f is negative and q_f is negative. Of course, p'_f cannot be negative because soil cannot sustain tension. Therefore, we cannot have a value of M_c greater than 3. Therefore, the region bounded by a slope $q/p' = 3$ originating from the origin and the deviatoric stress axis represents impossible soil states (Fig. 8.10a). For extension tests, the bounding slope is $q/p' = -3$. Also, you should recall from Chapter 6 that soil states to the right of the normal consolidation line are impossible (Fig. 8.10b).

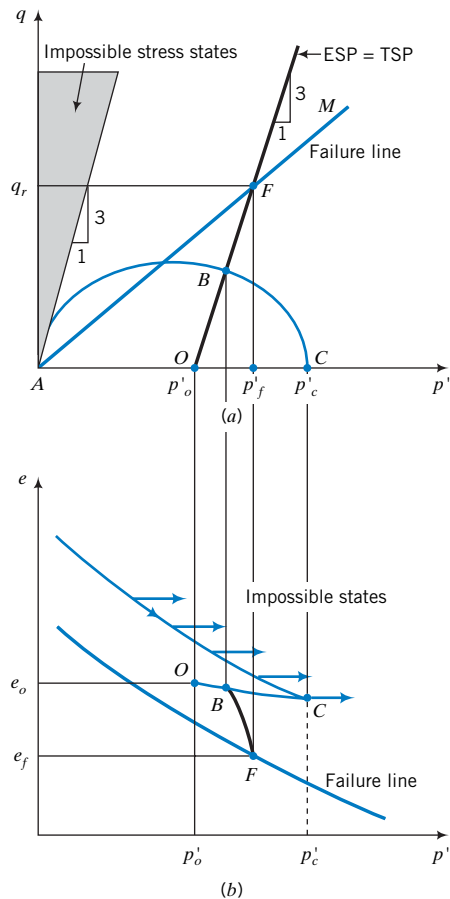


FIGURE 8.10 Failure in CD tests.

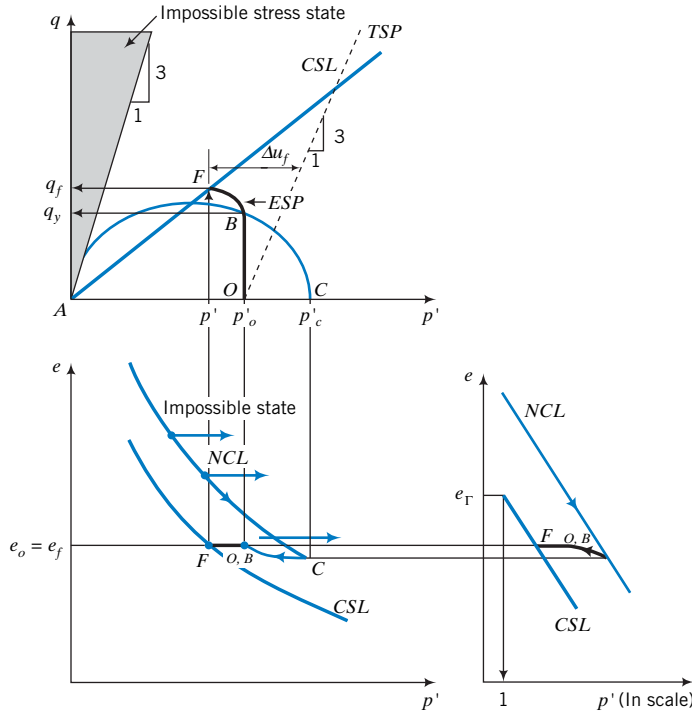


FIGURE 8.11 Failure in CU tests.

We have now delineated regions in stress space (q, p') and in void ratio versus mean effective stress space—that is, (e, p') space, that are possible for soils. Soil states cannot exist outside these regions.

8.5.2 Undrained Triaxial Test

In an undrained test, no volume change occurs—that is, $\Delta V = 0$ —which means that $\Delta \varepsilon_p = 0$ or $\Delta e = 0$ (Fig. 8.11) and, consequently,

$$e_f = e_o = e_\Gamma - \lambda \ln p'_f \quad (8.19)$$

By rearranging Eq. (8.19), we get

$$p'_f = \exp\left(\frac{e_\Gamma - e_o}{\lambda}\right) \quad (8.20)$$

Since $q_f = Mp'_f$, then

$$q_f = M \exp\left(\frac{e_\Gamma - e_o}{\lambda}\right) \quad (8.21)$$

For a CU test, the TSP has a slope of 3 (Fig. 8.11). For the elastic range of stress, the ESP is vertical ($\Delta p' = 0$) up to the yield stress and bends toward the critical state line as the porewater pressure increases considerably after yield.

The undrained shear strength, denoted by s_u , is defined as one-half the deviatoric stress at failure. That is,

$$s_u = \frac{M}{2} \exp\left(\frac{e_\Gamma - e_o}{\lambda}\right) \quad (8.22)$$

For a given soil, M , λ , and e_Γ are constants and the only variable in Eq. (8.22) is the initial void ratio. Therefore, the undrained shear strength of a particular saturated soil depends only on the initial void ratio or initial water content. You should recall that we discussed this in Chapter 7 but did not show any mathematical proof.

We can use Eq. (8.22) to compare the undrained shear strengths of two samples of the same soil tested at different void ratio or to predict the undrained shear strength of one sample if we know the undrained shear strength of the other. Consider two samples, A and B, of the same soil. The ratio of their undrained shear strength is

$$\frac{(s_u)_A}{(s_u)_B} = \frac{\left[\exp\left(\frac{e_\Gamma - e_o}{\lambda}\right)\right]_A}{\left[\exp\left(\frac{e_\Gamma - e_o}{\lambda}\right)\right]_B} = \exp\left(\frac{(e_o)_B - (e_o)_A}{\lambda}\right)$$

For a saturated soil, $e_o = wG_s$ and we can then rewrite the above equation as

$$\frac{(s_u)_A}{(s_u)_B} = \exp\left[\frac{G_s(w_B - w_A)}{\lambda}\right] \quad (8.23)$$

Let us examine the difference in undrained shear strength for a 1% difference in water content between samples A and B. We will assume that the water content of sample B is greater than sample A, that is, $(w_B - w_A)$ is positive, $\lambda = 0.15$ (a typical value for a silty clay), and $G_s = 2.7$. Putting these values into Eq. (8.23), we get

$$\frac{(s_u)_A}{(s_u)_B} = 1.20$$

That is, a 1% increase in water content causes a reduction in undrained shear strength of 20% for this soil. The implication on soil testing is that you should preserve the water content of soil samples, especially samples taken from the field, because the undrained shear strength can be significantly altered by even small changes in water content.

For highly overconsolidated clays ($R_o > 2$) or dense sands, the peak shear stress (q_p) is equal to the initial yield stress (Fig. 8.7). Recall that the CSM predicts that soils with $R_o > 2$ will behave elastically up to the peak shear stress (initial yield stress). By substituting $p' = p'_o$ and $q = q_p$ in the equation for the yield surface [Eq. (8.5)], we obtain

$$(p'_o)^2 - p'_o p'_c + \frac{q_p^2}{M^2} = 0$$

which simplifies to

$$q_p = Mp'_o \sqrt{\frac{p'_c}{p'_o} - 1} = Mp'_o \sqrt{R_o - 1}; \quad R_o > 2 \quad (8.24)$$

and

$$s_u = \frac{M}{2} p'_o \sqrt{R_o - 1}; \quad R_o > 2 \quad (8.25)$$

The excess porewater pressure at failure is found from the difference between the mean total stress and the corresponding mean effective stress at failure; that is,

$$\Delta u_f = p_f - p'_f$$

From the TSP,

$$p_f = p'_o + \frac{q_f}{3}$$

Therefore,

$$\Delta u_f = p'_o + \left(\frac{M}{3} - 1 \right) \exp\left(\frac{e_\Gamma - e_o}{\lambda} \right) \quad (8.26)$$

The essential points are:

1. The intersection of the ESP and the critical state line gives the failure stresses.
2. The undrained shear strength depends only on the initial void ratio.
3. Small changes in water content can significantly alter the undrained shear strength.

EXAMPLE 8.3 Predicting Yield and Failure Stresses and Excess porewater Pressures

Two specimens, A and B, of a clay were each isotropically consolidated under a cell pressure of 300 kPa and then unloaded isotropically to a mean effective stress of 200 kPa. A CD test is to be conducted on specimen A and a CU test is to be conducted on specimen B. Estimate, for each specimen, (a) the yield stresses, p'_y , q_y , $(\sigma'_1)_y$, and $(\sigma'_3)_y$; and (b) the failure stresses p'_f , q'_f , $(\sigma'_1)_f$, and $(\sigma'_3)_f$. Estimate for sample B the excess porewater pressure at yield and at failure. The soil parameters are $\lambda = 0.3$, $\kappa = 0.05$, $e_o = 1.10$, and $\phi'_{cs} = 30^\circ$. The cell pressure was kept constant at 200 kPa.

Strategy Both specimens have the same consolidation history but are tested under different drainage conditions. The yield stresses can be found from the intersection of the ESP and the initial yield surface. The initial yield surface is known since $p'_c = 300$ kPa, and M can be found from ϕ'_{cs} . The failure stresses can be obtained from the intersection of the ESP and the critical state line. It is always a good habit to sketch the q versus p' and the e versus p' graphs to help you solve problems using the critical state model. You can also find the yield and failure stresses using graphical methods as described in the alternative solution.

Solution 8.3

Step 1: Calculate M_c .

$$M_c = \frac{6 \sin 30^\circ}{3 - \sin 30^\circ} = 1.2$$

Step 2: Calculate e_Γ .

From Eq. (8.14), with $p'_o = 200$ kPa and $p'_c = 300$ kPa,

$$e_\Gamma = e_o + (\lambda - \kappa) \ln \frac{p'_c}{p'_o} + \kappa \ln p'_o = 1.10 + (0.3 - 0.05) \ln \frac{300}{200} + 0.05 \ln 200 = 2.62$$

Step 3: Make a sketch or draw scaled plots of q versus p' and e versus p' .

See Figs. E8.3a,b.

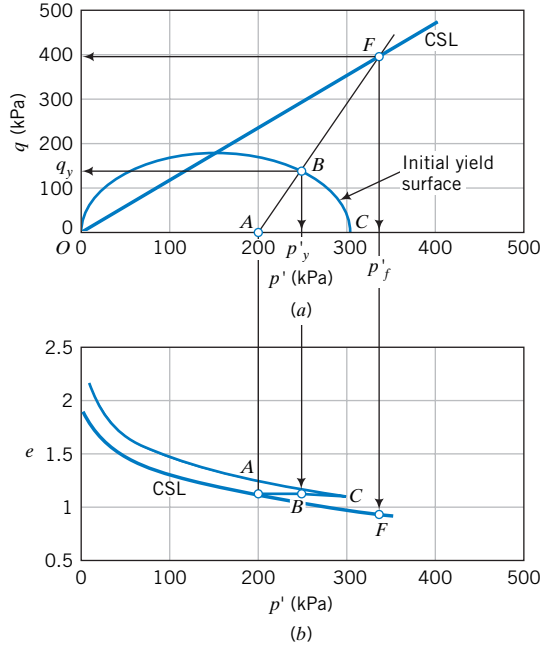


FIGURE E8.3a,b

Step 4: Find the yield stresses.

Drained Test Let p'_y and q_y be the yield stress (point B in Fig. E8.3a). From the equation for the yield surface [Eq. (8.5)],

$$(p'_y)^2 - 300p'_y + \frac{q_y^2}{(1.2)^2} = 0 \quad (1)$$

From the ESP,

$$q_y = 3(p'_y - p'_o) = 3p'_y - 600 \quad (2)$$

Solving Eqs. (1) and (2) for p'_y and q_y gives two solutions: $p'_y = 140.1$ kPa, $q_y = -179.6$ kPa and $p'_y = 246.1$ kPa, $q_y = 138.2$ kPa. Of course, $q_y = -179.6$ kPa is not possible because we are conducting a compression test. The yield stresses are then $p'_y = 246.1$ kPa, $q_y = 138.2$ kPa.

Now,

$$q_y = (\sigma'_1)_y - (\sigma'_3)_y = 138.2 \text{ kPa}; \quad (\sigma'_3)_f = 200 \text{ kPa}$$

Solving for $(\sigma'_1)_f$ gives

$$(\sigma'_1)_f = 138.2 + 200 = 338.2 \text{ kPa}$$

Undrained Test The ESP for the undrained test is vertical for the region of stress paths below the yield stress, that is, $\Delta p' = 0$. From the yield surface [Eq. (8.4)] for $p' = p'_y = p'_o$, we get

$$200^2 - 200 \times 300 + \frac{q_y^2}{1.2^2} = 0$$

$$\therefore q_y^2 = 1.2^2 \times 200 \times 100$$

and

$$q_y = 169.7 \text{ kPa}$$

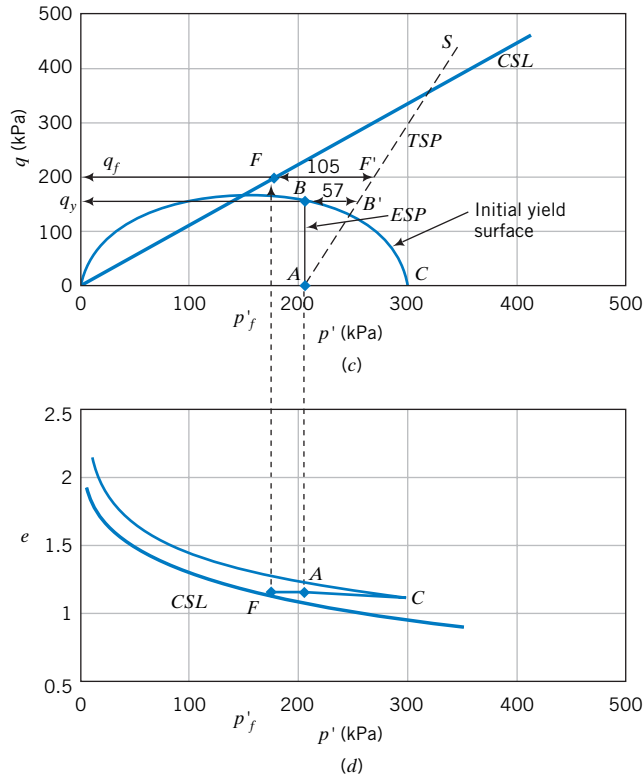


FIGURE E8.3c,d

From the TSP, we can find p_y (B' , Fig. E8.3c)

$$p_y = p'_o + \frac{q_y}{3} = 200 + \frac{169.7}{3} = 256.6 \text{ kPa}$$

The excess porewater pressure at yield is

$$\Delta u_y = p_y - p'_y = p_y - p'_o = 256.6 - 200 = 56.6 \text{ kPa}$$

Now

$$p'_y = p'_o = \frac{(\sigma'_1)_y + 2(\sigma'_3)_y}{3} = 200 \text{ kPa}$$

$$q_y = (\sigma'_1)_y - \sigma'_3 = 169.7 \text{ kPa}$$

Solving for $(\sigma'_1)_y$ and $(\sigma'_3)_y$ gives

$$(\sigma'_1)_y = 313.3 \text{ kPa}; \quad (\sigma'_3)_y = 143.4 \text{ kPa}$$

Check

$$(\sigma_3)_y = (\sigma'_3)_y + \Delta u_y = 143.4 + 56.6 = 200 \text{ kPa}$$

Step 5: Find the failure stresses.

Drained Test

$$\text{Equation (8.17): } p'_f = \frac{3 \times 200}{3 - 1.2} = 333.3 \text{ kPa}$$

$$\text{Equation (8.6): } q_f = 1.2 \times 333.3 = 400 \text{ kPa}$$

Now,

$$q_F = (\sigma'_1)_f - (\sigma'_3)_f = 400 \text{ kPa} \quad \text{and} \quad (\sigma'_3)_f = 200 \text{ kPa}$$

Solving for $(\sigma'_1)_f$, we get

$$(\sigma'_1)_f = 400 + 200 = 600 \text{ kPa}$$

Undrained Test

$$\text{Equation (8.20) : } p'_f = \exp\left(\frac{2.62 - 1.10}{0.3}\right) = 158.6 \text{ kPa}$$

$$\text{Equation (8.6) : } q_f = 1.2 \times 158.6 = 190.3 \text{ kPa}$$

Now,

$$p'_f = \frac{(\sigma'_1)_f + 2(\sigma'_3)_f}{3} = 158.6 \text{ kPa}$$

$$q_f = (\sigma'_1)_f - (\sigma'_3)_f = 190.4 \text{ kPa}$$

Solving for $(\sigma'_1)_f$ and $(\sigma'_3)_f$, we find

$$(\sigma'_1)_f = 285.5 \text{ kPa} \quad \text{and} \quad (\sigma'_3)_f = 95.1 \text{ kPa}$$

We can find the change in porewater pressure at failure from either Eq. (8.25)

$$\Delta u_f = 200 + \left(\frac{1.2}{3} - 1\right) \exp\left(\frac{2.62 - 1.10}{0.3}\right) = 104.9 \text{ kPa}$$

or

$$\Delta u_f = \sigma_3 - (\sigma'_3)_f = 200 - 95.1 = 104.9 \text{ kPa}$$

Graphical Method We need to find the equations for the normal consolidation line and the critical state lines.

Normal Consolidation Line Void ratio at preconsolidated mean effective stress:

$$e_c = e_o - \kappa \ln \frac{p'_c}{p'_o} = 1.10 - 0.05 \ln \frac{300}{200} = 1.08$$

Void ratio at $p' = 1 \text{ kPa}$ on NCL:

$$e_n = e_c + \lambda \ln p'_c = 1.08 + 0.3 \ln 300 = 2.79$$

The equation for the normal consolidation line is then

$$e = 2.79 - 0.3 \ln p'$$

The equation for the unloading/reloading line is

$$e = 1.08 + 0.05 \ln \frac{p'_c}{p'}$$

The equation for the critical state line in (e, p') space is

$$e = 2.62 - 0.3 \ln p'$$

Now you can plot the normal consolidation line, the unloading/reloading line, and the critical state line as shown in Fig. E8.3b.

Plot Initial Yield Surface The yield surface is

$$(p')^2 - 300p' + \frac{q^2}{(1.2)^2} = 0$$

$$\therefore q = \pm 1.2p' \sqrt{\frac{300}{p'} - 1}$$

For $p' = 0$ to 300, plot the initial yield surface as shown in Fig. E8.3a.

Plot Critical State Line The critical state line is

$$q = 1.2p'$$

and is plotted as OF in Fig. E8.3a.

Drained Test The ESP for the drained test is

$$p' = 200 + \frac{q}{3}$$

and is plotted as AF in Fig. E8.3a. The ESP intersects the initial yield surface at B and the yield stresses are $p'_y = 240$ kPa and $q_y = 138$ kPa. The ESP intersects the critical state line at F and the failure stresses are $p'_f = 333$ kPa and $q_f = 400$ kPa.

Undrained Test For the undrained test, the initial void ratio and the final void ratio are equal. Draw a horizontal line from A to intersect the critical state line in (e, p') space at F (Fig. E8.3d). Project a vertical line from F to intersect the critical state line in (q, p') space at F (Fig. E8.3c). The failure stresses are $p'_f = 159$ kPa and $q_f = 190$ kPa. Draw the TSP as shown by AS in Fig. E8.3c. The ESP within the elastic region is vertical as shown by AB . The yield stresses are $p'_y = 200$ kPa and $q_y = 170$ kPa. The excess porewater pressures are:

$$\begin{aligned} \text{At yield—horizontal line } BB' : \quad \Delta u_y &= 57 \text{ kPa} \\ \text{At failure—horizontal line } FF' : \quad \Delta u_f &= 105 \text{ kPa} \end{aligned}$$

EXAMPLE 8.4 Predicting s_u in Compression and Extension Tests

Determine the undrained shear strength in (a) a CU compression test and (b) a CU extension test for a soil with $R_o = 5$, $p'_o = 70$ kPa, and $\phi'_{cs} = 25^\circ$.

Strategy Since you are given ϕ'_{cs} , you should use Eqs. (8.7) and (8.9) to find M_c and M_e . Use Eq. (8.25) to solve the problem.

Solution 8.4

Step 1: Calculate M_c and M_e .

$$M_c = \frac{6 \sin \phi'_{cs}}{3 - \sin \phi'_{cs}} = \frac{6 \sin 25^\circ}{3 - \sin 25^\circ} = 0.98$$

$$M_e = \frac{6 \sin \phi'_{cs}}{3 + \sin \phi'_{cs}} = 0.74$$

Step 2: Calculate s_u .

Use Eq. (8.25).

$$\text{Compression : } s_u = \frac{0.98}{2} \times 70\sqrt{5-1} = 68.6 \text{ kPa}$$

$$\text{Extension : } s_u = \frac{0.74}{2} \times 70\sqrt{5-1} = 51.8 \text{ kPa}$$

Or, by proportion,

$$\text{Extension : } s_u = \frac{0.74}{0.98} \times 68.6 = 51.8 \text{ kPa}$$

EXAMPLE 8.5 Effects of Change of Water Content on s_u

The in situ water content of a soil sample is 48%. The water content decreases to 44% due to transportation of the sample to the laboratory and during sample preparation. What difference in undrained shear strength could be expected if $\lambda = 0.13$ and $G_s = 2.7$?

Strategy The solution to this problem is a straightforward application of Eq. (8.22).

Solution 8.5

Step 1: Determine the difference in s_u values.

Use Eq. (8.23).

$$\frac{(s_u)_{\text{lab}}}{(s_u)_{\text{field}}} = \exp \left[\frac{G_s(w_{\text{field}} - w_{\text{lab}})}{\lambda} \right] = \exp \left(\frac{2.7(0.48 - 0.44)}{0.13} \right) = 2.3$$

The laboratory undrained shear strength would probably show an increase over the in situ undrained shear strength by a factor greater than 2.

What's next... We have discussed methods to calculate the failure stresses. But failure stresses are only one of the technical criteria in the analysis of soil behavior. We also need to know the deformations or strains. But before we can get the strains from the stresses we need to know the elastic, shear, and bulk moduli. In the next section, we will use the CSM to determine these moduli.

8.6 PRACTICAL IMPLICATIONS



Concept Learning and Self-Assessment

Access Chapter 8, Section 8.6 to learn about the practical implications of the critical state soil model. Take Quiz 8.6 to test your understanding.

We considered a particular shape of the yield surface in developing the concept of critical state model. Other shapes of yield surfaces have been developed and used in predicting soil responses. For example, Schofield and Wroth (1968) developed a bullet-shaped yield surface using energy methods. For normally consolidated and lightly overconsolidated soils ($R_o < 2$), there is no peak shear strength. For overconsolidated soils ($R_o > 2$), the peak shear strength occurs on the initial yield surface, which in our case is an ellipse. Experimental data presented by Hvorslev (1937) reveal that the locus of peak shear

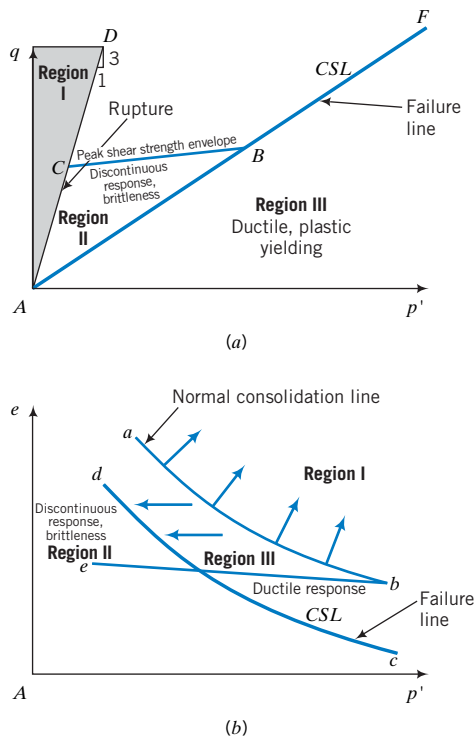


FIGURE 8.12 Regional description and expected responses of soil.

strength is approximately a straight line. We are going to use this approximation to interpret soil behavior for practical purposes as proposed by Schofield (2001).

In Figs. 8.12a and b, AD and ab are lines that separate possible from impossible states; ab is also the normal consolidation line; AF and cd are the critical state or failure lines; CB is Hvorslev's peak shear strength envelope; line be is the unloading/reloading line. We are going to describe three regions of soil behavior similar to Section 7.7.

Region I. Soil stress states within Region I are impossible.

Region II. Soil states within Region II will cause the soil to exhibit discontinuous responses and behave in a brittle manner (sudden failure or collapse). Soil strengths within this region are generally higher than Region III and deformations are lower. However, the higher shear strengths and lower deformations come with a price. In Region II, the soil tends to expand, but this expansion is not uniform. The implication is that some sections of the soil are looser (shear bands) than others. The permeability of the soil (especially along the shear bands) increases and if water is present this increased permeability can lead to sudden failure due to seepage (for example, flow slides on a slope). Loads that bring the soil to stress states within Region II present a high safety risk and should be avoided unless experience dictates otherwise. Soil states on the line CB are stress states of impending collapse. Soil states on line AC will cause the soil to rupture.

Region III. Soil stress states within Region III will cause the soil to behave in a ductile manner. It is desirable to design soil systems such that the soil will behave ductile under anticipated (design) loadings. Soil states on the CSL are failure states. Your design loads should not bring the soil state to failure under any anticipated loading condition.

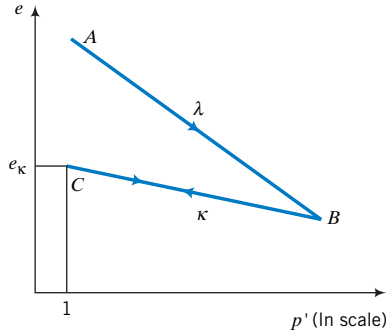


FIGURE 8.13 Loading and unloading/reloading (elastic) response of soils in $(e-p')$ space.

8.7 SOIL STIFFNESS



Concept Learning and Self-Assessment

Access Chapter 8, Section 8.7 on the CD to learn about the importance of soil stiffness and how to use the critical state concept to calculate the elastic shear and bulk moduli. Take Quiz 8.7 to test your understanding.

The elastic modulus, E' , or the shear modulus, G , and the bulk modulus, K' , characterize soil stiffness. In practice, E' or G , and K' are commonly obtained from triaxial or simple shear tests. We can obtain an estimate of E' or G and K' using the critical state model and results from axisymmetric, isotropic consolidation tests. The void ratio during unloading/reloading is described by

$$e = e_{\kappa} - \kappa \ln p' \quad (8.27)$$

where e_{κ} is the void ratio on the unloading/reloading line at $p' = 1$ unit of stress (Fig. 8.13). The unloading/reloading path BC (Fig. 8.13) is reversible, which is a characteristic of elastic materials. Differentiating Eq. (8.27) gives

$$de = -\kappa \frac{dp'}{p'} \quad (8.28)$$

The elastic volumetric strain increment is

$$d\varepsilon_p^e = -\frac{de}{1 + e_o} = \frac{\kappa}{1 + e_o} \frac{dp'}{p'} \quad (8.29)$$

But, from Eq. (5.109),

$$d\varepsilon_p^e = \frac{dp'}{K'}$$

Therefore,

$$\frac{dp'}{K'} = \frac{\kappa}{1 + e_o} \frac{dp'}{p'}$$

Solving for K' , we obtain

$$K' = \frac{p'(1 + e_o)}{\kappa} \quad (8.30)$$

From Eq. (5.110),

$$E' = 3K'(1 - 2\nu')$$

Therefore,

$$E' = \frac{3p'(1 + e_0)(1 - 2\nu')}{\kappa} \quad (8.31)$$

Also, from Eq. (5.112)

$$G = \frac{E'}{2(1 + \nu')}$$

Therefore,

$$G = \frac{3p'(1 + e_0)(1 - 2\nu')}{2\kappa(1 + \nu')} = \frac{1.5p'(1 + e_0)(1 - 2\nu')}{\kappa(1 + \nu')} \quad (8.32)$$

Equations (8.31) and (8.32) indicate that the elastic constants, E' and G , are proportional to the mean effective stress. This implies nonlinear elastic behavior and therefore calculations must be carried out incrementally. For overconsolidated soils, Eqs. (8.31) and (8.32) provide useful estimates of E' and G from conducting an isotropic consolidation test, which is a relatively simple soil test. However, it was argued (Zytynski et al., 1978) that relating E' , G and K' to the in situ stress state leads to unrealistically low values. Randolph et al. (1979) recommended that G should be related to the historically maximum value of K' , such that

$$K'_{\max} = \frac{1 + e_c}{\kappa} p'_c \quad (8.33)$$

$$G = 0.5 K'_{\max} \quad (8.34)$$

where p'_c is preconsolidated mean effective stress and e_c is the void ratio corresponding to p'_c .

Soil stiffness is influenced by the amount of shear strains applied. Increases in shear strains tend to lead to decreases in G and E' while increases in volumetric strains lead to decreases in K' . The net effect is that the soil stiffness decreases with increasing strains.

It is customary to identify three regions of soil stiffness based on the level of applied shear strains. At small shear strains (γ or ϵ_q usually $< 0.001\%$), the soil stiffness is approximately constant (Fig. 8.14)

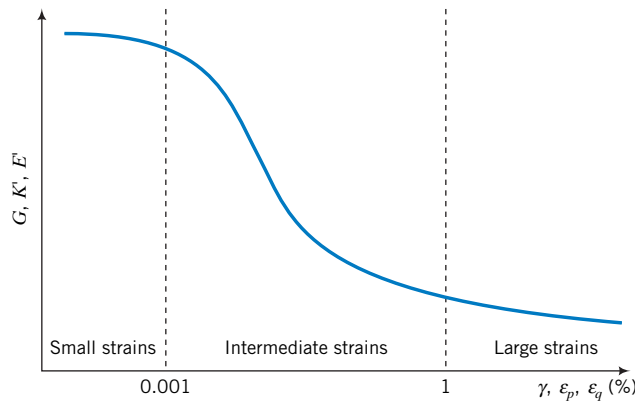


FIGURE 8.14 Schematic variation of shear, bulk, and Young's elastic moduli with strain levels.

and the soil behaves like a linearly elastic material. At intermediate shear strains between 0.001% and 1%, the soil stiffness decreases significantly and the soil behavior is elastoplastic (nonlinear). At large strains ($\gamma > 1\%$), the soil stiffness decreases slowly to an approximately constant value as the soil approaches critical state. At the critical state, the soil behaves like a viscous fluid.

In practical problems, the shear strains are in the intermediate range, typically $\gamma < 0.1\%$. However, the shear strain distribution within the soil is not uniform. The shear strains decrease with distance away from a structure and local shear strains near the edge of a foundation slab, for example, can be much greater than 0.1%. The implication of a nonuniform shear strain distribution is that the soil stiffness varies within the loaded region of the soil. Consequently, large settlements and failures are usually initiated in the loaded soil region where the soil stiffness is the lowest.

In conventional laboratory tests, it is not practical to determine the soil stiffness at shear strains less than 0.001% because of inaccuracies in the measurement of the soil displacements due to displacements of the apparatuses themselves and to resolution and inaccuracies of measuring instruments. The soil stiffness at small strains is best determined in the field using wave propagation techniques. In one such technique, vibrations are created at the soil surface or at a prescribed depth in the soil, and the shear wave velocity (v_{sh}) is measured. The shear modulus at small strains is calculated from

$$G = \frac{\gamma(v_{sh})^2}{g} \quad (8.35)$$

where γ is the bulk unit weight of the soil, and g is the acceleration due to gravity. In the laboratory, the shear modulus at small strains can be determined using a resonance column test (Drnevick, 1967). The resonance column test utilizes a hollow cylinder apparatus (Chapter 7) to induce resonance of the soil sample. Resonance column tests show that G depends not only on the level of shear strain but also on void ratio, overconsolidation ratio, and mean effective stress. Various empirical relationships have been proposed linking G to e , overconsolidation ratio, and p' . Two such relationships are presented below.

Jamiolkowski et al. (1991) for Clays

$$G = \frac{198}{e^{1.3}} (R_o)^a \sqrt{p'} \text{ MPa} \quad (8.36)$$

where G is the initial shear modulus, p' is the mean effective stress (MPa), and a is a coefficient that depends on the plasticity index as follows:

PI (%)	a
0	0
20	0.18
40	0.30
60	0.41
80	0.48
≥ 100	0.50

Seed and Idriss (1970) for sands

$$G = k_1 \sqrt{p'} \text{ MPa} \quad (8.37)$$

e	k_1	D_r (%)	k_1
0.4	484	30	235
0.5	415	40	277
0.6	353	45	298
0.7	304	60	360
0.8	270	75	408
0.9	235	90	484

What's next... Now that we know how to calculate the shear and bulk moduli, we can move on to determine strains, which we will consider next.

8.8 STRAINS FROM THE CRITICAL STATE MODEL



Concept Learning and Self-Assessment

Access Chapter 8, Section 8.8 on the CD to learn about strains from the critical state soil model. Take Quiz 8.8 to test your understanding.

8.8.1 Volumetric Strains

The total change in volumetric strains consists of two parts: the recoverable part (elastic) and the unrecoverable part (plastic). We can write an expression for the total change in volumetric strain as

$$\Delta \varepsilon_p = \Delta \varepsilon_p^e + \Delta \varepsilon_p^p \quad (8.38)$$

where the superscripts e and p denote elastic and plastic, respectively. Let us consider a soil sample that is isotropically consolidated to a mean effective stress p'_c and unloaded to a mean effective stress p'_o as represented by ACO in Figs. 8.15a,b. In a CD test, the soil will yield at D . Let us now consider a small increment of stress, DE , which causes the yield surface to expand as shown in Fig. 8.15a.

The change in void ratio for this stress increment is $\Delta e = |e_E - e_D|$ (Fig. 8.15b).

$$e_D = e_C + \kappa \ln \frac{p'_C}{p'_D}$$

$$e_E = e_G + \kappa \ln \frac{p'_G}{p'_E}$$

$$\text{but } e_G = e_C - \lambda \ln \frac{p'_G}{p'_C}$$

Therefore

$$|e_E - e_D| = \Delta e = (\lambda - \kappa) \ln \left(\frac{p'_G}{p'_C} \right) + \kappa \ln \left(\frac{p'_E}{p'_D} \right)$$

The corresponding total change in volumetric strain is

$$\Delta \varepsilon_p = \frac{\Delta e}{1 + e_o} = \frac{1}{1 + e_o} \left\{ (\lambda - \kappa) \ln \frac{p'_G}{p'_C} + \kappa \ln \frac{p'_E}{p'_D} \right\} \quad (8.39)$$

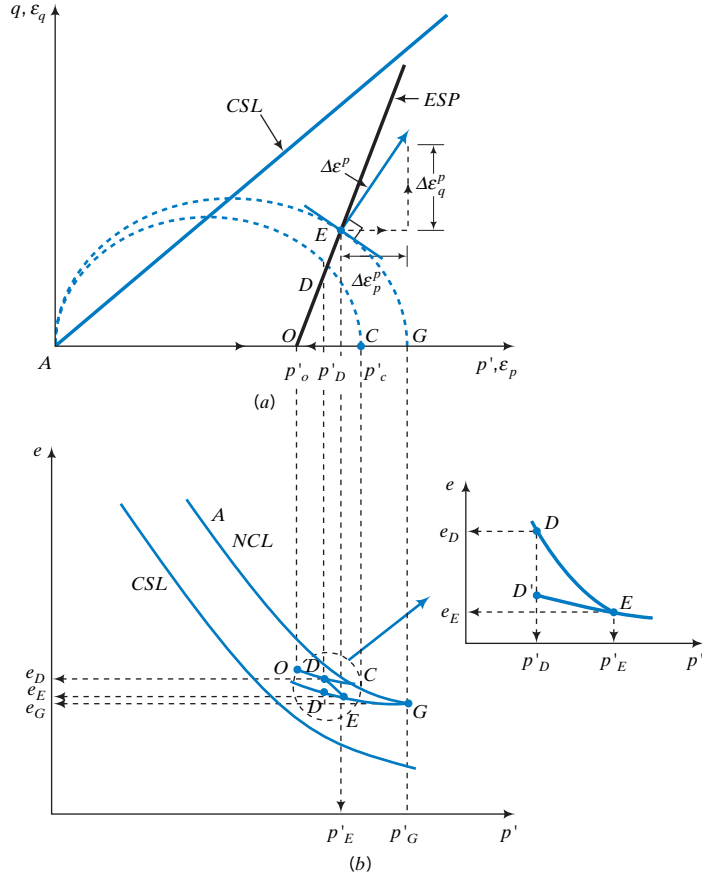


FIGURE 8.15 Determination of plastic strains.

The volumetric elastic strain component is represented by ED' . That is, if you were to unload the soil from E back to its previous stress state at D , the rebound will occur along an unloading/reloading line associated with the maximum mean effective stress for the yield surface on which unloading starts. The elastic change in volumetric strain from E to D' is

$$\Delta \epsilon_p^e = \frac{\Delta e}{1 + e_o} = \frac{|e_{D'} - e_E|}{1 + e_o} = \frac{\kappa}{1 + e_o} \ln \frac{p'_E}{p'_D} \quad (8.40)$$

We get a positive value of $\Delta \epsilon_p^e$ because rather than computing the rebound (expansion) from E to D' , we compute the compression from D' to E .

The volumetric elastic strains can also be computed from Eq. (5.109); that is,

$$\Delta \epsilon_p^e = \frac{\Delta p'}{K'} \quad (8.41)$$

The change in volumetric plastic strain is

$$\Delta \epsilon_p^p = \Delta \epsilon_p - \Delta \epsilon_p^e = \left(\frac{\lambda - \kappa}{1 + e_o} \right) \ln \frac{p'_G}{p'_C} \quad (8.42)$$

Under undrained conditions, the total volumetric change is zero. Consequently, from Eq. (8.38),

$$\Delta \varepsilon_p^e = -\Delta \varepsilon_p^p \quad (8.43)$$

8.8.2 Shear Strains

Let the yield surface be represented by

$$F = (p')^2 - p'p'_c + \frac{q^2}{M^2} = 0 \quad (8.44)$$

To find the shear or deviatoric strains, we will assume that the resultant plastic strain increment, $\Delta \varepsilon_p^p$, for an increment of stress is normal to the yield surface (Fig. 8.15a). Normally, the plastic strain increment should be normal to a plastic potential function but we are assuming here that the plastic potential function and the yield surface (yield function, F) are the same. A plastic potential function is a scalar quantity that defines a vector in terms of its location in space. Classical plasticity demands that the surfaces defined by the yield and plastic potential coincide. If they do not, then basic work restrictions are violated. However, advanced soil mechanics theories often use different surfaces for yield and potential functions to obtain more realistic stress-strain relationships. The resultant plastic strain increment has two components—a deviatoric or shear component, $\Delta \varepsilon_q^p$, and a volumetric component, $\Delta \varepsilon_p^p$, as shown in Fig. 8.15. We already found $\Delta \varepsilon_p^p$ in the previous section.

Since we know the equation for the yield surface [Eq. (8.44)], we can find the normal to it by differentiation of the yield function with respect to p' and q . The tangent or slope of the yield surface is

$$dF = 2p'dp' - p'_c dp' + 2q \frac{dq}{M^2} = 0 \quad (8.45)$$

Rearranging Eq. (8.45), we obtain the slope as

$$\frac{dq}{dp'} = \left(\frac{p'_c/2 - p'}{q/M^2} \right) \quad (8.46)$$

The normal to the yield surface is

$$-\frac{1}{dq/dp'} = -\frac{dp'}{dq}$$

From Fig. 8.15a, the normal, in terms of plastic strains, is $d\varepsilon_q^p/d\varepsilon_p^p$. Therefore,

$$\frac{d\varepsilon_q^p}{d\varepsilon_p^p} = -\frac{dp'}{dq} = -\frac{q/M^2}{p'_c/2 - p'} \quad (8.47)$$

which leads to

$$d\varepsilon_q^p = d\varepsilon_p^p \frac{q}{M^2(p' - p'_c/2)} \quad (8.48)$$

The elastic shear strains can be obtained from Eq. (5.111); that is,

$$\Delta \varepsilon_q^e = \frac{1}{3G} \Delta q \quad (8.49)$$

These equations for strains are valid only for small changes in stress. For example, you cannot use these equations to calculate the failure strains by simply substituting the failure stresses for p' and q .

You have to calculate the strains for small increments of stresses up to failure and then sum each component of strain separately. We need to do this because the critical state model considers soils as elastic–plastic materials and not linearly elastic materials.

EXAMPLE 8.6 Calculation of Strains

A sample of clay was isotropically consolidated to a mean effective stress of 225 kPa and was then unloaded to a mean effective stress of 150 kPa at which stress $e_o = 1.4$. A CD test is to be conducted. Calculate (a) the elastic strains at initial yield and (b) the total volumetric and deviatoric strains for an increase of deviatoric stress of 12 kPa after initial yield. For this clay, $\lambda = 0.16$, $\kappa = 0.05$, $\phi'_{cs} = 25.5^\circ$, and $\nu' = 0.3$.

Strategy It is best to sketch diagrams similar to Fig. 8.4 to help you visualize the solution to this problem. Remember that the strains within the yield surface are elastic.

Solution 8.6

Step 1: Calculate initial stresses and M_c .

$$p'_c = 225 \text{ kPa}, p'_o = 150 \text{ kPa}$$

$$R_o = \frac{225}{150} = 1.5$$

$$M_c = \frac{6 \sin \phi'_{cs}}{3 - \sin \phi'_{cs}} = \frac{6 \sin 25.5^\circ}{3 - \sin 25.5^\circ} = 1$$

Step 2: Determine the initial yield stresses.

The yield stresses are the stresses at the intersection of the initial yield surface and the effective stress path.

$$\text{Equation for the yield surface: } (p')^2 - p'p'_c + \frac{q^2}{M_c^2} = 0$$

$$\text{Equation of the ESP: } p' = p'_o + \frac{q}{3}$$

$$\text{At the initial yield point } D \text{ (Fig. 8.5): } p'_y = p'_o + \frac{q_y}{3} = 150 + \frac{q_y}{3}$$

Substituting $p' = p'_y$, $q = q_y$, and the values for M_c and p'_c into the equation for the initial yield surface [Eq. (8.5)] gives

$$\left(150 + \frac{q_y}{3}\right)^2 - \left(150 + \frac{q_y}{3}\right)225 + \frac{q_y^2}{1^2} = 0$$

Simplification results in

$$q_y^2 + 22.5q_y - 10125 = 0$$

The solution for q_y is $q_y = 90 \text{ kPa}$ or $q_y = -112.5 \text{ kPa}$. The correct answer is $q_y = 90 \text{ kPa}$ since we are applying compression to the soil sample. Therefore,

$$p'_y = 150 + \frac{q_y}{3} = 150 + \frac{90}{3} = 180 \text{ kPa}$$

Step 3: Calculate the elastic strains at initial yield.

Elastic volumetric strains

$$\text{Elastic volumetric strains: } \Delta \epsilon_p^e = \frac{\kappa}{1 + e_o} \ln \frac{p'_y}{p'_o} = \frac{0.05}{1 + 1.4} \ln \frac{180}{150} = 38 \times 10^{-4}$$

Alternatively, you can use Eq. (8.41). Take the average value of p' from p'_o to p'_y to calculate K' .

$$p'_{av} = \frac{p'_o + p'_y}{2} = \frac{150 + 180}{2} = 165 \text{ kPa}$$

$$K' = \frac{p'(1 + e_o)}{\kappa} = \frac{165(1 + 1.4)}{0.05} = 7920 \text{ kPa}$$

$$\Delta \epsilon_p^e = \frac{\Delta p'}{K'} = \frac{180 - 150}{7920} = 38 \times 10^{-4}$$

Elastic shear strains

$$G = \frac{3p'(1 + e_o)(1 - 2\nu')}{2\kappa(1 + \nu')} = \frac{3 \times 165(1 + 1.4)(1 - 2 \times 0.3)}{2 \times 0.05(1 + 0.3)} = 3655 \text{ kPa}$$

$$\Delta \epsilon_q^e = \frac{\Delta q}{3G} = \frac{90}{3 \times 3655} = 82 \times 10^{-4}$$

Alternatively:

Void ratio corresponding to p'_c is:

$$e_c = e_o + \kappa \ln \frac{p'_c}{p'_o} = 1.4 + 0.05 \ln \frac{225}{150} = 1.42$$

$$\text{Eq. (8.33): } K'_{\max} = \frac{1 + e_c}{\kappa} p'_c = \frac{1 + 1.42}{0.05} \times 225 = 10890 \text{ kPa}$$

$$\text{Eq. (8.34): } G = 0.5K'_{\max} = 5445 \text{ kPa}$$

The elastic shear strains computed using Eq. (8.34) would be $\left(\frac{5445 - 3655}{3655}\right) \times 100 = 33\%$ less than using Eq. (8.32). For high OCR (>2), Eq. (8.34) would give more realistic values of G than Eq. (8.32). For this problem, we will continue to use Eq. (8.32).

Step 4: Determine expanded yield surface.

$$\text{After initial yield: } \Delta q = 12 \text{ kPa}$$

$$\therefore \Delta p' = \frac{\Delta q}{3} = \frac{12}{3} = 4 \text{ kPa}$$

The stresses at E (Fig. 8.5) are $p'_E = p'_y + \Delta p = 180 + 4 = 184 \text{ kPa}$, and

$$q_E = q_y + \Delta q = 90 + 12 = 102 \text{ kPa}$$

The preconsolidated mean effective stress (major axis) of the expanded yield surface is obtained by substituting $p'_E = 184 \text{ kPa}$ and $q_E = 102 \text{ kPa}$ in the equation for the yield surface [Eq. (8.5)]:

$$(184)^2 - 184(p'_c)_E + \frac{102^2}{1^2} = 0$$

$$\therefore (p'_c)_E = 240.5 \text{ kPa}$$

Step 5: Calculate strain increments after yield.

$$\text{Eq. (8.39): } \Delta \varepsilon_p = \frac{1}{1 + e_o} \left\{ (\lambda - \kappa) \ln \frac{p'_G}{p'_C} + \kappa \ln \frac{p'_E}{p'_D} \right\} = \frac{1}{1 + 1.4} \left\{ (0.16 - 0.05) \ln \frac{240.5}{225} + 0.05 \ln \frac{184}{180} \right\} = 35 \times 10^{-4}$$

$$\text{Eq. (8.42): } \Delta \varepsilon_p^p = \frac{\lambda - \kappa}{1 + e_o} \ln \frac{p'_G}{p'_C} = \frac{0.16 - 0.05}{1 + 1.4} \ln \frac{240.5}{225} = 31 \times 10^{-4}$$

$$\begin{aligned} \text{Eq. (8.47): } \Delta \varepsilon_q^p &= \Delta \varepsilon_p^p \frac{q_E}{M_c^2 [p'_E - (p'_C)_E/2]} = 31 \times 10^{-4} \frac{102}{1^2 (184 - 240.5/2)} \\ &= 50 \times 10^{-4} \end{aligned}$$

Assuming that G remains constant, we can calculate the elastic shear strain from

$$\text{Eq. (8.49): } \Delta \varepsilon_q^e = \frac{\Delta q}{3G} = \frac{12}{3 \times 3655} = 11 \times 10^{-4}$$

Step 6: Calculate total strains.

$$\text{Total volumetric strains: } \varepsilon_q = \Delta \varepsilon_q^e + \Delta \varepsilon_p = (38 + 35)10^{-4} = 73 \times 10^{-4}$$

$$\text{Total shear strains: } \varepsilon_q = \Delta \varepsilon_q^e + \Delta \varepsilon_q^p = [(82 + 11) + 50]10^{-4} = 173 \times 10^{-4}$$

EXAMPLE 8.7 Expansion of Yield Surface

Show that the yield surface in an undrained test increases such that

$$p'_c = (p'_c)_{\text{prev}} \left(\frac{p'_{\text{prev}}}{p'} \right)^{\kappa/(\lambda - \kappa)}$$

where p'_c is the current value of the major axis of the yield surface, $(p'_c)_{\text{prev}}$ is the previous value of the major axis of the yield surface, p'_{prev} is the previous value of mean effective stress, and p' is the current value of mean effective stress.

Strategy Sketch an e versus $\ln p'$ diagram and then use it to prove the equation given.

Solution 8.7

Step 1: Sketch an e versus $\ln p'$ diagram.

See Fig. E8.7.

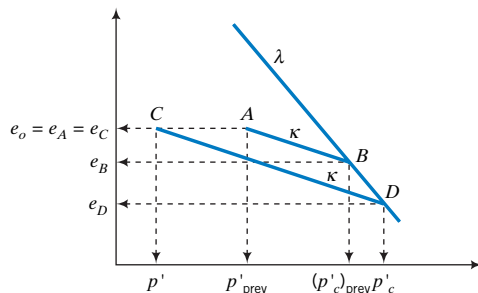


FIGURE E8.7

Step 2: Prove the equation.

Line AB

$$|e_B - e_A| = \kappa \ln \left(\frac{(p'_c)_{\text{prev}}}{p'_{\text{prev}}} \right) \quad (1)$$

Line CD

$$|e_D - e_C| = \kappa \ln \frac{p'_c}{p'} \quad (2)$$

Subtracting Eq. (2) from Eq. (1), noting that $e_A = e_C$, we obtain

$$|e_D - e_B| = \kappa \ln \left\{ \frac{(p'_c)_{\text{prev}}}{p'_{\text{prev}}} \right\} - \kappa \ln \frac{p'_c}{p'} \quad (3)$$

But, from the normal consolidation line BD , we get

$$|e_D - e_B| = \lambda \ln \left\{ \frac{p'_c}{(p'_c)_{\text{prev}}} \right\} \quad (4)$$

Substituting Eq. (4) into Eq. (3) and simplifying gives

$$p'_c = (p'_c)_{\text{prev}} \left(\frac{p'_{\text{prev}}}{p'} \right)^{\kappa/(\lambda-\kappa)} \quad \blacksquare$$

What's next... We have calculated the yield stresses, the failure stresses, and strains for a given stress increment. In the next section, a procedure is outlined to calculate the stress–strain, volume change, and excess porewater pressure responses of a soil using the critical state model.

8.9 CALCULATED STRESS–STRAIN RESPONSE



Computer Program Utility

Access Chapter 8, and click on Spreadsheet on the sidebar menu to view a spreadsheet program to calculate and plot the stress–strain response of drained and undrained compression tests. Select Example 8.8. You can modify a copy of this spreadsheet to solve the problems at the end of the chapter. Change input parameters and include your own lab data to explore the effects of various parameters, such as λ , κ , e_o , etc. and to compare the predicted stress–strain response with your data.

You can predict the stress–strain response, volume changes, and excess porewater pressure from the initial stress state to the failure stress state using the methods described in the previous sections. The required soil parameters are p'_o , e_o , p'_c or OCR, λ , κ , ϕ'_{cs} , and ν' . The procedures for a given stress path are as follows.

8.9.1 Drained Compression Tests

1. Determine the mean effective stress and the deviatoric stress at initial yield, that is, p'_y and q_y , by finding the coordinates of the intersection of the initial yield surface with the effective stress path. For a CD test,

$$p'_y = \frac{(M^2 p'_c + 18 p'_o) + \sqrt{(M^2 p'_c + 18 p'_o)^2 - 36(M^2 + 9)(p'_o)^2}}{2(M^2 + 9)} \quad (8.50)$$

$$q_y = 3(p'_y - p'_o) \quad (8.51)$$

2. Calculate the mean effective stress and deviatoric stress at failure by finding the coordinate of the intersection of the critical state line and the effective stress path, that is, p'_f and q_f . For a CD test, use Eqs. (8.17) and (8.18).
3. Calculate G using Eq. (8.32) or Eq. (8.34) or empirical equations (8.36) and (8.37). Use an average value of p' [$p' = (p'_o + p'_y)/2$] to calculate G if Eq. (8.32) is used.
4. Calculate the initial elastic volumetric strain using Eq. (8.40) and initial elastic deviatoric strain using Eq. (8.49).
5. Divide the ESP between the initial yield point and the failure point into a number of equal stress increments. Small increment sizes ($<5\%$ of the stress difference between q_f and q_y) tend to give a more accurate solution than larger increment sizes.

For each mean effective stress increment up to failure:

6. Calculate the preconsolidation stress, p'_c , for each increment; that is, you are calculating the major axis of the ellipse using Eq. (8.5), which gives

$$p'_c = p' + \frac{q^2}{M^2 p'} \quad (8.52)$$

where p' is the current mean effective stress.

7. Calculate the total volumetric strain increment using Eq. (8.39).
8. Calculate the plastic volumetric strain using Eq. (8.42).
9. Calculate the plastic deviatoric strain increment using Eq. (8.48).
10. Calculate the elastic deviatoric strain increment using Eq. (8.49).
11. Add the plastic and elastic deviatoric strain increments to give the total deviatoric strain increment.
12. Sum the total volumetric strain increments (ε_p).
13. Sum the total deviatoric shear strain increments (ε_q).
14. Calculate

$$\varepsilon_1 = \frac{3\varepsilon_q + \varepsilon_p}{3} = \varepsilon_q = \frac{\varepsilon_p}{3} \quad (8.53)$$

15. If desired, you can calculate

$$\sigma'_1 = \frac{2q}{3} + p' \quad \text{and} \quad \sigma'_3 = p' - \frac{q}{3}$$

The last value of mean effective stress should be about $0.99p'_f$ to prevent instability in the solution.

8.9.2 Undrained Compression Tests

1. Determine the mean effective stress and the deviatoric stress at initial yield, that is, p'_y and q_y . Remember that the effective stress path within the initial yield surface is vertical. Therefore, $p'_o = p_y$ and q_y are found by determining the intersection of a vertical line originating at p'_o with the initial yield surface. The equation to determine q_y for an isotropically consolidated soil is

$$q_y = Mp'_o \sqrt{\frac{p'_c}{p'_o} - 1} \quad (8.54)$$

If the soil is heavily overconsolidated, then $q_y = q_p$.

2. Calculate the mean effective and deviatoric stress at failure from Eqs. (8.19) and (8.20).
3. Calculate G using Eq. (8.32) or Eq. (8.34) or empirical equations (8.36) and (8.37).
4. Calculate the initial elastic deviatoric strain from Eq. (8.49).
5. Divide the horizontal distance between the initial mean effective stress, p'_o , and the failure mean effective stress, p'_f , in the e - p' plot into a number of equal mean effective stress increments. You need to use small stress increment size, usually less than $0.05(p'_o - p'_f)$.

For each increment of mean effective stress, calculate the following:

6. Determine the preconsolidation stress after each increment of mean effective stress from

$$p'_c = (p'_c)_{\text{prev}} \left(\frac{p'_{\text{prev}}}{p'} \right)^{\kappa/(\lambda-\kappa)}$$

where the subscript “prev” denotes the previous increment, p'_c is the current preconsolidation stress or the current size of the major axis of the yield surface, and p' is the current mean effective stress.

7. Calculate q at the end of each increment from

$$q = Mp' \sqrt{\frac{p'_c}{p'} - 1}$$

8. Calculate the volumetric elastic strain increment from Eq. (8.40).
9. Calculate the volumetric plastic strain increment. Since the total volumetric strain is zero, the volumetric plastic strain increment is equal to the negative of the volumetric elastic strain increment; that is, $\Delta \epsilon_p^p = -\Delta \epsilon_p^e$.
10. Calculate the deviatoric plastic strain increment from Eq. (8.48).
11. Calculate the deviatoric elastic strain increment from Eq. (8.49).
12. Add the deviatoric elastic and plastic strain increments to get the total deviatoric strain increment.
13. Sum the total deviatoric strain increments. For undrained conditions, $\epsilon_1 = \epsilon_d$.
14. Calculate the current mean total stress from the TSP. Remember you know the current value of q from Step 7. For a CU test, $p = p'_o + q/3$.
15. Calculate the change in excess porewater pressure by subtracting the current mean effective stress from the current mean total stress.

EXAMPLE 8.8 *Predicting Stress–Strain Responses for Drained and Undrained Conditions***Computer Program Utility**

Access Chapter 8 on the CD, click on Spreadsheet on the sidebar menu, and then select Example 8.8 for the solution of this example using a spreadsheet program.

Estimate and plot the stress–strain curve, volume changes (drained conditions), and excess porewater pressures (undrained conditions) for two samples of the same soil. The first sample, sample A, is to be subjected to conditions similar to a CD test and the second sample, sample B, is to be subjected to conditions similar to a CU test. The soil parameters are $\lambda = 0.25$, $\kappa = 0.05$, $\phi'_{cs} = 24^\circ$, $\nu' = 0.3$, $e_o = 1.15$, $p'_o = 200$ kPa, and $p'_c = 250$ kPa.

Strategy Follow the procedures listed in Section 8.9. A spreadsheet can be prepared to do the calculations. However, you should manually check some of the spreadsheet results to be sure that you entered the correct formulation. A spreadsheet will be used here but we will calculate the results for one increment for each sample.

Solution 8.8

$$\text{Calculate } M_c: \quad M_c = \frac{6 \sin \phi'_{cs}}{3 - \sin \phi'_{cs}} = \frac{6 \sin 24^\circ}{3 - \sin 24^\circ} = 0.94$$

$$\begin{aligned} \text{Calculate } e_\Gamma: \quad e_\Gamma &= e_o + (\lambda - \kappa) \ln \frac{p'_c}{2} + \kappa \ln p'_o \\ &= 1.15 + (0.25 - 0.05) \ln \frac{250}{2} + 0.05 \ln 200 = 2.38 \end{aligned}$$

Each step corresponds to the procedures listed in Section 8.9.

Sample A, Drained Test**Step 1:**

$$\begin{aligned} p'_y &= \frac{(M^2 p'_c + 18 p'_o) + \sqrt{(M^2 p'_c + 18 p'_o)^2 - 36(M^2 + 9)(p'_o)^2}}{2(M^2 + 9)} \\ &= \frac{(0.94^2 \times 250 + 18 \times 200) + \sqrt{(0.94^2 \times 250 + 18 \times 200)^2 - 36(0.94^2 + 9)(200)^2}}{2(0.94^2 + 9)} \\ &= 224 \text{ kPa} \\ q_y &= 3(p'_y - p'_o) = 3(224 - 200) = 72 \text{ kPa} \end{aligned}$$

Step 2:

$$\begin{aligned} p'_f &= \frac{3Mp'_o}{3 - M}; \\ p'_f &= \frac{3 \times 200}{3 - 0.94} = 291.3 \text{ kPa}, \quad q_f = Mp'_f = 0.94 \times 291.3 = 273.8 \text{ kPa} \end{aligned}$$

Step 3:

$$\begin{aligned} p'_{av} &= \frac{200 + 224}{2} = 212 \text{ kPa} \\ G &= \frac{3p'(1 + e_o)(1 - 2\nu')}{2\kappa(1 + \nu')} = \frac{3 \times 212(1 + 1.15) \times (1 - 2 \times 0.3)}{2 \times 0.05(1 + 0.3)} = 4207 \text{ kPa} \end{aligned}$$

Step 4:

$$(\Delta \varepsilon_q^e)_{\text{initial}} = \frac{\Delta q}{3G} = \frac{71.9}{3 \times 4207} = 5.7 \times 10^{-3}$$

$$(\Delta \varepsilon_p^e)_{\text{initial}} = \frac{\kappa}{1 + e_o} \ln \frac{p'_y}{p'_o} = \frac{0.05}{1 + 1.15} \ln \frac{224}{200} = 2.6 \times 10^{-3}$$

Step 5: Let $\Delta p' = 4$ kPa; then $\Delta q = 3 \times \Delta p' = 12$ kPa.

First stress increment after the initial yield follows.

Step 6: $p' = 224 + 4 = 228$ kPa, $q = 71.9 + 12 = 83.9$ kPa,

$$p'_c = p' + \frac{q^2}{M^2 p'} = 228 + \frac{83.9^2}{0.94^2 + 228} = 262.9 \text{ kPa}$$

Step 7:

$$\Delta \varepsilon_p = \frac{1}{1 + e_o} \left\{ (\lambda - \kappa) \ln \frac{p'_G}{p'_c} + \kappa \ln \frac{p'_E}{p'_D} \right\} = \frac{1}{1 + 1.15} \left\{ (0.25 - 0.05) \ln \frac{262.9}{250} + 0.05 \ln \frac{228}{224} \right\} = 5.1 \times 10^{-3}$$

Step 8:

$$\Delta \varepsilon_p^p = \frac{\lambda - \kappa}{1 + e_o} \ln \frac{p'_G}{p'_c} = \frac{(0.25 - 0.05)}{1 + 1.15} \ln \frac{262.9}{250} = 4.6 \times 10^{-3}$$

Step 9:

$$\Delta \varepsilon_q^p = \Delta \varepsilon_p^p \frac{q}{M^2 (p' - p'_c/2)} = 4.6 \times 10^{-3} \frac{83.9}{0.94^2 (228 - 262.9/2)} = 4.6 \times 10^{-3}$$

Step 10: Assuming G is constant,

$$\Delta \varepsilon_q^e = \frac{\Delta q}{3G} = \frac{12}{3 \times 4207} = 1.0 \times 10^{-3}$$

Step 11:

$$\Delta \varepsilon_q = \Delta \varepsilon_q^e + \Delta \varepsilon_q^p = (1.0 + 4.6) \times 10^{-3} = 5.6 \times 10^{-3}$$

Step 12:

$$\varepsilon_p = (\Delta \varepsilon_p^e)_{\text{initial}} + \Delta \varepsilon_p = (2.6 + 5.1) \times 10^{-3} = 7.7 \times 10^{-3}$$

Step 13:

$$\varepsilon_q = (\Delta \varepsilon_q^e)_{\text{initial}} + \Delta \varepsilon_q = (5.7 + 5.6) \times 10^{-3} = 11.3 \times 10^{-3}$$

Step 14:

$$\varepsilon_1 = \varepsilon_q + \varepsilon_p/3 = (11.3 + 7.7/3) \times 10^{-3} = 13.8 \times 10^{-3}$$

The spreadsheet program and the stress–strain plots are shown in the table below and Figs. E8.8a,b. There are some slight differences between the calculated values shown above and the spreadsheet because of number rounding.

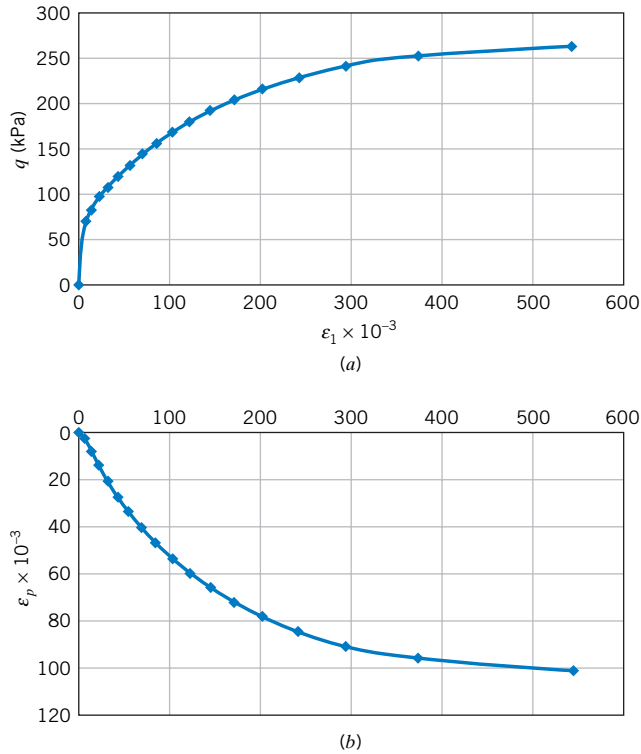


FIGURE E8.8a,b

Drained Case

Given data		Calculated values			
λ	0.25	M	0.94	$\Delta p'$	4 kPa
κ	0.05	R_o	1.25	Δq	12 kPa
ϕ'_{cs}	24	e_{cs}	2.38	G	4207.0 kPa
e_o	1.15	p'_f	291.4 kPa	$\Delta \varepsilon_p^a$	0.0026
p'_o	200 kPa	q'_f	274.2 kPa	$\Delta \varepsilon_p^a$	0.0057
p'_c	250 kPa	p'_y	224.0 kPa		
ν'	0.3	q_y	71.9 kPa		

^aSelected increment.**Tabulation**

p' kPa	$\Sigma \Delta q$ kPa	q kPa	p'_c kPa	$\Delta \varepsilon_p$ $\times 10^{-3}$	$\varepsilon_p = \Sigma \Delta \varepsilon_p$ $\times 10^{-3}$	$\Delta \varepsilon_p^p$ $\times 10^{-3}$	$\Delta \varepsilon_q^p$ $\times 10^{-3}$	$\Delta \varepsilon_q^e$ $\times 10^{-3}$	$\Delta \varepsilon_q$ $\times 10^{-3}$	$\varepsilon_q = \Sigma \Delta \varepsilon_q$ $\times 10^{-3}$	ε_1 $\times 10^{-3}$
0	0	0	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0'
224.0	0.0	71.9	250.0	2.6	2.6	0.0	0.0	5.7	5.7	5.7	6.6
228.0	12.0	83.9	262.8	5.1	7.7	4.6	4.6	1.0	5.5	11.2	13.8
232.0	24.0	95.9	276.7	6.4	14.1	4.8	5.5	1.0	6.5	17.7	22.4
236.0	36.0	107.9	291.6	6.5	20.6	4.9	6.6	1.0	7.6	25.3	32.1
240.0	48.0	119.9	307.6	6.6	27.2	4.9	7.8	1.0	8.7	34.0	43.0
244.0	60.0	131.9	324.4	6.6	33.8	5.0	9.1	1.0	10.0	44.0	55.2

(Continued)

Tabulation (continued)

p' kPa	$\Sigma \Delta q$ kPa	q kPa	p'_c kPa	$\Delta \varepsilon_p$ $\times 10^{-3}$	$\varepsilon_p = \Sigma \Delta \varepsilon_p$ $\times 10^{-3}$	$\Delta \varepsilon_p^p$ $\times 10^{-3}$	$\Delta \varepsilon_q^p$ $\times 10^{-3}$	$\Delta \varepsilon_q^e$ $\times 10^{-3}$	$\Delta \varepsilon_q$ $\times 10^{-3}$	$\varepsilon_q = \Sigma \Delta \varepsilon_q$ $\times 10^{-3}$	ε_1 $\times 10^{-3}$
248.0	72.0	143.9	342.2	6.6	40.3	5.0	10.5	1.0	11.4	55.4	68.9
252.0	84.0	155.9	360.8	6.5	46.9	4.9	12.1	1.0	13.1	68.5	84.1
256.0	96.0	167.9	380.3	6.5	53.3	4.9	14.1	1.0	15.0	83.5	101.3
260.0	108.0	179.9	400.5	6.4	59.7	4.8	16.4	1.0	17.3	100.8	120.8
264.0	120.0	191.9	421.4	6.3	66.0	4.7	19.3	1.0	20.3	121.1	143.1
268.0	132.0	203.9	443.1	6.2	72.2	4.7	23.1	1.0	24.1	145.2	169.2
272.0	144.0	215.9	465.4	6.1	78.3	4.6	28.4	1.0	29.4	174.6	200.6
276.0	156.0	227.9	488.4	5.9	84.2	4.5	36.3	1.0	37.3	211.9	239.9
280.0	168.0	239.9	512.0	5.8	90.0	4.4	49.7	1.0	50.6	262.5	292.5
284.0	180.0	251.9	536.2	5.7	95.7	4.3	77.1	1.0	78.1	340.5	372.4
288.0	192.0	263.9	561.0	5.6	101.3	4.2	167.7	1.0	168.7	509.2	543.0

Sample B, Undrained Test

Step 1:

$$q_y = Mp'_o \sqrt{\frac{p'_c}{p'_o} - 1} = 0.94 \times 200 \sqrt{\frac{250}{200} - 1} = 94 \text{ kPa}$$

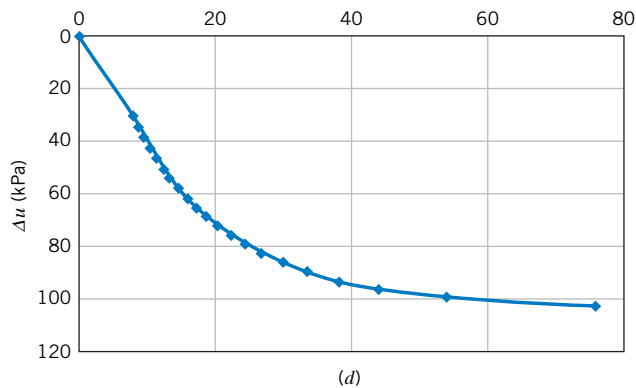
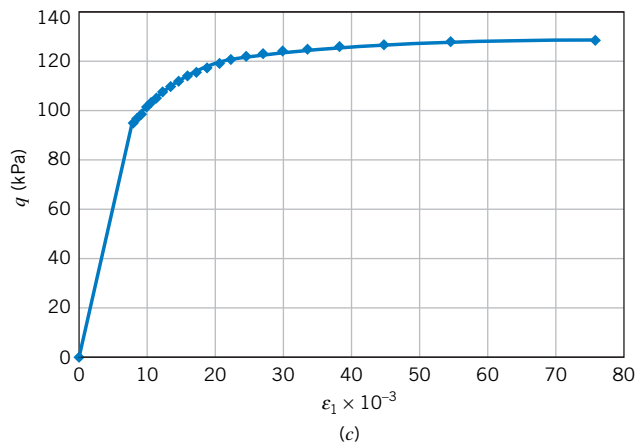


FIGURE E8.8c,d

Step 2:

$$p'_f = \exp\left(\frac{e_\Gamma - e_o}{\lambda}\right) = \exp\left(\frac{2.38 - 1.15}{0.25}\right) = 137 \text{ kPa}$$

$$q_f = Mp'_f = 0.94 \times 137 = 128.8 \text{ kPa}$$

Step 3:

$$G = \frac{3p'(1 + e_o)(1 - 2\nu')}{2\kappa(1 + \nu')} = \frac{3 \times 200(1 + 1.15) \times (1 - 2 \times 0.3)}{2 \times 0.05(1 + 0.3)} = 3969.2 \text{ kPa}$$

Step 4:

$$(\Delta\varepsilon_q^e)_{\text{initial}} = \frac{\Delta q}{3G} = \frac{94}{3 \times 3969.2} = 7.9 \times 10^{-3}$$

Step 5: Let $\Delta p' = 3 \text{ kPa}$.

First stress increment after the initial yield follows.

Step 6:

$$p' = p'_o - \Delta p' = 200 - 3 = 197 \text{ kPa}$$

$$p'_c = (p'_c)_{\text{prev}} \left(\frac{p'_{\text{prev}}}{p'}\right)^{\kappa/(\lambda-\kappa)} = 250 \left(\frac{200}{197}\right)^{0.05/(0.25-0.05)} = 250.9 \text{ kPa}$$

Step 7:

$$q = Mp' \sqrt{\frac{p'_c}{p'} - 1} = 0.94 \times 197 \sqrt{\frac{250.9}{197} - 1} = 97 \text{ kPa}$$

Step 8:

$$\Delta\varepsilon_p^e = \frac{\kappa}{1 + e_o} \ln \frac{p'_{\text{prev}}}{p'} = -\frac{0.05}{1 + 1.15} \ln \frac{200}{197} = -0.35 \times 10^{-3}$$

Step 9:

$$\Delta\varepsilon_p^p = -\Delta\varepsilon_p^e = 0.35 \times 10^{-3}$$

Step 10:

$$\Delta\varepsilon_q^p = \Delta\varepsilon_p^p \frac{q}{M^2(p' - p'_c/2)} = 0.35 \times 10^{-3} \frac{97}{0.94^2(197 - 250.9/2)} = 0.54 \times 10^{-3}$$

Step 11:

$$\Delta\varepsilon_q^e = \frac{\Delta q}{3G} = \frac{97 - 94.1}{3 \times 3969.2} = 0.24 \times 10^{-3}$$

Step 12: $\Delta\varepsilon_q = \varepsilon_q^e + \Delta\varepsilon_q^p = (0.24 + 0.54) \times 10^{-3} = 0.78 \times 10^{-3}$ **Step 13:** $\varepsilon_q = \varepsilon_1 = (\Delta\varepsilon_q^e)_{\text{initial}} + \Delta\varepsilon_q = (7.9 + 0.78) \times 10^{-3} = 8.7 \times 10^{-3}$ **Step 14:** $p = p'_o + q/3 = 200 + \frac{97}{3} = 232.3 \text{ kPa}$ **Step 15:** $\Delta u = p - p' = 232.3 - 197 = 35.3 \text{ kPa}$

The spreadsheet program and the stress-strain plots are shown in the table below and Figs. E8.8c,d.

Undrained Triaxial Test

Given data		Calculated values			
λ	0.25	M	0.94	Δp	3 kPa
κ	0.05	R_o	1.25	Δq	9 kPa
ϕ'_{cs}	24	e_{cs}	2.38	G	3969.2 kPa
e_o	1.15	p'_f	137.3 kPa	ε_p^e	0
p'_o	200 kPa	q'_f	129.2 kPa	ε_q^e	0.0079
p'_c	250 kPa	p'_y	200.0 kPa		
ν'	0.3	q_y	94.1 kPa		
		Δu_f	105.8 kPa		

Tabulation

p' kPa	p'_c kPa	q kPa	$\Delta\varepsilon_p^e$ $\times 10^{-3}$	$\Delta\varepsilon_p^p$ $\times 10^{-3}$	$\Delta\varepsilon_q^p$ $\times 10^{-3}$	$\Delta\varepsilon_q^e$ $\times 10^{-3}$	$\Delta\varepsilon_q$ $\times 10^{-3}$	$\varepsilon_q = \Sigma \Delta\varepsilon_q$ $\times 10^{-3}$	ε_1 $\times 10^{-3}$	p kPa	Δu kPa
0	0	0		0.0	0.0	0.0	0.0	0.0	0.0	0	0
200.0	250.0	94.1	0.0	0.0	0.0	7.9	7.9	7.9	7.9	231.4	31.4
197.0	250.9	97.0	-0.4	0.4	0.5	0.2	0.8	8.7	8.7	232.3	35.3
194.0	251.9	99.7	-0.4	0.4	0.6	0.2	0.8	9.5	9.5	233.2	39.2
191.0	252.9	102.3	-0.4	0.4	0.6	0.2	0.9	10.4	10.4	234.1	43.1
188.0	253.9	104.7	-0.4	0.4	0.7	0.2	0.9	11.3	11.3	234.9	46.9
185.0	254.9	107.0	-0.4	0.4	0.8	0.2	1.0	12.3	12.3	235.7	50.7
182.0	256.0	109.2	-0.4	0.4	0.9	0.2	1.0	13.3	13.3	236.4	54.4
179.0	257.0	111.2	-0.4	0.4	1.0	0.2	1.1	14.4	14.4	237.1	58.1
176.0	258.1	113.1	-0.4	0.4	1.1	0.2	1.2	15.7	15.7	237.7	61.7
173.0	259.2	114.9	-0.4	0.4	1.2	0.2	1.3	17.0	17.0	238.3	65.3
170.0	260.4	116.6	-0.4	0.4	1.3	0.1	1.5	18.5	18.5	238.9	68.9
167.0	261.5	118.2	-0.4	0.4	1.5	0.1	1.7	20.2	20.2	239.4	72.4
164.0	262.7	119.7	-0.4	0.4	1.7	0.1	1.9	22.0	22.0	239.9	75.9
161.0	263.9	121.1	-0.4	0.4	2.0	0.1	2.1	24.2	24.2	240.4	79.4
158.0	265.2	122.5	-0.4	0.4	2.4	0.1	2.5	26.7	26.7	240.8	82.8
155.0	266.4	123.7	-0.4	0.4	2.9	0.1	3.0	29.6	29.6	241.2	86.2
152.0	267.8	124.8	-0.5	0.5	3.5	0.1	3.6	33.3	33.3	241.6	89.6
149.0	269.1	125.9	-0.5	0.5	4.6	0.1	4.6	37.9	37.9	242.0	93.0
146.0	270.5	126.9	-0.5	0.5	6.3	0.1	6.4	44.3	44.3	242.3	96.3
143.0	271.9	127.8	-0.5	0.5	9.9	0.1	9.9	54.2	54.2	242.6	99.6
140.0	273.3	128.6	-0.5	0.5	21.4	0.1	21.5	75.7	75.7	242.9	102.9
138.0	274.3	129.1	-0.3	0.3	57.4	0.0	57.5	133.2	133.2	243.0	105.0

What's next... We have concentrated on isotropic consolidation of soils and axisymmetric conditions during shearing. The concepts and methodology developed are equally applicable to plane strain or other loading conditions. In nature, most soils are one-dimensionally consolidated, called K_o -consolidation. Next, we will consider K_o -consolidation using the critical state model.

8.10 K_o -CONSOLIDATED SOIL RESPONSE



Concept Learning and Self-Assessment

Access Chapter 8, Section 8.10 on the CD for a summary of correlations between soil strength and simple soils tests. Take Quiz 8.10 to test your understanding.

When a soil is one-dimensionally consolidated, anisotropy is conferred on the soil structure. The soil properties are no longer the same in all directions. We can use our simple critical state model to provide insights into K_o -consolidated soils although the model, as described, cannot handle anisotropy. We will assume that the yield surface is unaltered, that is, remains an ellipse, for K_o -consolidated soils. The normal consolidation line for a K_o -consolidated soil is shifted to the left of the normal consolidation line of an isotropically consolidated soil (Fig. 8.16b) because p' for a K_o -consolidated soil is

$$p' = \frac{1 + 2K_o}{3} \sigma'_z$$

compared with $p' = \sigma'_z$ for an isotropically consolidated soil. Recall that K_o is the lateral earth pressure coefficient at rest.

Let us compare the responses of two samples, sample I and sample II, of a soil. Sample I is K_o -consolidated while sample II is isotropically consolidated. Both soil samples are under axisymmetric conditions. You should recall from Chapter 5 that the stress path for isotropic consolidation is $\frac{q}{p'} = 0$

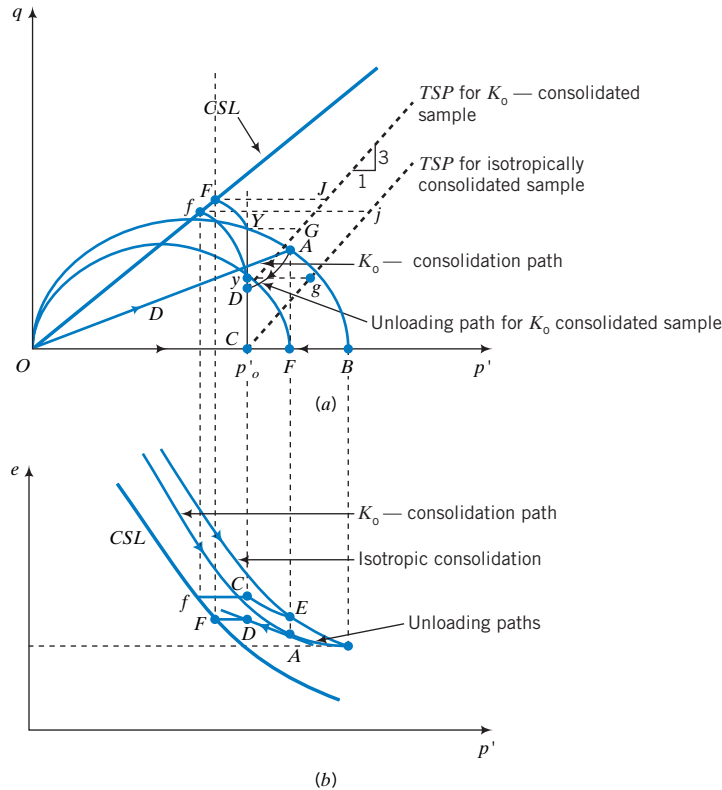


FIGURE 8.16 Comparison between a K_0 -consolidated soil and an isotropically consolidated soil.

and for K_0 -consolidation it is $\frac{q}{p'} = \frac{3(1-K_0^{nc})}{1+2K_0^{nc}}$. The stress path during consolidation of sample I is OA (Fig. 8.16a). The initial yield surface passes through point A and intersects the p' axis at B . You can find the mean effective stress at B (preconsolidated mean effective stress, p'_o) by substituting $p' = \frac{1+2K_0^{nc}}{3}\sigma'_{zc}$ and $q = (1-K_0^{nc})\sigma'_{zc}$ in Eq. (8.5). Let us unload sample I to an effective stress, p'_o , by reducing the vertical stress. The stress path during K_0 -unloading will not follow the loading path (OA) because upon unloading K_0 increases nonlinearly with mean effective stress as the soil sample becomes overconsolidated (Chapter 6). The unloading effective stress path for sample I is AD (Fig. 8.16a). Since A and B lie on the same yield surface, they must also lie on the same unloading/reloading line in the mean effective stress–void ratio plot as illustrated in Fig. 8.16b.

Sample II is isotropically consolidated to the same preconsolidated mean effective stress and isotropically unloaded to the same initial mean effective stress as sample I. Both samples then have the same overconsolidation ratio, R_o . The stress path for sample I is OEC (Fig. 8.16a). In the (p' vs. e) plot, the unloading path is EC .

Let us now conduct a CU test on each sample. The TSP for each sample has a slope of 3:1 as depicted in Fig. 8.16a. The effective stress paths within the initial yield surface for both samples are vertical. Recall from Chapter 3 that the mean effective stress during undrained loading of an elastic soil remains constant, so the ESP must be vertical. Sample I yields at Y while sample II yields at y . The excess porewater pressures at initial yield are shown by YG and yg for samples II and I respectively.

Beyond initial yield, the yield surfaces expand and excess porewater pressures increase significantly. The effective stress paths then bend toward the critical state line (Fig. 8.16a). In a CU test, the volume of the soil remains constant, so the paths to failure in the (p' vs. e) space are horizontal. These paths are shown as DF and Cf for samples II and I respectively. Failure for sample II occurs at F

while for sample I failure occurs at f . The excess porewater pressure at failure for sample I is ff and for sample it is FJ .

The implication is that although the overconsolidation ratios and the total stress paths are the same for both soils, the undrained shear strengths are different due to differences in stress history. Failure of soils is then dependent on the stress history.

Let us see whether we can develop an equation to estimate the undrained shear strength of a K_o -consolidated soil based on the ideas discussed in this chapter and using Skempton's porewater pressure coefficients (Chapter 7). Consider a saturated soil that has been K_o -consolidated and then subjected to total stresses $\Delta\sigma_1$ and $\Delta\sigma_3$ to bring it to failure. The initial stress conditions are $(\sigma'_1)_o > 0$ and $(\sigma'_3)_o = K_o(\sigma'_1)_o$. Upon application of the stresses, $\Delta\sigma_1$ and $\Delta\sigma_3$, the gross stresses on the soil are

$$\sigma_1 = (\sigma'_1)_o + \Delta\sigma_1 \quad (8.55)$$

$$\sigma'_1 = (\sigma'_1)_o + \Delta\sigma_1 - \Delta u \quad (8.56)$$

$$\sigma_3 = K_o(\sigma'_1)_o + \Delta\sigma_3 \quad (8.57)$$

$$\sigma'_3 = K_o(\sigma'_1)_o + \Delta\sigma_3 - \Delta u \quad (8.58)$$

For a saturated soil, Skempton's coefficient $B = 1$, and from Eq. (7.46)

$$\Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3) \quad (8.59)$$

Substituting Eq. (8.59) into Eq. (8.58) gives

$$\sigma'_3 = K_o(\sigma'_1)_o - A(\Delta\sigma_1 - \Delta\sigma_3)$$

Solving for $\Delta\sigma_1 - \Delta\sigma_3$, we obtain

$$\Delta\sigma_1 - \Delta\sigma_3 = \frac{K_o(\sigma'_1)_o - \sigma'_3}{A} \quad (8.60)$$

At failure,

$$\begin{aligned} s_u &= \left(\frac{\sigma_1 - \sigma_3}{2} \right)_f = \frac{1}{2} \{ [(\sigma'_1)_o + \Delta\sigma_1] - [K_o(\sigma'_1)_o + \Delta\sigma_3] \} \\ &= \frac{1}{2} [(\Delta\sigma_1 - \Delta\sigma_3) + (1 - K_o)(\sigma'_1)_o] \end{aligned} \quad (8.61)$$

Substituting Eq. (8.60) into Eq. (8.61) gives

$$s_u = \frac{1}{2} \left[\frac{K_o(\sigma'_1)_o - \sigma'_3}{A} + (1 - K_o)(\sigma'_1)_o \right] \quad (8.62)$$

At failure,

$$\frac{\sigma'_1}{\sigma'_3} = \frac{1 + \sin \phi'_{cs}}{1 - \sin \phi'_{cs}}$$

which by substitution into Eq. (8.62) leads to

$$\frac{s_u}{\sigma'_1} = \frac{s_u}{\sigma'_z} = \frac{\sin \phi'_{cs} [K_o + A(1 - K_o)]}{1 + (2A - 1) \sin \phi'_{cs}} \quad (8.63)$$

The essential points are:

1. A K_o -consolidated sample of a soil is likely to have a different undrained shear strength than an isotropically consolidated sample of the same soil even if the preconsolidation mean effective stresses and the initial confining pressures before shearing, and the slopes of the stress paths are the same.
2. Failure stresses in soils are dependent on the stress history of the soil.
3. Stress history does not influence the elastic response of soils.

What's next... We have established the main ideas behind the critical state model and used the model to estimate the response of soils to loading. The CSM can also be used with results from simple soil tests (e.g., Atterberg limits) to make estimates of the soil strengths. In the next section, we will employ the CSM to build some relationships among results from simple soil tests, critical state parameters, and soil strengths.

8.11 RELATIONSHIPS BETWEEN SIMPLE SOIL TESTS, CRITICAL STATE PARAMETERS, AND SOIL STRENGTHS



Self-Assessment

Access Chapter 8, Section 8.11 and take Quiz 8.11 to test your understanding.

Wood and Wroth (1978) and Wood (1990) used the critical state model to correlate results from Atterberg limit tests with various engineering properties of fine-grained soils. We are going to present some of these correlations. These correlations are very useful when limited test data are available during the preliminary design of geotechnical systems or when you need to evaluate the quality of test results. The correlations utilized water content, which at best is accurate to 0.1%. Most often water content results are reported to the nearest whole number and consequently significant differences can occur between the actual test results and the correlations, especially those involving exponentials. Since we are using CSM and index properties, the relationships only pertain to remolded or disturbed soils.

8.11.1 Undrained Shear Strength of Clays at the Liquid and Plastic Limits

Wood (1990), using test results reported by Youssef et al. (1965) and Dumbleton and West (1970), showed that

$$\frac{(s_u)_{PL}}{(s_u)_{LL}} = R \quad (8.64)$$

where R depends on activity (Chapter 3) and varies between 30 and 100, and the subscripts PL and LL denote plastic limit and liquid limit, respectively. Wood and Wroth (1978) recommend a value of $R = 100$ as reasonable for most soils. The recommended value of $(s_u)_{LL}$, culled from the published data, is 2 kPa (the test data showed variations between 0.9 and 8 kPa) and that for $(s_u)_{PL}$ is 200 kPa. Since most soils are within the plastic range these recommended values place lower (2 kPa) and upper (200 kPa) limits on the undrained shear strength of disturbed or remolded clays.

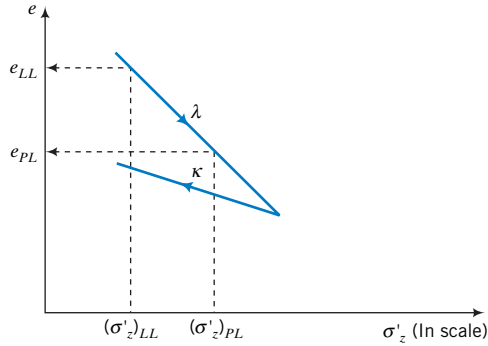
8.11.2 Vertical Effective Stresses at the Liquid and Plastic Limits

Wood (1990) used results from Skempton (1970) and recommended that

$$(\sigma'_z)_{LL} = 8 \text{ kPa} \quad (8.65)$$

The test results showed that $(\sigma'_z)_{LL}$ varies from 6 to 58 kPa. Laboratory and field data also showed that the undrained shear strength is proportional to the vertical effective stress. Therefore

$$(\sigma'_z)_{PL} = R(\sigma'_z)_{LL} \approx 800 \text{ kPa} \quad (8.66)$$

FIGURE 8.17 Illustrative graph of e versus σ'_z (ln scale).

8.11.3 Undrained Shear Strength–Vertical Effective Stress Relationship

Normalizing the undrained shear strength with respect to the vertical effective stress we get a ratio of

$$\frac{s_u}{\sigma'_z} = \frac{2}{8} \text{ or } \frac{200}{800} = 0.25 \quad (8.67)$$

Mesri (1975) reported, based on soil test results, that $S_u/\sigma'_{zc} = 0.22$, which is in good agreement with Eq. (8.65) for normally consolidated soils.

8.11.4 Compressibility Indices (λ and C_c) and Plasticity Index

The compressibility index C_c or λ is usually obtained from a consolidation test. In the absence of consolidation test results, we can estimate C_c or λ from the plasticity index. With reference to Fig. 8.17,

$$-(e_{PL} - e_{LL}) = \lambda \ln \frac{(\sigma'_z)_{PL}}{(\sigma'_z)_{LL}} = \lambda \ln R$$

Now, $e_{LL} = w_{LL} G_s$, $e_{PL} = w_{PL} G_s$, and $G_s = 2.7$. Therefore, for $R = 100$,

$$LL - PL = \frac{\lambda}{2.7} \ln R \approx 1.7\lambda$$

and

$$\lambda \approx 0.6 \text{ PI} \quad (8.68)$$

or

$$C_c = 2.3\lambda \approx 1.38 \text{ PI} \quad (8.69)$$

Equation (8.68) indicates that the compression index increases with plasticity index.

8.11.5 Undrained Shear Strength, Liquidity Index, and Sensitivity

Let us build a relationship between liquidity index and undrained shear strength. The undrained shear strength of a soil at a water content w , with reference to its undrained shear strength at the plastic limit, is obtained from Eq. (8.23) as

$$\frac{(s_u)_w}{(s_u)_{PL}} = \exp\left(G_s \frac{(PL - w)}{\lambda}\right)$$

Putting $G_s = 2.7$, $\lambda = 0.6$ PI in the above equation and recalling that

$$LI = \frac{w - PL}{PI}$$

we get

$$(s_u)_w = (s_u)_{PL} \exp(-4.6LI) \approx 200 \exp(-4.6LI) \quad (8.70)$$

Clays laid down in saltwater environments and having flocculated structure (Chapter 2) often have in situ (natural) water contents higher than their liquid limit but do not behave like a viscous liquid in their natural state. The flocculated structure becomes unstable when fresh water leaches out the salt. The undistributed or intact undrained shear strengths of these clays are significantly greater than their disturbed or remolded undrained shear strengths. The term sensitivity, S_t , is used to define the ratio of the intact undrained shear strength to the remolded undrained shear strength:

$$S_t = \frac{(s_u)_i}{(s_u)_r} \quad (8.71)$$

where i denotes intact and r denotes remolded. From Eq. (8.70) we can write

$$(s_u)_r \approx 200 \exp(-4.6LI) \quad (8.72)$$

For values of $S_t > 8$, the clay is called a quick clay. Quick clay, when disturbed, can flow like a viscous liquid ($LI > 1$). Bjerrum (1954) reported test data on quick clays in Scandinavia, which yield an empirical relationship between S_t and LI as

$$LI = 1.2 \log_{10} S_t \quad (8.73)$$

8.12 SUMMARY

In this chapter, a simple critical state model (CSM) was used to provide some insight into soil behavior. The model replicates the essential features of soil behavior but the quantitative predictions of the model may not match real soil values. The key feature of the critical state model is that every soil fails on a unique surface in (q, p', e) space. According to the CSM, the failure stress state is insufficient to guarantee failure; the soil must also be loose enough (reaches the critical void ratio). Every sample of the same soil will fail on a stress state that lies on the critical state line regardless of any differences in the initial stress state, stress history, and stress path among samples.

The model makes use of an elliptical yield surface that expands to simulate hardening or contracts to simulate softening during loading. Expansion and contraction of the yield surface are related to the normal consolidation line of the soil. Imposed stress states that lie within the initial yield surface will cause the soil to behave elastically. Imposed stress states that lie outside the initial yield surface will cause the soil to yield and to behave elastoplastically. Each imposed stress state that causes the soil to yield must lie on a yield surface and on an unloading/reloading line corresponding to the preconsolidation mean effective stress associated with the current yield surface.

The CSM is not intended to replicate all the details of the behavior of real soils but to serve as a simple framework from which we can interpret and understand the important features of soil behavior.

Practical Examples**EXAMPLE 8.9 Predicting Soil Response Under a Tank Foundation**

An oil tank foundation is to be located on a very soft clay, 6 m thick, underlain by a deep deposit of stiff clay. Soil tests at a depth of 3 m gave the following results: $\lambda = 0.32$, $\kappa = 0.06$, $\sigma'_{cs} = 26^\circ$, $\text{OCR} = 1.2$, and $w = 55\%$. The tank has a diameter of 8 m and is 5 m high. The dead load of the tank and its foundation is 350 kN. Because of the expected large settlement, it was decided to preconsolidate the soil by quickly filling the tank with water and then allowing consolidation to take place. To reduce the time to achieve the desired level of consolidation, sand drains were installed at the site. Determine whether the soil will fail if the tank is rapidly filled to capacity. What levels of water will cause the soil to yield and to fail? At the end of the consolidation, the owners propose to increase the tank capacity by welding a section on top of the existing tank. However, the owners do not want further preconsolidation or soil tests. What is the maximum increase in the tank height you would recommend so that the soil does not fail and settlement does not exceed 75 mm? The dead load per meter height of the proposed additional section is 40 kN. The unit weight of the oil is 8.5 kN/m^3 .

Strategy The soil is one-dimensionally consolidated before the tank is placed on it. The loads from the tank will force the soil to consolidate along a path that depends on the applied stress increments. A soil element under the center of the tank will be subjected to axisymmetric loading conditions. If the tank is loaded quickly, then undrained conditions apply and the task is to predict the failure stresses and then use them to calculate the surface stresses that would cause failure. After consolidation, the undrained shear strength will increase and you would have to find the new failure stresses.

Solution 8.9

Step 1: Calculate initial values.

$$\begin{aligned}
 e_o &= wG_s = 0.55 \times 2.7 = 1.49 \\
 K_o^{\text{nc}} &= 1 - \sin \phi'_{cs} = 1 - \sin 26^\circ = 0.56 \\
 K_o^{\text{nc}} &= K_o^{\text{nc}} (\text{OCR})^{1/2} = 0.56 \times (1.2)^{1/2} = 0.61 \\
 \gamma' &= \frac{G_s - 1}{1 + e_o} \gamma_w = \frac{2.7 - 1}{1 + 1.49} \times 9.8 = 6.69 \text{ kN/m}^2 \\
 \sigma'_{zo} &= \gamma' z = 6.69 \times 3 = 20.1 \text{ kPa} \\
 \sigma'_{xo} &= K_o^{\text{nc}} \sigma'_{zo} = 0.61 \times 20.1 = 12.3 \text{ kPa} \\
 \sigma'_{zc} &= \text{OCR} \times \sigma'_{zo} = 1.2 \times 20.1 = 24.1 \text{ kPa} \\
 M_c &= \frac{6 \sin \phi'_{cs}}{3 - \sin \phi'_{cs}} = \frac{6 \sin 26^\circ}{3 - \sin 26^\circ} = 1.03
 \end{aligned}$$

The stresses on the initial yield surface are:

$$\begin{aligned}
 (p'_c)_o &= \frac{1 + 2K_o^{\text{nc}}}{3} \sigma'_{zc} = \frac{1 + 2 \times 0.56}{3} \times 24.1 = 17 \text{ kPa} \\
 (q_c)_o &= (1 - K_o^{\text{nc}}) \sigma'_{zc} = (1 - 0.56) \times 24.1 = 10.6 \text{ kPa} \\
 p'_o &= \frac{1 + 2K_o^{\text{oc}}}{3} \sigma'_{zo} = \frac{1 + 2 \times 0.61}{3} \times 20.1 = 14.9 \text{ kPa} \approx 15 \text{ kPa} \\
 q_o &= (1 - K_o^{\text{oc}}) \sigma'_{zo} = (1 - 0.61) \times 20.1 = 7.8 \text{ kPa}
 \end{aligned}$$

You need to calculate the preconsolidated mean effective stress on the isotropic consolidation line (point *I*, Fig. E8.9). You should note that $\{(p'_c)_o, (q_c)_o\}$ lies on the initial yield surface (point *A*, Fig. E8.9).

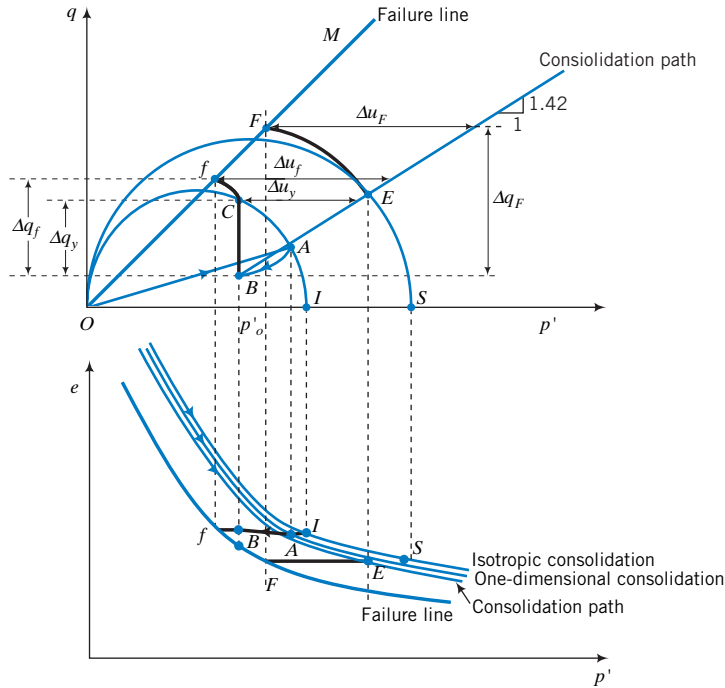


FIGURE E8.9

Find p'_c using Eq. (8.5), that is

$$(p'_c)_o^2 + (p'_c)_o p'_c + \frac{(q_c)_o^2}{M^2} = 0$$

Therefore, $17^2 + 17p'_c + (10.6)^2/(1.03)^2 = 0$ and solving for p'_c we get $p'_c = 23.2$ kPa.

$$e_\Gamma = e_o + (\lambda - \kappa) \ln \frac{p'_c}{2} + \kappa \ln p'_o = 1.49 + (0.32 - 0.06) \ln \frac{23.2}{2} + 0.06 \ln 15 = 2.29$$

Step 2: Calculate the stress increase from the tank and also the consolidation stress path.

$$\text{Area of tank: } A = \frac{\pi D^2}{4} = \frac{\pi \times 8^2}{4} = 50.27 \text{ m}^2$$

$$\text{Vertical surface stress from water: } \gamma_w h = 9.8 \times 5 = 49 \text{ kPa}$$

$$\text{Vertical surface stress from dead load: } \frac{350}{50.27} = 7 \text{ kPa}$$

$$\text{Total vertical surface stress: } q_s = 49 + 7 = 56 \text{ kPa}$$

$$\begin{aligned} \text{Vertical stress increase: } \Delta\sigma_z &= q_s \left[1 - \left(\frac{1}{1 + (r/z)^2} \right)^{3/2} \right] \\ &= q_s \left[1 - \left(\frac{1}{1 + (4/3)^2} \right)^{3/2} \right] = 0.78 q_s \end{aligned}$$

$$\begin{aligned} \text{Radial stress increase: } \Delta\sigma_r &= \frac{q_s}{2} \left((1 + 2\nu) - \frac{2(1 + \nu)}{[1 + (r/z)^2]^{1/2}} + \frac{1}{[1 + (r/z)^2]^{3/2}} \right) \\ &= \frac{q_s}{2} \left((1 + 2 \times 0.5) - \frac{2(1 + 0.5)}{[1 + (\frac{4}{3})^2]^{1/2}} + \frac{1}{[1 + (\frac{4}{3})^2]^{3/2}} \right) = 0.21 q_s \end{aligned}$$

$$\frac{\Delta\sigma_r}{\Delta\sigma_z} = \frac{0.21}{0.78} = 0.27$$

$$\Delta\sigma_z = 0.78 \times 56 = 43.7 \text{ kPa}, \quad \Delta\sigma_r = 0.21 \times 56 = 11.8 \text{ kPa},$$

$$\Delta p = \frac{43.7 + 2 \times 11.8}{3} = 22.4 \text{ kPa}$$

$$\Delta q = 43.7 - 11.8 = 31.9 \text{ kPa}$$

$$\text{Slope of TSP} = \text{ESP during consolidation} : \frac{\Delta q}{\Delta p} = \frac{31.9}{22.4} = 1.42$$

Step 3: Calculate the initial yield stresses and excess porewater pressure at yield.

The yield stresses (point C, Fig. E8.9) are found from Eq. (8.53); that is,

$$q_y = Mp'_o \sqrt{\frac{p'_c}{p'_o} - 1} = 1.03 \times 15 \sqrt{\frac{23.2}{15} - 1} = 11.4 \text{ kPa}$$

$$p'_y = p'_o = 15 \text{ kPa}, \quad \Delta q_y = q_y - q_o = 11.4 - 7.6 = 3.8 \text{ kPa}$$

The excess porewater pressure at yield is

$$\Delta u_y = \Delta p_y = \frac{\Delta q_y}{1.42} = \frac{3.8}{1.42} = 2.7 \text{ kPa}$$

The vertical effective stress and vertical total stresses are

$$(\Delta\sigma'_z)_y = \Delta p' + \frac{2}{3} \Delta q = 0 + \frac{2}{3} \times 3.8 = 2.5 \text{ kPa}$$

$$(\Delta\sigma_z)_y = (\Delta\sigma'_z)_y + \Delta u_y = 2.5 + 2.7 = 5.2 \text{ kPa}$$

Step 4: Calculate the equivalent surface stress.

$$\Delta q_s = \frac{(\Delta\sigma_z)_y}{0.78} = \frac{5.2}{0.78} = 6.7 \text{ kPa}$$

The vertical surface stress from the dead load of the tank is 7 kPa, which is greater than 6.7 kPa. Therefore, under the dead load the soil will yield.

Step 5: Calculate the failure stresses.

Failure occurs at point f, Fig. E8.9.

$$\Delta q_f = M \exp\left(\frac{(e_f - e_o)}{\lambda}\right) = 1.03 \exp\left(\frac{2.29 - 1.49}{0.32}\right) = 12.5 \text{ kPa}$$

$$\Delta p'_f = \frac{\Delta q_f}{M} = \frac{12.5}{1.03} = 12.1 \text{ kPa}$$

$$\Delta u_f = \Delta p'_f + \frac{\Delta q_f}{1.42} = 12.1 + \frac{12.5}{1.42} = 20.9 \text{ kPa}$$

$$(\Delta\sigma'_z)_f = \Delta p'_f + \frac{2}{3} \Delta q_f = 12.1 + \frac{2}{3} \times 12.5 = 20.4 \text{ kPa}$$

$$(\Delta\sigma_z)_f = (\Delta\sigma'_z)_f + \Delta u_f = 20.4 + 20.9 = 41.3 \text{ kPa}$$

Step 6: Calculate the height of water to bring the soil to failure.

$$\text{Equivalent surface stress: } \Delta q_s = \frac{(\Delta\sigma_z)_f}{0.78} = \frac{41.3}{0.78} = 52.9 \text{ kPa}$$

The vertical surface stress from the dead load of the tank is 7 kPa. Therefore, the equivalent vertical surface stress from water is $52.9 - 7 = 45.9 \text{ kPa}$.

$$\text{Height of water: } h_w = \frac{45.9}{\gamma_w} = \frac{45.9}{9.8} = 4.68 \text{ m}$$

Therefore, you cannot fill the tank to capacity. You will have to fill the tank with water to a height less than 4.68 m, allow the soil to consolidate, and then increase the height of water gradually.

Step 7: Determine the failure stresses after consolidation.

The soil is consolidated along a stress path of slope 1.42:1 up to point E , Fig. E8.9. Loading from E under undrained conditions (TSP has a slope of 1.42:1) will cause yielding immediately (E lies on the yield surface) and failure will occur at F (Fig. E8.9).

$$\begin{aligned}
 p'_E &= p'_o + \Delta p = 15 + 22.4 = 37.4 \text{ kPa} \\
 e_E &= e_o - \lambda \ln \frac{p'_E}{p'_o} = 1.49 - 0.32 \ln \frac{37.4}{15} = 1.20 \\
 \Delta q_F &= M \exp\left(\frac{e_F - e_E}{\lambda}\right) = 1.03 \exp\left(\frac{2.29 - 1.20}{0.32}\right) = 31.1 \text{ kPa} \\
 \Delta p'_F &= \frac{\Delta q_F}{M} = \frac{31.1}{1.03} = 30.2 \text{ kPa} \\
 \Delta u_F &= \Delta p'_F + \frac{\Delta q_F}{1.42} = 30.2 + \frac{31.1}{1.42} = 52.1 \text{ kPa} \\
 (\Delta \sigma'_z)_F &= \Delta p'_F + \frac{2}{3} \Delta q_F = 30.2 + \frac{2}{3} \times 31.1 = 50.9 \text{ kPa} \\
 (\Delta \sigma_z)_F &= (\Delta \sigma'_z)_F + \Delta u_F = 50.9 + 52.1 = 103 \text{ kPa}
 \end{aligned}$$

Step 8: Calculate the equivalent surface stress and load.

$$\begin{aligned}
 \text{Equivalent surface stress : } \Delta q_s &= \frac{(\Delta \sigma_z)_F}{0.78} = \frac{103}{0.78} = 132.1 \text{ kPa} \\
 \text{Surface load applied during consolidation : } &350 + H\gamma_w A = 350 + 5 \times 9.8 \times 50.27 \\
 &= 2813.2 \text{ kN} \\
 \text{Possible additional surface load : } &132.1 \times A = 132.1 \times 50.27 = 6640.7 \text{ kN} \\
 \text{Total surface load : } &2813.2 + 6640.7 = 9453.9 \text{ kN}
 \end{aligned}$$

Step 9: Find the additional height to bring the soil to failure after consolidation. Let Δh be the additional height.

$$\begin{aligned}
 (5 + \Delta h)\gamma_{\text{oil}} A + 350 + \text{additional load per meter} \times \Delta h &= 9453.9 \\
 \therefore (5 + \Delta h) \times 8.5 \times 50.27 + 350 + 40\Delta h &= 9453.9 \text{ and } \Delta h = 15.7 \text{ m}
 \end{aligned}$$

Step 10: Calculate the mean effective stress to cause 75 mm settlement.

$$\rho = \frac{\Delta e}{1 + e_o} H = \frac{H}{1 + e_E} \lambda \ln \frac{p'}{p'_E}$$

where H is the thickness of the very soft clay layer. Therefore,

$$\begin{aligned}
 75 &= \frac{6000}{1 + 1.20} \times 0.32 \ln \frac{p'}{37.4} \\
 \therefore p' &= 40.8 \text{ kPa} \\
 \Delta p' &= p' - p'_E = 40.8 - 37.4 = 3.4 \text{ kPa}, \quad \Delta q = 1.42 \times \Delta p' = 1.42 \times 3.4 = 4.8 \text{ kPa} \\
 \Delta u &= \Delta p' + \frac{\Delta q}{1.42} = 3.4 + \frac{4.8}{1.42} = 6.8 \text{ kPa} \\
 \Delta \sigma'_z &= \Delta p' + \frac{2}{3} \Delta q = 3.4 + \frac{2}{3} \times 4.8 = 6.6 \text{ kPa}, \\
 \Delta \sigma_z &= \Delta \sigma'_z + \Delta u = 6.6 + 6.8 = 13.4 \text{ kPa}
 \end{aligned}$$

Step 11: Calculate the height of oil for 75 mm settlement.

$$\text{Equivalent surface stress : } \Delta q_s = \frac{\Delta \sigma_z}{0.78} = \frac{13.4}{0.78} = 17.2 \text{ kPa}$$

$$\text{Additional height of tank : } \Delta h = \frac{17.2}{8.5} = 2.0 \text{ m}$$

Since the tank was preloaded with water and water is heavier than the oil, it is possible to get a further increase in height by $(9.8/8.5 - 1)5 = 0.76$ m. To be conservative, because the analysis only gives an estimate, you should recommend an additional height of 2.0 m. ■

EXAMPLE 8.10 Evaluating Soil Test Results Using Critical State Concepts

You requested a laboratory to carry out soil tests on samples of soils extracted at different depths from a borehole. The laboratory results are shown in Table E8.10a. The tests at depth 5.2 m were repeated and the differences in results were about 10%. The average results are reported for this depth. Are any of the results suspect? If so, which are?

TABLE E8.10a

Depth	w (%)	PL (%)	LL (%)	s_u (kPa)	λ
2.1	22	12	32	102	0.14
3	24	15	31	10	0.12
4.2	29	15	29	10	0.09
5.2	24	17	35	35	0.1
6.4	17	13	22	47	0.07
8.1	23	12	27	85	0.1

Strategy It appears that the results at depth 5.2 m are accurate. Use the equations in Section 8.11 to predict λ and s_u and then compare the predicted with the laboratory test results.

Solution 8.10

Step 1: Prepare a table and calculate λ and s_u .

Use Eq. (8.68) to predict λ and Eq. (8.70) to predict s_u . See Table E8.10b.

Step 2: Compare laboratory test results with predicted results.

The s_u value at 2.1 m is suspect because all the other values seem reasonable. The predicted value of s_u at depth 4.2 m is low in comparison with the laboratory test results. However, the water content at this depth is the highest reported but the plasticity index is about average. If the water content were about 24% (the average of the water content just above and below 4.2 m), the predicted s_u is 10.4 kPa compared with 10 kPa from laboratory tests. The water content at 4.2 m is therefore suspect.

The s_u value at 6.4 m, water content, and liquid limit appear suspicious. Even if the water content were taken as the average for the borehole, the s_u values predicted ($\cong 1$ kPa) would be much lower

TABLE E8.10b

Depth (m)	Laboratory results					Calculated results			
	w (%)	PL (%)	LL (%)	s_u (kPa)	λ	PI	LI	λ	s_u (kPa)
2.1	22	12	32	102	0.14	20	0.50	0.12	20.1
3	24	15	31	10	0.12	16	0.56	0.096	15.0
4.2	29	15	29	10	0.09	14	1.00	0.084	2.0
5.2	24	17	35	35	0.1	18	0.39	0.108	33.4
6.4	17	13	22	47	0.07	9	0.44	0.054	25.9
8.1	23	12	27	85	0.1	15	0.73	0.09	6.9
Average	23.2	14.0	29.3						
STD ^a	3.5	1.8	4.1						

^aSTD is standard deviation.

than the laboratory results. You should repeat the tests for the sample taken at 6.4 m. The s_u value at 8.1 m is suspect because all the other values seem reasonable at these depths. ■

EXERCISES

Assume $G_s = 2.7$, where necessary.

Theory

8.1 Prove that

$$R_o = \frac{1 + 2K_o^{\text{nc}}}{1 + 2K_o^{\text{oc}}} \text{OCR}$$

8.2 Prove that

$$K_o^{\text{nc}} = \frac{6 - 2M_c}{6 + M_c}$$

8.3 Show that the effective stress path in one-dimensional consolidation is

$$\frac{q}{p'} = \frac{3M_c}{6 - M_c}$$

8.4 Show, for an isotropically heavily overconsolidated clay, that $s_u = 0.5Mp'_o(0.5R_o)^{(\lambda-\kappa)/\lambda}$.

8.5 Show that $e_\Gamma = e_c - (\lambda - \kappa) \ln 2$, where e_Γ is the void ratio on the critical state line when $p' = 1$ kPa and e_c is the void ratio on the normal consolidation line corresponding to $p' = 1$ kPa.

8.6 The water content of a soil is 55% and $\lambda = 0.15$. The soil is to be isotropically consolidated. Plot the expected void ratio against mean effective stress if the load increment ratios are (a) $\Delta p'/p' = 1$ and (b) $\Delta p'/p' = 2$.

8.7 Plot the variation of Skempton's porewater pressure coefficient at failure, A_f , with overconsolidation ratio using the CSM for two clays: one with $\phi'_{\text{cs}} = 21^\circ$ and the other with $\phi'_{\text{cs}} = 32^\circ$. Assume $\kappa = 0.2\lambda$ and the peak shear stress is the failure shear stress.

8.8 A fill of height 5m with $\lambda_{\text{sat}} = 18 \text{ kN/m}^3$ is constructed to preconsolidate a site consisting of a soft normally consolidated soil. Test at a depth of 2 m in the soil gave the following results: $w = 45\%$, $\phi'_{\text{cs}} = 23.5^\circ$, $\lambda = 0.25$, and $\kappa = 0.05$. Groundwater is at the ground surface.

- (a) Show that the current stress state of the soil prior to loading lies on the yield surface given by

$$F = (p')^2 - p'p'_c + \frac{q^2}{M^2} = 0$$

- (b) The fill is rapidly placed in lifts of 1 m. The excess porewater pressure is allowed to dissipate before the next lift is placed. Show how the soil will behave in (q, p') space and in (e, p') space.

Problem Solving

Interactive Problem Solving

Access Chapter 8 on the CD, and click on Problem Solver to interactively solve problems similar to some of the problems in this exercise. You can use the spreadsheet on the sidebar to check your results.

- 8.9** The following data were obtained from a consolidation test on a clay soil. Determine λ and κ .

p' (kPa)	25	50	200	400	800	1600	800	400	200
e	1.65	1.64	1.62	1.57	1.51	1.44	1.45	1.46	1.47

- 8.10** The water content of a sample of saturated soil at a mean effective stress of 10 kPa is 85%. The sample was then isotropically consolidated using a mean effective stress of 150 kPa. At the end of the consolidation the water content was 50%. The sample was then isotropically unloaded to a mean effective stress of 100 kPa and the water content increased by 1%. (a) Draw the normal consolidation line and the unloading/reloading line and (b) draw the initial yield surface and the critical state line in (q, p') , (e, p') , and (e, p') spaces if $\phi'_{cs} = 25^\circ$.
- 8.11** Determine the failure stresses under (a) a CU test and (b) a CD test for the conditions described in Exercise 8.10.
- 8.12** A CU triaxial test was conducted on a normally consolidated sample of a saturated clay. The water content of the clay was 50% and its undrained shear strength was 22 kPa. Estimate the undrained shear strength of a sample of this clay if $R_o = 15$, $w = 30\%$, and the initial stresses were the same as the sample that was tested. The parameters for the normally consolidated clay are $\lambda = 0.28$, $\kappa = 0.06$, and $\phi'_{cs} = 25.3^\circ$.
- 8.13** A saturated sample of soil 38 mm diameter and 76 mm high was isotropically consolidated to 200 kPa in a triaxial cell. It was decided to stop the consolidation when the excess porewater pressure (Δu) was 20 kPa. The sample was subjected to a standard undrained test ($\sigma_3 = 200$ kPa is kept constant). Failure (critical state) was recorded when $q_f = 96$ kPa, and $\Delta u_f = 152$ kPa. The water content was 40%, $\lambda = 0.16$ and $\kappa = 0.03$. Determine M and e_Γ .
- 8.14** Two samples of a soft clay are to be tested in a conventional triaxial apparatus. Both samples were isotropically consolidated under a cell pressure of 250 kPa and then allowed to swell back to a mean effective stress of 175 kPa. Sample A is to be tested under drained conditions while sample B is to be tested under undrained conditions. Estimate the stress–strain, volumetric strain (sample A), and excess porewater pressure (sample B) responses for the two samples. The soil parameters are $\lambda = 0.15$, $\kappa = 0.04$, $\phi'_{cs} = 26.7^\circ$, $e_o = 1.08$, and $\nu' = 0.3$.
- 8.15** Determine and plot the stress–strain (q versus ε_1) and volume change (ε_p versus ε_1) responses for an overconsolidated soil under a CD test. The soil parameters are $\lambda = 0.17$, $\kappa = 0.04$, $\phi'_{cs} = 25^\circ$, $\nu' = 0.3$, $e_o = 0.92$, $p'_c = 280$ kPa, and $\text{OCR} = 8$.
- 8.16** Repeat Exercise 8.15 for an undrained triaxial compression (CU) test and compare the results with the undrained triaxial extension test.
- 8.17** A sample of a clay is isotropically consolidated to a mean effective stress of 300 kPa and is isotropically unloaded to a mean effective stress of 250 kPa. An undrained triaxial extension test is to be carried out by

keeping the axial stress constant and increasing the radial stress. Predict and plot the stress–strain (q versus ε_1) and the excess porewater pressure (Δu versus ε_1) responses up to failure. The soil parameters are $\lambda = 0.23$, $\kappa = 0.07$, $\phi'_{cs} = 24^\circ$, $\nu' = 0.3$, and $e_o = 1.32$.

Practical

- 8.18** A tank of diameter 5 m is to be located on a deep deposit of normally consolidated homogeneous clay, 25 m thick. The vertical stress imposed by the tank at the surface is 75 kPa. Calculate the excess porewater pressure at depths of 2, 5, 10, and 20 m if the vertical stress were to be applied instantaneously. The soil parameters are $\lambda = 0.26$, $\kappa = 0.06$, and $\phi'_{cs} = 24^\circ$. The average water content is 42% and groundwater level is at 1 m below the ground surface.

CHAPTER 9

BEARING CAPACITY OF SOILS AND SETTLEMENT OF SHALLOW FOUNDATIONS

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9.0 INTRODUCTION

In this chapter, we will consider bearing capacity of soils and settlement of shallow foundations. Loads from a structure are transferred to the soil through a foundation. A foundation itself is a structure, often constructed from concrete, steel, or wood. A geotechnical engineer must ensure that a foundation satisfies the following two stability conditions:

1. The foundation must not collapse or become unstable under any conceivable loading.
2. Settlement of the structure must be within tolerable limits.

Both requirements must be satisfied. Often, it is settlement that governs the design of shallow foundations.

We will also consider, in this chapter, the limit equilibrium method of analysis. This is the analysis that Coulomb performed in analyzing the lateral force on the fortresses from soil placed behind them (Chapter 1). The limit equilibrium method is used to find solutions for a variety of problems including bearing capacity of foundations, stability of retaining walls, and slopes. When you complete this chapter, you should be able to:

- Calculate the safe bearing capacity of soils.
- Estimate the settlement of shallow foundations.
- Estimate the size of shallow foundations to satisfy bearing capacity and settlement criteria.

You will use the following concepts learned from previous chapters and from your courses in mechanics.

- Statics
- Stresses and strains—Chapter 5
- Shear strength—Chapter 7

Sample Practical Situation The loads from a building are to be transferred to the soil by shallow foundations. You are required to recommend the sizes of shallow foundations so that there is a margin of safety against soil failure and settlements of the building are within tolerable limits. The construction of a shallow foundation is shown in Fig. 9.1.



FIGURE 9.1 Construction of a shallow foundation.

9.1 DEFINITIONS OF KEY TERMS

Foundation is a structure that transmits loads to the underlying soils.

Footing is a foundation consisting of a small slab for transmitting the structural load to the underlying soil. Footings can be individual slabs supporting single columns (Fig. 9.2a) or combined to support two or more columns (Fig. 9.2b), or be a long strip of concrete slab (Fig. 9.2c,d, width B to length L ratio is small, i.e., it approaches zero) supporting a load bearing wall, or a mat (Fig. 9.2e).

Embedment depth (D_f) is the depth below the ground surface where the base of the foundation rests.

Shallow foundation is one in which the ratio of the embedment depth to the minimum plan dimension, which is usually the width, is $D_f/B \leq 2.5$.

Ultimate bearing capacity is the maximum pressure that the soil can support.

Ultimate net bearing capacity (q_u) is the maximum pressure that the soil can support above its current overburden pressure.

Ultimate gross bearing capacity (q_{ult}) is the sum of the ultimate net bearing capacity and the overburden pressure above the footing base.

Allowable bearing capacity or safe bearing capacity (q_a) is the working pressure that would ensure a margin of safety against collapse of the structure from shear failure. The allowable bearing capacity is usually a fraction of the ultimate net bearing capacity.

Factor of safety or safety factor (FS) is the ratio of the ultimate net bearing capacity to the allowable net bearing capacity or to the applied maximum net vertical stress. In geotechnical engineering, a factor of safety between 3 and 5 is used to calculate the allowable bearing capacity.

Ultimate limit state defines a limiting stress or force that should not be exceeded by any conceivable or anticipated loading during the design life of a foundation or any geotechnical system.

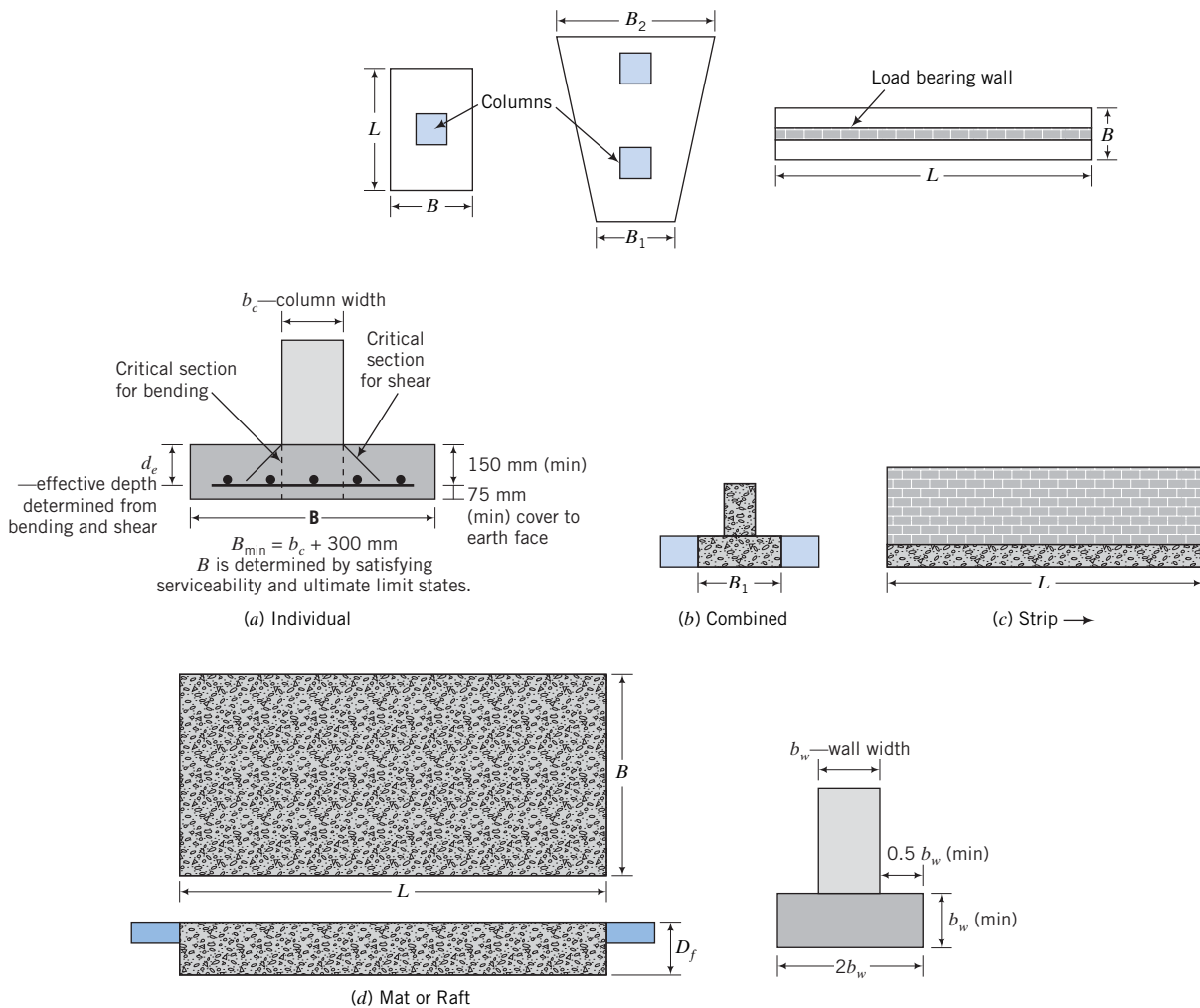


FIGURE 9.2 Types of shallow footings.

Serviceability limit state defines a limiting deformation or settlement of a foundation, which, if exceeded, will impair the function of the structure that it supports.

9.2 QUESTIONS TO GUIDE YOUR READING

1. What are the ultimate net bearing capacity and the allowable bearing capacity of shallow footings?
2. How do I determine the allowable bearing capacity for shallow footings?
3. What are the assumptions made in bearing capacity analyses?
4. What soil parameters are needed to calculate its bearing capacity?
5. What effects do groundwater and eccentric loads have on bearing capacity?
6. How do I determine the size of a footing to satisfy ultimate and serviceability limit states?

9.3 BASIC CONCEPTS



Interactive Concept Learning and Self-Assessment

Access Chapter 9, Section 9.3 on the CD to load a virtual footing resting on dense and loose soils and learn about the basic concepts of footing behavior through interactive simulations. Take Quiz 9.3 to test your understanding.

9.3.1 Collapse and Failure Loads

In developing the basic concepts we will use a generic friction angle, ϕ' , and later discuss whether to use ϕ'_p or ϕ'_{cs} . Failure in the context of bearing capacity means the ultimate gross bearing capacity. For dilating soils, failure corresponds to the peak shear stress, while for nondilating soils failure corresponds to the critical state shear stress. To distinguish these two states, we will refer to the failure load in dilating soils as the collapse load and reserve the term failure load for nondilating soils. Thus collapse load is the load at peak shear stress while failure load is the load at critical state. Collapse means a sudden decrease in the bearing capacity of a soil.

Let us consider two separate soil layers. One is a deep deposit of a dense sand; the other is a deep deposit of the same sand but in a loose state. We will now take two similar square blocks of concrete of width B and place one each on the surfaces of the two soil layers as shown in Fig. 9.3. We are going to add the same magnitude of vertical load at the center of each block and measure their settlements using displacement gauges.

Let us represent the concrete block on the dense sand layer as DS and that on the loose sand as LS. As we load the blocks, which block will settle more? Intuitively, and from your experience playing with sand, you may have guessed correctly that the loose sand would.

Here is a list of expected responses as we load the blocks.

1. Block LS will settle more than block DS for a given load before soil failure.
2. Block DS would collapse while block LS would not as illustrated in the graphs of settlement versus vertical load on the right side of Fig. 9.3.
3. Based on the Mohr–Coulomb failure criterion (Chapter 7), the soil will fail along slip planes at $\theta_1 = 45^\circ + \phi'/2$ to the horizontal plane and $\theta_2 = 45^\circ - \phi'/2$ to the vertical plane.

The pressure from the load on each block that brings the soil to collapse or failure is the soil's ultimate net bearing capacity. Would you, in practice, design a foundation so that the applied load is equal to the ultimate gross load, Q_{ult} ? Congratulations, if your answer is no. Because of spurious soil variations, uncertainty in soil test values and structural and environmental loads, and difficulties in quality control during construction, geotechnical engineers usually divide the ultimate bearing capacity by a factor of safety and call the resulting load the allowable bearing capacity.

9.3.2 Failure Surface

Prandtl (1920) showed theoretically that a wedge of material is trapped below a rigid plate when it is subjected to concentric loads. Terzaghi (1943) applied Prandtl's theory to a strip footing (length is much longer than its width) with the assumption that the soil is a semi-infinite, homogeneous, isotropic, weightless rigid-plastic material. Based on Prandtl's theory, failure of the footing occurs by a wedge of soil below the footing pushing its way downward into the soil.

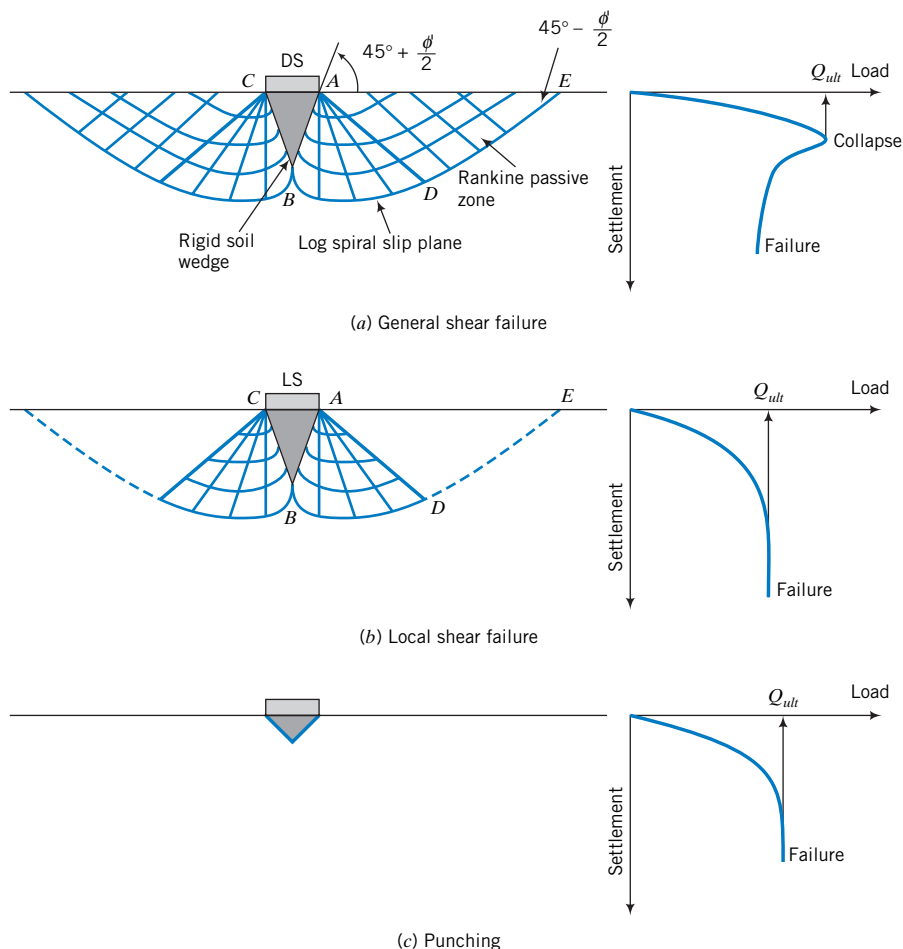


FIGURE 9.3 Failure mechanisms.

Why is the development of a wedge of soil under a shallow foundation a reasonable assumption for soils? You will recall from Chapter 7 that, according to the Mohr–Coulomb criterion, slip planes form when soils are sheared to failure. No slip plane, however, can pass through the rigid footing, so none can develop in the soil just below the footing. The slip planes form two zones around the rigid wedge—each zone symmetrical about a vertical plane (parallel to the length of the footing) through the center of the footing. One zone, ABD (Fig. 9.3), is a fan with radial slip planes stopping on a logarithm spiral slip plane. The other zone, ADE , consists of slip planes oriented at angles of $45^\circ + \phi'/2$ and $45^\circ - \phi'/2$ to the horizontal and vertical planes, respectively, as we found in Chapter 7. Zone ADE is called the Rankine passive zone. In Chapter 12, we will discuss Rankine passive zone and also Rankine active zone in connection with retaining walls. Surfaces AB and AD are frictional sliding surfaces and their actions are similar to rough walls being pushed into the soil. The pressure exerted is called passive lateral earth pressure. If we can find the value of this pressure, we can determine the ultimate gross bearing capacity by considering the equilibrium of wedge ABC .

For the dense sand, the slip planes are expected to reach the ground surface. But, for the loose sand, the slip planes, if they developed, are expected to lie within the soil layer below the base of the footing and extend laterally. The collapse in the dense sand is termed general shear failure, as the shear planes

are fully developed. The failure in the loose sand is termed local shear failure, as the shear planes are not fully developed. Another type of failure is possible. For very loose soil, the failure surfaces may be confined to the surfaces of the rigid wedge. This type of failure is termed punching shear.

The essential points are:

1. *Dense soils fail suddenly along well-defined slip planes resulting in a general shear failure.*
2. *Loose soils do not fail suddenly and the slip planes are not well defined, resulting in a local shear failure.*
3. *Very loose soils can fail by punching shear.*
4. *More settlement is expected in loose soils than in dense soils.*
5. *The expected failure surface for general shear failure consists of a rigid wedge of soil trapped beneath the footing bordering radial shear zones under Rankine passive zones.*

What's next... The bearing capacity equations that we will discuss in this chapter were derived by making alterations to the failure surface found by Prandtl. Before we consider these bearing capacity equations, we will find the collapse load for a strip footing resting on a clay soil using a popular analytical technique called the limit equilibrium method.

9.4 COLLAPSE LOAD USING THE LIMIT EQUILIBRIUM METHOD



Interactive Concept Learning and Self-Assessment

Access Chapter 9, Section 9.4 on the CD and learn the limit equilibrium method through an interactive animation and step-by-step procedures. Take Quiz 9.4 to test your understanding.

The bearing capacity equations that are in general use in engineering practice were derived using an analytical method called the limit equilibrium method. We will illustrate how to use this method by finding the collapse load (P_u) for a strip footing. The essential steps in the limit equilibrium method are (1) selection of a plausible failure mechanism or failure surface, (2) determination of the forces acting on the failure surface, and (3) use of the equilibrium equations that you learned in statics to determine the collapse or failure load.

Let us consider a strip footing of width B , resting on the surface of a homogeneous, saturated clay whose undrained strength is s_u (Fig. 9.4). For simplicity, we will neglect the weight of the soil. Step 1 of the limit equilibrium method requires that we either know or speculate on the failure mechanism. Since we do not know what the failure mechanism is because we have not done any testing, we will speculate that the footing will fail by rotating about the inner edge A (Fig. 9.4), so that the failure surface is a semicircle with radius B .

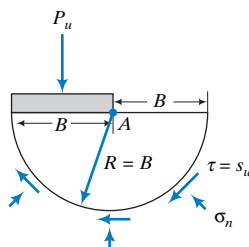


FIGURE 9.4 Circular failure mechanism.

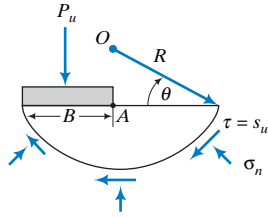


FIGURE 9.5 Circular arc failure mechanism.

Step 2 is to determine the forces on the failure surface. Along the circumference of the semicircle, there would be shear stresses (τ) and normal stresses (σ'_n). We do not know whether these stresses are uniformly distributed over the circumference, but we will assume that this is so; otherwise, we have to perform experiments or guess plausible distributions. Since failure occurred, the maximum shear strength of the soil is mobilized and therefore the shear stresses are equal to the shear strength of the soil. Now we are ready to move to Step 3. The moment due to the normal force acting on the semicircle about A is zero since its line of action passes through A. The moment equilibrium equation is then

$$P_u \times \frac{B}{2} - s_u \pi B \times B = 0 \quad (9.1)$$

and the collapse load is

$$P_u = 6.28 B s_u \quad (9.2)$$

We are unsure that the collapse load we calculated is the correct one since we guessed a failure mechanism. We can repeat the three steps above by choosing a different failure mechanism. For example, we may suppose that the point of rotation is not A but some point O above the footing such that the radius of the failure surface is R (Fig. 9.5).

Taking moments about O, we get

$$P_u(R \cos \theta - B/2) - s_u[(\pi - 2\theta)R]R = 0 \quad (9.3)$$

By rearranging Eq. (9.3), we get

$$P_u = \frac{s_u[(\pi - 2\theta)R]R}{(R \cos \theta - B/2)} = \frac{s_u(\pi - 2)R}{(\cos \theta - B/2R)} \quad (9.4)$$

The collapse load now depends on two variables, R and θ , and as such there is a family of failure mechanisms. We must then find the least load that will produce collapse. We do this by searching for extrema (minima and maxima in curves) by taking partial derivatives of Eq. (9.4) with respect to R and θ . Thus,

$$\frac{\partial P_u}{\partial R} = \frac{4s_u R(\pi - 2\theta)(R \cos \theta - B)}{(2R \cos \theta - B)^2} = 0 \quad (9.5)$$

and

$$\frac{\partial P_u}{\partial \theta} = \frac{4s_u R^2(B - 2R \cos \theta + \pi R \sin \theta - 2R \theta \sin \theta)}{(2R \cos \theta - B)^2} = 0 \quad (9.6)$$

The solutions of Eqs. (9.5) and (9.6) are $\theta = 23.2^\circ$ and $R = B \sec \theta$; that is, point O is directly above A. Substituting these values in Eq. (9.4), we obtain the collapse load as

$$P_u = 5.52 B s_u \quad (9.7)$$

This is a better solution because the collapse load is smaller than Eq. (9.2) but we need to investigate other possible mechanisms, which may yield yet a smaller value of P_u . The exact solution to our problem, using more complex analysis than the limit equilibrium method, gives

$$P_u = 5.14Bs_u \quad (9.8)$$

which is about 9% lower than our second mechanism.

What's next... We have just learned the rudiments of the limit equilibrium method. It is an iterative method in which you speculate on possible failure mechanisms and then use statics to find the collapse load. The bearing capacity equations that we will discuss next were derived using the limit equilibrium method. We will not derive the bearing capacity equations because no new concept will be learned. You can refer to the published literature mentioned later if you want to pursue the derivations.

9.5 BEARING CAPACITY EQUATIONS



Computer Program Utility and Self-Assessment

Access Chapter 9 on the CD, and click on Spreadsheet for a spreadsheet to calculate bearing capacity. Take Quiz 9.5 to test your understanding.

Terzaghi (1943) derived bearing capacity equations based on Prandtl (1920) failure mechanism and the limit equilibrium method for a footing at a depth D_f below the ground level of a homogeneous soil. For most shallow footings, the depth, D_f , called the embedment depth, accounts for frost action, freezing, thawing, and other environmental effects. Building codes provide guidance as to the minimum depth of embedment for footings. Terzaghi assumed the following:

1. The soil is a semi-infinite, homogeneous, isotropic, weightless, rigid-plastic material.
2. The embedment depth is not greater than the width of the footing ($D_f < B$).
3. General shear failure occurs.
4. The angle θ in the wedge (Fig. 9.6) is ϕ' . Later, it was found (Vesic, 1973) that $\theta = 45^\circ + \phi'/2$.
5. The shear strength of the soil above the footing base is negligible. Meyerhof (1951) Considered the shear above the footing base.

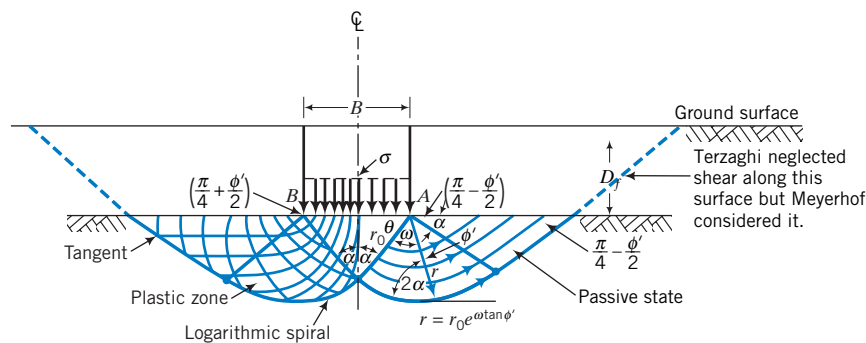


FIGURE 9.6 Failure surface assumed by Terzaghi.

6. The soil above the footing base can be replaced by a surcharge stress ($= \gamma D_f$).
7. The base of the footing is rough.

A plethora of bearing capacity equations, based on limiting equilibrium (e.g., Terzaghi, 1943; Meyerhof, 1963; Hansen, 1970; and Vesic, 1973) has been proposed. We will only consider a set of equations for general soil failure that has found general use in geotechnical practice. General shear failure is likely to occur in dense coarse-grained soils with relative density greater than 70% and in stiff fine-grained soils. Terzaghi (1943) suggested that for local shear failure ϕ'_p and s_u be reduced to 2/3 their values.

We will consider two limiting conditions. One is the short-term condition that requires a total stress analysis (TSA). TSA is applicable to fine-grained soils and the shear strength parameter is the undrained shear strength, s_u . The other is the long-term condition that requires an effective stress analysis (ESA). ESA is applicable to all soils and the shear strength parameter is the peak friction angle, ϕ'_p , from plane strain tests.

General Loading Conditions

Ultimate Net Bearing Capacity

The ultimate net bearing capacity equations for general failure can be written as

$$\text{TSA: } q_u = 5.14s_u s_c d_c i_c b_c g_c \quad (9.9)$$

$$\text{ESA: } q_u = \gamma D_f (N_q - 1) s_q d_q i_q b_q g_q + 0.5 \gamma B' N_\gamma s_\gamma d_\gamma b_\gamma g_\gamma \quad (9.10)$$

According to Eq. (9.9), the bearing capacity of fine-grained soils is not dependent on the size of the footing.

Ultimate Gross Bearing Capacity

The gross bearing capacity is

$$q_{\text{ult}} = q_u + \gamma D_f \quad (9.11)$$

Allowable Bearing Capacity

The allowable bearing capacity is

$$q_a = \frac{q_u}{\text{FS}} + \gamma D_f \quad (9.12)$$

where FS is a factor of safety ranging from 2 to 3; FS = 3 is most often used.

Vertical Centric Load Only

$$\text{TSA: } q_u = 5.14s_u s_c d_c \quad (9.13)$$

$$\text{ESA: } q_u = \gamma D_f (N_q - 1) s_q d_q + 0.5 \gamma B' N_\gamma s_\gamma d_\gamma \quad (9.14)$$

Inclined Load Only

$$\text{TSA: } q_u = 5.14s_u i_c \quad (9.15)$$

$$\text{ESA: } q_u = \gamma D_f (N_q - 1) i_q + 0.5 \gamma B' N_\gamma i_\gamma \quad (9.16)$$

where the factors, N_q and N_γ , are bearing capacity factors that are functions of ϕ'_p , s_c , s_q , and s_γ are shape factors, d_c , d_q , d_γ are embedment depth factors, i_c , i_q , i_γ are load inclination factors, b_c , b_q , b_γ are base inclination (base tilt) factors, B' is the equivalent footing width (see Fig. 9.9), and g_c , g_q , g_γ are ground inclination factors.

Bearing Capacity Factors The bearing capacity factor N_q is

$$N_q = e^{\pi \tan \phi'_p} \tan^2 \left(45^\circ + \frac{\phi'_p}{2} \right); \quad \phi'_p \text{ in degrees} \quad (9.17)$$

Various equations have been proposed for the N_γ in the literature. Among the popular equations are

$$\text{Vesic (1973): } N_\gamma = 2(N_q + 1) \tan \phi'_p; \quad \phi'_p \text{ in degrees} \quad (9.18)$$

$$\text{Meyerhof (1976): } N_\gamma = (N_q - 1) \tan(1.4\phi'_p); \quad \phi'_p \text{ in degrees} \quad (9.19)$$

$$\text{Davis and Booker (1971): } N_\gamma = 0.1054 \exp(9.6\phi'_p) \quad \text{for rough footing; } \phi'_p \text{ in radians} \quad (9.20)$$

$$N_\gamma = 0.0663 \exp(9.3\phi'_p) \quad \text{for smooth footing; } \phi'_p \text{ in radians} \quad (9.21)$$

The differences between these popular bearing capacity factors are shown in Fig. 9.7.

The bearing capacity factor, N_γ , proposed by Davis and Booker (1971) is based on a refined plasticity method and gives conservative values compared with Vesic (1973). Meyerhof's N_γ values are equal to Davis and Booker's N_γ for ϕ'_p less than about 35° . In this book, we will adopt the Davis and Booker (1971) values for N_γ .

The crucial parameter is ϕ'_p . The attainment of ϕ'_p depends on the ability of the soil to dilate, which can be suppressed by large normal effective stresses. Since neither the loads nor the stresses induced by the loads on the soil mass are certain, the use of ϕ'_p is then uncertain. We can write $\phi'_p = \phi'_{cs} + \alpha_p$ where ϕ'_{cs} is the friction angle at critical state (a fundamental soil parameter) and α_p is the peak dilation angle. The uncertainty in ϕ'_p comes mainly from the uncertainty in the value of α_p .

Plane strain conditions have been assumed in developing the theoretical bearing capacity equations. Thus, the values of ϕ'_p should come from plane strain tests such as the direct shear or direct simple shear tests. Triaxial tests on the same soil generally give values of ϕ'_p less than plane strain tests. Various conversions have been proposed. For practical purposes, the following conversion can be used

$$(\phi'_p)_{ps} = \frac{9}{8} (\phi'_p)_{tr} \quad (9.22)$$

where the subscripts ps and tr denote plane strain and triaxial, respectively.

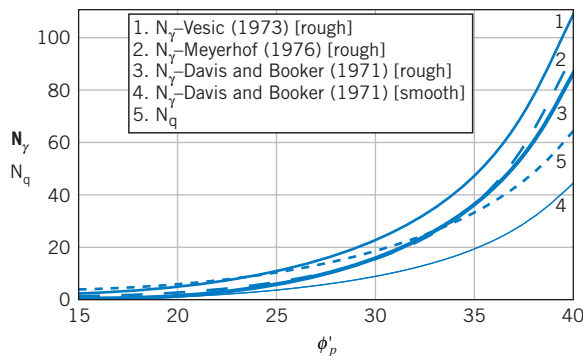


FIGURE 9.7 Comparison of some bearing capacity factors.

Rough footing denotes a footing in which full shear is developed at the footing–soil interface. It is common construction practice to compact a layer of coarse-grained soils for the footing to rest on. In this case, N_γ for rough footing is appropriate. When a footing rests directly on fine-grained soils, N_γ for smooth footing should be used.

Geometric Factors Geometric factors account for the shape and slope of the footing, the load inclination and eccentricity, and the ground slope (Fig. 9.8). Several equations have been proposed for these factors. In this book, we will adopt a set of factors proposed in the literature that are used in general practice or have been updated due to more refined analyses or experimental evidence. These factors are listed in Table 9.1 and must be regarded as estimates.

The bearing capacity equations apply for a single resultant load with normal, V_n , and horizontal components H_B parallel to the width, B (the short side), and horizontal components, H_L , parallel to the length, L (the long side). When investigating potential failure along the short side, use H_B . For failure along the long side, use H_L .

Eccentric Loads When the location of the resultant load (load center) is not coincident with the centroid (center of area) of the footing, the footing dimensions are theoretically adjusted to align the load center with the centroid. The distances from the center of area to the location of the vertical resultant load are eccentricities. Applied moments can be converted to a vertical resultant load at eccentricities, e_B and e_L , where

$$e_B = \frac{M_y}{V_n}; \quad e_L = \frac{M_x}{V_n} \quad (9.23)$$

V_n is the resultant vertical load, M_x and M_y are the moments about the X and Y axes as shown in Fig. 9.9c. Some possible cases of eccentric loads are shown in Fig. 9.9.

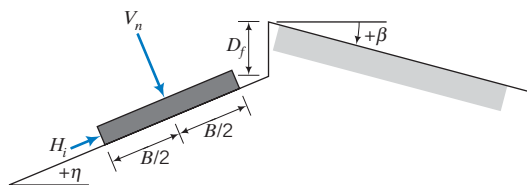


FIGURE 9.8 Footing on a slope.

TABLE 9.1 Geometric Factors for Use in Theoretical Bearing Capacity Equations

Geometric parameters for TSA				
s_c	d_c	i_c	b_c	g_c
$1 + 0.2 \frac{B'}{L'}$	$1 + 0.33 \tan^{-1} \frac{D_f}{B'}$ See note 1	$1 - \frac{nH}{5.14 s_u B' L'}$ See note 2	$\frac{\eta^\circ}{147}$ $\beta < \phi'_p; \eta^\circ + \beta^\circ < 90^\circ$ See Fig. 9.8	$\frac{\beta^\circ}{147}$ $\beta < \phi'_p; \eta^\circ + \beta^\circ < 90^\circ$ See Fig. 9.8
Geometric parameters for ESA				
s_q	d_q	i_q	b_q	g_q
$1 + \frac{B'}{L'} \tan \phi'_p$	$1 + 2 \tan \phi'_p (1 - \sin \phi'_p)^2 \tan^{-1} \left(\frac{D_f}{B'} \right)$	$\left(1 - \frac{H}{V_n} \right)^n$ See note 2	$b_q = b_\gamma$	$(1 - \tan \beta)^2$
s_γ	d_γ	i_γ	b_γ	g_γ
$1 - 0.4 \frac{B'}{L'}$	1	$\left(1 - \frac{H}{V_n} \right)^{n+1}$ See note 2	$(1 - \eta \tan \phi'_p)^2$ η is in radians	$(1 - \tan \beta)^2$

Note 1: If the shear strength of the soil above the footing is low compared with the soil below the footing, you should set all depth factors to 1. The term $\tan^{-1} \left(\frac{D_f}{B'} \right)$ is in radians.

Note 2: The depth and shape factors for inclined loads should be set to 1. For loading inclined in the direction of the width, B , $\theta = 90^\circ$ in Fig. 9.9d, $n = n_B = \left(2 + \frac{B'}{L'} \right) / \left(1 + \frac{B'}{L'} \right)$. For loading inclined in the direction of the length, L , $\theta = 0^\circ$ in Fig. 9.9d, $n = n_L = \left(2 + \frac{L'}{B'} \right) / \left(1 + \frac{L'}{B'} \right)$. For other loading conditions, $n = n_L \cos^2 \theta + n_B \sin^2 \theta$.

The stresses due to a vertical load at an eccentricity e (moment = $V_n e$) are (from your knowledge of mechanics):

$$\sigma = \frac{V_n}{A} \pm \frac{M y}{I} = \frac{V_n}{A} \pm \frac{V_n e y}{I} = \frac{V_n}{A} \pm \frac{V_n e}{Z} \quad (9.24)$$

where I is the second moment of area, y is the distance from the neutral axis to the outer edge, A is the cross-sectional area, and Z is the section modulus. For a rectangular section, $Z = \frac{I}{y} = \frac{\frac{B^3 L}{12}}{\frac{B}{2}} = \frac{B^2 L}{6}$ or $\frac{B L^2}{6}$ depending on the whether you are considering moment about the Y-axis or the X-axis.

The maximum and minimum vertical stresses along the X-axis are

$$\sigma_{\max} = \frac{V_n}{A} + \frac{V_n e}{Z} = \frac{V_n}{B L} \left(1 + \frac{6 e_B}{B} \right); \quad \sigma_{\min} = \frac{V_n}{A} - \frac{V_n e}{Z} = \frac{V_n}{B L} \left(1 - \frac{6 e_B}{B} \right) \quad (9.25)$$

and along the Y-axis are

$$\sigma_{\max} = \frac{V_n}{A} + \frac{V_n e}{Z} = \frac{V_n}{B L} \left(1 + \frac{6 e_L}{L} \right); \quad \sigma_{\min} = \frac{V_n}{A} - \frac{V_n e}{Z} = \frac{V_n}{B L} \left(1 - \frac{6 e_L}{L} \right) \quad (9.26)$$

The distribution of vertical stresses below an eccentrically loaded footing on a plane parallel to the direction of its width B and passing through the load point is shown in Fig. 9.10.

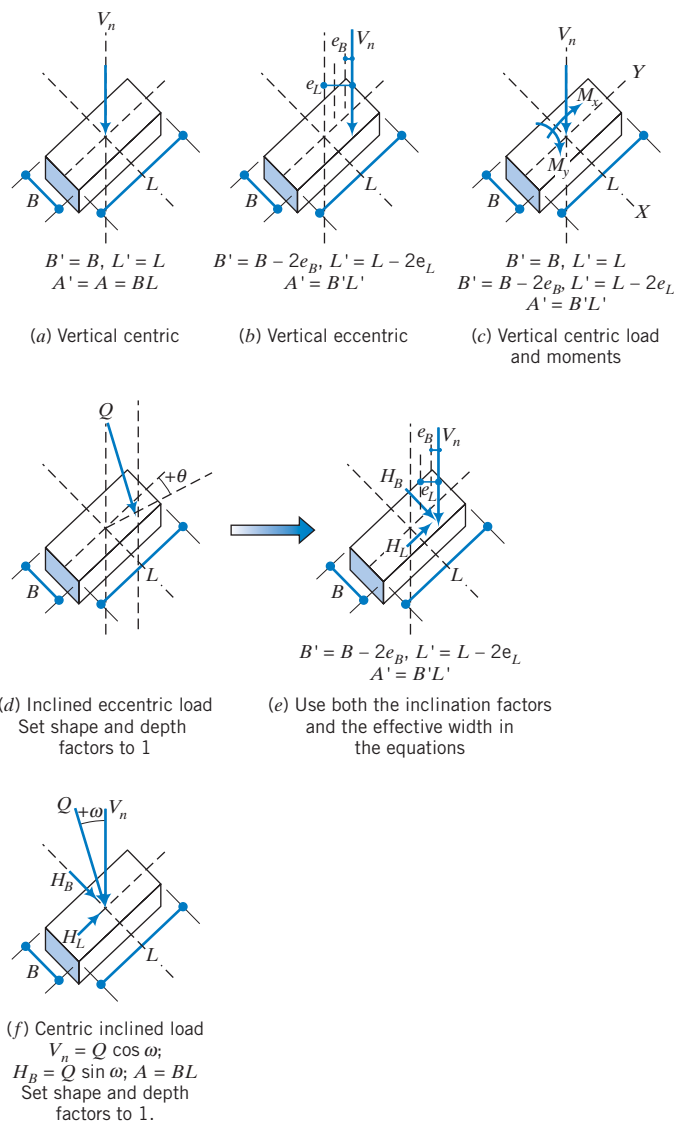


FIGURE 9.9 Some possible load cases.

Let us examine $\sigma_{\min}$. If $e_B = \frac{B}{6}$ or $e_L = \frac{L}{6}$, then $\sigma_{\min} = 0$. If, however, $e_B > \frac{B}{6}$ or $e_L > \frac{L}{6}$, then $\sigma_{\min} < 0$, and tension develops. Since the tensile strength of soil is approximately zero, part of the footing will not transmit loads to the soil. You should try to avoid this situation by designing the footing such that $e_B < \frac{B}{6}$ and $e_L < \frac{L}{6}$.

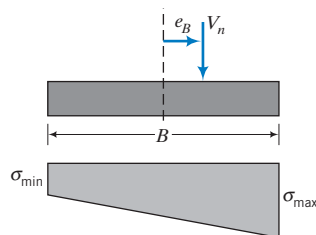
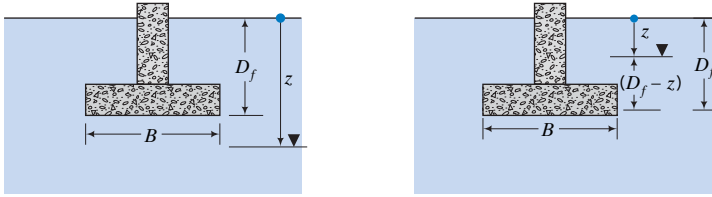


FIGURE 9.10 Vertical stress distribution below an eccentrically loaded footing.

(a) Groundwater within a depth B below base

(b) Groundwater within embedment depth

FIGURE 9.11 Groundwater effects below base of footing.

The ultimate net load is:

$$P_u = q_u B' L' \quad (9.27)$$

The effective width B' and effective length, L' , giving an effective area, $A' = B' L'$ must be used in the theoretical bearing capacity equations. If the load is centric, then $B = B'$ and $L = L'$. For circular footing, $B = L = D$ where D is diameter. The equivalent foundation area for a circular foundation subjected to an eccentric load is

$$A' = \frac{D^2}{2} \left[\cos^{-1} \frac{2e}{D} - \frac{2e}{D} \sqrt{1 - \left(\frac{2e}{D} \right)^2} \right] \quad (9.28)$$

Groundwater Effects You have to adjust the theoretical bearing capacity equations for groundwater condition. The term γD_f in the bearing capacity equations for an ESA refers to the vertical stress of the soil above the base of the foundation. The last term γB refers to the vertical stress of a soil mass of thickness, B , below the base of the footing. You need to check which one of three groundwater situations given below is applicable to your project.

Situation 1. Groundwater level at a depth greater than or equal to B below the base of the footing. No correction is required.

Situation 2. If the groundwater level is within a depth B below the base of the footing $D_f < z < (B + D_f)$ (Fig. 9.11a), then the term γB is $\gamma z + \gamma'(B - z)$ or $\gamma_{\text{sat}} z + \gamma'(B - z)$. The latter equation is used if the soil is saturated. The term γD_f remains unchanged.

Situation 3. If the groundwater level is within the embedment depth $0 \leq z \leq D_f$ (Fig. 9.11b), then the term γD_f is: $\gamma z + \gamma'(D_f - z)$ or $\gamma_{\text{sat}} z + \gamma'(D_f - z)$. The latter equation is used if the soil is saturated. The term γB becomes $\gamma' B$.

EXAMPLE 9.1 Allowable Bearing Capacity of a Sand

A footing 2 m square, subjected to a centric vertical load, is located at a depth of 1.0 m below the ground surface in a deep deposit of compacted sand ($\phi'_p = 35^\circ$, $\phi'_{\text{cs}} = 30^\circ$ and $\gamma_{\text{sat}} = 18 \text{ kN/m}^3$). The groundwater level is 5 m below the ground surface but you should assume that the soil above the groundwater is saturated. The friction angles were obtained from plain strain tests. Determine the allowable bearing capacity for a factor of safety of 3.

Strategy It is a good policy to sketch a diagram illustrating the conditions given (see Fig. E9.1). The groundwater level is located at $(5 \text{ m} - 1 \text{ m}) = 4 \text{ m}$ from the footing base. That is, the groundwater level is more than $B = 2 \text{ m}$ below the base. We can neglect the effects of groundwater.

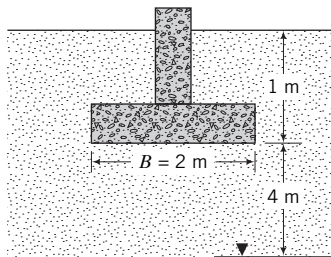


FIGURE E9.1

Solution 9.1

Step 1: Calculate the bearing capacity factors and geometric factors. Assume rough footing. Use $\phi' = \phi'_p = 35^\circ$.

$$N_q = e^{\pi \tan \phi'} \tan^2 \left(45^\circ + \frac{\phi'_p}{2} \right) = 33.3$$

$$N_q - 1 = 32.3$$

$$N_\gamma = 0.1054 \exp \left(9.6 \times \frac{35 \times \pi}{180} \right) = 37.1$$

$$s_q = 1 + \frac{B}{L} \tan \phi'_p = 1.70$$

$$s_\gamma = 1 - 0.4 \frac{B}{L} = 0.6$$

$$d_\gamma = 1.0$$

$$\begin{aligned} d_q &= 1 + 2 \tan \phi'_p (1 - \sin \phi'_p)^2 \tan^{-1} \left(\frac{D_f}{B'} \right) \\ &= 1 + 2 \tan(35^\circ) (1 - \sin 35^\circ)^2 \left[\tan^{-1} \left(\frac{1}{2} \times \frac{\pi}{180} \right) \right] = 1.13 \end{aligned}$$

Step 2: Calculate the ultimate net bearing capacity.

$$q_u = \gamma D_f (N_q - 1) s_q d_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma$$

$$\begin{aligned} q_u &= 18 \times 1 \times 32.3 \times 1.7 \times 1.13 + 0.5 \times 18 \times 2 \times 37.1 \times 0.6 \times 1.0 \\ &= 1515 \text{ kPa} \end{aligned}$$

$$\begin{aligned} q_a &= \frac{q_u}{\text{FS}} + \gamma D_f \\ &= \frac{1515}{3} + 18 \times 1 = 523 \text{ kPa} \end{aligned}$$

EXAMPLE 9.2 The Effects of Groundwater on Bearing Capacity

Compare the ultimate net bearing capacity for Example 9.1 using $\phi'_p = 35^\circ$ when the groundwater is located (a) at 5 m below the ground surface, (b) at the ground surface, (c) at the base of the footing, and (d) at 1 m below the base.

Strategy The trick here is to use the appropriate value of the unit weight in the bearing capacity equation.

Solution 9.2

Step 1: Calculate bearing capacity numbers and shape and depth factors.

These values are the same as an Example 9.1.

Step 2: Substitute values from Step 1 into Eq. (9.14).

- (a) *Groundwater level at 5 m below the surface.* The groundwater level is 4 m below the base, which is greater than the width of the footing. Therefore, groundwater has no effect.

$$\begin{aligned}\gamma &= \gamma_{\text{sat}} \\ q_u &= 1515 \text{ kPa} \quad \text{from Example 9.1}\end{aligned}$$

- (b) *Groundwater level at the ground surface.* In this case, the groundwater level will affect the bearing capacity. You should use

$$\begin{aligned}\gamma' &= \gamma_{\text{sat}} - \gamma_w = 18 - 9.8 = 8.2 \text{ kN/m}^3 \\ q_u &= \gamma' D_f (N_q - 1) s_q d_q + 0.5 \gamma' B' N_\gamma s_\gamma d_\gamma \\ &= 8.2 \times 1 \times 32.3 \times 1.7 \times 1.13 + 0.5 \times 8.2 \times 2 \times 37.1 \times 0.6 \times 1.0 \\ &= 691 \text{ kPa}\end{aligned}$$

Alternatively, since the change in the unit weight is the same for both terms of the bearing capacity equation, we can simply find q_u by taking the ratio γ'/γ , that is,

$$q_u = 1515 \times \frac{8.2}{18} = 690 \text{ kPa}$$

- (c) *Groundwater level at the base.* In this case, the groundwater level will affect the last term in the bearing capacity.

Thus,

$$\begin{aligned}q_u &= \gamma D_f (N_q - 1) s_q d_q + 0.5 \gamma' B' N_\gamma s_\gamma d_\gamma \\ &= 18 \times 1 \times 32.3 \times 1.7 \times 1.13 + 0.5 \times 8.2 \times 2 \times 37.1 \times 0.6 \times 1.0 \\ &= 1299 \text{ kPa}\end{aligned}$$

- (d) *Groundwater level at 1 m below the base.* In this case, the groundwater level is within a depth B below the base and will affect the last term in the bearing capacity, where you should use

$$\gamma' B' = \gamma_{\text{sat}} z + \gamma' (B' - z) = 18 \times 1 + 8.2 \times (2 - 1) = 26.2 \text{ kN/m}^2$$

Thus,

$$\begin{aligned}q_{\text{ult}} &= \gamma D_f (N_q - 1) s_q d_q + 0.5 (\gamma' B') N_\gamma s_\gamma d_\gamma \\ &= 18 \times 1 \times 32.3 \times 1.7 \times 1.13 + 0.5 \times 26.2 \times 37.1 \times 0.6 \times 1.0 \\ &= 1408 \text{ kPa}\end{aligned}$$

Step 3: Compare results.

We will compare the results by dividing (normalizing) each ultimate net bearing capacity by the ultimate net bearing capacity of case (a).

Groundwater level at	$\frac{q_u}{(q_u)_a} \times 100$
(b) Ground surface	$\frac{690}{1515} \times 100 \approx 46\%$
(c) Base	$\frac{1299}{1515} \times 100 = 86\%$
(d) 1 m below base	$\frac{1408}{1515} \times 100 = 93\%$

The groundwater level rising to the surface will reduce the bearing capacity by more than one-half. ■

EXAMPLE 9.3 Allowable Short-term Bearing Capacity of a Clay Soil

A footing $1.8\text{ m} \times 2.5\text{ m}$ is located at a depth of 1.5 m below the ground surface in a deep deposit of an overconsolidated clay. The groundwater level is 2 m below the ground surface. The undrained shear strength is 120 kPa and $\gamma_{\text{sat}} = 20\text{ kN/m}^3$. Determine the allowable bearing capacity, assuming a factor of safety of 3, for short-term condition. Neglect the effects of embedment.

Strategy Use the equation for the short-term bearing capacity. You do not need to consider the effect of groundwater when you are evaluating short-term condition.

Solution 9.3

Step 1: Calculate geometric factors.

$$s_c = 1 + 0.2 \frac{B'}{L'} = 1 + 0.2 \frac{1.8}{2.5} = 1.14, \quad d_c = 1$$

Step 2: Calculate q_u .

$$q_u = 5.14 s_u s_c d_c = 5.14 \times 120 \times 1.14 \times 1 = 703\text{ kPa}$$

Step 3: Calculate q_a

$$q_a = \frac{q_u}{\text{FS}} + \gamma D_f = \frac{703}{3} + 1.5 \times 20 = 264\text{ kPa} \quad \blacksquare$$

EXAMPLE 9.4 Sizing a Rectangular Footing

Determine the size of a rectangular footing to support a vertical centric load of 1800 kN on a granular soil. The friction angle obtained from a triaxial test was $\phi'_p = 28.4^\circ$ and $\gamma_{\text{sat}} = 18\text{ kN/m}^3$. The footing is to be located at 1 m below the ground surface. Groundwater level is 6 m below the ground surface. Assume $\text{FS} = 3$.

Strategy Neither the footing width nor the length is given. Both of these are required to find q_a . You can fix a length to width ratio and then assume a width (B). Solve for q_a and if it is not satisfactory [$q_a \geq (\sigma)_{\text{max}}$] then reiterate using a different B value. You have to convert the triaxial friction angle to an equivalent plane strain value using Eq. (9.22).

Solution 9.4

Step 1: Calculate bearing capacity numbers, shape and depth factors.

Assume, $L/B = \frac{L'}{B'} = 1.5$; that is, $B/L = \frac{B'}{L'} = 0.67$, and $B = B' = 1.5\text{ m}$.

$$(\phi'_p)_{\text{ps}} = \frac{9}{8} (\phi'_p)_{\text{tr}} = \frac{9}{8} \times 28.4^\circ = 32^\circ$$

$$N_q = e^{\pi \tan 32^\circ} \tan^2(45^\circ + 32^\circ/2) = 23.2$$

$$N_q - 1 = 23.2 - 1 = 22.2$$

Assume rough footing.

$$\begin{aligned}
 N_\gamma &= 0.1054 \exp(9.6\phi'_p) = 0.1054 \exp\left(9.6 \times \frac{32 \times \pi}{180}\right) \\
 &= 22.5 \\
 s_q &= 1 + \frac{B'}{L'} \tan \phi'_p = 1 + 0.67 \tan 32^\circ = 1.42 \\
 s_\gamma &= 1 - 0.4 \frac{B'}{L'} = 0.73 \\
 d_q &= 1 + 2 \tan \phi'_p (1 - \sin \phi'_p)^2 \tan^{-1} \frac{D_f}{B'} \\
 &= 1 + 2 \tan 32^\circ (1 - \sin 32^\circ)^2 \left[\tan^{-1} \left(\frac{1}{1.5} \times \frac{\pi}{180} \right) \right] \\
 &= 1.16 \\
 d_\gamma &= 1.0
 \end{aligned}$$

Step 2: Substitute the above values into the bearing capacity equation, Eq. (9.14). The groundwater level is located more than B below the base. Therefore, groundwater will not affect the bearing capacity.

$$\begin{aligned}
 q_u &= \gamma D_f (N_q - 1) s_q d_q + 0.5 \gamma B' N_\gamma s_\gamma d_\gamma \\
 &= 18 \times 1 \times 22.2 \times 1.42 \times 1.16 + 0.5 \times 18 \\
 &\quad \times 1.5 \times 22.5 \times 0.73 \times 1.0 = 880 \text{ kPa} \\
 q_a &= \frac{q_u}{\text{FS}} + \gamma D_f = \frac{880}{3} + 18 \times 1 = 311 \text{ kPa} \\
 (\sigma)_{\max} &= \frac{\text{Applied load}}{\text{Area}} = \frac{1800}{1.5 \times (1.5 \times 1.5)} = 533 \text{ kPa} > q_a (= 311 \text{ kPa})
 \end{aligned}$$

Unacceptable; try another footing size.

Step 3: Try another width and recalculate.

We need to try a larger B . Try $B = B' = 2$ m. The depth factor for this case changes to 1.13. s_γ and s_q have the same values.

The new allowable bearing capacity is

$$\begin{aligned}
 q_a &= \frac{\gamma D_f (N_q - 1) s_q d_q + 0.5 \gamma B' N_\gamma s_\gamma d_\gamma}{\text{FS}} + \gamma D_f \\
 &= \frac{18 \times 1 \times 22.2 \times 1.42 \times 1.13 + 0.5 \times 18 \times 2 \times 22.5 \times 0.73 \times 1.0}{3} + 18 \times 1 \\
 &= 330 \text{ kPa} \\
 (\sigma)_{\max} &= \frac{\text{Applied load}}{\text{Area}} = \frac{1800}{2 \times (1.5 \times 2)} = 300 \text{ kPa} < q_a.
 \end{aligned}$$

The footing size is then 2 m $\times$ 3 m. ■

EXAMPLE 9.5 Allowable Bearing Capacity with an Inclined Load

Using the footing geometry of Example 9.1, determine q_a with the load of 2000 kN inclined at 20° to the vertical along the footing width.

Strategy You need to use Eq. (9.16) for inclined loads. You only need to calculate the inclination factors, since shape and depth factors are not included for load inclination.

Solution 9.5

Step 1: Calculate the inclination factors and depth factors.

$$B = B'; \quad \omega = 20^\circ$$

$$n = n_B = \left(2 + \frac{B'}{L'}\right) / \left(1 + \frac{B'}{L'}\right) = (2 + 1) / (1 + 1) = 1.5$$

$$i_q = \left(1 - \frac{H}{V_n}\right)^n = (1 - \tan \omega)^n = (1 - \tan 20^\circ)^{1.5} = 0.51$$

$$i_\gamma = \left(1 - \frac{H}{V_n}\right)^{n-1} = (1 - \tan \omega)^{n-1} = (1 - \tan 20^\circ)^{1.5-1} = 0.32$$

Step 2: Calculate the ultimate net bearing capacity.

Use Eq. (9.16).

$$q_u = \gamma D_f (N_q - 1) i_q + 0.5 \gamma B N_\gamma i_\gamma = 18 \times 1 \times 32.3 \times 0.51 \\ + 0.5 \times 18 \times 2 \times 37.1 \times 0.32 = 510 \text{ kPa}$$

$$q_a = \frac{q_{ult}}{FS} + \gamma D_f = \frac{510}{3} + 18 \times 1 = 188 \text{ kPa}$$

EXAMPLE 9.6 Factor of Safety of a Footing Subjected to a Vertical Load and a Moment

The footing in Example 9.1 is subjected to a vertical load of 500 kN and a moment about the Y axis of 125 kN·m. Calculate the factor of safety.

Strategy Since we are only given the moment about the Y axis, we only need to find the eccentricity, e_B . The bearing capacity factors are the same as those in Example 9.2.

Solution 9.6

Step 1: Draw a sketch of the problem and calculate e_B .

See Fig. E9.6 for a sketch.

$$e_B = \frac{125}{500} = 0.25 \text{ m}$$

Step 2: Check if tension develops.

$$\frac{B}{6} = \frac{2}{6} = 0.33 \text{ m} > 0.25 \text{ m}$$

Therefore, tension will not occur.

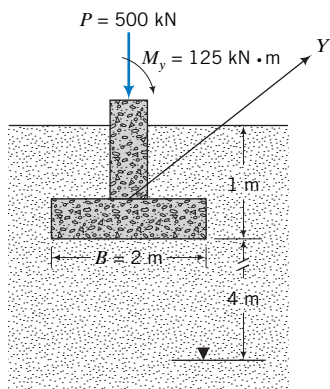


FIGURE E9.6

Step 3: Calculate the maximum vertical stress.

$$\sigma_{\max} = \frac{P}{BL} \left(1 + \frac{6e_B}{B} \right) = \frac{500}{2 \times 2} \left(1 + \frac{6 \times 0.25}{2} \right) = 219 \text{ kPa}$$

Step 4: Calculate reduced footing size.

$$B' = 2 - 2(0.25) = 1.5 \text{ m}$$

Step 5: Calculate the shape and depth factors.

$$s_q = 1 + \frac{B'}{L} \tan \phi'_p = 1 + \frac{1.5}{2} \tan 35^\circ = 1.53$$

$$s_\gamma = 1 - 0.4 \frac{B'}{L} = 1 - 0.4 \frac{1.5}{2} = 0.7$$

$$d_q = 1 + 2 \tan \phi'_p (1 - \sin \phi'_p)^2 \tan^{-1} \left(\frac{D_f}{B'} \right) = 1 + 2 \tan 35^\circ (1 - \sin 35^\circ)^2 \left[\tan^{-1} \left(\frac{1}{1.5} \times \frac{\pi}{180} \right) \right] = 1.15$$

$$d_\gamma = 1$$

Step 6: Substitute the appropriate values into the bearing capacity equation.

$$\begin{aligned} q_u &= \gamma D_f (N_q - 1) s_q d_q + 0.5 \gamma B' N_\gamma s_\gamma d_\gamma \\ &= 18 \times 1 \times 32.3 \times 1.53 \times 1.15 + 0.5 \times 18 \times 15 \times 37.1 \times 0.7 \times 1.0 \\ &= 1373 \text{ kPa} \end{aligned}$$

Step 7: Calculate the factor of safety.

$$FS = \frac{q_u}{(\sigma_a)_{\max} - \gamma D_f} = \frac{1373}{219 - 1 \times 18} = 6.8$$

What's next... In some projects, spread or individual foundations may overlap each other or differential settlements may be intolerable. One solution is to use a foundation that covers the entire loaded area. These foundations are called mat or raft foundations. Next, we will introduce mat foundations.

9.6 MAT FOUNDATIONS



Concept Learning and Self-Assessment

Access Chapter 9, Section 9.6 to learn about mat foundations. Take Quiz 9.6 to test your understanding.

A mat foundation is conventionally a concrete slab used when

- Spread or individual footings cover over 50% of the foundation area because of large column loads and/or because the soil is soft with a low bearing capacity.
- Pockets of soft soils are present.
- The structure is sensitive to differential settlement.

A mat foundation can be located at the surface or buried deep into the soil to compensate for all or part of the applied loads. Mat foundations are described frequently as raft foundations because they act like rafts when part or all of the loads are compensated by embedment. Let us suppose that the average vertical stress at the ground surface from a structure including the self-weight of the raft is 50 kPa and the unit weight of the soil is 20 kN/m³. Then, if the raft is buried at a depth $D_f = 50/20 = 2.5$ m we get a fully compensated raft. If the raft is buried say 2 m, then the compensation is $\frac{2 \times 20}{50} \times 100 = 80\%$. The raft

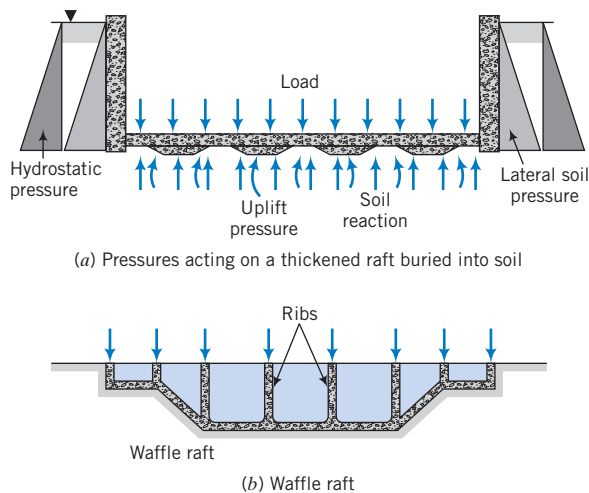


FIGURE 9.12 Two types of mats (rafts).

is then partially compensated. Mat foundations may also be supported on piles to transfer the load to stronger soils. These are called piled rafts. Two types of rafts are shown in Fig. 9.12.

Mats are complex soil-structure interaction problems and require advanced analyses that are beyond the scope of this textbook. The bearing capacity of a mat is calculated similar to a spread foundation. However, the settlement of the mat is much more complex. The settlement depends on the rigidity of the mat, type of mat, the type of soil, the homogeneity of the soil, groundwater condition, and construction method. In designing a mat foundation, you should consider

1. The loads to be supported. Very heavy loads may require a fully or partially compensated raft foundation.
2. The sensitivity of the structure and any machines to settlement.
3. The stability of the mat particularly if it is embedded in the ground. If groundwater level is below the possible depth of excavation, the theoretical depth of excavation in a fine-grained soil is given by Bjerrum and Eide (1956) as

$$(D_f)_{cr} = N_c \frac{s_u}{\gamma} \quad (9.29)$$

and the factor of safety against bottom heave is

$$FS = N_c \frac{s_u}{\gamma D_f + q_s} \quad (9.30)$$

where $N_c \approx 6 \left(1 + 0.2 \frac{D_f}{B}\right) I_c$ for $\frac{D_f}{B} \leq 2.5$, $N_c = 9 I_c$ for $\frac{D_f}{B} > 2.5$, $I_c = (0.84 + 0.16 \frac{B}{L})$, s_u is the undrained shear strength, q_s is the average foundation vertical stress, B and L are the width and length of the raft respectively. If the groundwater level is within the possible depth of excavation and the excavation is done under water, replace γ by $(\gamma_{sat} - \gamma_w)$ in Eq. (9.30).

4. Heaving. When a fine-grained soil is excavated to embed the raft, it will swell. This will be reversed (reconsolidation) when the foundation is constructed resulting in settlement.

EXAMPLE 9.7 Allowable Bearing Capacity for a Mat Foundation

A tank foundation 10 m diameter is required to support a vertical centric load of 15,700 kN on the surface of a deep deposit of a stiff clay with $s_u = 80$ kPa, $\phi'_p = 25^\circ$, and $\gamma_{sat} = 20.8$ kN/m³. Groundwater

is at the surface. A 75 mm thick compacted granular base will be placed on the clay surface prior to the construction of the tank foundation. Calculate the factor of safety.

Strategy For this mat, you need to consider both short-term and long term conditions. The granular base course will impart a rough condition at the soil–foundation interface. The groundwater is at the surface. Therefore, the effective unit weight should be used in the bearing capacity equations. For a surface footing, $D_f = 0$; the first term in the long-term bearing capacity equation is zero. Therefore, do not calculate N_q .

Solution 9.7

Step 1: Calculate the bearing capacity and geometric factors.

$$N_\gamma = 0.1054 \exp(9.6\phi'_p) = 0.1054 \exp\left(9.6 \times 25 \frac{\pi}{180}\right) = 7$$

The foundation is at the surface. Set all depth factors to 1.

$$D = B = L = L' = B' = 10 \text{ m}$$

$$s_c = 1 + 0.2 \frac{B'}{L'} = 1.2$$

$$s_\gamma = 1 + \frac{B'}{L'} \tan \phi'_p = 1 + \tan 35^\circ = 1.7$$

Step 2: Calculate the short-term bearing capacity.

$$q_u = 5.14 s_u s_c d_c = 5.14 \times 80 \times 1.2 \times 1 = 493 \text{ kPa}$$

Step 3: Calculate the long-term bearing capacity.

$$q_u = \gamma D_f (N_q - 1) s_q d_q + 0.5 \gamma' B' N_\gamma s_\gamma d_\gamma = 0 + 0.5 \times 10 \times 10 \times 7 \times 1.7 \times 1 = 595 \text{ kPa}$$

Step 4: Calculate factor of safety.

Short-term condition controls.

$$\sigma = \frac{\text{load}}{\text{area}} = \frac{15700}{\pi \frac{10^2}{4}} = 200 \text{ kPa}$$

$$\text{FS} = \frac{493}{200} \cong 2.5$$

9.7 BEARING CAPACITY OF LAYERED SOILS

No simple, satisfactory, analytical method is currently available to determine the bearing capacity of layered soils. If the thickness, H_1 , of the soil below the footing in the top layer (Fig. 9.13) is greater than

$$H_{\text{cr}} = \frac{B}{2 \cos\left(45^\circ + \frac{\phi'_p}{2}\right)} \exp[A \tan \phi'_p] ; A = \left(45^\circ - \frac{\phi'_p}{2}\right) \text{ in radians} \quad (9.31a)$$

the failure surface will be confined in the top layer and it is sufficiently accurate to calculate the bearing capacity based on the properties of the soil in the top layer. Otherwise, the failure surface would be influenced by the bottom layer and may extend into it. Alternatively, H_{cr} can be calculated from

$$H_{\text{cr}} = \frac{3B \ln \frac{(q_u)_t}{(q_u)_b}}{2 \left(1 + \frac{B}{L}\right)} \quad (9.31b)$$

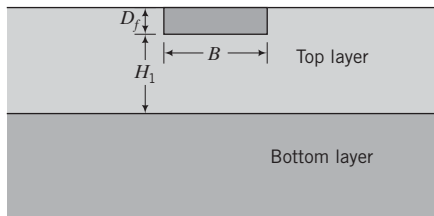


FIGURE 9.13 Footing on a two layer soil.

where $(q_u)_t$ is the ultimate net bearing capacity of the top layer and the $(q_u)_b$ is the ultimate net bearing capacity of the bottom layer with a fictitious footing of the same size and shape but resting on the surface of the bottom layer. A geotechnical engineer can apply a set of practical guidelines or use numerical tools such as the finite element method for analyzing layered soils. The basic problem lies in determining and defining the soil properties for layered soils. We will resort to some practical guidelines for three common cases—a soft clay over a stiff clay, a stiff clay over a soft clay, and thinly stratified soils.

Soft clay over stiff clay: In general, shallow foundations on soft clays should be avoided except lightly loaded structures such as houses and one-story buildings. You should investigate removing the soft clay and replacing it with compacted fills. An inexperienced geotechnical engineer should calculate the bearing capacity using the methods described previously before making a decision to replace the soft clay.

Stiff clay over soft clay: The bearing capacity for this case is the smaller value of (1) treating the stiff clay as if the soft clay layer does not exist and (2) assuming that the footing punches through the stiff clay and is supported on the soft clay. The bearing capacity is the sum of the shear required to punch through a vertical plane in the stiff clay and the bearing capacity of the soft clay layer. Only a fraction, about $\frac{2}{3}$ to $\frac{1}{2}$, of the undrained shear strength should be used in computing the shear resistance on the vertical plane in the stiff clay layer.

Another method is to place an imaginary footing on the soft clay layer with dimensions $(B + t_{sc}) \times (L + t_{sc})$, where B and L are the real width and length of the footing and t_{sc} is the thickness of the stiff clay layer below the base, and then calculate the ultimate net bearing capacity using the bearing capacity equations.

Thinly stratified soils: In this type of deposit, deep foundations should be used. If deep foundations are uneconomical, then the bearing capacity can be calculated by using the shear strength parameters for the weakest layer. Alternatively, harmonic mean values for s_u and ϕ' can be calculated (see Chapter 6) and then these values can be used to calculate the bearing capacity.

EXAMPLE 9.8 *Bearing Capacity of a Stiff Clay Over a Soft Clay*

The soil profile at a site is shown in Fig. E9.8. A square footing 5 m wide is located at 1.0 m below ground level in the stiff clay. Determine the safety factor for short-term loading for an applied load of 6875 kN. Neglect effects of embedment.

Strategy Check critical height below footing. If the critical height is less than the soil thickness below the base, check the factor of safety of the stiff clay assuming that the soft clay layer does not exist. We can then use an artificial footing on top of the soft clay and calculate the factor of safety.

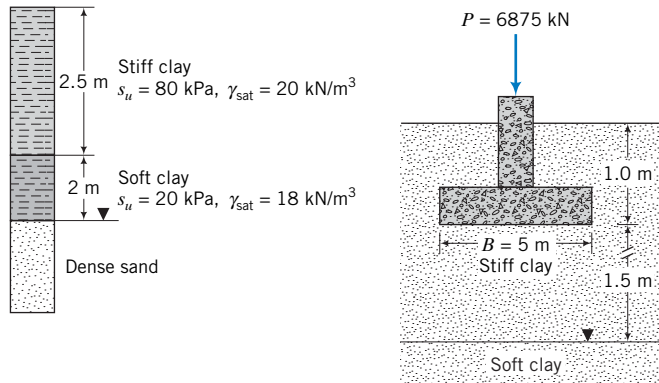


FIGURE E9.8

Solution 9.8**Step 1:** Check critical height.

$$H_{cr} = \frac{B}{2 \cos\left(45^\circ + \frac{\phi'}{2}\right)} \exp[A \tan \phi'_p] = \frac{B}{2 \cos 45^\circ} = \frac{5}{2 \cos 45^\circ} = 1.77 \text{ m}$$

The height of soil below the footing to the top of the soft clay is $2.5 - 1 = 1.5 \text{ m} < H_{cr}$. Use stiff clay over soft clay guidelines.

Step 2: Calculate the factor of safety for the stiff clay.

$$\begin{aligned} s_c &= 1 + 0.2 \frac{B'}{L'} = 1.2 \\ d_c &= 1 \\ q_u &= 5.14 s_u s_c d_c = 5.14 \times 80 \times 1.2 \times 1 \\ &= 493 \text{ kPa} \\ (\sigma_a)_{\max} &= \frac{P}{A} = \frac{3875}{5 \times 5} = 155 \text{ kPa} \\ \text{FS} &= \frac{q_u}{(\sigma_a)_{\max} - \gamma D_f} = \frac{493}{155 - 20 \times 1.0} = 3.7 \end{aligned}$$

Step 3: Check the safety factor for the soft clay.

Artificial footing size: $B + t_{sc} = 5 + 1.5 = 6.5 \text{ m}$; $L + t_{sc} = 5 + 1.5 = 6.5 \text{ m}$

$$\begin{aligned} q_u &= 5.14 s_u s_c d_c = 5.14 \times 20 \times 1.2 \times 1 = 123 \text{ kPa} \\ (\Delta \sigma_a)_{\max} &= \frac{P}{A} = \frac{3875}{6.5 \times 6.5} = 92 \text{ kPa} \\ \text{FS} &= \frac{123}{92 - 2.5 \times 20} = 2.9 \end{aligned}$$

9.8 BUILDING CODES BEARING CAPACITY VALUES

Building codes usually provide recommended values of bearing capacity for local conditions. You can use these values for preliminary design but you should check these values using soil test data and the

TABLE 9.2 Allowable Bearing Capacity (IBC, 2003)

Soil type	q_a (kPa)
Sandy gravel/gravel (GW and GP)	145
Sand, silty sand, clayey sand, silty gravel (SW, SP, SM, SC, GM, GC)	96
Clay, sandy clay, silty clay, and clayey silt (CL, ML, MH, and CH)	72

bearing capacity equations. Table 9.2 shows the allowable bearing capacity values for general soil types (IBC, 2003).

What's next... The size of many shallow foundations is governed by settlement rather than bearing capacity considerations. That is, serviceability limit state governs the design rather than ultimate limit state. Next, we will consider how to determine settlement for shallow foundations.

9.9 SETTLEMENT

It is practically impossible to prevent settlement of shallow foundations. At least, elastic settlement will occur. Your task as a geotechnical engineer is to prevent the foundation system from reaching a serviceability limit state. A description of some serviceability limit states is given in Table 9.3.

Foundation settlement can be divided in three basic types: rigid block or uniform settlement (Fig. 9.14a), tilt or distortion (Fig. 9.14b) and nonuniform settlement (Fig. 9.14c). Most damage from uniform settlement is limited to surrounding drainage systems, attached buildings, and utilities. Distortion and nonuniform settlements are caused by differential movement and may cause serious problems especially in tall buildings. Distortion produces bending and is the cause of most cracking in structures. Distortion is quantified by δ/ℓ , where δ is the maximum differential settlement, and ℓ is the length over which the settlement occurs. This distortion is an angular measurement (radians). If we were able to accurately model soil behavior and be certain of the loading conditions and soil variations, then we would be able to calculate distortions that would cause damage to structures. Unfortunately, methods of calculating settlements have been found to be unreliable and we have to resort to experiments using models and collecting and analyzing data from field observations.

Skempton and MacDonald (1956) surveyed 98 buildings and found 40 to be damaged. They then correlated the occurrence or nonoccurrence of damage with the maximum distortion and recommended the limits shown in Table 9.4.

The data of Skempton and MacDonald (1956) do not apply to buildings that settled from the presence of deep seated clay layers (e.g., a clay layer overlaid by a thick deposit of sand). All buildings

TABLE 9.3 Serviceability Limit States

Serviceability limit state	Examples
Architectural damage (damage to appearance)	Tilting of structures (chimneys, retaining walls) and cracking in walls
Loss of serviceability	Cracked floors, misalignment of machinery, dislocation of pipe joints, jammed doors and windows
Structural damage (collapse)	Severe differential settlement of footings causing buckling of columns and overstressing beams

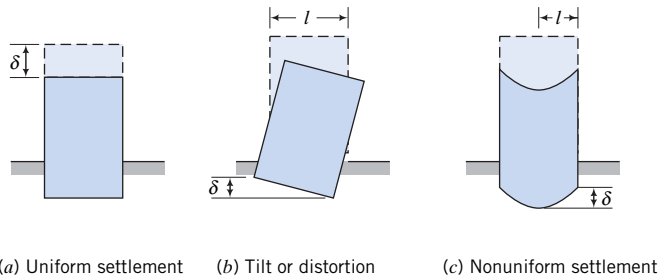


FIGURE 9.14 Types of settlement.

TABLE 9.4 Distortion Limits

Type of damage	Distortion, $\frac{\delta}{\ell}$
Architectural	$\frac{1}{300}$
Structural	$\frac{1}{150}$

surveyed by Skempton and MacDonald (1956) had steel or reinforced concrete frames with load bearing walls.

It is desirable to get zero distortion but this is practically impossible because (1) the properties of building materials and the loading conditions are not known accurately, and (2) the variability of the soils at a site and the effects of construction methods are uncertain. Even if we do know items (1) and (2), the calculations would be very complex. One practical solution is to correlate distortion to the maximum settlement. Skempton and MacDonald (1956) proposed that the maximum tolerable settlement could be calculated from

$$\rho_{\max} = R \frac{\delta}{\ell} \quad (9.32)$$

where R is a correlation factor that depends on the foundation and soil types. They also suggested limiting values of $\rho_{\max}$. The values for R and $\rho_{\max}$ suggested by Skempton and MacDonald (1956) are listed in Table 9.5.

You should use the limits in Table 9.5 with caution. For important structures, a thorough site investigation should be conducted to uncover soil inhomogeneities that may result in intolerable differential settlement.

TABLE 9.5 R Values and Maximum Allowable Settlement^a

Soil type	Values	Foundation type	
		Isolated footing	Rafts or mats
Clay	R	22,500	30,000
	$\rho_{\max}$ (mm)	75	100
Sand	R	15,000	18,000
	$\rho_{\max}$ (mm)	50	60

^aThe original R and $\rho_{\max}$ values were for use in English units. The author converted these values for SI and adjusted them for ease of use in practice.

The essential points are:

- 1. Distortion caused by differential settlement is crucial in design because it is responsible for cracking and damage to structures.*
- 2. The distortion limits were observed for old structures and may not be applicable to modern structures.*
- 3. To estimate the total maximum allowable settlement, multiply the appropriate **R** values in Table 9.5 by the distortion value.*

What's next... In the next section, we are going to discuss methods to calculate settlement of foundations.

9.10 SETTLEMENT CALCULATIONS



Concept Learning

Access Chapter 9, Section 9.10 on the CD to learn about the settlement of shallow footings. Take Quiz 9.10 to test your understanding.

The settlement of shallow foundations is divided into three segments—immediate or elastic settlement, primary consolidation settlement, and secondary consolidation settlement (creep). We have already considered elastic settlement (Chapter 5) and consolidation settlement (Chapter 6). However, we have to make some modifications to the methods described in those chapters for calculating settlement of shallow foundations. These modifications are made to the method of calculating elastic and primary consolidation settlements.

9.10.1 Immediate Settlement

We can use the theory of elasticity to determine the immediate or elastic settlement of shallow foundations. In the case of a uniform rectangular flexible load, we can use Eq. (5.93). However, the elastic equations do not account for the shape of the footing (not just L/B ratio) and the depth of embedment, which significantly influence settlement. An embedded foundation has the following effects in comparison with a surface footing:

1. Soil stiffness generally increases with depth, so the footing loads will be transmitted to a stiffer soil than the surface soil. This can result in smaller settlements.
2. Normal stresses from the soil above the footing level have been shown (Eden, 1974; Gazetas and Stoke, 1991) to reduce the settlement by providing increased confinement on the deforming halfspace. This is called the trench effect or embedment effect.
3. Part of the load on the footing may also be transmitted through the side walls depending on the amount of shear resistance mobilized at the soil–wall interface. The accommodation of part of the load by side resistance reduces the vertical settlement. This has been called the side wall–soil contact effect.

Gazetas et al. (1985) considered an arbitrarily shaped rigid footing embedded in a deep homogeneous soil (Fig. 9.15) and proposed the following equation for the elastic settlement:

$$\rho_e = \frac{P}{E_u L} (1 - \nu_u^2) \mu_s \mu_{\text{emb}} \mu_{\text{wall}} \quad (9.33)$$

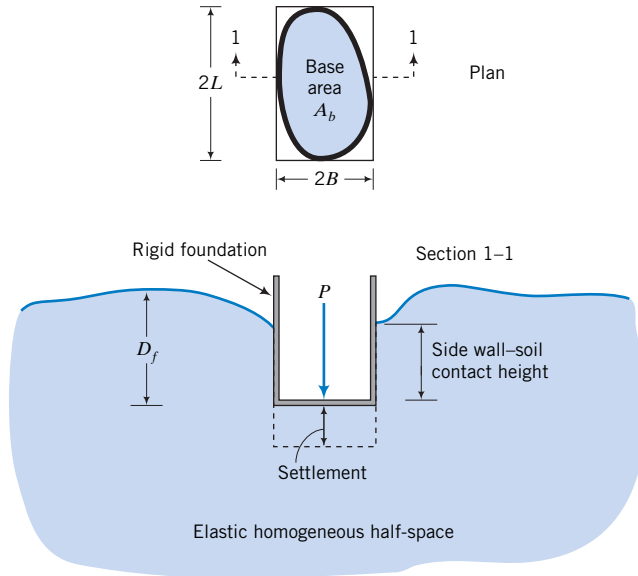


FIGURE 9.15 Geometry to calculate elastic settlement of shallow footings. (On behalf of the Institution of Civil Engineers.)

where P is total vertical load, E_u is the undrained elastic modulus of the soil, L is one-half the length of a circumscribed rectangle, ν_u is Poisson's ratio for the undrained condition, and μ_s , μ_{emb} , and μ_{wall} are shape, embedment (trench), and side wall factors given as

$$\mu_s = 0.45 \left(\frac{A_b}{4L^2} \right)^{-0.38} \quad (9.34)$$

$$\mu_{\text{emb}} = 1 - 0.04 \frac{D_f}{B} \left[1 + \frac{4}{3} \left(\frac{A_b}{4L^2} \right) \right] \quad (9.35)$$

$$\mu_{\text{wall}} = 1 - 0.16 \left(\frac{A_w}{A_b} \right)^{0.54} \quad (9.36)$$

A_b is the actual area of the base of the foundation and A_w is the actual area of the wall in contact with the embedded portion of the footing. The length and width of the circumscribed rectangle are $2L$ and $2B$, respectively. The dimensionless shape parameter, $A_b/4L^2$, has the values for common footing geometry shown in Table 9.6.

The equations proposed by Gazetas et al. (1985) apply to a foundation of arbitrary shape on a deep homogeneous soil. There is no clear definition of what signifies "deep." The author suggests that the equations of Gazetas et al. (1985) can be used when the thickness of the soil layer is such that 90% of the applied stresses are distributed within it. For a rectangular area of actual width B_r , the thickness of the soil layer should be at least $2B_r$.

Equation (5.90) can be modified to account for embedment as

$$\rho_e = \frac{q_s B_r (1 - \nu_u^2)}{E_u} I_s \mu'_{\text{emb}} \quad (9.37)$$

TABLE 9.6 Values of $A_b/4L^2$ for Common Footing Shapes

Footing shape	$\frac{A_b}{4L^2}$
Square	1
Rectangle	B/L
Circle	0.785
Strip	0

where

$$\mu'_{\text{emb}} = 1 - 0.08 \frac{D_f}{B_r} \left(1 + \frac{4 B_r}{3 L_r} \right) \quad (9.38)$$

where B_r and L_r are the actual width and length, respectively.

The accuracy of any elastic equation for soils depends particularly on the accuracy of the elastic modulus. It is common laboratory practice to determine a secant E_u from undrained triaxial tests or unconfined compression tests at a deviatoric stress equal to one-half the maximum shear strength. However, for immediate settlement it is better to determine E_u over the range of deviatoric stress pertaining to the problem. In addition, the elastic modulus is strongly dependent on depth while Eqs. (9.33) and (9.37) are cast in terms of a single value of E_u . One possible solution is to divide the soil into sublayers and use a weighted harmonic mean value of E_u (Chapter 6).

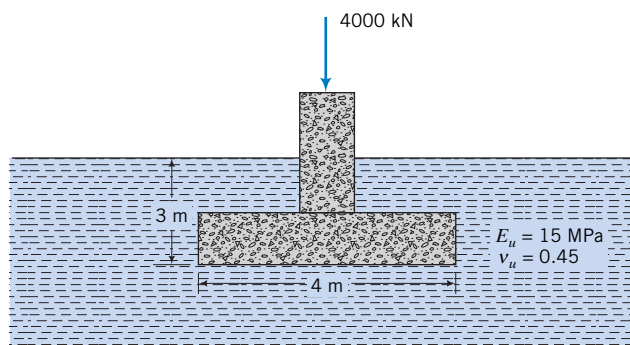
The full wall resistance will only be mobilized if sufficient settlement occurs. It is difficult to ascertain the quality of the soil–wall adhesion. Consequently, you should be cautious in relying on the reduction of settlement resulting from the wall factor. If wall friction and embedment are neglected, then $\mu_{\text{wall}} = 1$ and $\mu_{\text{emb}} = 1$.

Equations (9.33) and (9.37) strictly apply to fine-grained soils under short-term loading. For long-term loading in fine-grained soils and for coarse-grained soils, you should use E' and ν' instead of E_u and ν_u .

EXAMPLE 9.9 Elastic (Immediate) Settlement of a Footing on a Clay Soil

Determine the immediate settlement of a rectangular footing $4 \text{ m} \times 6 \text{ m}$ embedded in a deep deposit of homogeneous clay as shown in Fig. E9.9.

Strategy You have sufficient information to directly apply Eq. (9.33) or Eq. (9.37). The side wall effect should not be considered, that is, $\mu_{\text{wall}} = 1$, since there really is no wall.

**FIGURE E9.9**

Solution 9.9

Step 1: Determine geometric parameters.

$$A_b = 4 \times 6 = 24 \text{ m}^2, \quad L = \frac{6}{2} = 3 \text{ m}, \quad B = \frac{4}{2} = 2 \text{ m}$$

$$\frac{A_b}{4L^2} = 0.67 \quad \left(\text{Proof: } \frac{A_b}{4L^2} = \frac{2B \times 2L}{4 \times L^2} = \frac{B}{L} = \frac{4}{6} = 0.67 \right)$$

Step 2: Calculate the shape and embedment factors.

$$\mu_s = 0.45 \left(\frac{A_b}{4L^2} \right)^{-0.38} = 0.45 (0.67)^{-0.38} = 0.52$$

$$\mu_{\text{emb}} = 1 - 0.04 \frac{D_f}{B} \left[1 + \frac{4}{3} \left(\frac{A_b}{4L^2} \right) \right] = 1 - 0.04 \times \frac{3}{2} \left[1 + \frac{4}{3} (0.67) \right] = 0.89$$

Step 3: Calculate the immediate settlement.

$$\rho_e = \frac{P}{E_u L} (1 - \nu_u^2) \mu_s \mu_{\text{emb}} \mu_{\text{wall}} = \frac{4000}{15,000 \times 3} (1 - 0.45^2)$$

$$\times 0.52 \times 0.89 \times 1 = 0.033 \text{ m} = 33 \text{ mm}$$

EXAMPLE 9.10 Immediate Settlement When Elastic Modulus Varies with Depth

Determine the immediate settlement of the foundation shown in Fig. E9.10. The undrained elastic modulus varies with depth as shown in the figure and $\nu_u = 0.45$.

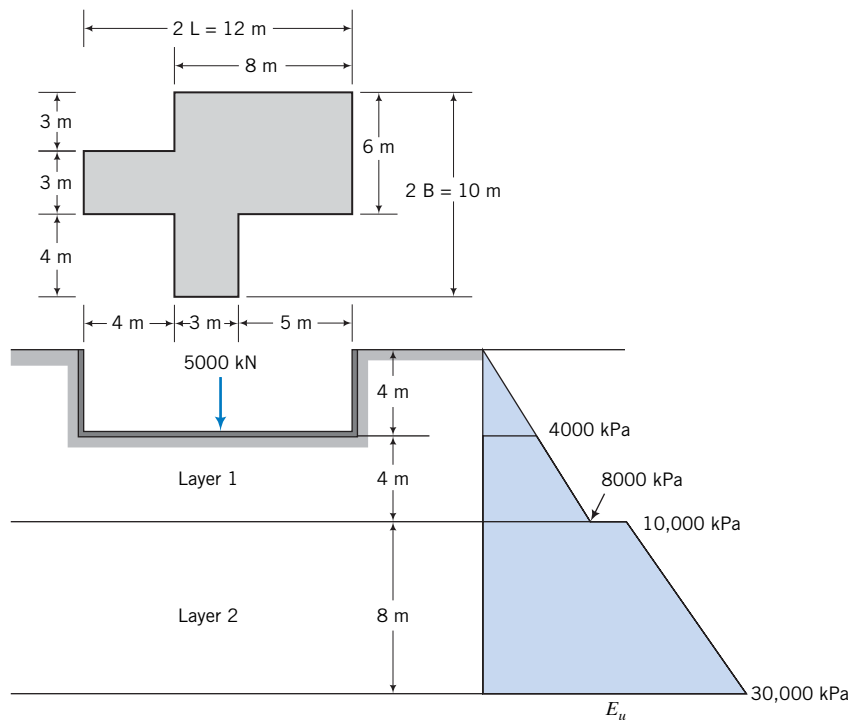


FIGURE E9.10

Strategy You have to determine the length ($2L$) and width ($2B$) of a circumscribed rectangle. The undrained elastic modulus varies with depth, so you need to consider the average value of E_u for each of the layers and then find the harmonic mean. You also need to find the shape parameter $A_b/4L^2$.

Solution 9.10

Step 1: Determine the length and width of the circumscribed rectangle.

$$\begin{aligned} 2L &= 8 + 4 = 12 \text{ m}; \quad L = 6 \text{ m} \\ 2B &= 3 + 3 + 4 = 10 \text{ m}; \quad B = 5 \text{ m} \end{aligned}$$

Step 2: Determine E_u .

Layer 1

$$\begin{aligned} E_u \text{ at base level} &= \frac{4}{8} \times 8000 = 4000 \text{ kPa}; \quad E_u \text{ at bottom of layer} = 8000 \text{ kPa} \\ (E_u)_{\text{avg}} &= \frac{4000 + 8000}{2} = 6000 \text{ kPa} \end{aligned}$$

Layer 2

$$\begin{aligned} E_u \text{ at top of layer} &= 10,000 \text{ kPa}; \quad E_u \text{ at bottom of layer} = 30,000 \text{ kPa} \\ (E_u)_{\text{avg}} &= \frac{10,000 + 30,000}{2} = 20,000 \text{ kPa} \end{aligned}$$

Step 3: Find the weighted harmonic mean E_u .

$$E_u = \frac{2(6000) + 1(20,000)}{3} = 10,667 \text{ kPa}$$

Step 4: Find the shape parameter $A_b/4L^2$.

$$A_b = (3 \times 4) + (3 \times 10) + (6 \times 5) = 72 \text{ m}^2; \quad \frac{A_b}{4L^2} = \frac{72}{4 \times 6^2} = 0.5$$

Step 5: Find the shape, embedment, and wall factors.

$$\begin{aligned} \mu_s &= 0.45(0.5)^{-0.38} = 0.59; \quad \mu_{\text{emb}} = 1 - 0.04 \frac{4}{5} \left(1 + \frac{4}{3} \times 0.5 \right) = 0.94 \\ A_w &= \text{Perimeter} \times \text{depth} = (3 + 4 + 5 + 6 + 8 + 3 + 4 + 3 + 4 + 4) \times 4 \\ &= 176 \text{ m}^2 \\ \frac{A_w}{A_b} &= \frac{176}{72} = 2.44; \quad \mu_{\text{wall}} = 1 - 0.16(2.44)^{0.54} = 0.74 \end{aligned}$$

Step 6: Calculate the immediate settlement.

$$\begin{aligned} \rho_e &= \frac{P}{E_u L} (1 - \nu_u^2) \mu_s \mu_{\text{emb}} \mu_{\text{wall}} \\ &= \frac{5000}{10667 \times 6} (1 - 0.45^2) \times 0.59 \times 0.94 \times 0.74 = 0.026 \text{ m} = 26 \text{ mm} \end{aligned}$$

9.10.2 Primary Consolidation Settlement

The method described in Chapter 6 can be used to calculate the primary consolidation settlement of clays below the footing. However, these equations were obtained for one-dimensional consolidation where the lateral strain is zero. In practice, lateral strains are significant except for very thin layers of clays or for situations when the ratio of the layer thickness to the lateral dimension of the loaded area

is small (approaches zero). We also assumed that the initial excess porewater pressure is equal to the change in applied stress at the instant the load is applied. Theoretically, this is possible if the lateral stresses are equal to the vertical stresses. If the lateral strains are zero, then under undrained condition (at the instant the load is applied), the vertical settlement is zero.

Skempton and Bjerrum (1957) proposed a method to modify the one-dimensional consolidation equation to account for lateral stresses but not lateral strains. They proposed the following equation:

$$(\rho_{pc})_{SB} = \int_0^{H_o} m_v \Delta u dz \quad (9.39)$$

where Δu is the excess porewater pressure and H_o is the thickness of the soil layer. Skempton and Bjerrum (1957) suggested that the error in neglecting the lateral strains could lead to an error of up to 20% in the estimation of the consolidation settlement. Skempton's equation (Eq. 7.44) for the excess porewater pressure in a saturated soil under axisymmetric loading can be algebraically manipulated to yield

$$\Delta u = \Delta \sigma_1 \left(A + \frac{\Delta \sigma_3}{\Delta \sigma_1} (1 - A) \right) \quad (9.40)$$

By substituting Eq. (9.40) into Eq. (9.39) we get

$$(\rho_{pc})_{SB} = \int_0^{H_o} m_v \Delta \sigma_1 \left(A + \frac{\Delta \sigma_3}{\Delta \sigma_1} (1 - A) \right) dz = \Sigma (m_v \Delta \sigma_z H_o) \mu_{SB} = \Sigma \rho_{pc} \mu_{SB} \quad (9.41)$$

where ρ_{pc} is the one-dimensional primary consolidation settlement (Chapter 6), $\mu_{SB} = A + \alpha_{SB}(1 - A)$ is a settlement coefficient to account for the effects of the lateral stresses, and $\alpha_{SB} = (\int \Delta \sigma_3 dz) / (\int \Delta \sigma_1 dz)$. The values of μ_{SB} typically vary from 0.6 to 1.0 for soft clays and from 0.3 to 0.8 for overconsolidated clays. Values of μ_{SB} for circular and strip footings are shown in Fig. 9.16. For square

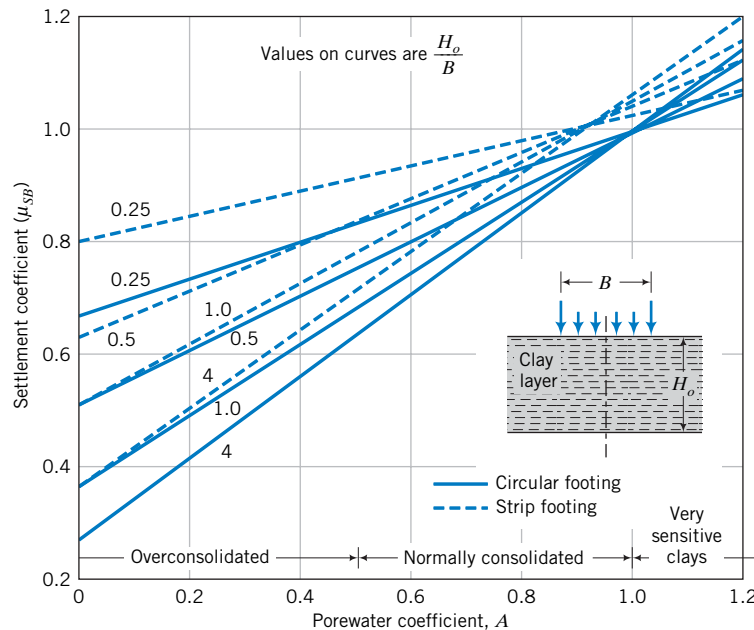


FIGURE 9.16 Values of μ_{SB} for circular and strip footings. (Redrawn from Scott, 1963.)

or rectangular footings use an equivalent circular footing of diameter $D = 2\sqrt{A/\pi}$, where A is area of the rectangle or the square. Equation (9.41) must be used appropriately. It is obtained from triaxial conditions and only applies to situations where axial symmetry occurs, such as under the center of a circular footing.

EXAMPLE 9.11 Primary Consolidation Settlement of a Footing

Determine the primary consolidation settlement under the square footing shown in Fig. E9.11 using the Skempton–Bjerrum method.

Strategy First you have to calculate the one-dimensional primary consolidation settlement and then determine μ_{SB} from the chart. Since the clay layer is finite, you should calculate the vertical stress increase using the coefficients in Appendix B.

Solution 9.11

Step 1: Calculate applied stress increase at the center of each layer below the base of the footing.

$$\frac{L}{B} = 1, \quad \frac{H_o}{B} = 2, \quad \frac{z}{B} = 0.5$$

Interpolation from Table B1.2 gives $I_{zp} = 0.72$.

$$\Delta\sigma_z = I_{zp}q_s = 0.72 \times 100 = 72 \text{ kPa}$$

Step 2: Determine μ_{SB} .

$$\text{Area of base} = 2 \times 2 = 4 \text{ m}^2$$

$$\text{Equivalent diameter of circular footing: } D = 2\sqrt{\frac{4}{\pi}} = 2.26 \text{ m}$$

$$\frac{H_o}{B} = \frac{4}{2.26} = 2.21, \quad A = 0.2; \quad \text{from Fig. 9.16, } \mu_{SB} = 0.43$$

Step 3: Calculate the primary consolidation settlement.

$$(\rho_{pc})_{SB} = m_v \Delta\sigma_z H_o \mu_{SB} = 0.0001 \times 72 \times 4 \times 0.43 = 0.003 \text{ m} = 3 \text{ mm}$$

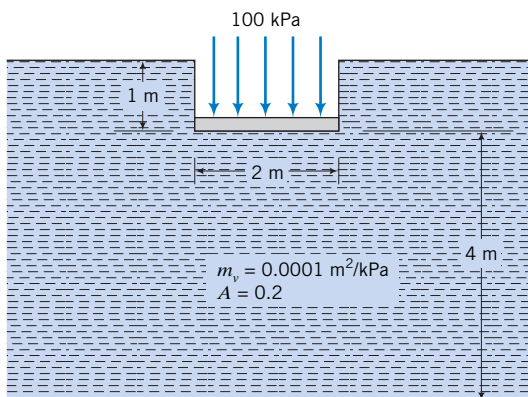


FIGURE E9.11

What's next... Often, the recovery of soils, especially coarse-grained soils, for laboratory testing is difficult and one has to use results from field tests to determine the bearing capacity and settlement of shallow foundations. Some of the field methods used for coarse-grained soils are presented in the next section.

9.11 DETERMINATION OF BEARING CAPACITY AND SETTLEMENT OF COARSE-GRAINED SOILS FROM FIELD TESTS



Self-Assessment

Access Chapter 9, Section 9.11 and take Quiz 9.11 to test your understanding of calculating settlement from field test data.

We are going to consider the SPT and CPT results in estimating the bearing capacity and settlement of shallow foundations. You need to be extra careful in selecting the N or q_c values to use in the empirical equations given below. You should inspect the results and then eliminate excessively high values of either N or q_c that appear to be spurious. These high values may be due to an obstruction such as a boulder. A continuous set of low values of N or q_c may indicate a weak soil layer. Depending on the location of this layer, for example, within a depth B below the foundation, it may control the performance of the foundation.

9.11.1 Standard Penetration Test (SPT)

It is difficult to obtain undisturbed samples of coarse-grained soils for testing in the laboratory. Consequently, the allowable bearing capacity and settlement of footings on coarse-grained soils are often based on empirical methods using test data from field tests. One popular method utilizes results from the standard penetration test (SPT). It is customary to correct the N values for overburden pressure. Various correction factors have been suggested by a number of investigators. Energy and other corrections were considered in Chapter 2. Two suggestions for correcting N values for overburden pressure are included in this text. These are

$$c_N = \left(\frac{95.8}{\sigma'_{z0}} \right)^{1/2}; \quad C_N \leq 2 \quad (\text{Liao and Whitman, 1985}) \quad (9.42)$$

$$c_N = 0.77 \log_{10} \left(\frac{1916}{\sigma'_{z0}} \right); \quad C_N \leq 2, \quad \sigma'_{z0} > 24 \text{ kPa} \quad (\text{Peck et al., 1974}) \quad (9.43)$$

where c_N is a correction factor for overburden pressures, and σ'_{z0} is the effective overburden pressure in kPa. A further correction factor is imposed on N values if the groundwater level is within a depth B below the base of the footing. The groundwater correction factor is

$$c_w = \frac{1}{2} + \frac{z}{2(D_f + B)} \quad (9.44)$$

where z is the depth to the groundwater table, D_f is the footing depth, and B is the footing width. If the depth of the groundwater level is beyond B from the footing base, $c_w = 1$.

The corrected N value is

$$N_1 = c_N c_w N \quad (9.45)$$

The ultimate bearing capacity for a shallow footing under vertical loads is

$$q_{ult} = 32N_1B \text{ (kPa)} \quad (9.46)$$

where B is the width in m. In practice, each value of N in a soil layer up to a depth $1.5B$ below the footing base is corrected and an average value of N_1 is used in Eq. (9.46).

Meyerhof (1965) proposed that no correction should be applied to N values for the effects of groundwater as these are already incorporated in the measurement. Furthermore, he suggested that q_{ult} calculated from Eq. (9.46) using $N_1 = c_N N$ be increased by 50%. In using Eq. (9.46), the settlement is assumed to be less than 25 mm.

Burland and Burbridge (1985) did a statistical analysis of settlement records from 200 footings located in sand and in gravel. They proposed the following equation for the settlement of a footing in a normally consolidated sand at the end of construction:

$$\rho = f_s f_1 \sigma_a B^{0.7} I_c \quad (9.47)$$

where ρ is the settlement (mm),

$$f_s = \text{Shape factor} = \left(\frac{1.25L/B}{L/B + 0.25} \right)^2$$

$f_1 = (H_o/z_1)(2 - H_o/z_1)$ is a correction factor if the thickness (H_o) of the sand stratum below the footing base is less than an influence depth z_1 , σ_a is the vertical stress applied by the footing or allowable bearing capacity (kPa), B and L are the width and length of the footing (m), respectively,

$$I_c = \text{Compressibility index} = \frac{1.71}{N^{1.4}} \quad (9.48)$$

and N is the uncorrected N value. The influence depth is the depth below the footing that will influence the settlement and bearing capacity. If N increases with depth or N is approximately constant, the influence depth is taken as $z_1 = B^{0.763}$. If N tends to decrease with depth, the influence depth is $z_1 = 2B$.

If the sand is overconsolidated,

$$\rho = f_1 f_s (\sigma_a - \frac{2}{3} \sigma'_{zc}) B^{0.7} I_c, \quad \text{if } \sigma_a > \sigma'_{zc} \quad (9.49)$$

$$\rho = q_a B^{0.7} \frac{I_c}{3}, \quad \text{if } \sigma_a < \sigma'_{zc} \quad (9.50)$$

The procedure for the Burland–Burbridge method is as follows:

1. Determine the influence depth z_1 .
2. Find the average N value within the depth z_1 below the footing.
3. Calculate I_c from Eq. (9.48).
4. Determine ρ from the appropriate equation [Eq. (9.47) or (9.49) or (9.50)] or, if ρ is specified, you can determine σ_a .

EXAMPLE 9.12 *Allowable Bearing Capacity Using SPT Data***Computer Program Utility**

Access Chapter 9, click on Spreadsheet to see the calculations for this example.

The SPT results at various depths in a soil are shown in Table E9.12a.

TABLE E9.12a

Depth (m)	0.6	0.9	1.2	1.5	2.1	2.7	3	3.3	4.2
<i>N</i> (blows/ft)	25	28	33	29	28	29	31	35	41

Determine the allowable bearing capacity for a square footing 2 m wide located at 0.6 m below the surface. The tolerable settlement is 25 mm. The groundwater level is deep and its effects can be neglected.

Strategy The question that arises is what value of *N* to use. We will estimate the thickness of the soil ($\approx 2B$) below the footing that will be stressed significantly ($>10\%$ of applied stress) and take an average value of *N* within that layer. The unit weight is not given so we have to estimate this based on the description and the *N* values (see Chapter 7, Table 7.4).

Solution 9.12**Computer Program Utility**

Access Chapter 9, and click on Spreadsheet on the sidebar to view the solution for this example. You can change the copy of this spreadsheet to solve similar types of problems.

TABLE E9.12B**Bearing capacity from SPT**

q_a		1033		kPa		
Width of footing		2		m		
Depth of footing		0.9		m		
Groundwater		5		m		
FS		3				
Depth (m)	Unit weight kN/m ³	Vertical effective stress (kPa)	C_n Calc.	C_n use	N	N_1
0	0	0	0	0	0	0
0.6	18.5	11.1	2.9	2	25	50
0.9	19	16.8	2.4	2.0	28	56
1.2	20	22.8	2.0	2.0	33	66
1.5	19	28.5	1.8	1.8	29	53
2.1	19	39.9	1.5	1.5	28	43
2.7	19	51.3	1.4	1.4	29	40
3	20	57.3	1.3	1.3	31	40
3.3	20.5	63.45	1.2	1.2	35	43
4.2	20.5	81.9	1.1	1.1	41	44
				Avg	31	48

Step 1: Determine N_1

Calculate σ'_{z0} and the correction factor c_N using either Eq. (9.42) or (9.43). Use a spreadsheet to do the calculation as shown in Table E9.12b.

Step 2: Calculate q_a .

$$\text{Eq. (9.46)} \quad q_{\text{ult}} = 32N_1 B = 32 \times 48 \times 2 = 3072 \text{ kPa}$$

$$q_a = \frac{q_{\text{ult}}}{\text{FS}} = \frac{3072}{3} = 1024 \text{ kPa}$$

The spreadsheet gives $q_a = 1033 \text{ kPa}$ because it uses more significant figures in doing the calculations. ■

EXAMPLE 9.13 Allowable Bearing Capacity and Settlement Using SPT

Redo Example 9.12 using the Burland–Burbridge method for a footing $3 \text{ m} \times 4 \text{ m}$.

Strategy You have to determine whether the sand is normally consolidated or overconsolidated. No direct evidence is provided to allow you to make a decision as to the consolidation state of the sand. One way around this problem is to use Table 7.4 to make an estimate of the consolidation state.

Solution 9.13

Step 1: Determine the consolidation state and find z_1 .

Within a depth equal to B (3 m), the average N value is 29. From Table 7.4, the sand can be classified as medium (N in the range 10–30). A reasonable estimate of the consolidation state is normally consolidated.

$$z_1 = B^{0.763} = 3^{0.763} = 2.3 \text{ m}$$

Step 2: Find an average N for a depth 2.3 m below the base.

Average N value over a depth of 2.3 m below the base is 29. (*Note:* 2.3 m below the base is equivalent to a depth of 2.9 m, so use the N values up to 3 m.)

Step 3: Calculate I_c .

$$I_c = \frac{1.71}{N^{1.4}} = \frac{1.71}{29^{1.4}} = 0.015$$

Step 4: Calculate q_a .

$$\frac{L}{B} = \frac{4}{3} = 1.33; \quad f_s = \left(\frac{1.25L/B}{L/B + 0.25} \right)^2 = \left(\frac{1.25 \times 1.33}{1.33 + 0.25} \right)^2 = 1.11$$

$$f_1 = 1 \quad (\text{thickness of sand stratum greater than } 2.3 \text{ m})$$

$$\sigma_a = \frac{\rho}{f_s f_1 B^{0.7} I_c} = \frac{25}{1.11 \times 1 \times 3^{0.7} \times 0.015} = 696 \text{ kPa} \quad \blacksquare$$

9.11.2 Cone Penetration Test (CPT)

Schmertmann (1970) and Schmertmann et al. (1978) proposed a methodology to determine settlement from the quasi-static cone test data for sands. They assumed that the sand is a linearly elastic material and only stress changes within depths of $2B$ for axisymmetric conditions and $4B$ for plane strain conditions influence the settlement. Settlement is calculated by integrating the vertical

strains; that is,

$$\rho = \int \varepsilon_z dz$$

The equation proposed for settlement (mm) by Schmertmann et al. (1978) is

$$\rho = \frac{c_D c_t}{\beta} q_{\text{net}} \sum_{i=1}^n \frac{(I_{\text{co}})_i}{(q_c)_i} \Delta z_i \quad (9.51)$$

where

$$c_D = \text{Depth factor} = 1 - 0.5 \frac{\sigma'_{z_0}}{q_{\text{net}}} \geq 0.5 \quad (9.52)$$

$$c_t = \text{Creep factor} = 1.0 + A \log_{10} \left| \frac{t}{0.1} \right|, \quad (9.53)$$

β is cone factor [$\beta = 2.5$ for square footing (axisymmetric condition), $\beta = 3.5$ for strip footing (plane strain condition $\frac{L}{B} > 10$)], q_{net} is the net footing pressure in kPa (applied stress minus soil pressure above the base of footing), σ'_{z_0} is the original vertical effective stress in kPa at the depth of the footing, t is time in years ($t \geq 0.1$), A is an empirical factor taken as 0.2, Δz_i is the thickness of the i th layer, $(I_{\text{co}})_i$ is the influence factor of the i th layer given as:

Axisymmetric: $L = B$

$$I_{\text{co}} = 0.1 + 2(I_{\text{cp}} - 0.1) \frac{z}{B} \quad \text{for} \quad \frac{z}{B} \leq \frac{1}{2} \quad (9.54)$$

$$I_{\text{co}} = I_{\text{cp}} \left[1 - \frac{2}{3} \left(\frac{z}{B} - \frac{1}{2} \right) \right] \quad \text{for} \quad 2 \geq \frac{z}{B} > \frac{1}{2} \quad (9.55)$$

Plane strain: $L > 10B$

$$I_{\text{co}} = 0.2 + (I_{\text{cp}} - 0.2) \frac{z}{B} \quad \text{for} \quad \frac{z}{B} \leq 1 \quad (9.56)$$

$$I_{\text{co}} = I_{\text{cp}} \left[1 - \frac{1}{3} \left(\frac{z}{B} - 1 \right) \right] \quad \text{for} \quad 4 \geq \frac{z}{B} > 1 \quad (9.57)$$

where $I_{\text{cp}} = 0.5 + 0.1 \sqrt{\frac{q_{\text{net}}}{\sigma'_{zp}}}$, $(q_c)_i$ is the cone tip resistance for the i th layer, σ'_{zp} is original vertical effective stress at the depth where I_{cp} occurs, which is $\frac{B}{2}$ for axisymmetric condition and B for plane strain, and n is the number of sublayers. The units of B is meter.

The procedure to determine the settlement from cone data is as follows:

1. Divide the soil below the footing into a number of sublayers. For square footings, the total depth of the sublayers is $2B$ and a reasonable number of sublayers is four. For strip footing, the total depth is $4B$ and a reasonable number of sublayers is eight.
2. Determine the average value of $(q_c)_i$ for each sublayer from the field data of q_c versus depth.
3. Find I_{co} at the center of each sublayer.
4. Estimate ρ using Eq. (9.51).

The bearing capacity from the CPT test is estimated by taking a weighted average of the cone resistance over a depth of $2B$ for axisymmetric condition and $4B$ for plane strain condition.

EXAMPLE 9.14 *Allowable Bearing Capacity and Settlement Using CPT Data***Computer Program Utility**

Access Chapter 9, and click on Spreadsheet on the sidebar to view the solution for this example using a spreadsheet. You can change the copy of this spreadsheet to solve similar types of problems.

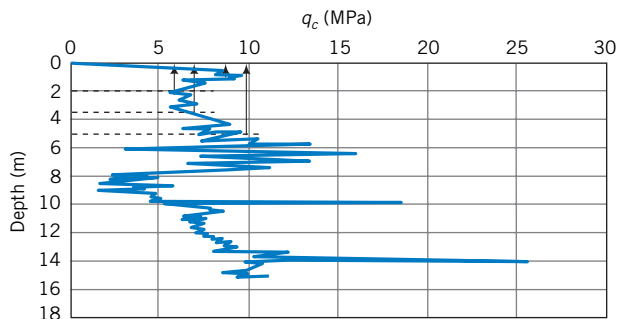
A representative set of cone data at a site is shown in Fig. E9.14a. A square footing 3 m wide imposing an applied stress of 217 kPa is to be located 1 m below ground level at this site. Determine (1) the bearing capacity and (2) the settlement of the footing one year after construction. The bulk unit weight of the sand is 17 kN/m³. Groundwater level is 8 m below the ground surface.

Strategy For a square footing, the influence depth is $2B$. You need to divide this depth into soil layers and then use Eq. (9.51).

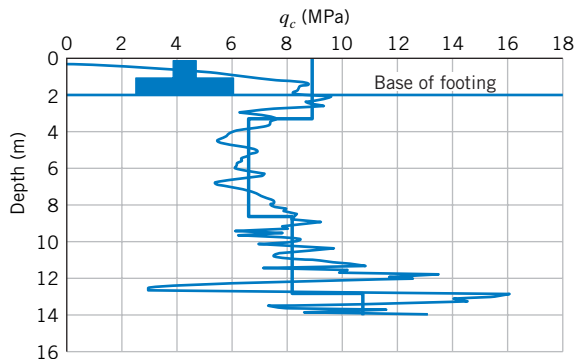
Solution 9.14

Step 1: Determine the influence depth below base of footing.

$$\text{Influence depth} = 2B = 6 \text{ m}$$

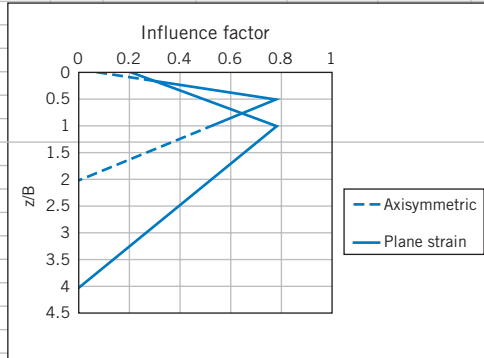
**FIGURE E9.14a**

Step 2: Inspect q_c values over the influence depth below the base of the footing. Ignore excessively large q_c values and sketch a composite distribution as shown in Fig. E9.14b. Find the average value of q_c , I_{co} , and ρ over the influence depth.

**FIGURE E9.14b**

Use a spreadsheet program.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Muni Budhu "Soil Mechanics and Foundations", John Wiley & Sons, NY, 2003														
2	Settlement of shallow footings using cone penetrometer results														
3	Condition	1	Axisymmetric		HELP										
4	Settlement		26.3	mm											
5	Bearing capacity		7.74	MPa											
6															
7	q		217	kPa		c _D	0.96								
8	D _f		1	m		c _t	1.20								
9	h		17	kN/m ²											
10	q _{net}		200	kPa											
11	B		3	m											
12	L		3	m											
13	I _{sp}		0.78												
14	t		1	yr											
15	Layer	z	Δz	z/B	I _{co}	q _c	(I _{co} /q _c)Δz	q _c Δz							
16	m	m	m			mPa		MPa.m							
17			0												
18	1	0.20	0.20	0.07	0.19	8.40	0.005	1.68							
19	2	2.90	2.70	0.97	0.54	6.30	0.230	17.01							
20	3	4.60	1.70	1.53	0.24	8.00	0.052	13.60							
21	4	6.00	1.40	2.00	0.00	10.10	0.000	14.14							
22						Sum	0.286	46.43							
23															



Since this is an axisymmetric case, the depth at which I_{cp} occurs is $B/2$.

$$\sigma'_{zp} = 17 \times \frac{3}{2} = 25.5 \text{ kPa}; \quad I_{cp} = 0.5 + 0.1 \sqrt{\frac{200}{25.5}} = 0.78$$

$$q_{net} = 217 - 1 \times 17 = 200 \text{ kPa}$$

$$\sigma'_z = 1 \times 17 = 17 \text{ kPa}; \quad c_D = 1 - 0.5 \times \frac{17}{200} = 0.96;$$

$$c_t = 1 + 0.2 \log_{10} \left| \frac{1}{0.1} \right| = 1.2$$

Step 3: Calculate the bearing capacity.

$$q_{ult} = \frac{\sum q_c \Delta z}{\sum \Delta z} = \frac{46.4}{6} = 7.73 \text{ MPa}$$

Step 4: Calculate the settlement.

$$\rho = \frac{c_D c_t}{\beta} q_{net} \sum_{i=1}^n \frac{(I_{co})_i}{(q_c)_i} \Delta z = \frac{0.96 \times 1.2}{2.5} \times 200 \times 0.286 = 26.3 \text{ mm}$$

9.11.3 Plate Load Test (PLT)

Tests on full sized footings are desirable but expensive. The alternative is to carry out plate load tests (Fig. 9.17) to simulate the load–settlement behavior of a real footing. The plates are made from steel with sizes varying from 150 to 760 mm. Two common plate sizes are used in practice. One is a square plate of width 300 mm and the other is a circular plate of diameter 300 mm. The test is carried out in a pit of depth at least 1.5 m. Loads are applied in increments of 10% to 20% of the estimated allowable bearing capacity. Each load increment is held until settlement ceases. The final settlement at the end of each loading increment is recorded. Loading is continued until the soil fails or settlements are in excess

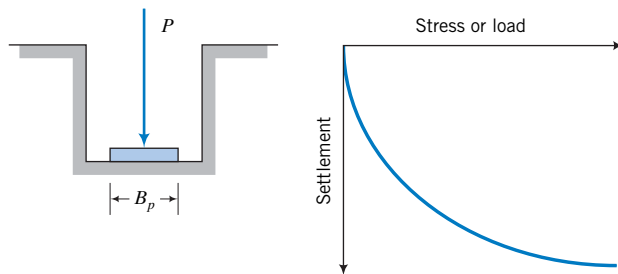


FIGURE 9.17 Plate load test.

of 10% of the plate diameter. The maximum load should be at least 1.5 times the estimated allowable bearing capacity.

If the sand were to behave like an elastic material, then the settlement can be calculated from

$$\rho_p = q_{ap} B_p \frac{1 - (v')^2}{E'} I_p \quad (9.58)$$

where ρ_p is the plate settlement, σ_{ap} is the applied stress, B_p is the width or diameter of the plate, v' is Poisson's ratio, E' is the elastic modulus, and I_p is an influence factor (0.82 for a rigid plate). The settlement of the real footing (ρ) of width B is related to the plate settlement by

$$\rho = \rho_p \left(\frac{2}{1 + B_p/B} \right)^2 \quad (9.59)$$

In the limit $B_p/B \rightarrow \infty$, $\rho/\rho_p \rightarrow 4$.

Equation (9.58) is only valid if the strains are small (infinitesimal). There are several problems associated with the plate load test.

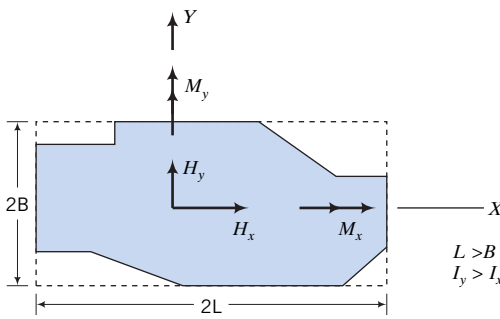
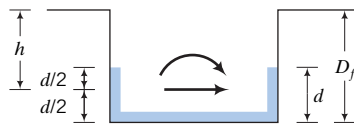
1. The test is reliable only if the sand layer is thick and homogeneous.
2. The depth of sand that is stressed below the plate is significantly lower than the real footing. A weak soil layer below the plate may not affect the test results because it may be at a depth at which the stresses imposed on the weak layer by the plate loads may be insignificant. However, this weak layer can have significant effect on the bearing capacity and settlement of the real footing.
3. Local conditions such as a pocket of weak soil near the surface of the plate can affect the test results but these may have no significant effect on the real footing.
4. The correlation between plate load test results and the real footing is problematic. Settlement in a sand depends on the size of the plate. Settlement increases with increases in the plate size. Bjerrum and Eggstad (1963) found that there is significant scatter in the relationship between plate size and settlement for a given applied stress. Bjerrum and Eggstad (1963) also reported that field evidence indicates that the limit of ρ/ρ_p ranges between 3 and 5 rather than a fixed value of 4.
5. Performance of the test is difficult. On excavating sand to make a pit, the soil below the plate invariably becomes looser and this has considerable influence on the test results. Good contact must be achieved between the plate and the sand surface but this is often difficult. If the plate were just above the groundwater, your results would be affected by negative porewater pressure.

What's next... We have considered only vertical settlement under vertical and inclined loads. Horizontal displacements and rotations are important for structures subjected to significant horizontal loads. Next we will consider horizontal displacements and rotations.

9.12 HORIZONTAL ELASTIC DISPLACEMENT AND ROTATION

Structures such as radar towers and communication transmission towers are subjected to significant horizontal loads from wind, which can lead to intolerable lateral displacement and rotation of their foundations. Gazetas and Hatzikonstantinou (1988) proposed equations based on an isotropic linearly elastic soil to determine the elastic horizontal displacement and rotation of an arbitrarily loaded foundation. The equations were obtained by curve fitting theoretical elastic solutions for $\frac{D_f}{B} \leq 2$. A summary of these equations are presented in Table 9.7. You must be cautious (see Section 9.10.1) in using the equations in Table 9.7.

TABLE 9.7 Equations for Estimating the Horizontal Displacement and Rotation of Arbitrarily Shaped Foundations (Source: Gazetas and Hatzikonstantinou, 1988).

Loading	Surface foundation		Foundation placed in open trench
			
	Lateral direction y Longitudinal direction x		Both directions
Horizontal	$\rho_e = \frac{H_i}{EL} (2 - \nu)(1 + \nu) \mu_s$ $\mu_{sy} \approx 0.5 - 0.28 \left[\frac{A_b}{4L^2} \right]^{0.45}$ $\mu_{sx} \approx \left[1 + 1.12 \left(\frac{1 - B/L}{1 - \nu} \right) \right] \mu_{sy}$		$(\rho_e)_{\text{emb}} \approx \rho_e \mu_{\text{emb}}$ $\mu_{\text{emb}} \approx 1 - 0.14 \left(\frac{D_f}{B} \right)^{0.35}$
Moment	$\theta_{ry} = \frac{M 2(1 - \nu^2)}{E I_y^{0.75}} (\mu_s)_{ry}$ $(\mu_s)_{ry} \approx 0.43 - 0.10 \frac{B}{L}$ $\theta_{rx} = \frac{M 2(1 - \nu^2)}{E I_x^{0.75}} \left(\frac{B}{L} \right)^{0.25} (\mu_s)_{rx}$ $(\mu_s)_{rx} \approx 0.33 \left(\frac{B}{L} \right)^{0.15}$		$\theta_{\text{emb}} \approx \theta_r$
	Embedded foundation with sidewall-soil contact surface of total areas A_w		
			
Loading	Lateral direction y		Longitudinal direction x
Horizontal	$\rho_e = (\rho_e)_{\text{emb}} \mu_{\text{wall}}$ $(\mu_{\text{wall}})_y \approx 1 - 0.35 \left[\frac{h A_w}{B L^2} \right]^{0.2} \approx (\mu_{\text{wall}})_x$		
Moment	$\theta = \theta_{\text{emb}} (\mu_{\text{wall}})_r$ $(\mu_{\text{wall}})_{ry} \approx \left\{ 1 + 0.92 \left(\frac{d}{L} \right)^{0.6} \left[1.5 + \left(\frac{d}{L} \right)^{1.9} \left(\frac{D_f}{d} \right)^{0.6} \right] \right\}^{-1}$ $(\mu_{\text{wall}})_{rx} \approx \left\{ 1 + 1.26 \frac{d}{B} \left[1 + \frac{d}{B} \left(\frac{D_f}{d} \right)^{0.2} \left(\frac{B}{L} \right)^{0.5} \right] \right\}^{-1}$		

NOTES: Substitute the appropriate values for H_i and M in the above equations. For example, if you are considering the X direction, $H_i = H_x$ and $M = M_x$. Also use the appropriate values for μ_s and θ_r . For example, in the X direction, $\mu_s = \mu_{sx}$ and $\theta_r = \theta_{rx}$. For short term loading use the undrained values of E and ν while for long term loading use the effective values. The terms I_x and I_y are the second moment of areas about the X and Y axes respectively and θ is the rotation caused by the moment.

9.13 SUMMARY

In this chapter, we described ways in which you can calculate the bearing capacity and settlement of shallow foundations. The critical criterion for design of most shallow foundations is the serviceability limit state. Because of sampling difficulties, the bearing capacity and settlement of coarse-grained soils are often determined from field tests. We showed how to use the results of the SPT, the cone test, and the plate test to estimate the bearing capacity and settlement of shallow foundations on coarse-grained soils.

Practical Examples

EXAMPLE 9.15 Design (Sizing) a Square Footing

Determine the size of a square footing to carry a vertical load of 500 kN. The soil at the site is shown in Fig. E9.15. The tolerable total settlement is 20 mm. The groundwater level is 3 m below the ground surface and the depth of the footing is 1.5 m. However, the groundwater level is expected to seasonally rise to the surface. You may assume that the clay layer is thin so that one-dimensional consolidation takes place.

Strategy The presence of the soft clay layer gives a clue that settlement may govern the design. In this case, we should determine the width required to satisfy settlement and then check the bearing capacity. One can assume a width, calculate the settlement, and reiterate until the settlement criterion is met. Since this is a multilayer soil profile, it is more accurate to calculate the increase in vertical stress for multilayer soils. However, for simplicity, we will use Boussinesq's method for a uniform soil.

Solution 9.15

Step 1: Assume a width and a shape.

Try $B = 3$ m, and assume a square footing.

Step 2: Calculate the elastic settlement.

Sand

Neglect side wall effects; that is, $\mu_{\text{wall}} = 1$

$$\mu'_{\text{emb}} = 1 - 0.08 \frac{D_f}{B} \left(1 + \frac{4B}{3L} \right) = 1 - 0.08 \frac{15}{1.5} \left(1 + \frac{4}{3} \times 1 \right) = 0.81$$

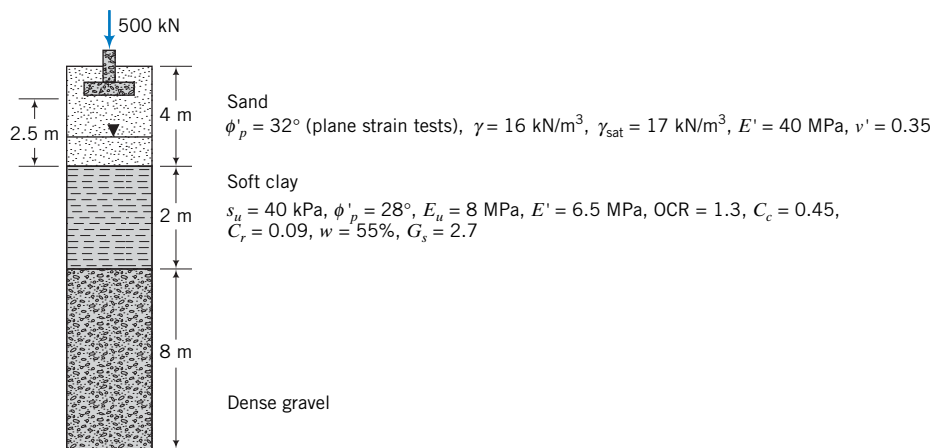


FIGURE E9.15

$$\begin{aligned}
 I_s &= 0.62 \ln(L/B) + 1.12 = 1.12 \\
 \rho_e &= \frac{q_s B [1 - (\nu')^2]}{E'} I_s \mu'_{\text{emb}} = \frac{P [1 - (\nu')^2]}{E' L} I_s \mu'_{\text{emb}} \\
 &= \frac{500(1 - 0.35^2)}{40 \times 10^3 \times 3} \times 1.12 \times 0.81 = 3.3 \times 10^{-3} = 3.3 \text{ mm}
 \end{aligned}$$

Clay

Find the equivalent footing size at the top of the clay layer. Let z_1 be the depth from the base of the footing to the top of the clay layer.

Equivalent width and length of footing at top of clay: $B + z_1 = 3 + 2.5 = 5.5 \text{ m}$

$$\mu'_{\text{emb}} = 1 - 0.08 \frac{4}{2.75} \left(1 + \frac{4}{3} \times 1 \right) = 0.73$$

For immediate settlement in clays, use undrained conditions with $\nu = \nu_u = 0.5$.

$$\rho_e = \frac{P(1 - \nu_u^2)}{E_u L} I_s \mu'_{\text{emb}} = \frac{500(1 - 0.5)}{8000 \times 2.75} \times 1.12 \times 0.73 = 9.3 \times 10^{-3} = 9.3 \text{ mm}$$

Step 3: Calculate the consolidation settlement of the clay.

$$\begin{aligned}
 e &= w G_s = 0.55 \times 2.7 = 1.49 \\
 \gamma_{\text{sat}} &= \frac{G_s + e}{1 + e} \gamma_w = \frac{2.7 + 1.49}{1 + 1.49} 9.8 = 16.5 \text{ kN/m}^3
 \end{aligned}$$

Calculate the current vertical effective stress (overburden pressure) at the center of the clay layer.

$$\sigma'_{z0} = 3 \times 16 + 1(17 - 9.8) + 1(16.5 - 9.8) = 61.9 \text{ kPa}$$

Calculate the stress increase at the center of the clay layer ($z = 3.5 \text{ m}$).

$$\begin{aligned}
 m &= n = \frac{\left(\frac{3}{2}\right)}{3.5} = 0.43; \quad \text{from stress chart (Fig. 5.26), } I_z = 0.068 \\
 \Delta\sigma_z &= 4 \times \frac{500}{3^2} \times 0.068 = 15.1 \text{ kPa} \\
 \sigma'_{z0} + \Delta\sigma_z &= 61.9 + 15.1 = 77 \text{ kPa} \\
 \sigma'_{zc} &= \text{OCR} \times \sigma'_{z0} = 1.3 \times 61.9 = 80.5 \text{ kPa} > \sigma'_{z0} + \Delta\sigma_z \\
 \rho_{\text{pc}} &= \frac{H_o}{1 + e_o} C_r \log \frac{\sigma'_{z0} + \Delta\sigma_z}{\sigma'_{z0}} \\
 &= \frac{2}{1 + 1.49} \times 0.09 \log \frac{77}{61.9} = 6.9 \times 10^{-3} \text{ m} = 6.9 \text{ mm}
 \end{aligned}$$

Step 4: Find the total settlement.

$$\begin{aligned}
 \text{Total settlement : } \rho &= (\rho_e)_{\text{sand}} + (\rho_e)_{\text{clay}} + \rho_{\text{pc}} \\
 &= 3.3 + 9.3 + 6.9 = 19.5 \text{ mm} < 20 \text{ mm}
 \end{aligned}$$

Step 5: Check the bearing capacity.

The groundwater table is less than $B = 3 \text{ m}$ below the footing base so groundwater effects must be taken into account.

Step 6: Check critical height.

$$H_{cr} = \frac{B}{2 \cos\left(45^\circ + \frac{\phi'_p}{2}\right)} \exp[A \tan \phi'_p]$$

$$A = \left(45^\circ - \frac{\phi'_p}{2}\right) = \left(45^\circ - \frac{32}{2}\right) \frac{\pi}{180} = 0.506 \text{ rad}$$

$$H_{cr} = \frac{3}{2 \cos\left(45^\circ + \frac{32}{2}\right)} \exp[0.506 \tan 32] = 4.2 \text{ m}$$

The height of soil below the footing to the top of the soft clay is $2.5 \text{ m} < H_{cr}$. Therefore, you must consider the bearing capacity of the soft layer.

ESA (sand)

$$d_\gamma = 1, \quad s_q = 1 + \frac{B'}{L'} \tan \phi'_p = 1 + \tan 32^\circ = 1.62$$

$$s_\gamma = 1 - 0.4 \frac{B'}{L'} = 0.6$$

$$d_q = 1 + 2 \tan \phi'_p (1 - \sin \phi'_p)^2 \tan^{-1}\left(\frac{D_f}{B'}\right) = 1 + 2 \tan 32^\circ (1 - \sin 32^\circ)^2 \left[\tan^{-1}\left(\frac{1.5}{3} \times \frac{\pi}{180}\right)\right] = 1.13$$

$$N_q = e^{\pi \tan 32^\circ} \tan^2\left(45 + \frac{32}{2}\right) = 23.2; \quad N_q - 1 = 23.2 - 1 = 22.2$$

$$N_\gamma = 0.1054 \exp\left(9.6 \times 32 \times \frac{\pi}{180}\right) = 22.5$$

Calculate the bearing capacity for the worst-case scenario—groundwater level at surface.

$$q_u = \gamma D_f (N_q - 1) s_q d_q + 0.5(\gamma B) N_\gamma s_\gamma d_\gamma; \quad \text{use } \gamma = \gamma' = 17 - 9.8 = 7.2 \text{ kN/m}^3$$

$$q_u = 7.2 \times 1.5 \times 22.2 \times 1.62 \times 1.13 + 0.5$$

$$\times 7.2 \times 3 \times 22.5 \times 0.6 \times 1.0 = 595 \text{ kPa}$$

$$(\sigma_a)_{\max} = \frac{\text{Applied load}}{\text{Area}} = \frac{500}{3^2} = 55.6 \text{ kPa}$$

$$\text{FS} = \frac{q_u}{(\sigma_a)_{\max} - \gamma D_f} = \frac{595}{55.6 - 7.2 \times 1.5} = 13 > 1.5 \quad (\text{okay})$$

TSA (clay)

Check the bearing capacity of the clay.

$$\text{Equivalent footing width: } B + z_1 = 3 + 2.5 = 5.5 \text{ m}$$

$$(\Delta\sigma)_{\max} = \frac{550}{5.5^2} = 16.5 \text{ kPa}$$

$$q_u = 5.14 s_u s_c d_c$$

$$s_c = 1 + 0.2 \frac{B'}{L'} = 1.2$$

$d_c = 1.0$ because the footing is assumed to be a surface footing on the clay.

$$q_u = 5.14 \times 40 \times 1.2 \times 1.0 = 247 \text{ kPa}$$

$$\text{FS} = \frac{247}{16.5} = 15 > 3 \quad (\text{okay})$$

ESA (clay)**Surface Footing**

$$s_\gamma = 1 - 0.4 \frac{B'}{L'} = 0.6$$

$$d_q = d_\gamma = 1$$

Smooth Surface

$$N_\gamma = 0.0663 \exp(9.3\phi'_p) = 0.0663 \exp\left(9.3 \times 28 \times \frac{\pi}{180}\right)$$

$$= 6.8$$

$$\gamma' = \gamma_{\text{sat}} - \gamma_w = 16.5 - 9.8 = 6.7 \text{ kN/m}^3$$

$$q_u = 0.5\gamma' B' N_\gamma s_\gamma$$

$$= 0.5 \times 6.7 \times 5.5 \times 6.8 \times 0.6$$

$$= 75 \text{ kPa}$$

$$\text{FS} = \frac{75}{16.5} = 4.6 > 3 \quad (\text{okay})$$

Settlement governs the design.

EXAMPLE 9.16 Design (Sizing) of Footings to Limit Differential Settlement

Figure E9.16 shows two isolated footings at the two ends of a building. Your local code requires a maximum distortion of $\delta/\ell = 350$ and a maximum total settlement of 50 mm. Determine the most economical size for each footing to satisfy ultimate and serviceability limit states. Groundwater level is 5 m below the ground surface. Creep effects in the soil are negligible. The embedment depth is 1 m.

Strategy The clay layer is very thick and only a certain thickness below each footing would be stressed. From Chapter 5, you know that the vertical stress increase below a depth of $2B$ for a square footing is less than 10%. So you can use a thickness of $2B$ or $3B$ as the effective thickness of the clay. The Skempton–Bjerrum method for calculating consolidation settlement is suitable for this problem.

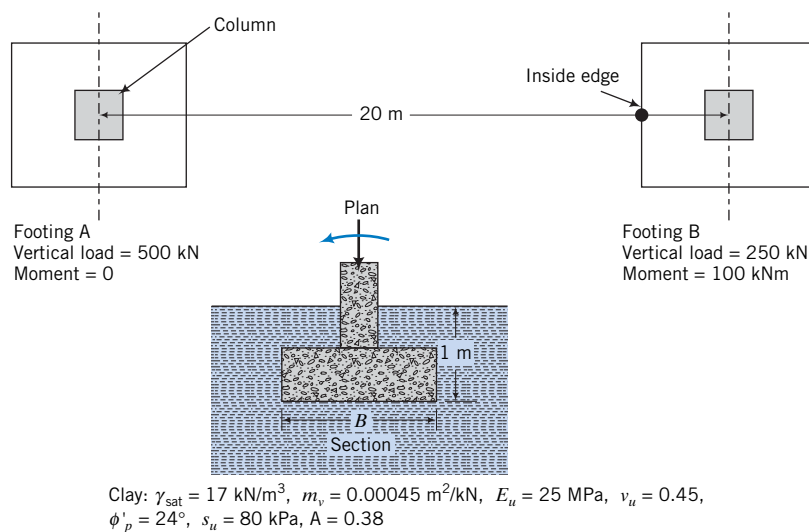


FIGURE E9.16

Solution 9.16

Footing A

Step 1: Assume a width.

Assume a square footing of width $B = 3$ m.

Step 2: Calculate the elastic settlement.

$$\begin{aligned} \text{From Eqs. (9.34) to (9.36): } \frac{A}{4L^2} &= 1, \quad \mu_{\text{wall}} = 1, \quad \mu_s = 0.45 \left(\frac{A_b}{4L^2} \right)^{0.38} \\ &= 0.45 \times 1 = 0.45 \end{aligned}$$

$$\begin{aligned} \mu_{\text{emb}} &= 1 - 0.04 \frac{D_f}{B} \left[1 + \frac{4}{3} \left(\frac{A_b}{4L^2} \right) \right] \\ &= 1 - 0.04 \frac{1}{1.5} \left[1 + \frac{4}{3} (1) \right] = 0.94 \end{aligned}$$

$$\begin{aligned} \text{From Eq. (9.33): } \rho_e &= \frac{P}{E_u L} (1 - \nu_u^2) \mu_s \mu_{\text{emb}} \mu_{\text{wall}} = \frac{500}{25,000 \times 1.5} (1 - 0.45^2) \\ &\times 0.45 \times 0.94 \times 1 = 4.5 \times 10^{-3} = 4.5 \text{ mm} \end{aligned}$$

Step 3: Compute the consolidation settlement.

Determine the stress increase over a depth $3B = 9$ m (effective depth) below the footing. Divide the effective depth into three layers of 3 m each and find the stress increase at the center of each layer under the center of the footing. Stress overlap is unlikely as the footings are greater than B apart.

$$\sigma_a = \frac{500}{3^2} = 55.6 \text{ kPa}$$

Layer	Depth to center of layer, z (m)	$m = n = \frac{(3/2)}{z}$	I_z	$\Delta\sigma_z = 4q_s I_z$ (kPa)
1	1.5	1	0.175	38.9
2	4.5	0.33	0.045	10.0
3	7.5	0.2	0.018	4.0
				<u>$\Sigma 52.9$</u>

$$\text{Equivalent diameter of footing: } D = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{9}{\pi}} = 3.38 \text{ m}$$

$$\frac{H_o}{B} = \frac{H_o}{D} = \frac{9}{3.38} = 2.7$$

$H_o/B = 5.3$ is outside the plotted limits in Fig. 9.16. We will use $H_o/B = 4$, which will result in an overestimation of the primary consolidation settlement. From Fig. 9.16, $\mu_{\text{SB}} = 0.58$ for $A = 0.38$.

$$\begin{aligned} \rho_{\text{pc}} &= \Sigma m_v \Delta\sigma_z H_o \mu_{\text{SB}} = 0.00045 \times 52.9 \times 3 \times 0.58 \\ &= 41.4 \times 10^{-3} \text{ m} = 41.4 \text{ mm} \end{aligned}$$

$$\text{Total settlement} = 4.5 + 41.4 = 45.9 \text{ mm} < 50 \text{ mm}$$

Total settlement of footing A is okay.

Footing B

Step 4: Calculate the eccentricity.

$$e = \frac{M}{P} = \frac{100}{250} = 0.4 \text{ m}$$

Step 5: Assume a width.

For no tension $e/B < 6$; that is, $B > 6e$.

$$B_{\min} = 6 \times 0.4 = 2.4 \text{ m}; \quad \text{try } B = 2.5 \text{ m}$$

Step 6: Calculate the elastic settlement

$$\mu_{\text{emb}} = 1 - 0.04 \times \frac{1}{1.25} \left(1 + \frac{4}{3}(1) \right) = 0.93$$

$$\rho_e = \frac{500}{25,000 \times 1.25} (1 - 0.45^2) 0.45 \times 0.93 \times 1 = 5.3 \times 10^{-3} \text{ m} = 5.3 \text{ mm}$$

Step 7: Calculate the consolidation settlement.

Because of the eccentric loading, the vertical stress distribution under footing B would be nonuniform. The maximum vertical stress, which occurs at the inside edge of magnitude (Eq. 9.25), is

$$(\sigma_a)_{\max} = \frac{250}{2.5^2} \left(1 + \frac{6 \times 0.4}{2.5} \right) = 78.4 \text{ kPa}$$

We need then to find the stress increase under the center of the inside edge of footing B (shown by a circle in Fig. E9.16). A table is used for ease of computation and checking.

Layer	Depth to center of layer, z (m)	$m = \frac{B}{z} = \frac{2.5}{z}$	$n = \frac{L}{z} = \frac{1.25}{z}$	I_z	$\Delta\sigma_z = 2q_s I_z$
1	3	0.83	0.42	0.10	15.7
2	4.5	0.56	0.28	0.055	8.6
3	7.5	0.33	0.17	0.025	3.9
					<u>$\Sigma 28.2$</u>

Equivalent diameter of footing: $D = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{2.5^2}{\pi}} = 2.8 \text{ m}$

$$\frac{H_o}{B} = \frac{H_o}{D} = \frac{9}{2.8} = 3.2$$

From Fig. 9.16, $\mu_{\text{SB}} = 0.57$.

$$\rho_{\text{pc}} = \Sigma m_v \Delta\sigma_z H_o \mu_{\text{SB}} = 0.00045 \times 28.2 \times 3 \times 0.57$$

$$= 22 \times 10^{-3} \text{ m} = 22 \text{ mm}$$

Step 8: Calculate the total settlement.

$$\rho = \rho_e + \rho_{\text{pc}} = 5.3 + 22 = 27.3 \text{ mm} < 50 \text{ mm}$$

Total settlement of footing B is okay.

Step 9: Calculate the distortion.

$$\delta = 45.9 - 27.4 = 18.5 \text{ mm}$$

$$\frac{\delta}{L} = \frac{18.5}{(20 - 1.25) \times 10^3} = \frac{1}{1014} < \frac{1}{350}; \quad \text{therefore, distortion is okay}$$

Step 10: Check the bearing capacity.

Use Meyerhof's method because of the eccentric loading of footing B.

Footing A

Factors

Neglect effects of depth of embedment.

$$B' = B, \quad L' = L$$

$$s_c = 1 + 0.2 \frac{B'}{L'} = 1.2$$

$$s_\gamma = 1 - 0.4 \frac{B'}{L'} = 0.6$$

$$s_q = 1 + \frac{B'}{L'} \tan \phi'_p = 1 + \tan 24^\circ = 1.45$$

$$d_c = d_q = d_\gamma = 1$$

TSA

$$q_u = 5.14 s_u s_c d_c = 5.14 \times 80 \times 1.2 \times 1.0 = 493 \text{ kPa}$$

$$\text{FS} = \frac{493}{55.6 - 17 \times 1} = 13 > 3 \quad (\text{okay})$$

ESA

Assume smooth surface.

$$N_q = e^{\pi \tan 24^\circ} \tan^2(45^\circ + 24^\circ/2) = 9.6$$

$$N_\gamma = 0.0663 \exp(9.3 \phi'_p) = 0.0663 \exp\left(9.3 \times 24 \times \frac{\pi}{180}\right) = 3.3$$

$$q_u = \gamma D_f (N_q - 1) s_q d_q + 0.5 \gamma B' N_\gamma s_\gamma d_\gamma$$

$$q_u = (17 \times 1 \times 8.6 \times 1.45 \times 1.0) + (0.5 \times 17 \times 3 \times 3.3 \times 0.6 \times 1.0) = 262 \text{ kPa}$$

$$\text{FS} = \frac{262}{55.6 - 17 \times 1} = 6.8 > 1.5 \quad (\text{okay})$$

Footing B

$$B' = B - 2e = 2.5 - 2 \times 0.4 = 1.7 \text{ m}$$

TSA

Same as Footing A.

ESA

$$q_u = \gamma D_f (N_q - 1) s_q d_q + 0.5 \gamma B' N_\gamma s_\gamma d_\gamma$$

$$= (17 \times 1 \times 8.6 \times 1.45 \times 1.0)$$

$$+ (0.5 \times 17 \times 1.7 \times 3.3 \times 0.6 \times 1.0) = 241 \text{ kPa}$$

$$\text{FS} = \frac{241}{78.4 - 17 \times 1} = 3.9 > 1.5 \quad (\text{okay})$$

Step 11: Recommend footing sizes.

Footing A: $3 \text{ m} \times 3 \text{ m}$; $\rho \approx 46 \text{ mm}$

Footing B: $2.5 \text{ m} \times 2.5 \text{ m}$; $\rho \approx 27 \text{ mm}$

EXAMPLE 9.17 Sizing Footings for a Building Using SPT Data

The column layout for part of a building is shown in Fig. E9.17a. Three boreholes are located within this part of the building and the SPT data are shown in Figs. E9.17b,c,d. Determine satisfactory sizes for the foundations under columns B2 and B4 using ASD. The loads on these columns are given in Fig. E9.17a. The design criteria based on local codes are maximum settlement = 25 mm, minimum depth of footing = 300 mm, and the finished grade elevation is 780 m. Groundwater was not reached in any borehole.

Strategy It is best to draw an estimated soil profile based on the three borehole data items. It is customary to size the footing under the largest load and then use the maximum vertical stress under this footing to size the other footings. This may not always be satisfactory as will be shown in this example. Other conditions such as a footing with a lower load located on a weaker soil layer than the footing with the highest load may govern the design. This example illustrates the iteration process required in design.

Solution 9.17

Step 1: Draw a composite soil profile based on nearby boreholes.

- (1) Map boreholes in a graph of elevation versus distance using column line ④–④ as reference line (see Fig. E9.17e).
- (2) Inspect soil type and N values in each borehole.
- (3) Sketch lines linking similar soil types.
- (4) Divide each soil type, if necessary, into different density groups (dense, medium, loose) based on N values and soil description (see Fig. E9.17e). Assign values of unit weight based on Table 7.4.

Step 2: Decide on the depth for the footing with highest load.

Assume $D_f = 0.5$ m (Elevation: 779.5) for footing B4.

This footing is just below the top of soil layer 2.

Step 3: Assume a footing size for B4.

Assume a square footing 1 m $\times$ 1 m.

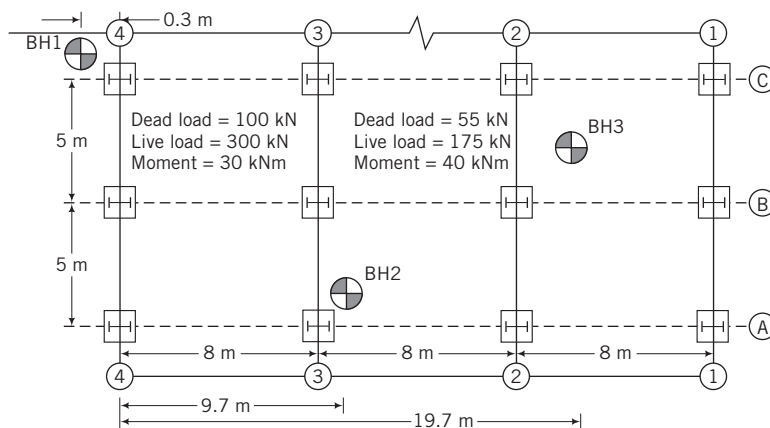


FIGURE E9.17a

														BORING LOG: BH-1							
														SHEET 1 of 1 PROJECT #:							
PROJECT: CLIENT: CLIENT PROJECT:														CONTRACTOR: DRILLER: INSPECTOR:							
COORDINATES N: E: LOCATION: Tucson														REF. ALIGNMENT: STATION: OFFSET:				RIG TYPE: DRILLING METHOD: HAMMER TYPE:			
COMMENTS:														SURFACE ELEV.: 781.5 TOTAL DEPTH: 23 START DATE: TIME: FINISH DATE: TIME:							
		Casing	Split Spoon	Ring Sampler	Shelby Tube	Cuttings	Core Barrel	GROUNDWATER DATA													
Type/Symbol		S	☒	R	☐	U	☐	CU	☐	C	☒	Date	Time	Water Depth (m)	Casing Depth (m)	Hole Depth (m)	Symbol				
I.D.		34.93mm	63.50mm																		
O.D.		50.80mm	76.20mm																		
Length		457.20mm	457.20mm																		
Hammer WT.		622.75 N	Drill Rod Size																		
Hammer Fall		762mm	I.D. (O.D.)																		
DEPTH BELOW SURFACE (m)	ELEVATION (m)	GRAPHIC	SOIL SAMPLE			BLOWS			N-VALUE	RECOVERY (m)	VISUAL MATERIAL CLASSIFICATION AND REMARKS	MOISTURE, %	DRY DENSITY	SAMPLES SENT TO LAB							
			TYPE	NUMBER	SYMBOL	DEPTH (m) FROM TO	0-152.4 mm	152.4-304.8 mm							304.8-457.2 mm						
5	780		S	S1	☒	0.5	0.957	4	5	9	14	.457	Silty sand with gravel (SM)								
													Poorly graded sand with gravel and boulders (SP)								
	775		S	S2	☒	6	6.457	20	21	17	38	.457									
	770		S	S3	☒	10	10.457	12	14	10	24	.457	Lean clay, hard, dark brown to gray (CL)								
	765		S	S4	☒	14	14.457	25	30	41	71	229	Well graded sand, very dense (SW)								
	760																				
	25												EOB @ 23m. No ground water encountered.								

FIGURE E9.17b

										BORING LOG: BH-2									
										SHEET 1 of 1 PROJECT #:									
PROJECT: CLIENT: CLIENT PROJECT:										CONTRACTOR: DRILLER: INSPECTOR:									
COORDINATES N: E: LOCATION: Tucson										REF. ALIGNMENT: STATION: OFFSET:									
COMMENTS:										SURFACE ELEV.: 781 TOTAL DEPTH: 20 START DATE: TIME: FINISH DATE: TIME:									
		Casing	Split Spoon	Ring Sampler	Shelby Tube	Cuttings	Core Barrel	GROUNDWATER DATA											
Type/Symbol		S		R		CU		C		Date	Time	Water Depth (m)	Casing Depth (m)	Hole Depth (m)	Symbol				
I.D.		34.93mm	63.50mm																
O.D.		50.80mm	76.20mm																
Length		457.20mm	457.20mm																
Hammer WT.		622.75 N	Drill Rod Size																
Hammer Fall		762mm	I.D. (O.D.)																
DEPTH BELOW SURFACE (m)	ELEVATION (m)	SOIL SAMPLE				BLOWS				N-VALUE	RECOVERY (m)	VISUAL MATERIAL CLASSIFICATION AND REMARKS				MOISTURE, %	DRY DENSITY	SAMPLES SENT TO LAB	
		TYPE	NUMBER	SAMPLE	DEPTH (m)	FROM	TO	0-152.4 mm	152.4-304.8 mm	304.8-457.2 mm									
780		S	S1		1	1.457	6	9	10	19	.457	Silty sand with gravel (SM)							
		S	S2		2	2.457	11	12	10	22	.457	Poorly graded sand with trace of gravel (SP)							
5																			
775		S	S3		7	7.457	20	28	20	48	.457								
10		S	S4		9	9.457	10	13	15	28	.457	Lean clay, hard, dark brown to gray (CL)							
770																			
15		S	S5		14	14.457	25	35	31	66	.229	Well graded sand, very dense							
765																			
20																			
760												EOB @ 20m. No ground water encountered.							
25																			

FIGURE E9.17c

[illegible]

FIGURE E9.17d

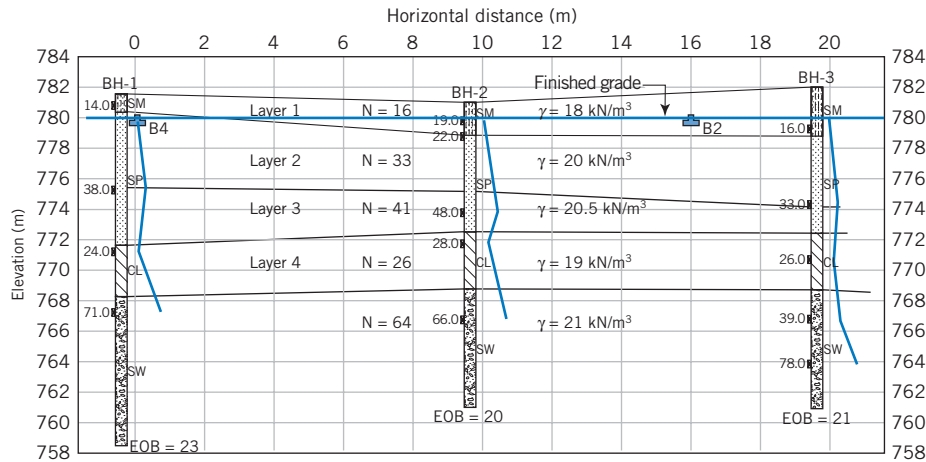


FIGURE E9.17e

Step 4: Correct N values.

Use an average depth of $\frac{1.5B}{2}$ below the footing, i.e., $\frac{1.5 \times 1}{2} = 0.75$ m (Elevation: 778.75)
 Original ground level at B4: $\simeq$ elevation 781.5 m.

$$\sigma'_{zo} = \gamma z = 18(781.5 - 780.5) + 20(780.5 - 778.75) = 53 \text{ kPa}$$

$$c_N = \left(\frac{95.8}{\sigma'_{zo}} \right)^{\frac{1}{2}} = \left(\frac{95.8}{53} \right)^{\frac{1}{2}} = 1.34 < 2 \quad (\text{okay})$$

$$N_1 = c_N N = 1.34 \times 33 = 44$$

Step 5: Estimate q_{ult} .

$$q_{ult} = 32N_1B = 32 \times 44 \times 1 = 1408 \text{ kPa}$$

Step 6: Calculate maximum imposed vertical stress.

$$V = \text{Total vertical load} = 400 \text{ kN}; \quad M = \text{Moment} = 30 \text{ kN} \cdot \text{m}$$

$$\text{eccentricity, } e = \frac{30}{400} = 0.075 \text{ m}$$

$$\frac{B}{6} = \frac{1.0}{6} = 0.17 > e; \quad \therefore \text{No tension}$$

$$\sigma_{\max} = \frac{V}{A} \left(1 + \frac{6e}{B} \right) = \frac{400}{1 \times 1} \left(1 + \frac{6 \times 0.075}{1} \right) = 580 \text{ kPa}$$

Step 7: Check factor of safety.

$$FS = \frac{q_{ult}}{\sigma_{\max}} = \frac{1664}{580} = 2.4$$

Adjust B so that $FS \simeq 3$. Using a spreadsheet makes this easier as shown in Table E9.17a.
 With a square footing $1.1 \text{ m} \times 1.1 \text{ m}$, $FS = 3.3$.

Step 8: Sizing footing B2.

Assume the depth of footing B2 is at the same elevation as B4. Footing B2 is now located in soil layer 1 with $N = 16$. The key point is that in order for the settlement of footings B2 and B4 to be about equal (< 25 mm), the maximum vertical stress should be the same.

TABLE E9.17a

Finished grade elevation	780	m	Dead load	100	kN		
Width of footing	1.1	m	Live load	300	kN		
Depth of footing	0.5	m	Moment	30.00	kN.m		
Length of footing	1.1	m	Total load	400.00	kN		
Groundwater	100	m	e	0.08	m	no tension	
FS	3.3		$\sigma_{\max}$	465.8	kPa		

Elevation m	Thickness m	Unit weight kN/m ³	Vertical effective stress kPa	C_N use	N	N_1	q_{ult} kPa
781.5	0	0	0.0	0	0	0	0
780.5	1	18	18.0	2.0	16	32	1126
778.7	1.8	20	54.5	1.3	33	44	1540

Step 9: Calculate size of footing B2.

$$\text{Total load} = 55 + 175 = 230 \text{ kN}; \quad \text{Moment} = 40 \text{ kN} \cdot \text{m}$$

$$e = \frac{40}{230} = 0.17 \text{ m}$$

$$B_{\min} \text{ for no tension} = 6e = 6 \times 0.17 = 1.02 \text{ m}$$

$$\text{Assume } B = 1.1 \text{ m}$$

$$\sigma_{\max} = \frac{V}{A} \left(1 + \frac{6e}{B} \right)$$

$$\sigma_{\max} = \frac{230}{1.1^2} \left(1 + \frac{6 \times 0.17}{1.1} \right) = 370 \text{ kPa} > 465.8 \text{ kPa} \quad (\text{Table E9.17a})$$

To get $\sigma_{\max} = 465.8 \text{ kPa}$, B must be reduced. But this cannot be done because tension would develop. Therefore the maximum vertical stress under footing B2 controls the design.

Step 10: Resize footing B2.

It is best to set up a spreadsheet to carry out the calculations as shown in Table E9.17b. With a footing size of $1.2 \text{ m} \times 1.2 \text{ m}$, the factor of safety is 3.1 and the maximum vertical stress is 298.6 kPa.

TABLE E9.17b

Finished grade elevation	780	m	Dead load	55	kN		
Width of footing	1.2	m	Live load	175	kN		
Depth of footing	0.5	m	Moment	40.00	kN.m		
Length of footing	1.2	m	Total load	230.00	kN		
Groundwater	100	m	e	0.17	m	no tension	
FS	3.1		$\sigma_{\max}$	298.6	kPa		

Elevation m	Thickness m	Unit weight kN/m ³	Vertical effective stress kPa	C_N use	N	N_1	q_{ult} kPa
781.0	0	0	0.0	0	0	0	0
778.6	2.4	18	43.2	1.5	16	24	915

TABLE E9.17c

Finished grade elevation	780	m	Dead load	100	kN		
Width of footing	1.35	m	Live load	300	kN		
Depth of footing	0.5	m	Moment	30.00	kN.m		
Length of footing	1.35	m	Total load	400.00	kN		
Groundwater	100	m	e	0.08	m	no tension	
FS	6.2		$\sigma_{\max}$	292.6	kPa		

Elevation m	Thickness m	Unit weight kN/m ³	Vertical effective stress kPa	C_N use	N	N_1	q_{ult} kPa
781.5	0	0	0.0	0	0	0	0
780.5	1	18	18.0	2.0	16	32	1382
778.5	2.0	20	58.3	1.3	33	42	1828

Step 11: Resize footing B4.

Use the maximum vertical stress due to footing B2 as allowable stress for B4. The size of footing for B4 is 1.35 m $\times$ 1.35 m with a maximum vertical stress of 292.6 kPa and a factor of safety of 6.2 (see Table E9.17c) from spreadsheet.

Step 12: Assess design.

The settlement of the two footings will be the same only if the N value and the applied stress distribution are the same. Since footing B2 is located in a weaker soil (lower N value) than footing B4, the settlement is unlikely to be the same.

A better alternative is to locate both footings in layer 2 ($N = 33$) at an elevation of 778.5 m. Additional expenses will occur for excavation to construct the footings but the settlement of the two footings is now likely to be similar and the footing sizes may be smaller.

Resize footing B2

Try $B = L = 1.1$ m and $D_f = 1.5$ m. From Table E9.17d, $\sigma_{\max} = 370.4$ kPa and FS = 3.9.

Resize footing B4

TABLE E9.17d

Finished grade elevation	780	m	Dead load	55	kN		
Width of footing	1.1	m	Live load	175	kN		
Depth of footing	1.5	m	Moment	40.00	kN.m		
Length of footing	1.1	m	Total load	230.00	kN		
Groundwater	100	m	e	0.17	m	no tension	
FS	3.9		$\sigma_{\max}$	370.4	kPa		

Elevation m	Thickness m	Unit weight kN/m ³	Vertical effective stress kPa	C_N use	N	N_1	q_{ult} kPa
781	0	0	0.0	0	0	0	0
779	2	18	36.0	1.6	16	26	919
777.7	1.3	20	62.5	1.2	33	41	1438

TABLE E9.17e

Finished grade elevation	780	m	Dead load	100	kN		
Width of footing	1.21	m	Live load	300	kN		
Depth of footing	1.5	m	Moment	30.00	kN.m		
Length of footing	1.21	m	Total load	400.00	kN		
Groundwater	100	m	e	0.08	m	no tension	
FS	3.8		$\sigma_{\max}$	374.8	kPa		

Elevation m	Thickness m	Unit weight kN/m ³	Vertical effective stress kPa	C_N use	N	N_1	q_{ult} kPa
781.5	0	0	0.0	0	0	0	0
780.5	1	18	18.0	2.0	16	32	1239
777.6	2.9	20	76.2	1.1	33	37	1433

With $B = L = 1.21$ m, $\sigma_{\max} = 374.8$ kPa and FS = 3.8 (see Table E9.17e).

EXERCISES

Theory

- 9.1 Show that the ultimate load for a strip footing under long-term conditions using the two triangle failure surfaces shown in Fig. P9.1 is $P_u = \frac{1}{2} \gamma B^2 N_\gamma$, where

$$N_\gamma = \frac{2 \tan \phi' (\tan^2 \phi' - 3)}{[(1 + \tan^2 \phi') - 2](1 - \tan \phi')} = \frac{2 \sin 3\phi'}{\cos \phi' - \sin 3\phi'}$$

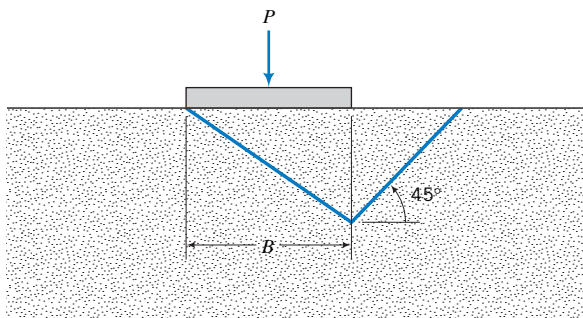


FIGURE P9.1

- 9.2 A strip footing, 5 m wide, is founded on the surface of a deep deposit of clay. The undrained shear strength of the clay increases linearly from 3 kPa at the surface to 10 kPa at a depth of 5 m. Estimate the vertical ultimate load assuming that the load is applied at an eccentricity of 0.75 m from the center of the footing's width. [Hint: Try a circular failure surface, determine the equation for the distribution of shear strength with depth, and integrate the shear strength over the radius to find the shear force.]
- 9.3 The centroid of a square foundation of side 5 m is located 10 m away from the edge of a vertical cut of depth 4 m. The soil is a stiff clay whose undrained strength is 20 kPa and whose unit weight is 16 kN/m³. Calculate the vertical ultimate load. Assume a circular failure surface for the footing and a planar surface for the cut.

Problem Solving

- 9.4** Calculate the ultimate net bearing capacity of (a) a strip footing 2 m wide, (b) a square footing 3 m $\times$ 3 m, and (c) a circular footing 3 m in diameter. All footings are located on the ground surface and the groundwater level is at the ground surface. The soil is medium-dense coarse-grained with $\gamma_{\text{sat}} = 17 \text{ kN/m}^3$, $\phi'_p = 30^\circ$.
- 9.5** A strip footing, founded on dense sand ($\phi'_p = 35^\circ$ and $\gamma_{\text{sat}} = 17 \text{ kN/m}^3$), is to be designed to support a vertical load of 400 kN per meter length. Determine a suitable width for this footing for FS = 3. The footing is located 1 m below the ground surface. The groundwater level is 10 m below the ground surface.
- 9.6** A square footing, 3 m wide, is located 1.5 m below the surface of a stiff clay. Determine the allowable bearing capacity for short-term condition if $s_u = 100 \text{ kPa}$, and $\gamma_{\text{sat}} = 20 \text{ kN/m}^3$. If the footing were located on the surface, what would be the allowable bearing capacity? Use FS = 3.
- 9.7** A column carrying a load of 750 kN is to be founded on a square footing at a depth of 2 m below the ground surface in a deep clay stratum. What will be the size of the footing for FS = 3 for ESA and TSA? The soil properties are $\phi'_p = 28^\circ$, $\gamma_{\text{sat}} = 18.5 \text{ kN/m}^3$, and $s_u = 55 \text{ kPa}$. The groundwater level is at the base of the footing but it is expected to rise to the ground surface during rainy seasons.
- 9.8** Repeat Exercise 9.7 with a moment of 250 kN.m about an axis parallel to the length in addition to the vertical load.
- 9.9** A square footing located on a dense sand is required to carry a load of 500 kN inclined at 15° to the vertical plane. The building code requires an embedment depth of 1.2 m. Groundwater level is at 1 m below the ground surface. Calculate the size of the footing for $\phi'_p = 37^\circ$, $\gamma_{\text{sat}} = 18.5 \text{ kN/m}^3$ and FS = 3. Assume the soil above the groundwater level to be saturated.
- 9.10** The footing for a bridge pier is to be founded in sand, as shown in Fig. P9.10. The clay layer is normally consolidated with $C_c = 0.25$. Determine the factor of safety against bearing capacity failure and the total settlement (elastic compression and primary consolidation) of the pier.

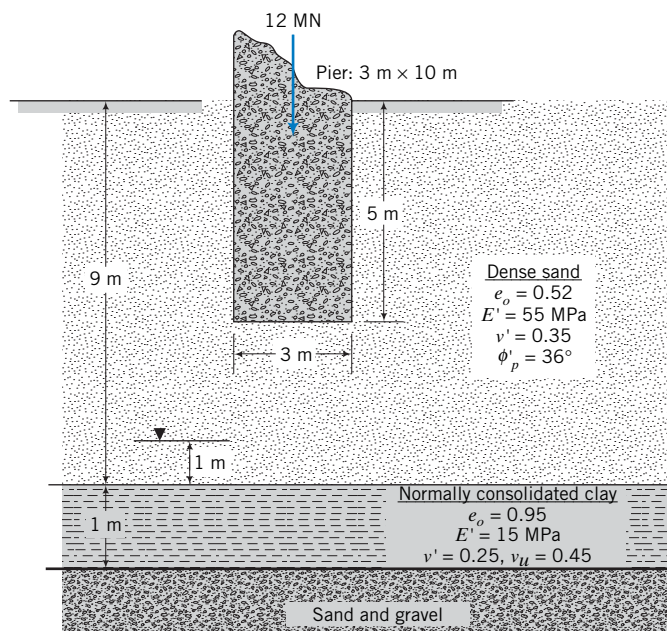


FIGURE P9.10

- 9.11** A multilevel building is supported on a footing 58 m wide $\times$ 75 m long $\times$ 3 m thick resting on a very stiff deposit of saturated clay. The footing is located at 3 m below ground level. The average stress at the base of

the footing is 350 kPa. Groundwater level is at 12 m below the surface. Field and laboratory tests gave the following results:

Depth (m)	0.5	6	25
s_u (kPa)	58	122	156

$e_0 = 0.57$, $C_c = 0.16$, $C_r = 0.035$, $\text{OCR} = 10$, $\phi'_p = 28^\circ$, $\phi'_{cs} = 24^\circ$, $E_u = 100$ MPa, $\nu_u = 0.45$, $E' = 90$ MPa, and $\nu' = 0.3$. Determine the total settlement and the safety factor against bearing capacity failure.

Practical

- 9.12** A circular foundation of diameter 8 m supports a tank. The base of the foundation is at 1 m from the ground surface. The vertical load is 20 MN. The tank foundation was designed for short-term loading conditions ($s_u = 80$ kPa and $\gamma_{\text{sat}} = 19$ kN/m³). The groundwater level when the tank was initially designed was at 4 m below the ground surface. It was assumed that the groundwater level was stable. Fourteen months after the tank was constructed and during a week of intense rainfall, the tank foundation failed. It was speculated that failure occurred by bearing capacity failure. Establish whether this is so or not. The friction angle is $\phi'_p = 25^\circ$.
- 9.13** The foundation (base) of a long retaining wall is required to support the load (including self-weight of the base and the wall) and moment (this comes from the lateral loads on the wall) shown in Fig. P9.13. Determine the factor of safety against bearing capacity failure. Would the settlement of the base be uniform? Discuss your answer.

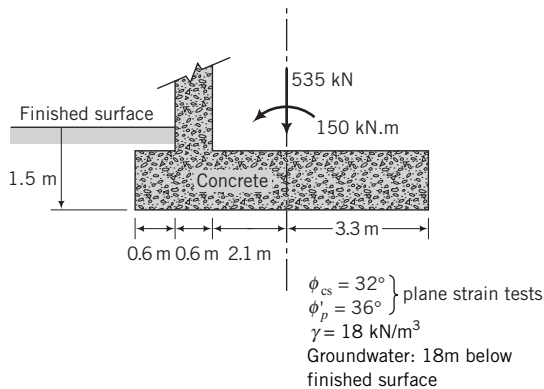


FIGURE P9.13

- 9.14** Calculate the size of a square footing on a soil profile consisting of a medium-dense layer with N values as shown in the table below.

Depth (m)	0	5	10	15
N	0	28	31	37

The design load is 700 kN and an $\text{FS} = 3$ is desired. The footing is embedded 0.5 m into the sand. Groundwater is 4 m below the surface.

CHAPTER 10

PILE FOUNDATIONS

10.0 INTRODUCTION

In the previous chapter, we studied the analysis of shallow foundations. In some cases, shallow foundations are inadequate to support the structural loads and deep foundations are required. When the term deep foundations is used, it invariably means pile foundations. A pile is a slender, structural member installed in the ground to transfer the structural loads to soils at some significant depth below the base of the structure. Structural loads include axial loads, lateral loads, and moments.

Pile foundations are used when:

- The soil near the surface does not have sufficient bearing capacity to support the structural loads.
- The estimated settlement of the soil exceeds tolerable limits (i.e., settlement greater than the serviceability limit state).
- Differential settlement due to soil variability or nonuniform structural loads is excessive.
- The structural loads consist of lateral loads and/or uplift forces.
- Excavations to construct a shallow foundation on a firm soil layer are difficult or expensive.

In this chapter, you will study the bearing capacity (load capacity) and settlement of single and group piles under axial loads. When you complete this chapter, you should be able to:

- Determine the allowable axial load capacity of single piles and pile groups.
- Determine the settlement of single piles and pile groups.

You would need to recall the following:

- Effective stresses—Chapter 5
- Consolidation—Chapter 6
- Statics

Sample Practical Situation A structure is to be constructed on a deposit of soft soil. Shallow foundations were ruled out because the estimated settlement exceeded the tolerable settlement. The structure is to be supported on piles. You are required to determine the type of piles to be used, the configuration of the piles (single piles or pile groups) and the length of piles required to safely support the structure without exceeding ultimate and serviceability limit states. An example of a pile foundation under construction is shown in Fig. 10.1.



FIGURE 10.1 Concrete piles used for the foundation of a skyscraper. (© Fritz Henle/Photo Researchers.)

10.1 DEFINITIONS OF KEY TERMS

Displacement pile —a pile that displaces a large volume of soil. Driven piles with solid sections are displacement piles.

Nondisplacement pile —a pile that displaces only a small volume of soil relative to its external volume. Steel H-piles and open-ended pipe piles are nondisplacement piles.

Pile —a slender, structural member consisting of steel or concrete or timber.

Skin friction stress or shaft friction stress or adhesive stress (f_s) is the frictional or adhesive stress on the shaft of a pile.

End bearing stress or point resistance stress (f_b) is the stress at the base or tip of a pile.

Ultimate load capacity (Q_{ult}) is the maximum load that a pile can sustain before soil failure occurs.

Ultimate group load capacity [$(Q_{ult})_g$] is the maximum load that a group of piles can sustain before soil failure occurs.

Skin friction or shaft friction or side shear (Q_f) is the frictional force generated on the shaft of a pile.

End bearing or point resistance (Q_b) is the resistance generated at the base or tip of a pile.

End bearing or point bearing pile is one that transfers almost all the structural load to the soil at the bottom end of the pile.

Friction pile is one that transfers almost all the structural load to the soil by skin friction along a substantial length of the pile.

Floating pile is a friction pile in which the end bearing resistance is neglected.

10.2 QUESTIONS TO GUIDE YOUR READING

1. What are the differences between the different types of piles?
2. How is a pile installed in the ground?
3. How do I estimate the allowable load capacity of a single pile and pile groups?
4. How do I estimate the settlement of a single pile and pile groups?
5. Is the analysis of the load capacity of a pile exact or is it an approximation?

10.3 TYPES OF PILES AND INSTALLATION



Concept Learning and Self-Assessment

Access Chapter 10, Section 10.3 on the CD to learn about the different pile types. Take Quiz 10.3 to test your understanding.

Piles are made from concrete or steel or timber (Fig. 10.2). The selection of the type of pile required for a project depends on what type is readily available, the magnitude of the loading, the soil type, and

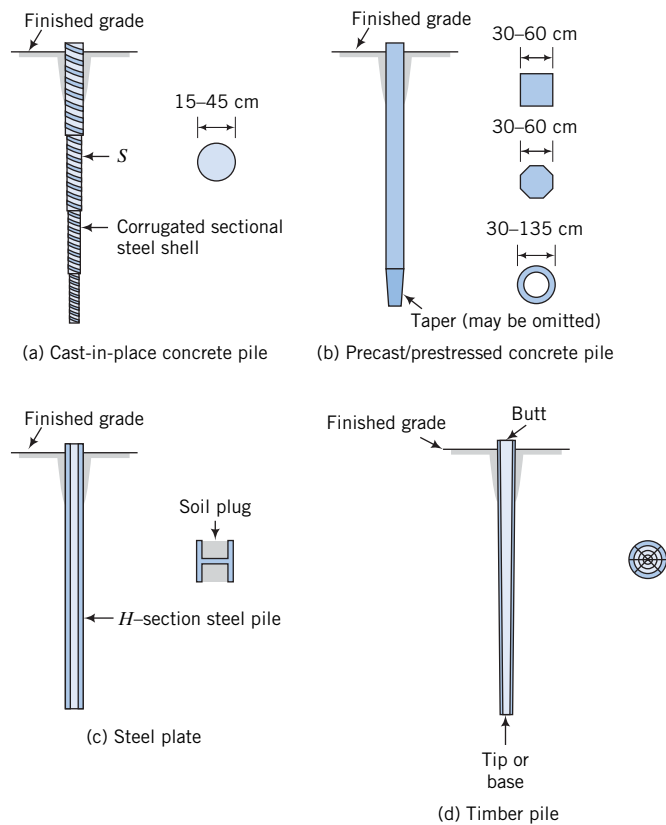


FIGURE 10.2 Pile types.

the environment in which the pile will be installed, for example, a corrosive environment or a marine environment.

10.3.1 Concrete Piles

There are several types of concrete piles that are commonly used. These include cast-in-place concrete piles, precast concrete piles, and drilled shafts. Cast-in-place concrete piles are formed by driving a cylindrical steel shell into the ground to the desired depth and then filling the cavity of the shell with fluid concrete. They are called displacement piles. The steel shell is for construction convenience and does not contribute to the load transfer capacity of the pile. Its purpose is to open a hole in the ground and keep it open to facilitate the construction of the concrete pile. Plain concrete is used when the structural load is only compressive. If moments and lateral loads are to be transferred, then a steel reinforcement cage is used in the upper part of the pile. Vigilant quality control and good construction practice are necessary to ensure the integrity of cast-in-place piles.

Precast concrete piles usually have square or circular or octagonal cross sections and are fabricated in a construction yard from reinforced or prestressed concrete. They are preferred when the pile length is known in advance. The disadvantages of precast piles are problems in transporting long piles, cutting, and lengthening. A very popular type of precast concrete pile is the Raymond cylindrical prestressed pile. This pile comes in sections, and lengths up to 70 m can be obtained by stacking the sections. Typical design loads are greater than 2 MN.

10.3.2 Steel Piles

Steel piles come in various shapes and sizes and include cylindrical, tapered, and H-piles. Steel H-piles are rolled steel sections and are called nondisplacement piles. Steel pipe piles are seamless pipes that can be welded to yield lengths up to 70 m. They are usually driven with open ends into the soil. A conical tip is used where the piles have to penetrate boulders and rocks. To increase the load capacity of steel pipe piles, the soil plug (Fig. 10.2c) is excavated and replaced by concrete. These piles are called concrete-filled steel piles. The soil plug may adhere to the pile surface and moves down with it during driving. This is called plugging.

10.3.3 Timber Piles

Timber piles have been used since ancient times. The lengths of timber piles depend on the types of trees used to harvest the piles, but common lengths are about 12 m. Longer lengths can be obtained by splicing several piles. Timber piles are susceptible to termites, marine organisms, and rot within zones exposed to seasonal changes. Timber piles are displacement piles.

A comparative summary of the different pile types is given in Table 10.1.

10.3.4 Pile Installation

Piles can either be driven into the ground (driven piles) or be installed in a predrilled hole (bored piles or drilled shafts). A variety of driving equipment is used in pile installations. The key components are the leads and the hammer. The leads are used to align the hammer to strike the pile squarely (Fig. 10.3). Hammers can be simple drop hammers of weights between 2.5 and 15 kN or modern steam/pneumatic hammers. Two popular types of steam/pneumatic hammers are shown in Fig. 10.4. The single-acting hammer utilizes steam or air to lift the ram and its accessories (cushion, drive caps, etc.). The double-acting hammer is used to increase the number of blows/minute and utilizes steam/air to lift the ram and force it down.

TABLE 10.1 Comparisons of Different Piles

Pile type	Section (m)	Common lengths (m)	Average load (kN)	Allowable stress (MPa)	Advantages	Disadvantages
Cast-in-place concrete	0.15–0.45 (diameter)	≤35	600	4.5–8.5	Can sustain hard driving, resistant to marine organisms, easily inspected, length can be changed easily, easy to handle and ship	Concrete can arch during placement, can be damaged if adjacement piles are driven before concrete sets
Precast concrete	0.15–0.25 (width)	≤35	750	4.5–7	Economical for specified length, higher capacity than timber	Cutting and lengthening of piles can be expensive, handling is a problem, shipping long piles is expensive, may crack during driving
Raymond cylinder	0.6–2.3 (diameter)	<60	2000	40–70	Can ship in sections, high capacity, long length	Cost (expensive)
Steel pipe	0.2–1 (diameter)	<35	900	59–83	High axial and lateral capacity, can take hard driving, easy inspection and handling, length can be changed easily, resistant to deterioration	Needs treatment for corrosive environment
Concrete-filled pipe	0.2–1 (diameter)	<35	900	Concrete: 4.5–8.5 Steel: 62–83	Similar to steel pile	Similar to steel pile
Steel H-pile	Webs: 1–3 Flange: 0.2–0.35	<60	900	59–83	“Nondisplacement,” can take hard driving, easy handling, high axial and lateral capacity, length can be changed easily	Needs treatment for corrosive environment
Timber	0.125–0.45 (diameter)	12–35	250	5.5–8.5	Low cost, availability	Low capacity, can deteriorate if not protected, cannot take hard driving, cannot be inspected after driving

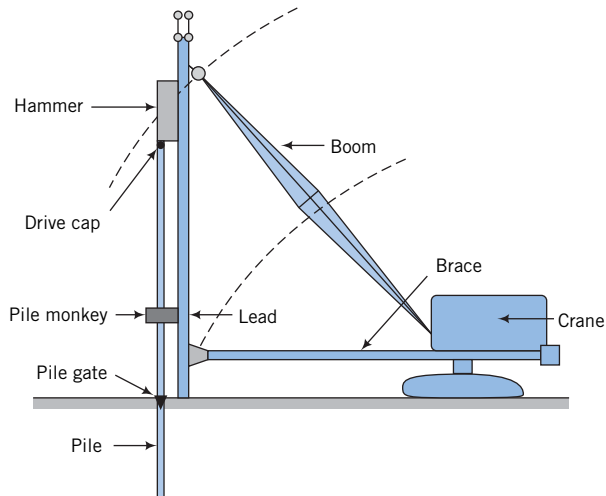


FIGURE 10.3 Key components of pile-driving operation.

The maximum for piles driven from the top must not exceed the compressive or tensile strength of the pile material.

The essential points are:

- 1. The selection of a pile type depends on the structural loads, availability, and the environment at the site.***
- 2. Piles can be installed using simple drop hammers but, most often, they are installed using steam or pneumatic hammers.***

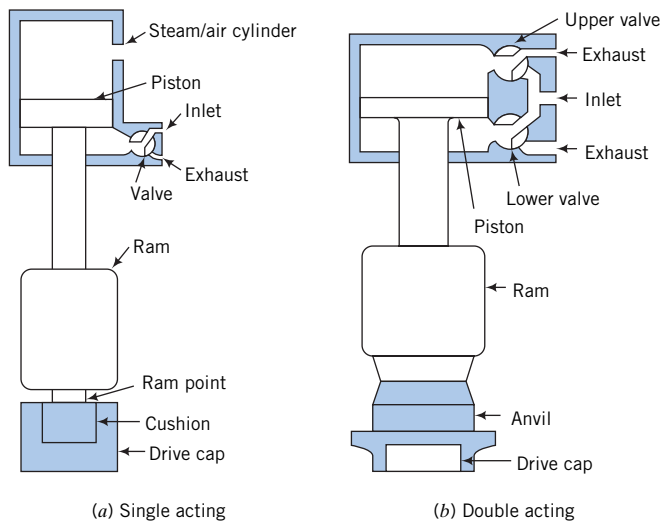


FIGURE 10.4 Two types of hammer.

What's next... We have described the various types of piles that are in common use. In the next section, methods of estimating the load capacity are presented. The load capacity of piles has been studied extensively. But no single satisfactory method of determining the load capacity has evolved. Pile load capacity is mostly based on empirical equations, experience, and judgment.

10.4 LOAD CAPACITY OF SINGLE PILES

Analysis of the bearing capacity, called load capacity, of piles can be classified into three categories depending on the degree of sophistication required and the importance of the structure. These three categories are summarized in Table 10.2. In this book, we will deal only with category 1 and category 2.

All analyses of pile load capacity are approximations because it is difficult, if not impossible, to account for the variability of soil types and the differences in the quality of construction practice. Even if the highest quality construction practice is followed, the installation of piles invariably changes the soil characteristics in ways that we cannot, at least simply, take into account.

The ultimate load capacity, Q_{ult} , of a pile consists of two parts. One part is due to friction (Fig. 10.5), called skin friction or shaft friction or side shear, Q_f , and the other is due to end bearing at the base or tip of the pile, Q_b . If the skin friction is greater than about 80% of the end bearing load capacity, the pile is deemed a friction pile and, if the reverse, an end bearing pile. If the end bearing is neglected, the pile is called a floating pile.

TABLE 10.2 Categories of Analysis/Design Procedures

Category	Subdivision	Characteristics	Method of parameter determination
1		Empirical—not based on soil mechanics principles.	Simple in situ or laboratory tests with correlations
2	2A	Based on simplified theory or charts; uses soil mechanics principles; amenable to hand calculation. Theory is linear elastic (deformation) or rigid plastic (stability).	Routine, relevant in situ tests—may require some correlations
	2B	As for 2A, but theory is nonlinear (deformation) or elastoplastic (stability).	
3	3A	Based on theory using site-specific analysis, uses soil mechanics principles. Theory is linear elastic (deformation) or rigid plastic (stability).	Careful laboratory and/or in situ tests that follow the appropriate stress paths
	3B	As for 3A, but nonlinearity is allowed for in a relatively simple manner.	
	3C	As for 3A, but nonlinearity is allowed for by way of proper constitutive models of soil behavior.	

SOURCE: On behalf of the Institution of Civil Engineers.

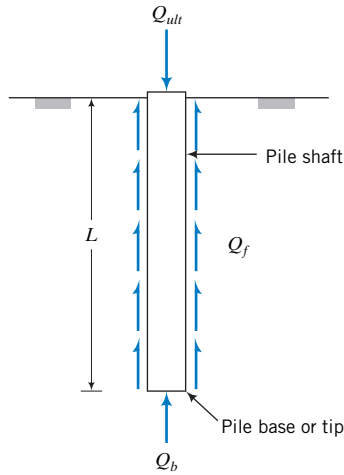


FIGURE 10.5 Pile shaft and end bearing resistance.

From statics,

$$Q_{ult} = Q_f + Q_b \quad (10.1)$$

The conventional allowable load capacity is expressed as

$$Q_a = \frac{Q_{ult}}{FS} \quad (10.2)$$

where FS is a gross factor of safety usually greater than 2.

The manner in which skin friction is transferred to the adjacent soil depends on the soil type. In fine-grained soils, the load transfer is nonlinear and decreases with depth, as illustrated in Fig. 10.6.

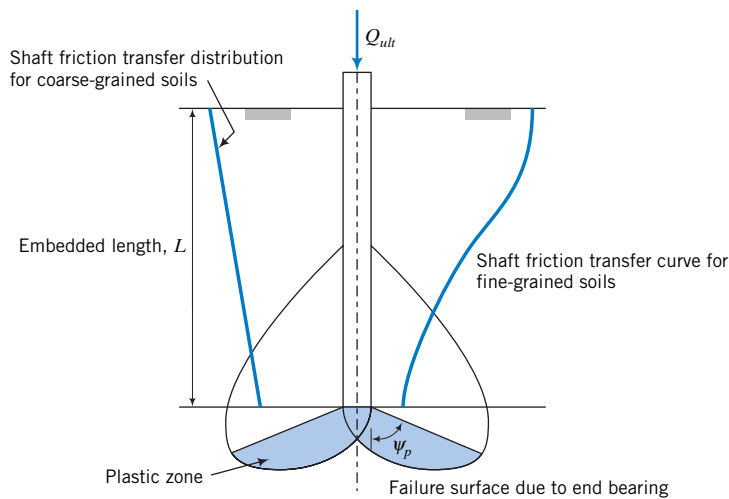


FIGURE 10.6 Load transfer characteristics.

As a result, elastic compression of the pile is not uniform; more compression occurs on the top part than on the bottom part of the pile. For coarse-grained soils, the load transfer is approximately linear with depth (higher loads at the top and lower loads at the bottom).

In order to mobilize skin friction and end bearing, some movement of the pile is necessary. Field tests have revealed that to mobilize the full skin friction in a driven pile a vertical displacement of 2.5 to 10 mm is required. The actual vertical displacement depends on the strength of the soil and is independent of the pile length and pile diameter. The full end bearing resistance is mobilized in driven piles when the vertical displacement is about 8 to 10% of the pile tip diameter. For bored piles or drilled shafts, a vertical displacement of about 30% of the pile tip diameter is required. The full end bearing resistance is mobilized when slip or failure zones similar to shallow foundations are formed (Fig. 10.6). The end bearing resistance can then be calculated by analogy with shallow foundations. The important bearing capacity factor is N_q .

The full skin friction and full end bearing are not mobilized at the same displacement. The skin friction is mobilized at about one-tenth of the displacement required to mobilize the end bearing resistance. This is important in deciding on the factor of safety to be applied to the ultimate load. Depending on the tolerable settlement, different factors of safety can be applied to skin friction and to end bearing.

Generally, piles driven into loose, coarse-grained soils tend to densify the adjacent soil. When piles are driven into dense, coarse-grained soils, the soil adjacent to the pile becomes loose. Pile driving usually remolds fine-grained soils near the pile shaft. The implication of pile installation is that the intact shear strength of the soil is changed and you must account for this change in your estimations of the load capacity.

Except for loose, coarse-grained soils, the soil mass adjacent to a pile reaches the critical state or close to it. You should then use critical state shear strength parameters in determining the long-term load capacity of piles. Loose, coarse-grained soils adjacent to driven piles are densified and $\phi' = \phi'_p$. The actual value of ϕ'_p depends on the magnitude of the normal effective stress. Since the magnitude of the normal effective stress is uncertain, you should be cautious on using ϕ'_p in estimating the long-term load capacity. It would be prudent to use $\phi' = \phi'_{cs}$ in all cases except for some overconsolidated clays with a predominance of flat particles, where it is advisable to use $\phi' = \phi'_r$. The formulas for load capacity are developed or stated for a generic ϕ' . You must, however, use the appropriate value of ϕ' in problem solving.

The essential points are:

- 1. Pile load capacity depends on the soil type, method of installation, and construction practice.***
- 2. The critical state friction angle should be used in estimating the long-term load capacity of piles, except you should use the residual friction angle for overconsolidated clays with a predominance of flat particles.***

What's next... A variety of methods are available to determine Q_f and Q_b . We will deal with four methods:

- Pile load test
- Statics— α - and β -methods
- Pile-driving formulas
- Wave analysis

We begin with the pile load test.

10.5 PILE LOAD TEST



Concept Learning and Self-Assessment

Access Chapter 10, Section 10.5 on the CD to view an animation of a pile test. Take Quiz 10.5 to test your understanding of pile load tests.

The purposes of a pile load test are:

- To determine the axial load capacity of a single pile
- To determine the settlement of a single pile at working loads
- To verify estimated axial load capacity
- To obtain information on load transfer in skin friction and end bearing

In a typical pile load test, the test pile is driven to the desired depth, loads are applied incrementally, and the settlement of the pile is recorded. The axial loads can be applied by stacking sand bags on a loading frame attached to the pile or, more popularly, by jacking against a reaction beam and reaction piles (Fig. 10.7). If load transfer information is required, the pile must be instrumented to ascertain the internal load of the pile shaft.

The interpretation of the load capacity depends on the method of loading. Two loading methods are popular. In one method, called the constant rate of penetration (CRP) test, the load is applied at a constant rate of penetration of 0.75 mm/min in fine-grained soils and 1.5 mm/min in coarse-grained soils. In the other method, called the quick maintained load (QML), increments of load, about 15% of the design load, are applied at intervals of about 2.5 min. At the end of each load

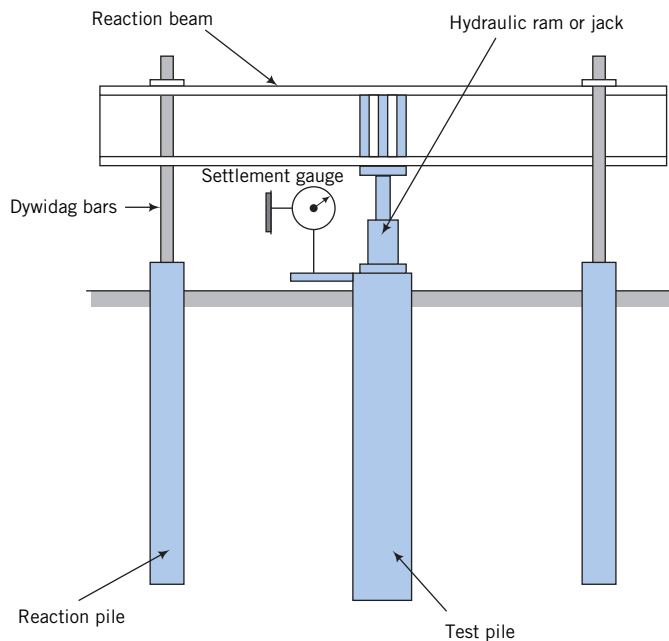


FIGURE 10.7 A pile load test setup.

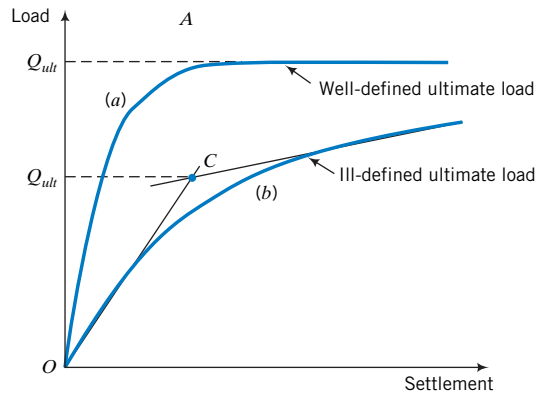


FIGURE 10.8 Load-settlement curves.

increment, the load and settlement are recorded. Schematic variations of pile load test plots are shown in Fig. 10.8.

The ultimate load is not always well defined. Load-settlement curve (a) in Fig. 10.8 shows a well-defined ultimate load while curve (b) does not. To obtain the ultimate load from curve (b) various empirical procedures have been suggested. One simple method is to find the intersection of the tangents of the two parts of the curve. The value at the ordinate of the intersection (C in Fig. 10.8) is Q_{ult} . The allowable load capacity is found by dividing the ultimate load by a factor of safety, usually 2. An alternative criterion is to determine the allowable pile load capacity for a desired serviceability limit state, for example, a settlement of 10% of the pile diameter. The settlement at the allowable (working) load capacity is readily determined from the load-settlement plot (Fig. 10.8).

The essential points are:

1. A pile load test provides the load capacity and settlement of a pile at the working load at a particular location in a job site.
2. Various criteria and techniques are used to determine the allowable load capacity from pile load tests.

EXAMPLE 10.1 Interpreting Pile Load Test Data

The results of a load test on a 0.45 m diameter pile are shown in the table below. Determine (a) the ultimate pile load capacity, (b) the allowable load for a factor of safety of 2, and (c) the allowable load capacity at 10% pile displacement.

Displacement (mm)	0.0	1.3	2.5	5.1	7.6	10.2	12.7	15.2	17.8	20.3	22.9	25.4	27.9	30.5	33.0
Load (kN)	0	200	350	670	870	1070	1250	1400	1500	1600	1700	1750	1780	1810	1830

Displacement (mm)	35.6	38.1	40.6	43.2	45.7	47.0
Load (kN)	1860	1870	1890	1890	1900	1905

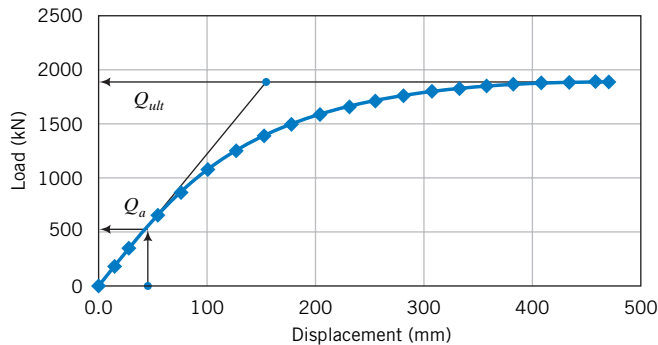


FIGURE E10.1

Strategy Plot a graph of displacement versus load and then follow the procedure in Section 10.4.

Solution 10.1

Step 1: Plot a displacement–load graph.

See Fig. E10.1.

Step 2: Determine the ultimate pile load capacity.

The failure load is ill defined. Locate the intersection of the tangents at the beginning and the end of the curve. The ordinate of this intersection is the ultimate pile load capacity.

$$Q_{ult} = 1780 \text{ kN}$$

Step 3: Determine the allowable pile load capacity.

$$FS = 2$$

$$Q_a = \frac{Q_{ult}}{FS} = \frac{1780}{2} = 890 \text{ kN}$$

Step 4: Determine the pile load capacity at 10% pile diameter.

$$\text{Displacement} = 450 \times 0.01 = 4.5 \text{ mm}$$

$$\text{From Fig. E10.1: } Q_a = 510 \text{ kN}$$

What's next... Pile load tests are expensive and are not often conducted in the preliminary stages of a design. To get an estimate of the pile load capacity, at least in the preliminary design stages, recourse is made to statics and to correlations using soil test results. In the next section, we will examine how statics is applied to obtain an estimate of pile load capacity and some correlations between field soil test results and measured pile load capacity.

10.6 METHODS USING STATICS



Concept Learning and Self-Assessment

Access Chapter 10, Section 10.6 on the CD to learn about estimating pile load capacity using statics. Take Quiz 10.6 to test your understanding.

10.6.1 α -Method

The α -method is based on a total stress analysis (TSA) and is normally used to estimate the short-term load capacity of piles embedded in fine-grained soils. In the α -method, a coefficient, α_u , is used to relate the undrained shear strength, s_u , to the adhesive stress (f_s) along the pile shaft. The skin friction, Q_f , over the embedded length of the pile is the product of the adhesive stress ($f_s = \alpha_u s_u$) and the surface area of the shaft (perimeter $\times$ embedded length). Thus,

$$Q_f = \sum_{i=1}^j (\alpha_u)_i (s_u)_i \times (\text{Perimeter}) \times (\text{Length})_i \quad (10.3)$$

where j is the number of soil layers within the embedded length of the pile. For a cylindrical pile of uniform cross section and diameter, D , penetrating a homogeneous soil, Q_f is given by

$$Q_f = \alpha_u s_u \pi D L \quad (10.4)$$

where L is the embedded length of the pile.

The value of α_u to use in determining the load capacity of piles is a subject of much debate and testing. Most tests to determine α_u are laboratory tests on model piles installed in a uniform deposit of soil. The major problems with these laboratory tests are:

1. It is difficult to scale up the laboratory model test results to real piles.
2. The soils in the field are mostly nonuniform compared with carefully prepared uniform soils in the laboratory.
3. Pile installation in the field strongly influences α_u , which cannot be accurately duplicated in the laboratory.

Full-scale field tests on real piles are preferred, but such tests are expensive and the results may apply only to the site where the tests are performed. The results from cone penetrometers and the SPT have been linked to α_u but these are found from statistical correlations with rather low coefficient of correlation. We will adopt a few of the popular expressions for α_u as given in Table 10.3 and Fig. 10.9 for driven piles and Table 10.5 for drilled shafts.

$$\alpha_u = 1.82 s_u^{-0.4}; \quad 20 \text{ kPa} < s_u < 90 \text{ kPa} \quad (10.5)$$

For $s_u < 20 \text{ kPa}$, $\alpha_u = 0.55$. For $\alpha_u > 90 \text{ kPa}$, the soil is treated as rock.

You should use α_u from at least two sources to estimate Q_f . As you gain field experience and correlate your estimated Q_f with field tests on full-scale piles, you should be able to judge which source of α_u best represents your field condition.

The soil near the top of the drilled shaft is subjected to environmental and construction effects while the soil just above the base may develop tensile cracking. Consequently, the upper 1.5 m of the shaft and one pile diameter above the base are ignored in calculating skin friction for drilled shafts.

The end bearing capacity is found by analogy with shallow foundations and is expressed as

$$Q_b = f_b A_b = N_c (s_u)_b A_b \quad (10.6)$$

where f_b is the base resistance; N_c is a bearing capacity coefficient, usually 9 for driven piles; $(s_u)_b$ is the undrained shear strength of the soil at the base of the pile; and A_b is the cross-sectional area of the base of the pile. Selected expressions for N_c are shown in Table 10.4. The s_u value must be obtained from field or laboratory tests within 2 pile diameters below the pile tip. If $s_u < 25 \text{ kPa}$, the value of N_c should be reduced by one-third.

TABLE 10.3 Skin Friction Factors α_u and β for Driven Piles from Laboratory Tests

Soil type	Equation	Skin friction factors	Reference
Clay	$f_s = \alpha_u s_u$	$\alpha_u = 1.0 (s_u \leq 25 \text{ kPa})$	API (1984)
		$\alpha_u = 0.5 (s_u \geq 70 \text{ kPa})$	
		$\alpha_u = 1 - \left(\frac{s_u - 25}{90} \right)$ for $25 \text{ kPa} < s_u < 70 \text{ kPa}$	Burland (1973) Esrib and Kirly (1979)
	$f_s = \beta \sigma'_z$	$\beta = (1 - \sin \phi') (\tan \phi') (\text{OCR})^{0.5}$	Burland (1973)
		$\beta = 0.11 (\text{OCR}) + 0.24$; $\text{PI} = 20\%$, $\beta \leq 2$	
		$\beta = 0.13 (\text{OCR}) + 0.17$; $\text{PI} = 60\%$, $\beta \leq 2$	Esrib and Kirly (1979)
Sand	$f_s = \beta \sigma'_z$	$\beta = (1 - \sin \phi') \tan \phi'$ $\beta = 0.44$ for $\phi' = 28^\circ$ 0.75 for $\phi' = 35^\circ$ 1.2 for $\phi' = 37^\circ$	Burland (1973) Meyerhof (1976)
Uncemented calcareous sand	$f_s = \beta \sigma'_z$	$\beta = 0.05 - 0.1$	Poulos (1988)

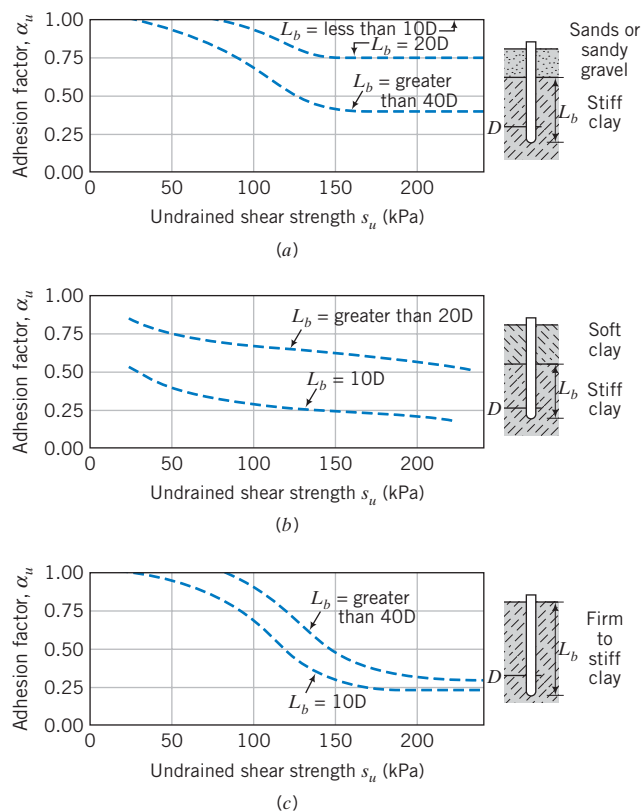
**FIGURE 10.9** α_u values for fine-grained soils. (After Tomlinson, 1987.)

TABLE 10.4 End Bearing or Base Resistance Factors, N_c and N_q for Driven Piles and Drilled Shafts

Soil type	Equation	End bearing factors	Reference
Clay	$f_b = N_c (s_u)_b$	$N_c = 9$ for $L/D \geq 3$ (driven piles) $N_c = 6.44 \ln(s_u)_b - 0.63 [\ln(s_u)_b]^2 - 7.55$ (drilled shafts); $(s_u)_b$ in the average undrained shear strength over two diameters below the base. $25 \leq (s_u)_b \leq 190$ kPa $f_b \leq 4.0$ MPa	Skempton (1959)
Clay or sand	$f_b = N_q (\sigma'_z)_b$	$N_q = (\tan \phi' + \sqrt{1 + \tan^2 \phi'})^2 \exp(2\Psi_p \tan \phi')$ $\Psi_p = \pi/3$ to 0.58π lower values in the range are for soft, fine-grained soils, higher values are for dense, coarse-grained soils and overconsolidated fine-grained soils	Janbu (1976)
Sand	$f_b = N_q (\sigma'_z)_b \leq f_{blim}$ $f_{blim} = 10 - 15$ MPa	$N_q = 40$ N_q plotted against ϕ' (applicable to dense sand) —see Fig. 10.10	API (1984) Berezantzev et al. (1961)
Uncemented calcareous sand	$f_b = N_q (\sigma'_z)_b$	Typical range of $N_q = 8-20$	Poulos (1988)

10.6.2 β -Method

The β -method is based on an effective stress analysis and is used to determine the short-term and long-term pile load capacities. The friction along the pile shaft is found using Coulomb's friction law, where the frictional stress is given by $f_s = \mu \sigma'_x = \sigma'_x \tan \phi'_i$, and where μ is the coefficient of friction, σ'_x is the lateral effective stress, and ϕ'_i is the interfacial effective friction angle. The skin friction is

TABLE 10.5 Skin Friction Factors α_u and β for Bored Piles or Drilled Shafts

Soil type	Equation	Skin friction factors	Reference
Clay	$f_s = \alpha_u s_u$	$\alpha_u = 0.55; \frac{s_u}{p_a} \leq 1.5$ $\alpha_u = 0.55 - 0.1 \left(\frac{s_u}{p_a} - 1.5 \right); 1.5 < \frac{s_u}{p_a} \leq 2.5$ p_a = atmospheric pressure = 101.3 kPa	O'Neill and Reese (1999)
Sand	$f_s = \beta \sigma'_z$	$\beta = 0.1$ for $\phi' = 33^\circ$ 0.2 for $\phi' = 35^\circ$ 0.35 for $\phi' = 37^\circ$	Meyerhof (1976)
	$(f_s \leq 200 \text{ kPa})$	$\beta = 1.5 - 0.245\sqrt{z}; 1.2 \leq \beta \leq 0.25,$ $z(m)$ = depth; $N > 15$ $\beta = \frac{N}{15} (1.5 - 0.245\sqrt{z}; N \leq 15, 0 < \beta < 1.2)$ $N = N_{60}$ (uncorrected)	O'Neill and Reese (1988)
Uncemented calcareous sand	$f_s = \beta \sigma'_z$ $60 \text{ kPa} \leq f_s \leq 100 \text{ kPa}$	$\beta = 0.5 - 0.8$	Poulos (1988)

expressed as

$$Q_f = \sum_{i=1}^j (\sigma'_x)_i \tan \phi'_i \times (\text{Perimeter})_i \times (\text{Length})_i \quad (10.7)$$

The lateral effective stress is proportional to the vertical effective stress (σ'_z) by a coefficient K (Chapter 5). Therefore, we can write Eq. (10.7) as

$$Q_f = \sum_{i=1}^j K_i (\sigma'_z)_i \tan \phi'_i \times (\text{Perimeter})_i \times (\text{Length})_i \quad (10.8)$$

We can replace the two coefficients K and $\tan \phi'_i$ by a single factor β to yield

$$Q_f = \sum_{i=1}^j \beta_i (\sigma'_z)_i \times (\text{Perimeter})_i \times (\text{Length})_i \quad (10.9)$$

Recall from Chapter 5 that for normally consolidated soils

$$K = K_o^{\text{nc}} = 1 - \sin \phi'_{\text{cs}}$$

and for overconsolidated soils

$$K = K_o^{\text{oc}} = (1 - \sin \phi'_{\text{cs}})(\text{OCR})^{0.5}$$

Three expressions for β that are popular for fine-grained soils are:

$$\beta = K \tan \phi'_i = K_o^{\text{oc}} \tan \phi'_i = (1 - \sin \phi'_{\text{cs}})(\text{OCR})^{0.5} \tan \phi'_i \quad (10.10)$$

$$\beta = 0.11(\text{OCR}) + 0.24; \quad \text{PI} = 20\%, \quad \beta \leq 2 \quad (10.11)$$

$$\beta = 0.13(\text{OCR}) + 0.17; \quad \text{PI} = 60\%, \quad \beta \leq 2 \quad (10.12)$$

Eq. (10.10) was proposed by Burland (1973) while Eqs. (10.11) and (10.12) were curve fitted by the author from results published by Esrig and Kirby (1979).

The value of β is also a subject of many debates, especially for coarse-grained soils. The selected expressions for β are given in Table 10.3 for driven piles and Table 10.5 for drilled shafts. For drilled shafts in clays, use β values from Table 10.3. Because of the remolding or loosening of the soil adjacent to piles during installation, the value of ϕ'_i to use in determining β should not be greater than ϕ'_{cs} or ϕ'_r . The latter friction value should be used for overconsolidated clays with a predominance of flat platy particles. The vertical effective stress, σ'_z , is normally calculated at the center of each soil layer. You should estimate Q_f using at least two sources of β .

The end bearing capacity is calculated by analogy with the bearing capacity of shallow footings and is determined from

$$Q_b = f_b A_b = N_q (\sigma'_z)_b A_b \quad (10.13)$$

where $f_b = N_q (\sigma'_z)_b$ is the base resistance, N_q is a bearing capacity coefficient that is a function of ϕ' , $(\sigma'_z)_b$ is the vertical effective stress at the base, and A_b is the cross-sectional area of the base.

Janbu (1976) proposed an equation for N_q as a function of ϕ' and the angle ψ_p (called the angle of pastification, as shown in Fig. 10.6). Janbu's equation is

$$N_q = (\tan \phi' + \sqrt{1 + \tan^2 \phi'})^2 \exp(2\psi_p \tan \phi') \quad (10.14)$$

The value of ψ_p varies from $\psi_p \leq \pi/3$ for soft, fine-grained soils to $\psi_p \leq 0.58\pi$ for dense, coarse-grained soils and overconsolidated fine-grained soils. For soft, compressible soils, the pile tip may easily penetrate the soil without causing significantly large plastic zones. For these types of soils, ψ_p should not exceed $\pi/3$. In dense, coarse-grained soils, the plastic zones could be substantial but it is recommended (Janbu, 1976) that for these soils ψ_p should not exceed $\pi/2$. Berezantzev et al. (1961) proposed a relationship between N_q and ϕ' as shown in Fig. 10.10; N_q is corrected by a factor α_t , which depends on the length to diameter ratio (see inset diagram, Fig. 10.10).

Vesic (1975) proposed another equation for N_q from theoretical considerations of the expansion of a cylindrical cavity. Vesic's equation for N_q is

$$N_q = \frac{3}{3 - \sin \phi'} \left\{ \exp \left[\left(\frac{\pi}{2} - \phi' \right) \tan \phi' \right] \tan^2 \left(\frac{\pi}{4} + \frac{\phi'}{2} \right) I_{rr}^{\frac{4 \sin \phi'}{1 + \sin \phi'}} \right\} \quad (10.15)$$

where I_{rr} is a rigidity index expressed as

$$I_{rr} = \frac{I_r}{1 + \varepsilon_p I_r} \quad (10.16)$$

and

$$I_r = \frac{G}{(\sigma'_z)_b \tan \phi'} \quad (10.17)$$

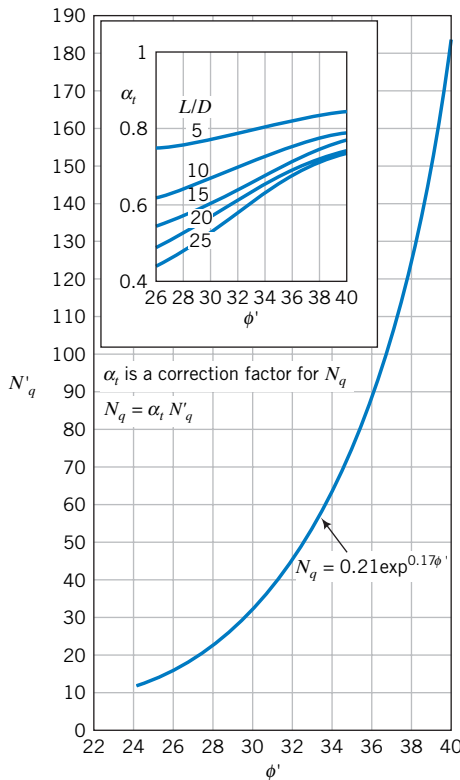


FIGURE 10.10 N_q values proposed by Berezantzev et al. (1961).

TABLE 10.6 Correlations^a Between Skin Frictional Stress, f_s and SPT Values

Pile type	Soil type	A	B	Remarks	Reference
Driven displacement	Sand	0	1.9	f_s = average value over shaft N = average SPT along shaft; $f_s \leq 100$ kPa Halve f_s for nondisplacement and small displacement piles; ($f_s \leq 50$ kPa)	Meyerhof (1956)
Drilled shaft	Sand	Use expression given by O'Neill and Reese (1988) in Table 10.3			

^aCorrelation equation is $f_s = A + BN$ (kPa); $N = N_{60}$; values in blows/ft or blows/0.31 m. and not corrected for overburden pressure.

where G is the shear modulus, ε_p is the volumetric strain, and $(\sigma'_z)_b$ is the vertical effective stress at the base. The original equation for N_q proposed by Vesic (1975) contained s_u in the denominator of Eq. (10.17). The author removed s_u from the original equation since we are considering an effective stress analysis. A list giving values for N_q proposed by various investigators for use in this textbook is shown in Table 10.4.

Coarse-grained soils are difficult to sample. A number of correlations between field soil tests and pile load capacities have been proposed for coarse-grained soils. Some of these correlations for the SPT are shown in Tables 10.6 and 10.7. Other correlations exist for cone penetrometers and pressuremeters.

The pile load capacity predicted by the α and β methods and those using SPT values can vary significantly. The prediction is generally worse for coarse-grained soils. Settlements to fully mobilize skin friction and end bearing (Section 10.4) are different and are often greater than serviceability limit states for most structures. Evaluation and judgment based on experience with similar soils and construction methods are necessary to decide whether the predicted pile load capacity is reasonable. In the absence of experience, you should use more than one method and conduct pile load tests, when possible, to help you to make judgment on the appropriate pile load capacity. In this textbook, we will use the expressions given in Tables 10.3 to 10.5 to estimate the pile load capacity. The expression that will be used will depend on the soil parameter given. For example, if ϕ'_{cs} and OCR are given for a clay

TABLE 10.7 Correlations^a Between End Bearing Resistance, f_b and SPT Values

Pile type	Soil type	C	Remarks	Reference
Driven displacement	Sand	$38 \frac{L_s}{D}$	L_s = length of pile in sand, D = diameter or width. N is in vicinity of base; $C \leq 380$	Meyerhof (1976)
	Residual sandy silts	250		Decourt (1982)
	Residual clayey silts	200		Decourt (1982)
	Clay	200		Martin et al. (1987)
Drilled shaft	Sand	100	$L \geq 10m$; $f_b \leq 2900$ kPa	Shioi and Fukui (1982)
		$\frac{57.5L}{10}$	$L \leq 10m$; $f_b \leq 2900$ kPa	Quiros and Reese (1977)
		57.5	$L > 10m$; $f_b \leq 2900$ kPa	
	Chalk	250	$N < 30$	Hobbs (1977)
		200	$N > 40$	

^aCorrelation equation is $f_b = CN$ (kPa); $N = N_{60}$ and not corrected for overburden pressure.

then we will use Burland's expression for β for driven piles. If, however, OCR and PI are given, we will use Esrib and Kirbg's expression for β .

The essential points are:

1. *The α -method is based on a TSA and is used to estimate the short-term pile load capacity in fine-grained soils.*
2. *The β -method is based on an ESA and is used to estimate the short-term and long-term pile load capacities in all soil types.*
3. *The actual values of α_u , β , and N_q are uncertain.*

EXAMPLE 10.2 Pile Load Capacity in a Clay Soil

A cylindrical pile of diameter 400 mm is driven to a depth of 10 m into a clay with $s_u = 40$ kPa, $\phi'_{cs} = 28^\circ$, OCR = 2 and $\gamma_{sat} = 18$ kN/m³. Groundwater level is at the surface. Estimate the allowable load capacity for a factor of safety of 3. Is the pile a friction pile?

Strategy The solution is a straightforward application of the pile load capacity equations.

Solution 10.2

Step 1: Select α_u and β .

$$\text{Table 10.3: } (\text{API})\alpha_u = 1 - \left(\frac{s_u - 25}{90}\right) = 1 - \left(\frac{40 - 25}{90}\right) = 0.83$$

$$\beta = (1 - \sin \phi'_{cs})(\text{OCR})^{0.5} \tan \phi'$$

For OCR = 2 and $\phi' = \phi'_{cs}$, we get

$$\beta = (1 - \sin 28^\circ)(2)^{0.5} \tan 28^\circ = 0.4$$

Step 2: Calculate Q_a using a TSA.

$$L = 10 \text{ m, Perimeter} = \pi D = \pi \times 0.4 = 1.26 \text{ m,}$$

$$A_b = \frac{\pi D^2}{4} = \frac{\pi \times 0.4^2}{4} = 0.126 \text{ m}^2$$

$$Q_f = \alpha_u s_u (\pi D) L = 0.83 \times 40 \times (1.26) \times 10 = 418 \text{ kN}$$

$$Q_b = 9 s_u A_b = 9 \times 40 \times 0.126 = 45 \text{ kN}$$

$$Q_{ult} = Q_f + Q_b = 418 + 45 = 463 \text{ kN}$$

$$Q_a = \frac{Q_{ult}}{\text{FS}} = \frac{463}{3} = 154 \text{ kN}$$

Step 3: Calculate Q_a using an ESA.

$$Q_f = \beta \sigma'_z (\pi D L)$$

Use an average value of σ'_z over the embedded length.

$$\sigma'_z = \gamma' \frac{L}{2} = (18 - 9.8) \times \frac{10}{2} = 41 \text{ kPa}$$

$$Q_f = 0.4 \times 41 \times 1.26 \times 10 = 207 \text{ kN}$$

$$Q_b = N_q (\sigma'_z)_b A_b = N_q \gamma' L A_b = N_q \times (18 - 9.8) 10 \times 0.126 = 10.3 N_q$$

Use Janbu's equation for N_q .

$$N_q = (\tan \phi'_{cs} + \sqrt{1 + \tan^2 \phi'_{cs}})^2 \exp(2\psi_p \tan \phi'_{cs})$$

For $\phi'_{cs} = 28^\circ$ and assuming $\psi_p = \pi/3$ (soft clay), we find

$$N_q = \{\tan 28^\circ + \sqrt{1 + \tan^2(28^\circ)}\}^2 \exp\left(\frac{2\pi}{3} \tan 28^\circ\right) = 8.4$$

$$Q_b = 10.3 \times 8.4 = 87 \text{ kN}$$

$$Q_{ult} = Q_f + Q_b = 207 + 87 = 294 \text{ kN}$$

$$Q_a = \frac{Q_{ult}}{3} = \frac{294}{3} = 98 \text{ kN}$$

$$\frac{Q_f}{Q_{ult}} = \frac{207}{294} = 0.7; \quad \text{pile not solely a friction pile.}$$

The load capacity from an ESA is less than that from a TSA. Therefore, use the allowable pile load capacity from the ESA, that is, $Q_a = 98 \text{ kN}$. ■

EXAMPLE 10.3 Pile Load Capacity in a Layered Soil Profile

A square concrete pile $0.3 \text{ m} \times 0.3 \text{ m}$ is required to support a load of 150 kN with a factor of safety of 3. The soil stratification consists of 5 m of medium clay ($s_u = 40 \text{ kPa}$, $\phi'_{cs} = 26^\circ$, $\text{OCR} = 2$, $\gamma_{\text{sat}} = 18 \text{ kN/m}^3$) underlain by a deep deposit of overconsolidated clay ($s_u = 80 \text{ kPa}$, $\phi'_{cs} = 24^\circ$, $\text{OCR} = 4$, $\gamma_{\text{sat}} = 18.5 \text{ kN/m}^3$). Groundwater level is at 2 m below the ground surface. You may assume that the soil above the groundwater level is saturated. Estimate the length of pile required.

Strategy You are given Q_a and FS, so you can calculate $Q_{ult} = Q_a \times \text{FS}$. Assume an embedment depth, L_1 , into the stiff clay and then solve for L_1 since you know Q_{ult} . You should consider both short-term and long-term conditions and select the value of L_1 that is larger.

Solution 10.3

Step 1: Determine the ultimate load.

$$Q_a = 150 \text{ kN}$$

$$Q_{ult} = Q_a \times \text{FS} = 150 \times 3 = 450 \text{ kN}$$

Step 2: Draw a sketch of the soil profile and pile, and calculate the required coefficients.

See Fig. E10.3.

Table 10.3: $s_u = 40 \text{ kPa}$, $\alpha_u = 0.83$ [see Example 10.2]
 $s_u = 80 \text{ kPa}$, $\alpha_u = 0.5$ (stiff clay)

For $\phi' = \phi'_i = \phi'_{cs}$, we find the following:

$$\begin{aligned} \text{Medium clay: } \beta &= (1 - \sin \phi'_{cs})(\text{OCR})^{0.5} \tan \phi'_{cs} \\ &= (1 - \sin 26^\circ)(2)^{0.5} \tan 26^\circ = 0.39 \end{aligned}$$

$$\text{Stiff clay: } \beta = (1 - \sin 24^\circ)(4)^{0.5} \tan 24^\circ = 0.53$$

$$\text{Perimeter} = 2(B + B) = 2(0.3 + 0.3) = 1.2 \text{ m}$$

$$A_b = 0.3 \times 0.3 = 0.09 \text{ m}^2$$

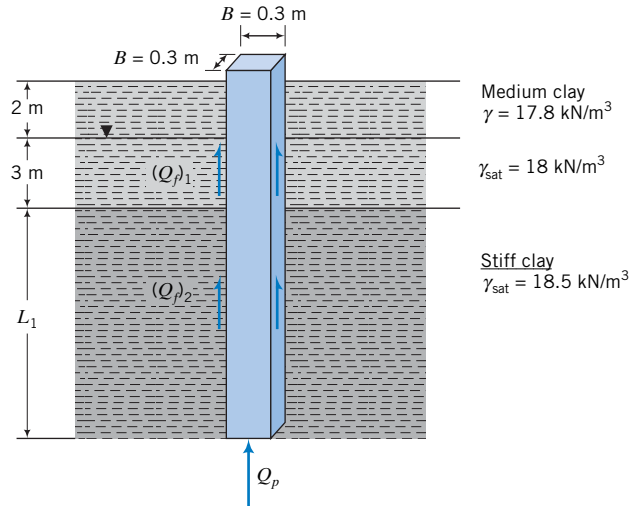


FIGURE E10.3

Step 3: Calculate the required length using a TSA.

Medium clay

Length in medium clay = 5 m

$$\begin{aligned}(Q_f)_{\text{medium clay}} &= \alpha_u s_u \times \text{Perimeter} \times \text{Length} \\ &= 0.83 \times 40 \times 200 \times 5 = 200 \text{ kN}\end{aligned}$$

Stiff clay

$$\begin{aligned}(Q_f)_{\text{stiff clay}} &= 0.5 \times 80 \times 1.2 \times L_1 = 48L_1 \text{ kN} \\ Q_p &= 9s_u A_b = 9 \times 80 \times 0.09 = 65 \text{ kN}\end{aligned}$$

Ultimate load capacity

$$Q_{ult} = (Q_f)_{\text{medium clay}} + (Q_f)_{\text{stiff clay}} + Q_p$$

That is,

$$450 = 200 + 48L_1 + 65$$

Solving, we get

$$L_1 = 3.9 \text{ m}$$

Step 4: Calculate the required length using an ESA.

Medium clay: Average value of $\sigma'_z = (2 \times 18) + 0.5(18 - 9.8) = 40.1 \text{ kPa}$

Stiff clay: Average value of σ'_z over length L_1 :

$$\sigma'_z = (2 \times 18) + 3(18 - 9.8) + \frac{L_1}{2}(18.5 - 9.8) = 60.6 + 4.35L_1$$

$$Q_f = \beta \sigma'_z \times \text{Perimeter} \times \text{Length}$$

Medium clay: $(Q_f)_{\text{medium clay}} = 0.39 \times 40.1 \times 1.2 \times 5 = 94 \text{ kN}$

$$\begin{aligned}\text{Stiff clay: } (Q_f)_{\text{stiff clay}} &= 0.53 \times (60.6 + 4.35L_1) \times 1.2 \times L_1 \\ &= 2.8(L_1)^2 + 38.5L_1\end{aligned}$$

End bearing resistance in stiff clay

Using Janbu's equation with $\phi' = \phi'_{cs} = 24^\circ$ and $\psi_p = \pi/2$ gives

$$N_q = [\tan 24^\circ + \sqrt{1 + \tan^2(24^\circ)}]^2 \exp\left(\frac{2\pi}{2} \tan 24^\circ\right) = 9.6$$

$$(\sigma'_z)_b = 2 \times 18 + 3(18 - 9.8) + L_1(18.5 - 9.8) = 60.6 + 8.7L_1$$

$$Q_p = N_q(\sigma'_z)_b A_b = 9.6 \times (60.6 + 8.7L_1) \times 0.09 = 52.4 + 7.5L_1$$

Ultimate load capacity

$$Q_{ult} = (Q_f)_{\text{soft clay}} + (Q_f)_{\text{stiff clay}} + Q_p$$

That is,

$$450 = 94 + 2.8(L_1)^2 + 38.5L_1 + (52.4 + 7.5L_1)$$

Rearranging and simplifying, we get

$$L_1^2 + 16.4L_1 - 108.4 = 0$$

Solution is $L_1 = 5$ m.

Step 5: Decide on the length required.

$$\text{TSA: } L_1 = 3.9 \text{ m}$$

$$\text{ESA: } L_1 = 5 \text{ m}$$

Use the larger of the two values,

$$L_1 = 5 \text{ m}$$

$$\text{Total length} = 5 + L_1 = 10 \text{ m}$$

EXAMPLE 10.4 Pile Load Capacity in a Clay with s_u Varying Linearly with Depth

A pile 450 mm in diameter and 15 m long is driven into a clay. The undrained strength of the soil varies linearly with depth such that $s_u = 0.22\sigma'_z$. Determine the allowable pile load capacity using a TSA. The factor of safety required is 2 and $\gamma_{\text{sat}} = 17 \text{ kN/m}^3$. Groundwater is at the surface.

Strategy Consider an element of thickness dz and perform integration to find the skin friction. You need to check if $s_u > 25 \text{ kPa}$. If it is not, then $\alpha_u = 1$ or else you would have to use α_u as a linear variable of s_u .

Solution 10.4

Step 1: Make a sketch of the problem.

See Fig. E10.4.

Step 2: Calculate the skin friction.

$$\gamma' = 17 - 9.8 = 7.2 \text{ kN/m}^3$$

$$(s_u)_b = 0.22\gamma' L = 0.22 \times 7.2 \times 15 = 23.8 \text{ kPa}$$

$$\text{Table 10.3: } \alpha_u = 1 \text{ for } (s_u)_b < 25 \text{ kPa}$$

Consider an element of thickness dz . The surface area is $2\pi r dz$ and the skin friction is

$$Q_f = 2\pi r \alpha_u \int_0^L (0.22\gamma' z) dz = 2.24 \left[\frac{z^2}{2} \right]_0^{15} = 252 \text{ kN}$$

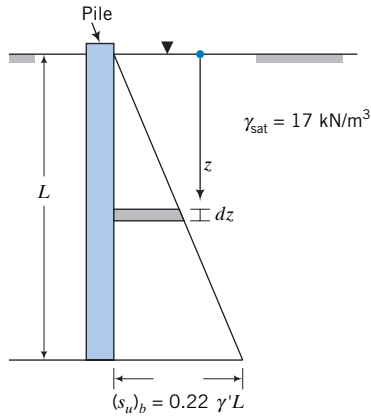


FIGURE E10.4

Step 3: Calculate the end bearing capacity.

$$A_b = \frac{\pi \times D^2}{4} = \frac{\pi \times 0.45^2}{4} = 0.159 \text{ m}^2$$

$$Q_b = 9(s_u)_b A_b = 9 \times 23.8 \times 0.159 = 34 \text{ kN}$$

Step 4: Calculate the allowable load capacity.

$$Q_{ult} = Q_f + Q_b = 252 + 34 = 286 \text{ kN}$$

$$Q_s = \frac{Q_{ult}}{\text{FS}} = \frac{286}{3} = 95 \text{ kN}$$

What's next... SPT and CPT are popular field tests that are used extensively in estimating pile capacities. We will now consider how to estimate pile load capacities using SPT and CPT results.

10.7 PILE LOAD CAPACITY BASED ON SPT AND CPT RESULTS



Self-Assessment

Access Chapter 10 and take Quiz 10.7 to test your understanding.

Various correlations have been proposed among results from SPT and CPT and pile load capacity. A few of these correlations to estimate the pile load capacity are given below.

SPT Tables 10.6 and 10.7 list some correlations for skin frictional stress and end bearing resistance based on SPT. The ultimate skin friction (kPa) is

$$Q_f = (A + BN) \times \text{perimeter (m)} \times \text{length (m)} \quad (10.18)$$

where A and B are constants (Table 10.6), and N is the arithmetic average SPT value of each soil layer along the pile shaft; $N = N_{60}$ is not corrected for overburden pressure. The ultimate end bearing

resistance (kPa) is

$$Q_b = CNA_b \quad (10.19)$$

where C is a constant (Table 10.7). We will use Meyerhof (1976) values of $A = 0$, $B = 1.9$, and $C = 38 \frac{L_s}{D}$ in this textbook; L_s is the length of pile driven in sand.

For drilled shafts, we will use the value of C (Table 10.7) recommended by Quiros and Reese (1977).

CPT The cone penetrometer discussed in Chapter 2 was originally developed to estimate the end bearing capacity of piles. Recall that the resistance of the cone tip, q_c , to axial thrust and the resistance of the sleeve, q_{s1} , are measured. The cone resistance, q_c , is a measure of the end bearing capacity and the sleeve resistance, q_{s1} , is a measure of the skin or shaft friction. The pile end bearing resistance, f_b , has been observed to vary from about $0.4q_c$ to $2q_c$. However, it is generally assumed that $f_b = q_c$.

The end bearing capacity of a single pile is estimated from

$$Q_b = q_c A_b \quad (10.20)$$

where A_b is the area of the pile base.

The determination of q_c from cone penetrometer results to use in equation (10.20) is controversial and several methods have been suggested. The differences in the methods come from the influence zone over which the cone values are extracted and the averaging procedures used. One method, proposed by Fleming and Thorburn (1983), is to average cone values over an influence zone of 8 pile diameters above the pile base and 2 pile diameters below the pile base. The average cone resistance is calculated from

$$q_c = \frac{q_{c1} + q_{c2} + 2q_{c3}}{4} \quad (10.21)$$

where q_{c1} is the arithmetic average of cone resistance values over 2 pile diameters below the pile base, q_{c2} is the minimum cone resistance value over 2 pile diameters below the pile base, and q_{c3} is the arithmetic average of minimum cone resistance values below q_{c2} over 8 pile diameters above the pile base.

Another method (Eslami and Fellenius, 1997) is to consider the cone resistances over an influence zone 8 pile diameters above the base and 4 pile diameters below the base—a total of 12 pile diameters—for piles penetrating a weak soil and resting on a dense soil. For piles penetrating a dense soil and resting in a weak soil, the cone resistance is considered over an influence zone of 2 pile diameters above the pile base. The geometric mean cone resistance is

$$(q_c)_{ag} = \{q_{c1}q_{c2}q_{c3} \dots q_{cn}\}^{\frac{1}{n}} \quad (10.22)$$

where q_{c1} to q_{cn} are discrete cone resistances over a distance 12 pile diameters or 2 pile diameters depending on the soil layering, and n is the number of q_c values.

Where the sleeve resistance is measured, either an arithmetic or a geometric mean value of cone resistance over the embedded pile depth is used as the shaft or skin friction. Cone sleeve friction is affected by the relative density of the soil and soil compressibility while skin friction on a pile is affected by the relative density of the soil, method of installation, soil compressibility, pile geometry and roughness. In the case of fine-grained soils, the cone sleeve friction value is strongly influenced by soil consolidation around the sleeve. Thus, significant differences between short-term and long-term load capacity can be expected using cone penetrometer results. Pile test results show that the short-term load capacity estimated from cone penetrometer results can be as low as 20% of the long-term load capacity. If the cone sleeve resistance is not measured, an estimate can be made from one of several equations proposed in the literature. Some of these are:

$$\text{Eslami and Fellenius (1997): } f_s = C_s q_{cs} \quad (10.23)$$

TABLE 10.8 Values of C_s

Soil Type	C_s
Soft sensitive soils	0.08
Clay	0.05
Stiff clay and mixture of clay and silt	0.025
Mixture of silt and sand	0.01
Sand	0.004

where q_{cs} is the cone resistance after adjustments for porewater pressure measured at the cone shoulder and C_s is a coefficient that depends on soil type as shown in Table 10.8.

$$\text{Vesic (1977)—coarse-grained soils: } f_s = 0.11 \exp(-3 \tan \phi'_{cs}) q_c \quad (10.24)$$

$$\text{Jardine et. al. (1998)—coarse-grained soil: } f_s = \sigma'_{rc} \tan \phi_i \quad (10.25)$$

where σ'_{rc} is the radial effective stress on the shaft and is empirically related to the cone resistance as $\sigma'_{rc} = 0.029 q_c \left(\frac{\sigma'_{zo}}{p_a} \right)^{0.13} \left(\frac{h}{R^*} \right)^{-0.38}$, σ'_{zo} = vertical effective stress at a depth z , p_a = atmospheric pressure, h = depth from pile base to the depth at which the shaft friction is considered, ϕ_i is the interface friction angle ($\phi_i \leq \phi'_{cs}$), $R^* = R_o$ for closed-end piles, R_o is outer radius, $R^* = (R_o^2 - R_i^2)^{0.5}$ for open-ended pile, and R_i is inner radius.

$$\text{Tumay and Fakhroo (1984)—Stiff clays: } f_s = 0.5 q_c \quad (10.26)$$

EXAMPLE 10.5 Pile Load Capacity Using SPT Data

A 450 mm diameter pile is driven into a sand profile to depth of 10 m. The SPT results are shown in the table below. Estimate the allowable load capacity for a factor of safety of 3.

Depth (m)	1	3	5	6	8	10	11	13
N (blows/ft)	22	18	25	20	30	36	39	45

Strategy The N values are blows/ft not blows/m. Use one of the correlations shown in Tables 10.6 and 10.7.

Solution 10.5

Step 1: Determine the average value of N over the depth.

You can treat each N value as representing a layer or you can take an average value.

$$N = N_{av} = \frac{22 + 18 + 25 + 20 + 30 + 36}{6} = 25$$

Step 2: Determine the load capacity.

Skin friction

$$\text{Perimeter} = \pi D = \pi \times 0.45 = 1.41$$

$$\text{Table 10.6: } Q_f = (A + BN) \times \text{Perimeter} \times \text{Length}$$

Using Meyerhof's method, $A = 0$, $B = 1.9$

$$f_s = A + BN = (0 + 1.9 \times 25) = 47.5 \text{ kPa} < 100 \text{ kPa; okay}$$

$$Q_f = 47.5 \times 1.41 \times 10 = 670 \text{ kN}$$

End bearing

$$A_b = \frac{\pi D^2}{4} = \frac{\pi \times 0.45^2}{4} = 0.159 \text{ m}^2$$

Using Table 10.7 and Meyerhof's method,

$$C = 38 \quad \frac{L_s}{D} = 38 \times \frac{10}{0.45} = 844 > 380$$

Use $C = 380$.

$$Q_b = CNA_b$$

where N is the SPT value at the base or the average value over a length $3D$ below the base, whichever is the lesser.

$$Q_b = 380 \times 36 \times 0.159 = 2175 \text{ kN}$$

$$Q_{ult} = Q_f + Q_b = 670 + 2175 = 2845 \text{ kN}$$

$$Q_a = \frac{Q_{ult}}{\text{FS}} = \frac{2845}{3} = 948 \text{ kN}$$

EXAMPLE 10.6 Pile Load Capacity Using CPT Data

The results of cone tip resistance and sleeve resistances from a CPT are shown in Fig. E.10.6a. Estimate the ultimate load capacity of a driven pile 0.6 m diameter and 10 m long.

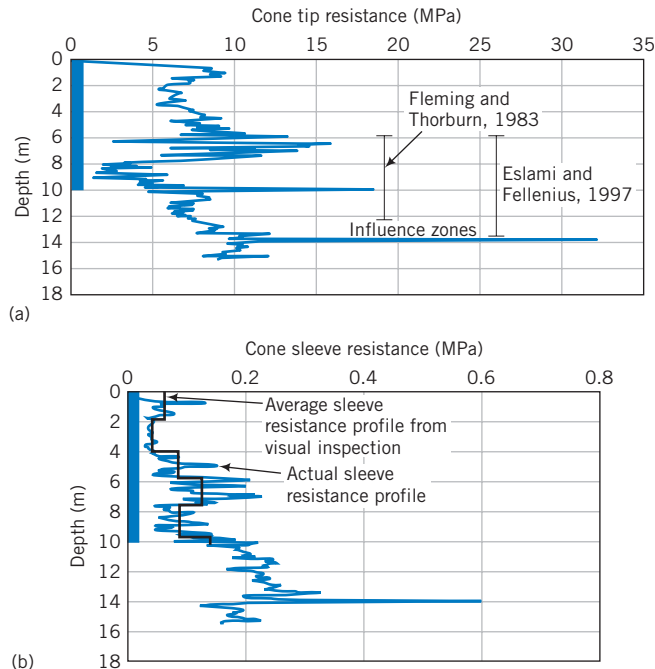


FIGURE E10.6

Strategy It is best to select two or more methods to determine the cone resistance and sleeve resistance and then make a decision on the pile load capacity after conducting the calculations. We are going to select the methods proposed by Fleming and Thorburn (1983) and Eslami and Fellenius (1997).

Solution 10.6

Step 1: Determine the influence zone.

We will assume the pile is penetrating a weak soil and resting on a strong soil since the cone tip resistance is increasing just below the pile base.

Eslami and Fellenius (1997): Above base = $8D = 8 \times 0.6 = 4.8$ m

Below base = $4D = 4 \times 0.6 = 2.4$ m

Fleming and Thorburn (1983): Above base = $8D = 8 \times 0.6 = 4.8$ m

Below base = $2D = 2 \times 0.6 = 1.2$ m

Step 2: Determine cone tip resistance.

Eslami and Fellenius (1997): Geometric mean value $(q_c)_{ag} = 6.39$ MPa from depth 4.8 m above the base to 2.4 m below the base after neglecting excessively large values of q_c . You can use GEOMEAN function in Excel to find the geometric mean.

$$\text{Fleming and Thorburn (1983): } q_c = \frac{7.5 + 4.77 + 2(3.07)}{4} = 4.6 \text{ MPa}$$

Step 3: Determine average sleeve resistance.

You can inspect the sleeve resistance profile and visually draw an average sleeve resistance profile as shown in Fig. E10.6b.

Average sleeve resistance = 0.075 MPa over a depth of 10 m.

Step 4: Calculate load capacity

$$\text{Area of base, } A_b = \frac{\pi D^2}{4} = \frac{\pi \times 0.6^2}{4} = 0.28 \text{ m}^2$$

Eslami and Fellenius (1997): $Q_b = (q_c)_{ag} A_b = 6.39 \times 0.28 = 1.79$ MN

Fleming and Thorburn (1983): $Q_b = q_c A_b = 4.6 \times 0.28 = 1.29$ MN

Skin friction, $Q_f = f_s \pi D L = 0.075 \times \pi \times 0.6 \times 10 = 1.41$ MN

Eslami and Fellenius (1997): $Q_{ult} = 1.79 + 1.41 = 3.2$ MN

Fleming and Thorburn (1983): $Q_{ult} = 1.29 + 1.41 = 2.7$ MN

There is about 15% difference between the two methods. You can take an average value as an estimate of the ultimate pile load capacity, i.e., $(3.2 + 2.7)/2 = 2.95$ MN. ■

What's next... Piles are rarely used singly. They are often clustered into groups. Next, we will discuss how to determine the load capacity of pile groups.

10.8 PILE GROUPS

**Concept Learning and Self-Assessment**

Access Chapter 10, Section 10.8 on the CD to learn about pile groups. Take Quiz 10.8 to test your understanding.

In most practical situations, piles are used in groups. They are arranged in geometric patterns (squares, rectangles, circles, and octagons) at a spacing, s (center to center distance), not less than $2D$ (where D is the diameter or width of the pile). The piles are connected at their heads by a concrete pile cap, which may or may not be in contact with the ground (Fig. 10.11). If the pile cap is in contact with the ground, part of the load will be transferred directly to the soil.

The load capacity for a pile group is not necessarily the load capacity of a single pile multiplied by the number of piles. In fine-grained soils, the outer piles tend to carry more loads than the piles in the center of the group. In coarse-grained soils, the piles in the center take more loads than the outer piles.

The ratio of the load capacity of a pile group, $(Q_{ult})_g$, to the total load capacity of the piles acting as individual piles (nQ_{ult}) is called the efficiency factor, η_e ; that is,

$$\eta_e = \frac{(Q_{ult})_g}{nQ_{ult}} \quad (10.27)$$

where n is the number of piles in the group and Q_{ult} is the ultimate load capacity of a single pile. The efficiency factor is usually less than 1. However, piles driven into a loose, coarse-grained soil tend to densify the soil around the piles and η_e could exceed 1.

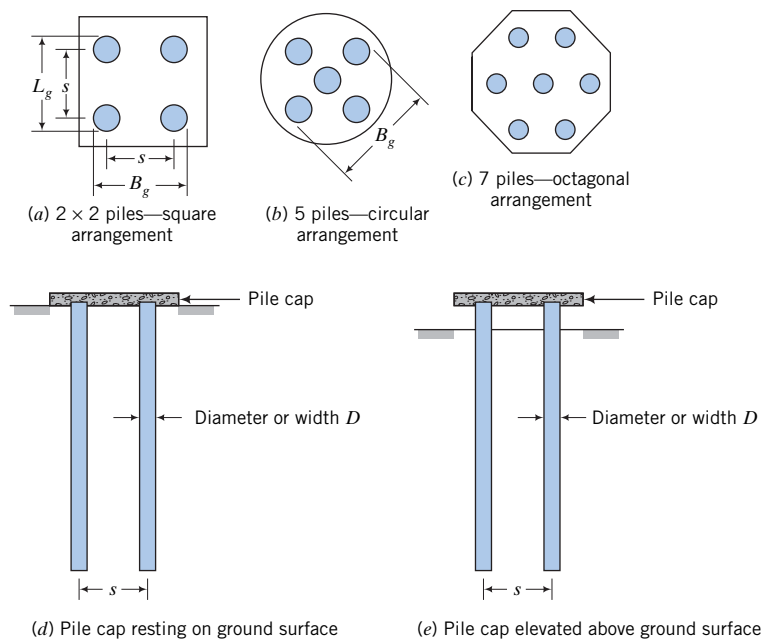


FIGURE 10.11 Pile groups.

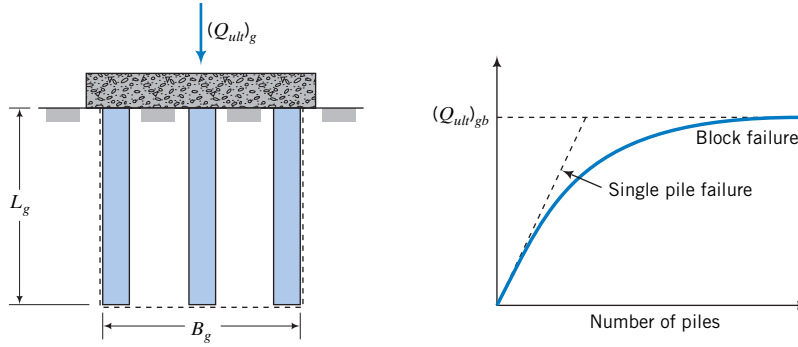


FIGURE 10.12 Block failure mode.

Two modes of soil failure are normally investigated to determine the load capacity of a pile group. One mode, called block failure (Fig. 10.12), may occur when the spacing of the piles is small enough to cause the pile group to fail as a unit. The group load capacity for block failure mode is

ESA

$$(Q_{ult})_{gb} = \sum_{i=1}^j \{ \beta_i (\sigma'_z)_i \times (\text{Perimeter})_{ig} \times L_i \} + N_q (\sigma'_z)_b (A_b)_g \quad (10.28)$$

TSA

$$(Q_{ult})_{gb} = \sum_{i=1}^j \{ (\alpha_u)_i (s_u)_i \times (\text{Perimeter})_{ig} \times L_i \} + N_c (s_u)_b (A_b)_g \quad (10.29)$$

where the subscript *gb* denotes block mode of failure for the group.

The other failure mode is failure of individual piles called single pile failure mode or punching failure mode. The key assumption in the single pile failure mode is that each pile mobilizes its full load capacity. Thus, the group load capacity is

$$(Q_{ult})_g = n Q_{ult} \quad (10.30)$$

The group efficiency in fine-grained soils is defined as

$$\eta_e = (Q_{ult})_{gb} [(Q_{ult})_{gb}^2 + (n Q_{ult})^2]^{-1/2} \quad (10.31)$$

The values of ψ_p to use in determining N_q in Janbu's equation depend on the s/D ratio and the friction angle. Janbu (1976) showed that

$$\frac{s}{D} = 1 + 2 \sin \psi_p (\tan \phi' + \sqrt{1 + \tan^2 \phi'}) \exp(\psi_p \tan \phi') \quad (10.32)$$

The value of ψ_p is not significantly affected by $s/D \leq 2.5$.

The essential points are:

1. *The ultimate load capacity of a pile group is not necessarily the ultimate load capacity of a single pile multiplied by the number of piles in the group.*
2. *A pile group can either fail by the group failing as a single unit, called block failure mode, or as individual piles, called single pile failure mode.*

EXAMPLE 10.7 Pile Group Load Capacity in Layered Clays

A pile group consisting of 9 piles, each 0.4 m in diameter, is arranged in a 3×3 matrix at a spacing of 1.2 m. The piles penetrate a medium clay soil ($s_u = 40$ kPa, $\phi'_{cs} = 30^\circ$, $\gamma_{sat} = 18$ kN/m³, OCR = 2) of thickness 8 m and are embedded 2 m in a stiff clay ($s_u = 90$ kPa, $\phi'_{cs} = 28^\circ$, $\gamma_{sat} = 18.5$ kN/m³, OCR = 5). Calculate the group allowable load capacity for a factor of safety of 3 and the efficiency factor. Groundwater level (GWL) is at 2 m below the surface but can rise to the surface due to seasonal changes.

Strategy You need to calculate the ultimate load capacity assuming (a) block failure mode and (b) single pile failure mode. Use a sketch to illustrate the problem.

Solution 10.7

Step 1: Draw a sketch and calculate the geometric properties.

See Fig E10.7.

$$\text{Single pile: } D = 0.4 \text{ m; Perimeter} = \pi D = \pi \times 0.4 = 1.26 \text{ m;}$$

$$A_b = \frac{\pi D^2}{4} = \pi \times \frac{0.4^2}{4} = 0.126 \text{ m}^2$$

$$\text{Group: Perimeter} = 4(2s + D) = 4[2(1.2) + 0.4] = 11.2 \text{ m;}$$

$$\text{Base area} = (A_b)_g = (2s + D)^2 = 2.8^2 = 7.84 \text{ m}^2$$

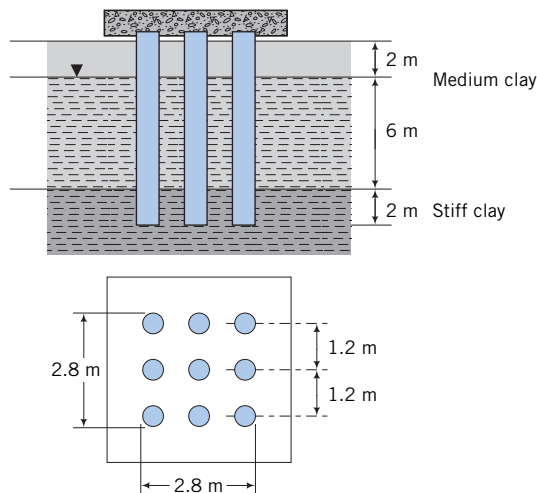


FIGURE E10.7

Step 2: Calculate the ultimate load capacity using TSA.

TSA—Block Failure Mode

Soft clay—skin friction

Table 10.3: $\alpha_u = 0.83$ for $s_u = 40$ kPa [see Example 10.2]

$$\begin{aligned}(Q_f)_{\text{soft clay}} &= \alpha_u s_u \times (\text{Perimeter})_g \times \text{Length} \\ &= 0.83 \times 40 \times 11.2 \times 8 = 2975 \text{ kN}\end{aligned}$$

Stiff clay—skin friction

Table 10.3: $\alpha_u = 0.5$ for $s_u = 90$ kPa

$$(Q_f)_{\text{stiff clay}} = 0.5 \times 90 \times 11.2 \times 2 = 1008 \text{ kN}$$

Stiff clay—end bearing

$$(Q_p)_{\text{stiff clay}} = 9s_u(A_b)_g = 9 \times 90 \times 7.84 = 6350 \text{ kN}$$

Group load capacity

$$\begin{aligned}(Q_{ult})_{gb} &= (Q_f)_{\text{medium clay}} + (Q_f)_{\text{stiff clay}} + (Q_p)_{\text{stiff clay}} \\ (Q_{ult})_{gb} &= 2975 + 1008 + 6350 = 10333 \text{ kN}\end{aligned}$$

TSA—Single Pile Failure Mode

Medium clay—skin friction

$$\begin{aligned}(Q_f)_{\text{medium clay}} &= \alpha_u s_u \times \text{Perimeter} \times \text{Length} \\ &= 0.83 \times 40 \times 1.26 \times 8 = 335 \text{ kN}\end{aligned}$$

Stiff clay—skin friction

$$(Q_f)_{\text{stiff clay}} = 0.5 \times 90 \times 1.26 \times 2 = 113 \text{ kN}$$

Stiff clay—end bearing

$$(Q_b)_{\text{stiff clay}} = 9s_u A_b = 9 \times 90 \times 0.126 = 102 \text{ kN}$$

Single pile load capacity

$$\begin{aligned}Q_{ult} &= (Q_f)_{\text{medium clay}} + (Q_f)_{\text{stiff clay}} + (Q_p)_{\text{stiff clay}} \\ Q_{ult} &= 335 + 113 + 102 = 550 \text{ kN}\end{aligned}$$

Group load capacity

Number of piles: $n = 9$

$$(Q_{ult})_g = nQ_{ult} = 9 \times 550 = 4950 \text{ kN}$$

Step 3: Calculate the ultimate load capacity using an ESA.

ESA—Block Failure Mode

Assume GWL will rise to the surface and $\phi'_i = \phi'_{cs}$.

$$\text{Table 10.3: } \beta = (1 - \sin \phi'_{cs})(\text{OCR})^{0.5} \tan \phi'_{cs}$$

$$\text{Medium clay: } \gamma' = 18 - 9.8 = 8.2 \text{ kN/m}^3$$

$$\text{Stiff clay: } \gamma' = 18.5 - 9.8 = 8.7 \text{ kN/m}^3$$

$$\text{Medium clay: } \beta = (1 - \sin 30^\circ)(2)^{0.5} \tan 30^\circ = 0.41$$

$$\text{Stiff clay: } \beta = (1 - \sin 28^\circ)(5)^{0.5} \tan 28^\circ = 0.63$$

Skin friction

$$Q_f = \beta \sigma'_z \times (\text{Perimeter})_g \times L; \quad \sigma'_z \text{ is calculated at mid-depth of the layer}$$

$$\text{Medium clay: } Q_f = 0.41 \times 8.2 \times \frac{8}{2} \times 11.2 \times 8 = 1205 \text{ kN}$$

$$\text{Stiff clay: } Q_f = 0.63 \times (8.2 \times 8 + 8.7 \times 1) \times 11.2 \times 2 = 1049 \text{ kN}$$

End bearing—stiff clay

$$Q_b = N_q(\sigma'_z)_b(A_b)_g$$

Use Janbu's equation with $\phi' = \phi'_{cs}$ and $\psi_p = \pi/2$.

$$N_q = [\tan 28^\circ + \sqrt{1 + \tan^2(28^\circ)}]^2 \exp\left(2 \frac{\pi}{2} \tan 28^\circ\right) = 14.7$$

$$(\sigma'_z)_b = 8 \times 8.2 + 2 \times 8.7 = 83 \text{ kPa}$$

$$Q_b = 14.7 \times 83 \times 7.84 = 9566 \text{ kN}$$

Group load capacity

$$(Q_{ult})_{gb} = 1205 + 1049 + 9566 = 11820 \text{ kN}$$

ESA—Single Pile Failure Mode

Skin friction

Use proportion since the only difference in the calculation is the perimeter.

$$\text{Medium clay: } Q_f = 1205 \times \frac{136}{11.2} = 136 \text{ kN}$$

$$\text{Stiff clay: } Q_f = 1049 \times \frac{1.26}{11.2} = 118 \text{ kN}$$

End bearing—stiff clay

$$Q_b = 9566 \times \frac{0.126}{7.84} = 154 \text{ kN}$$

Group load capacity

$$Q_{ult} = 136 + 118 + 154 = 408 \text{ kN}$$

$$(Q_{ult})_g = nQ_{ult} = 9 \times 408 = 3672 \text{ kN}$$

Step 4: Decide which failure mode and conditions govern.

Analysis	Load capacity (kN)	
	Block mode	Single pile mode
TSA	10333	4950
ESA	11820	3672

The lowest ultimate load capacity is 3672 kN for an ESA.

$$\therefore Q_a = \frac{(Q_{ult})_g}{FS} = \frac{3672}{3} = 1224 \text{ kN}$$

What's next... We have discussed methods to determine the ultimate load capacity, that is, the ultimate limit state, which is only one of two limit states required in analyses and design of geotechnical systems. The other limit state is serviceability limit state (settlement). Next, we will examine settlement of piles.

10.9 ELASTIC SETTLEMENT OF PILES



Concept Learning and Self-Assessment

Access Chapter 10, Section 10.9 to learn about pile settlement. Take Quiz 10.9 to test your understanding.

The elastic settlement of a single pile depends on the relative stiffness of the pile and the soil, the length to diameter ratio of the pile, the relative stiffness of the soil at the base and of the soil over the pile length, and the distribution of elastic modulus of the soil along the pile length. The relative stiffness of the pile to the soil is $K_{ps} = E_p/E_{so}$, where E_p is the elastic modulus of an equivalent solid cross section of the pile and E_{so} is the elastic modulus of the soil. The elastic modulus of the soil at the base or tip of the pile will be denoted by E_{sb} . Usually, the secant elastic modulus is used in design practice. For short-term loading in fine-grained soils, $(E_{so})_u$ is used.

Various analyses have been proposed to calculate the settlement of single piles and pile groups. Poulos (1989) provided an excellent discussion on the various numerical procedures to calculate settlement of piles. The settlement consists of three components—skin friction, end bearing, and elastic shortening.

Skin friction tends to deform the soil near the shaft, as illustrated in Fig. 10.13. The deformation mode near the shaft is analogous to simple shear strain (Chapter 5), and the shear strain, γ_{zx} , is

$$\gamma_{zx} = \frac{\tau}{G} = \frac{f_s}{G} \quad (10.33)$$

where G is the shear modulus, τ is shear stress and f_s is the skin frictional stress. The shear strains can be integrated over the pile length to give the elastic settlement (ρ_{es}) resulting from skin friction; that is,

$$\rho_{es} = \frac{1}{G(z)} \int_0^L \tau(z) dz \quad (10.34)$$

where (z) means that the parameter in front of it varies with depth. To solve Eq. (10.34), we need to know how G and τ vary with depth but this we do not know. Therefore, we have to speculate on their

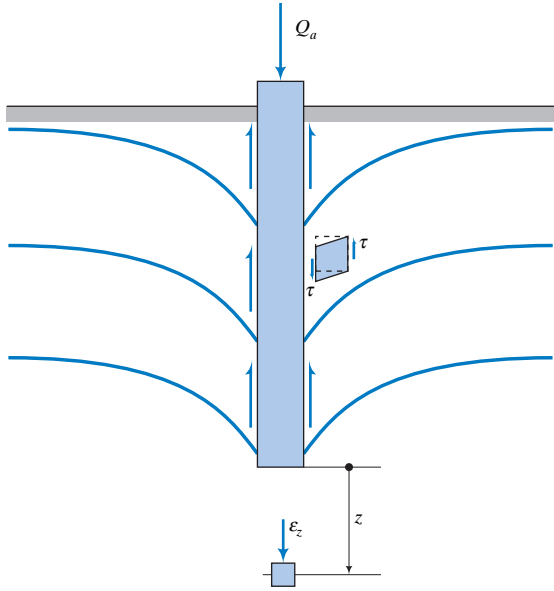


FIGURE 10.13 Shear deformation from skin friction and compression from end bearing.

variations and then solve Eq. (10.34). A further complication arises in that the boundary conditions for a pile problem are complex. Thus, we have to solve Eq. (10.31) using numerical procedures. For example, we can assume that

$$\tau(z) = \int F(z, i) \tau(i) di \quad (10.35)$$

where $F(z, i)$ is a stress function and $\tau(i)$ is the shear stress at an ordinate i . We can use finite element or boundary element to solve Eqs. (10.34) and (10.35). Stress functions for a point load within a half-space were developed by Mindlin (1936).

For a homogeneous soil (E_{so} is constant with depth), a solution of Eq. (10.34) using Eq. (10.35) for the elastic settlement of a single pile is

$$\rho_{es} = \frac{Q_{af}}{E_{so}L} I \quad (10.36)$$

where I is an influence factor and Q_{af} is the design load transferred as skin friction. An approximate equation for I is

$$I = 0.5 + \log\left(\frac{L}{D}\right) \quad (10.37)$$

Soft soils tend to have elastic moduli that vary linearly with depth; that is,

$$E_{so} = mz \quad (10.38)$$

where m is the rate of increase of E_{so} with depth (units: kPa/m). For soft soils, the elastic settlement is

$$(\rho_{es})_{so} = \frac{Q_{af}}{mL^2} I_{so} \quad (10.39)$$

where

$$I_{so} = 2.0 \log \left(\frac{L}{D} \right) \quad (10.40)$$

Poulos (1989) developed another solution for the elastic settlement. He showed that ρ_{es} for a floating pile can be determined from

$$\rho_{es} = \frac{Q_{af}}{E_{so}D} I_p \quad (10.41)$$

where I_p is an influence factor that depends on the L/D ratio and K_{ps} as shown in Fig. 10.14.

The soil mass under a pile is subjected to compression from end bearing. We can use elastic analyses (described in Chapter 5) to determine the elastic compression under the pile. For friction piles, the settlement due to end bearing is small in comparison to skin friction and is often neglected.

The elastic shortening of the pile (ρ_p) is found for column theory, which for a soil embedded in a homogeneous soil is

$$\rho_p = C \frac{Q_{af}L}{E_p A_p} \quad (10.42)$$

where C is a reduction factor to account for the fact that the vertical strain reduces with the embedded length of the pile. For most soils, $C \approx 0.5$ except for soft clay soils for which $C \approx 0.7$. Elastic shortening is only significant for slender piles ($E_p/E_{so} < 500$). The total elastic settlement for a floating pile is $\rho_{et} = \rho_{es} + \rho_p$.

Field observations indicate that at design loads, the elastic settlement of a single driven pile is within the range 0.9% to 1.25% of the pile diameter. Hull (1987) found, from numerical analyses, that embedding a pile beyond a certain critical length does not reduce the settlement. The critical length normalized to the diameter (width) is

$$\frac{L_c}{D} = \sqrt{\frac{\pi E_p A_p}{E_{so} D^2}} = \sqrt{\frac{\pi K_{ps} A_p}{D^2}} \quad (10.43)$$

where L_c is the critical length, A_p is the area of the cross section of the pile and

$$K_{ps} = \frac{E_p}{E_{so}}$$

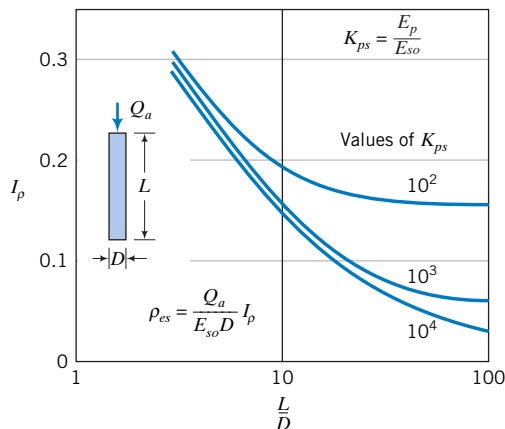


FIGURE 10.14 Influence factor for vertical settlement of a single floating pile. (After Poulos, 1989.)

Piles in a group tend to interact with each other depending on the spacing between them. The smaller the spacing, the greater the interaction and the larger the settlement. Pile group settlement is influenced by spacing to diameter (or width) ratio (s/D), the number of piles (n) in the group, and the length to diameter ratio (L/D). For convenience, pile group settlement is related to a single pile settlement through a group settlement factor R_s defined as

$$R_s = \frac{\text{Settlement of group}}{\text{Settlement of single pile at same average load}} \quad (10.44)$$

An empirical relationship between R_s and n (Fleming et al., 1985) is

$$R_s = n^\Phi \quad (10.45)$$

where the exponent Φ is between 0.4 and 0.6.

The settlement due to end bearing can be calculated by assuming the pile base is a rigid punch on the surface of the soil relocated a depth, L . The settlement (Timoshenko and Goodier, 1970) is:

$$\rho_b = \frac{Q_b}{r_b G_b} \frac{1 - \nu}{4} \quad (10.46)$$

where r_b and G_b are the radius and shear modulus at the base respectively and ν is Poisson's ratio. The settlement for group piles in coarse-grained soils from SPT and CPT can be estimated from:

$$\text{SPT: } \rho_{es} = \frac{360 \Delta \sigma_z I \sqrt{B_g}}{N_1} \text{ mm} \quad [\text{Units: } \Delta \sigma_z \text{ in MPa, } B_g \text{ in mm}] \quad (10.47)$$

$$\text{CPT: } \rho_{es} = \frac{\Delta \sigma_z I B_g}{2 q_c} \text{ mm} \quad [\text{Units: } \Delta \sigma_z \text{ in MPa, } B_g \text{ in mm, } q_c \text{ in MPa}] \quad (10.48)$$

where $I = 1 - 0.08 \frac{L}{B_g} \geq 0.5$, $\Delta \sigma_z$ is the vertical stress increase with the load acting at a depth of $\frac{2L}{3}$, i.e.,

$$\left(\Delta \sigma_z \approx \frac{Q_a}{\left(B_g + \frac{L}{z} \right) (L_g + z)} \right) \text{ MPa} \quad [\text{Units: } Q_a \text{ in MN, } B_g \text{ and } L_g \text{ in m}]$$

B_g and L_g are the width and length of the pile group, L is the embedded length of the pile, z is the depth below $\frac{2L}{3}$ usually taken as B_g , N_1 is the average N value corrected for overburden pressure over a depth z below $\frac{2L}{3}$, and q_c is the arithmetic average of the cone resistance over two pile diameters below the cone tip.

10.10 CONSOLIDATION SETTLEMENT UNDER A PILE GROUP



Self-Assessment

Access Chapter 10, and take Quiz 10.10 to test your understanding.

Sometimes, a pile group may be embedded above a soft clay layer and transfer sufficient load to it (soft clay) to cause consolidation settlement. To estimate the consolidation settlement, the full design load is

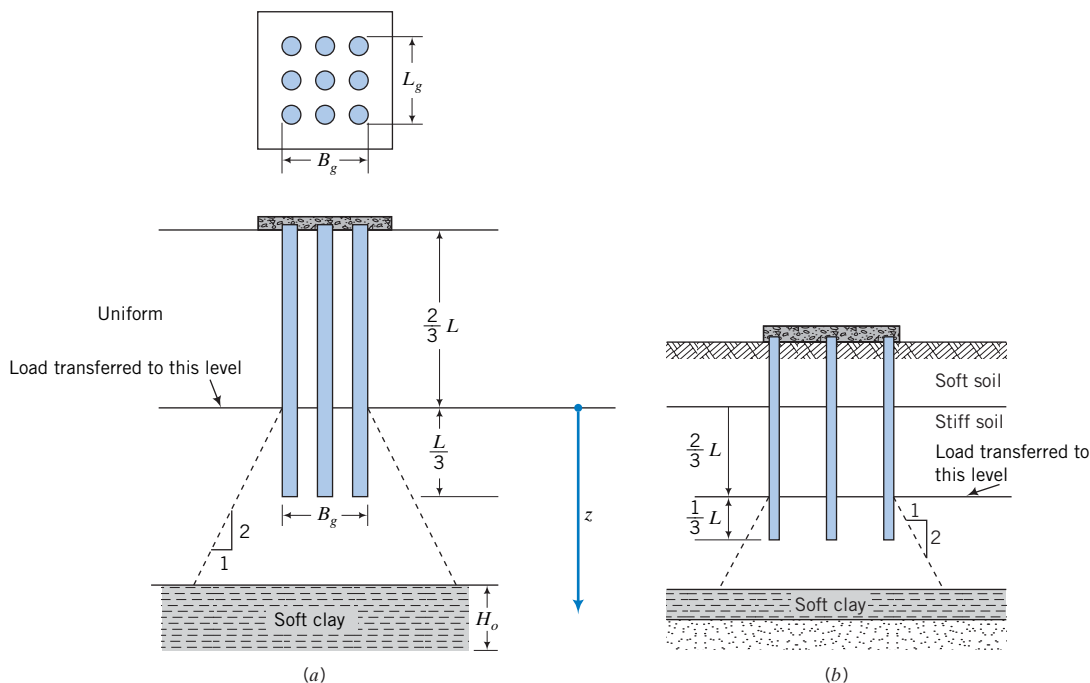


FIGURE 10.15 Assumed distribution of load for calculating settlement of a pile group.

assumed to act a depth of $\frac{2}{3}L$ and is then distributed in the ratio of 2:1 (vertical:horizontal). The increase in vertical stress at a depth z in the soft clay layer shown in Fig. 10.15 is

$$\Delta\sigma_z = \frac{Q_{ag}}{(B_g + z)(L_q + z)} \quad (10.49)$$

where Q_{ag} is the design group load, B_g is the width of the group, L_g is the length of the group, and z is the depth from the load transfer point on the pile group to the location in the clay layer at which the increase in vertical stress is desired. The primary consolidation settlement is then calculated using the procedure described in Section 6.4.

10.11 PROCEDURE TO ESTIMATE SETTLEMENT OF SINGLE AND GROUP PILES

The procedure to estimate the settlement of single piles and pile group is:

1. Obtain the required parameters Q_{af} or Q_{ag} , E_{so} , and E_p .
2. Calculate the elastic settlement, ρ_{es} , for a single pile using Eq. (10.36) or (10.39) or (10.41).
3. Calculate the elastic shortening (ρ_p) using Eq. (10.41).
4. Calculate the total elastic settlement, $\rho_{et} = \rho_{es} + \rho_p$.

For group piles, skip procedure 4 and continue as follows:

5. Calculate R_s using an estimated value of Φ .

6. Calculate the group settlement from $(\rho_e)_g = R_s \rho_{es} + \rho_p$.
7. Calculate the consolidation settlement, ρ_{pc} , if necessary.
8. Add the elastic and the consolidation settlement to obtain the total group settlement.

The essential points are:

1. *Settlement of piles is determined using numerical analyses assuming the soil is an elastic material.*
2. *The equations given for pile settlement will only give an estimated settlement.*

EXAMPLE 10.8 Settlement of a Single Pile

Determine the settlement of the pile in Example 10.2 using $E_p = 20 \times 10^6$ kPa, $E_{so} = 5000$ kPa, and a design shaft friction load $Q_a = 70$ kN.

Strategy This is a straightforward application of the procedure in Section 10.11.

Solution 10.8

Step 1: Determine the influence factor.

$$\frac{L}{D} = \frac{10}{0.4} = 25$$

$$\text{Equation (10.37): } I = 0.5 + \log\left(\frac{L}{D}\right) = 0.5 + \log(25) = 1.9$$

$$K_{ps} = \frac{E_p}{E_{so}} = \frac{20 \times 10^6}{5000} = 4000$$

From Fig. 10.14, $I_p = 0.09$ for $K_{ps} = 4000$ and $L/D = 25$.

Step 2: Calculate the elastic settlement.

$$\begin{aligned} \text{Equation (10.36): } \rho_{es} &= \frac{Q_{af}}{E_{so}L} I = \frac{70}{5000 \times 10} \times 1.9 \\ &= 2.7 \times 10^{-3} \text{ m} = 2.7 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Equation (10.41): } \rho_{es} &= \frac{Q_{af}}{E_{so}D} I_p = \frac{70}{5000 \times 0.4} \times 0.09 \\ &= 3.2 \times 10^{-3} \text{ m} = 3.2 \text{ mm} \end{aligned}$$

Difference between the two solutions is 0.5 mm. Use an average value of 3 mm.

Step 3: Calculate the elastic shortening of the pile.

$$\text{Equation (10.42): } \rho_p = C \frac{Q_{af}L}{E_p A_p}$$

$$A_p = \frac{\pi \times 0.4^2}{4} = 0.126 \text{ m}^2; \quad C = 0.7$$

$$\begin{aligned} \rho_p &= 0.7 \frac{70 \times 10}{20 \times 10^6 \times 0.126} \\ &= 277.8 \times 10^{-6} \text{ m} \cong 278 \times 10^{-3} \text{ mm} \end{aligned}$$

The elastic shortening of the pile is extremely small and can be neglected.

Step 4: Determine the total elastic settlement.

$$\rho_{et} = \rho_{es} + \rho_p = 3.0 + (\approx 0) = 3.0 \text{ mm}$$

The estimated total elastic settlement is approximately 3 mm.

$$\frac{\rho_t}{D} = \frac{3}{400} \times 100 = 0.75\%$$

EXAMPLE 10.9 Settlement of Group Piles

A 3×3 pile group with a pile spacing of 1 m and pile diameter of 0.4 m supports a load of 3 MN (Fig. E10.9). (a) Determine the factor of safety of the pile group. (b) Calculate the total settlement of the pile group.

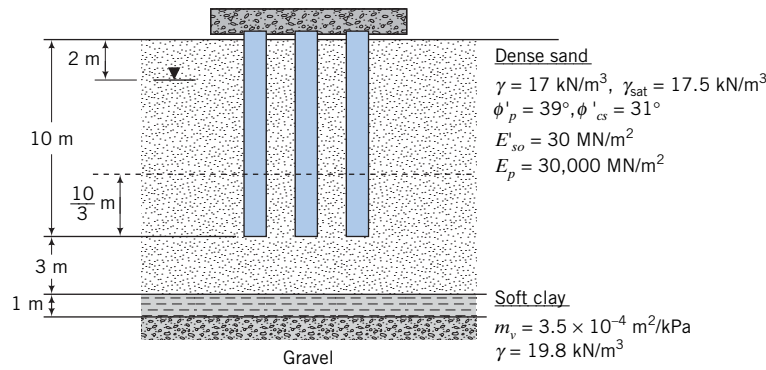


FIGURE E10.9

Strategy Since the sand is dense, driving of the piles will likely loosen it around the piles. You should use ϕ'_{cs} in calculating the pile load capacity.

Solution 10.9

Step 1: Determine the geometric parameters and β and N_q .

$$D = 0.4 \text{ m}, \quad \frac{L}{D} = \frac{10}{0.4} = 25; \quad n = 9 \text{ piles}, \quad s = 1 \text{ m};$$

$$K_{ps} = \frac{E_p}{E'_{so}} = \frac{30 \times 10^6}{30,000} = 1000$$

$$\text{Single pile: Perimeter} = \pi D = \pi \times 0.4 = 1.26 \text{ m};$$

$$A_b = \frac{\pi D^2}{4} = \frac{\pi \times 0.4^2}{4} = 0.126 \text{ m}^2$$

$$\text{Group piles: } L_g = B_g = 2s + D = 2 \times 1 + 0.4 = 2.4;$$

$$\text{Perimeter for group} = 2.4 \times 4 = 9.6 \text{ m}; \quad (A_b)_g = 2.4^2 = 5.76 \text{ m}^2$$

$$\text{Table 10.3: } \beta = 0.44 + \left(\frac{0.75 - 0.44}{7} \times 3 \right) = 0.57$$

[by linear interpolation of Meyerhof (1976) values]

$$\text{Figure 10.10: } N'_q = 40; \quad \alpha_i = 0.55 \quad \text{for } \frac{L}{D} = 25;$$

$$N_q = \alpha_i N'_q = 0.55 \times 40 = 22$$

Step 2: Determine the single pile failure mode load capacity.

$$\text{Sand: } \gamma' = 17.5 - 9.8 = 7.7 \text{ k/m}^3$$

Center of sand layer within embedment length of the pile:

$$\sigma'_z = 2 \times 17 + 3 \times 7.7 = 57.1 \text{ kPa}$$

$$\text{At base: } (\sigma'_z)_b = 2 \times 17 + 8 \times 7.7 = 95.6 \text{ kPa}$$

$$\text{Skin friction: } Q_f = \beta \sigma'_z \times \text{Perimeter} \times \text{Length}$$

$$= 0.57 \times 57.1 \times 1.26 \times 10 = 410 \text{ kN}$$

$$\text{End bearing: } Q_b = N_q (\sigma'_z)_b A_b = 22 \times 95.6 \times 0.126 = 265 \text{ kN}$$

$$\text{Ultimate load capacity: } Q_{ult} = Q_f + Q_b = 410 + 265 = 675 \text{ kN}$$

Assume $\eta_e = 1$.

$$(Q_{ult})_g = n Q_{ult} = 9 \times 675 = 6075 \text{ kN}$$

Step 3: Determine the block failure mode load capacity by proportion.

$$\text{Skin friction: } (Q_f)_g = 410 \times \frac{9.6}{1.26} = 3125 \text{ kN}$$

$$\text{End bearing: } (Q_b)_g = 265 \times \frac{5.76}{0.126} = 12114 \text{ kN}$$

$$\text{Ultimate load capacity: } (Q_{ult})_g$$

$$= (Q_f)_g + (Q_b)_g = 3125 + 12114 = 15239 \text{ kN}$$

Step 4: Calculate the factor of safety.

The single pile failure mode governs.

$$\therefore \text{FS} = \frac{6075}{3000} \cong 2$$

Step 5: Calculate the elastic settlement of the pile.

Assume the full design load is carried by skin friction and the load is equally shared by each pile.

$$Q_{af} = \frac{3000}{9} \approx 333 \text{ kN}$$

$$\text{Equation (10.37): For } \frac{L}{D} = 25, \quad I = 0.5 + \log(25) = 1.9$$

$$\text{Equation (10.36): } \rho_{es} = \frac{Q_{af}}{E_{so} L} \quad I = \frac{333}{30,000 \times 10} \times 1.9 = 2.1 \times 10^{-3} = 2.1 \text{ mm}$$

Neglect elastic shortening of the pile since $K_{ps} > 500$.

Assume $\Phi = 0.5$.

$$R_s = n^\Phi = 9^{0.5} = 3$$

$$(\rho_{es})_g = 2.1 \times 3 = 6.3 \text{ mm}$$

Step 6: Calculate the consolidation settlement.

The design load is transferred to $\frac{2}{3}L$ from the surface. The distance of the load transfer plane to the center of the clay is

$$\begin{aligned}\frac{10}{3} + 3 + \frac{1}{2} &= 6.83 \text{ m} \\ \Delta\sigma_z &= \frac{Q_{ag}}{(B_g + z)^2} = \frac{3000}{(2.4 + 6.83)^2} = 35.2 \text{ kPa} \\ \rho_{pc} &= m_v H_o \quad \Delta\sigma_z = 3.5 \times 10^{-4} \times 1 \times 35.2 \\ &= 123.2 \times 10^{-4} \text{ m} = 12.3 \text{ mm}\end{aligned}$$

Step 7: Compute the total settlement.

$$\rho_t = (\rho_{es})_g + \rho_{pc} = 6.3 + 12.3 = 18.6 \text{ mm} = 19 \text{ mm}$$

10.12 SETTLEMENT OF DRILLED SHAFTS

The short-term settlement of drilled shafts based on field data are shown in Fig. 10.16. The load-settlement curves for fine-grained soils due to skin friction show plateau values, between 0.2% and 0.8%, of the pile diameter but for end bearing, the plateau values are between 2% and 5%. For coarse-grained soils, the load-settlement curves show plateau values between 0.1% and 1% for skin friction but for end-bearing no plateau value was observed. Rather, the load-settlement curve shows increasing settlement for end bearing in excess of the ultimate end bearing load capacity. These latter results illustrate the difficulty in predicting the load-settlement response of piles in coarse-grained soils. Consequently, confidence in the prediction depends on experience with similar soil and construction conditions.

10.13 PILES SUBJECTED TO NEGATIVE SKIN FRICTION

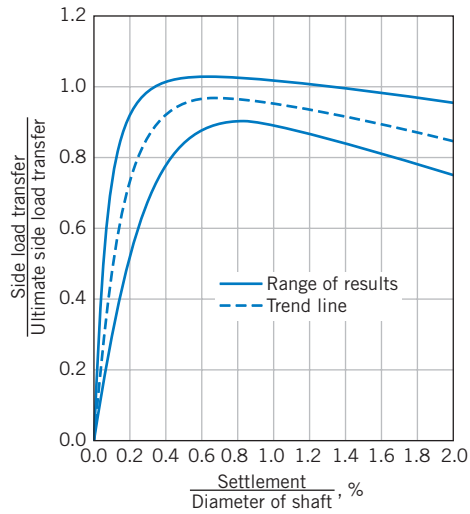


Concept Learning and Self-Assessment

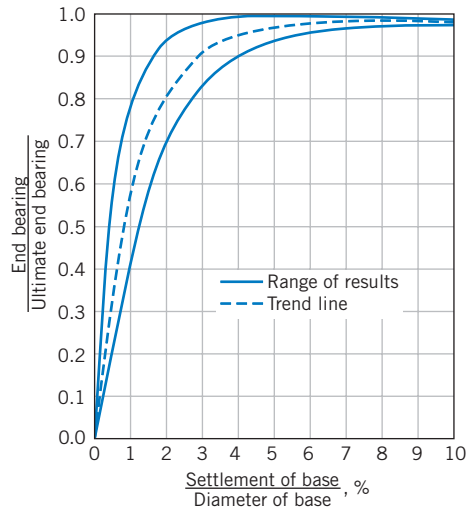
Access Chapter 10, Section 10.13 on the CD to learn about negative skin friction. Take Quiz 10.11 to test your understanding.

Piles located in setting soil layers (e.g., soft clays or fills) are subjected to negative skin friction called downdrag (Fig. 10.17). The settlement of the soil layer causes the friction forces to act in the same direction as the loading on the pile. Rather than providing resistance, the negative friction imposes additional loads on the pile. The net effect is that the pile load capacity is reduced and pile settlement increases. The allowable load capacity is given, with reference to Fig. 10.17, as

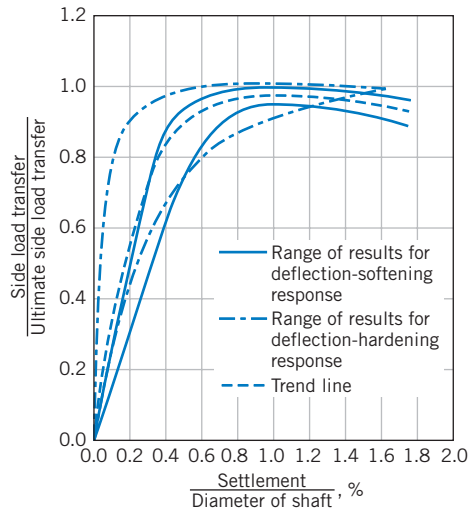
$$Q_a = \frac{Q_b + Q_f}{FS} - Q_{nf} \quad (10.50)$$



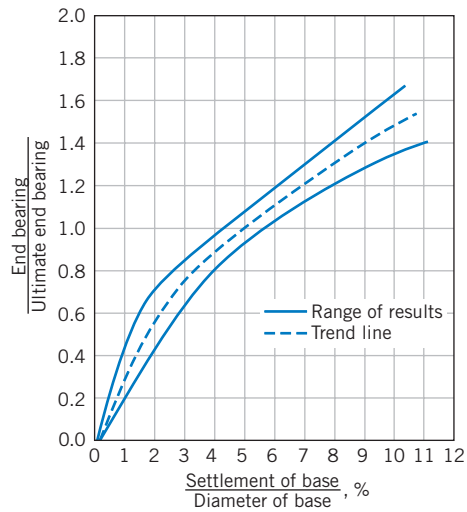
(a) Load transfer in skin friction in fine-grained soils



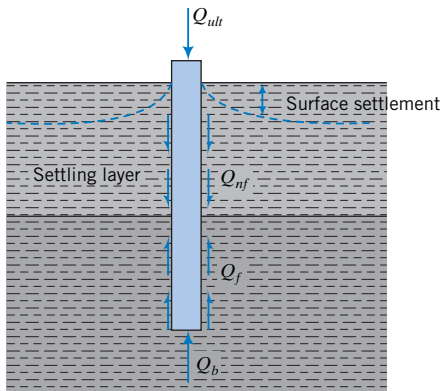
(b) Load transfer in end bearing in fine-grained soils



(c) Load transfer in skin friction in coarse-grained soils



(d) Load transfer in end bearing in coarse-grained soils

FIGURE 10.16 Load transfer–settlement curves for drilled shaft. (Source: Reese and O'Neill, 1988.)**FIGURE 10.17** Negative skin friction.

For a soft, normally consolidated soil, the negative skin friction is usually calculated over one-half its thickness. Negative skin friction should be computed for long-term condition; that is, you should use an ESA.

EXAMPLE 10.10 *Negative Skin Friction Due to a Fill*

Determine the allowable load capacity of the 0.4 m diameter pile shown in Fig. E10.10. The fill is recent and unconsolidated. To eliminate negative skin friction, a steel shell is proposed around the pile within the fill. The groundwater level is at 1 m below the fill in the soft clay but is expected to rise to the surface. A factor of safety of 3 is required.

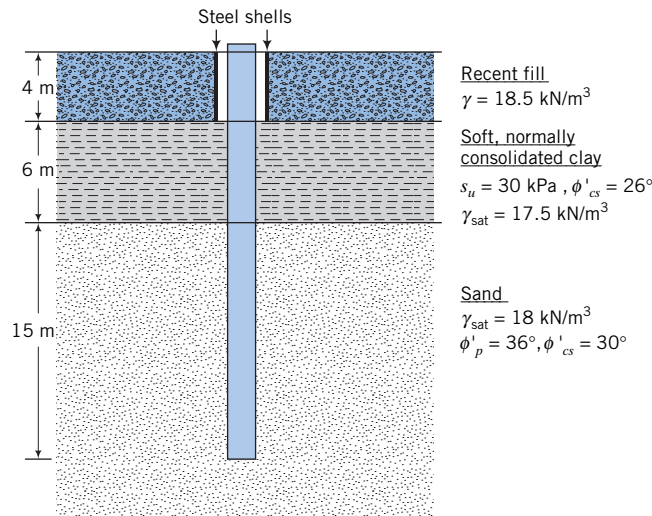


FIGURE E10.10

Strategy The trick here is to think about what would happen or is happening to the soft clay. Under the load of the fill, the soft clay will settle, dragging the pile down. Therefore, we have to consider negative skin friction imposed by the soft clay. You should use an ESA. Since the pile would likely loosen the dense sand adjacent to it, you should use ϕ'_{cs} rather than ϕ'_p .

Solution 10.10

Step 1: Determine β , N_q , and other relevant parameters.

$$\frac{L}{D} = \frac{25}{0.4} = 62.5; \quad \text{Perimeter} = \pi D = 0.4\pi = 1.26 \text{ m}$$

$$A_b = \frac{\pi D^2}{4} = \frac{\pi \times 0.4^2}{4} = 0.126 \text{ m}^2$$

Clay

$$\beta = (1 - \sin \phi'_{cs}) \tan \phi'_{cs} (\text{OCR})^{0.5} = (1 - \sin 26^\circ) \tan(26^\circ) (1)^{0.5} = 0.27$$

Sand

Table 10.3: Use Meyerhof's (1976) proposed values of β

For $\phi'_{cs} = 30^\circ$; $\beta \approx 0.52$ (linear interpolation)

From Fig. 10.10, $N'_q = 32$, for $\phi'_{cs} = 30^\circ$. The inset curve for α_t stops at $L/D = 25$; therefore, you have to make an estimate for α_i . It is unsafe to extrapolate results, especially experimental results. You should make a conservative estimate of α_t . Using $\alpha_t = 0.4$, we get $N_q = \alpha_t N'_q = 0.4 \times 32 = 12.8$.

Step 2: Calculate the negative skin friction for the clay layer.

Assume the groundwater will rise to the surface and the top one-half of the soft clay will be subjected to negative skin friction.

$$\gamma'_{\text{fill}} = 18.5 - 9.8 = 8.7 \text{ kN/m}^3$$

$$\gamma'_{\text{clay}} = 17.5 - 9.8 = 7.7 \text{ kN/m}^3$$

$$\text{At center of the upper half of the clay layer: } \sigma'_z = 8.7 \times 4 + 7.7 \times 3/2 = 46.4 \text{ kPa}$$

$$Q_{\text{nf}} = \beta \sigma'_z \times \text{Perimeter} \times \text{Length} = 0.27 \times 46.4 \times 1.26 \times 3 = 47 \text{ kN}$$

Step 3: Calculate skin friction of lower half of soft clay.

At center of the lower half of the clay layer

$$\sigma'_z = 8.7 \times 4 + 7.7 \times 4.5 = 69.5 \text{ kPa}$$

$$(Q_f)_{\text{clay}} = \beta \sigma'_z \times \text{Perimeter} \times \text{Length} = 0.27 \times 69.5 \times 1.26 \times 3 = 71 \text{ kN}$$

Step 4: Calculate the skin friction and end bearing in sand.

$$\gamma'_{\text{sand}} = 18 - 9.8 = 8.2 \text{ kN/m}^3$$

$$\text{At center of sand layer: } \sigma'_z = 8.7 \times 4 + 7.7 \times 6 + 8.2 \times 7.5 = 142.5 \text{ kPa}$$

$$Q_f = 0.52 \times 142.5 \times 1.26 \times 15 = 1400 \text{ kN}$$

$$(\sigma'_z)_b = 8.7 \times 4 + 7.7 \times 6 + 8.2 \times 15 = 204 \text{ kPa}$$

$$Q_b = N_q (\sigma'_z)_b A_b = 12.8 \times 204 \times 0.126 = 329 \text{ kN}$$

Step 5: Calculate the allowable load capacity.

$$Q_a = (Q_f)_{\text{clay}} + \frac{(Q_f)_{\text{sand}} + (Q_b)_{\text{sand}}}{\text{FS}} - Q_{\text{nf}} = \frac{71 + 1400 + 329}{3} - 47 = 553 \text{ kN}$$

10.14 PILE-DRIVING FORMULAS AND WAVE EQUATION

A number of empirical equations have been proposed to relate the energy delivered by a hammer during pile driving and the pile load capacity. One of the earliest equations is the ENR (*Engineering News Record*) equation, given as

$$Q_{\text{ult}} = \frac{W_R h}{s_1 + C_1} \quad (10.51)$$

where W_R is the weight of the ram, h is the height of fall, s_1 is the penetration per blow, and C_1 is a constant. For drop hammers, $C_1 \cong 25 \text{ mm}$; for steam hammers, $C_1 \cong 2.5 \text{ mm}$. The units of h and s_1 must be the same. The ENR equation was developed for timber piles using drop hammers for installation but is used to estimate or check the ultimate load capacity of all types of piles. Equation (10.51) does not

TABLE 10.9 Hammer Efficiency

Hammer type	η_1
Drop hammer	0.75–1.0
Single-acting hammer	0.75–0.85
Double-acting hammer	0.85
Diesel hammer	0.85–1.0

account for energy losses due to the elastic compression of the pile. This equation can be modified using an efficiency factor, η_1 ; that is,

$$Q_{ult} = \frac{\eta_1 W_R h}{s_1 + C_1} \quad (10.52)$$

Suggested ranges of values for η_1 are shown in Table 10.9.

When a hammer strikes a pile the shock sets up a stress wave, which moves down the pile at the speed of sound. From elastic theory, the change in force on the pile over an infinitesimal length Δz , assuming the pile to be a rod, is

$$\frac{\partial F}{\partial z} = E_p A_p \frac{\partial}{\partial z} (\epsilon_z) = E_p A_p \frac{\partial^2 u}{\partial z^2} \quad (10.53)$$

where $\epsilon_z = \partial u / \partial z$ is the vertical strain, E_p is Young's modulus of the pile, A_p is the cross-sectional area of the pile, and u is the pile displacement. From Newton's second law,

$$\frac{\partial F}{\partial z} = A_p \frac{\gamma}{g} \frac{\partial^2 u}{\partial t^2} \quad (10.54)$$

where g is the acceleration due to gravity and t is time. Setting Eq. (10.54) equal to Eq. (10.53), we get

$$E_p A_p \frac{\partial^2 u}{\partial z^2} = \frac{\gamma}{g} A_p \frac{\partial^2 u}{\partial t^2} \quad (10.55)$$

which, by simplification, leads to

$$\frac{\partial^2 u}{\partial t^2} = V_c^2 \frac{\partial^2 u}{\partial z^2} \quad (10.56)$$

where

$$V_c = \sqrt{\frac{E_p g}{\gamma}} \quad (10.57)$$

is the vertical wave propagation velocity in the pile.

The solution of Eq. (10.56) is found using appropriate boundary conditions and can be modified to account for soil resistance. Computer programs (e.g., WEAP) have been developed for routine wave analysis of pile-driving operations. These programs are beyond the scope of this book.

Driving records can provide useful information on the consistency of a soil at a site. For example, if the number of blows to drive a pile at a certain depth is A blows and B blows at another location at the same site, then the soil stratification is different. You should re-examine your design and the soil boring records and make the necessary adjustments, for example, increase or decrease the pile length.

The essential points are:

1. *A number of empirical equations are available to estimate the pile load capacity from driving records.*
2. *Driving records can provide some useful information regarding the character of the soil at a site.*
3. *Careful judgment and significant experience are required to rely on pile load capacity from pile-driving operations.*

What's next... Piles are often used to resist lateral loads and moments in addition to vertical loads. The analysis of laterally loaded piles is complex and only a brief introduction is presented in the next section.

10.15 LATERALLY LOADED PILES



Computer Program Utility

Access Chapter 10 and run program APILES to predict the response of single laterally loaded piles.

Structures founded on piles are often subjected to lateral loads and moments in addition to vertical loads. Lateral loads may come from wind, traffic, seismic events, waves, docking ships, and earth pressures. Moments may come from the eccentricity of the vertical force, fixity of the superstructure to the piles, and the location of the lateral forces on the pile with reference to the ground surface.

When a pile is subjected to lateral forces and moments, the pile tends to bend or deflect as illustrated in Fig. 10.18. The deflection of the pile causes strains in the soil mass. To satisfy equilibrium, the soil must provide reactions along the length of the pile to balance the applied loads and moments.

Because soil is a nonlinear material, the soil reaction is not linearly related to the pile deflection. Consequently at every point along the length of the pile, a nonlinear relationship between soil resistance (p) and pile deflection (y) exists as illustrated in Fig. 10.18.

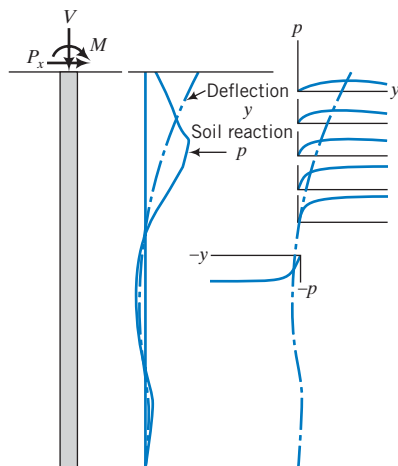


FIGURE 10.18 Pile-soil response to lateral loads and moments.

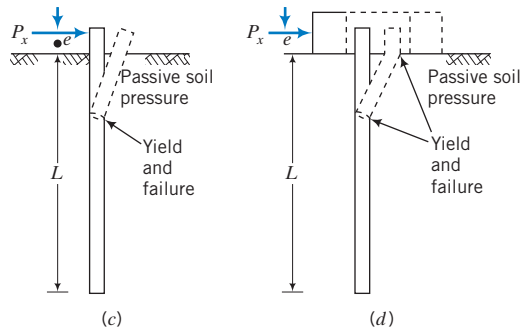


FIGURE 10.19 Possible failure modes for long piles ($L/D > 10$): (a) free head, (b) fixed head.

In designing laterally loaded piles, we need to know the pile deflection, particularly the pile head deflection, to satisfy serviceability requirement and the bending moments for sizing the pile. The pile head deflection depends on soil type, pile installation, pile flexibility (or pile stiffness), loading condition, and on how the pile is attached to the superstructure and pile cap. A pile that is attached to the pile cap such that no rotation occurs is called a fixed head pile (Fig. 10.19a). A pile that is attached to the pile cap such that rotation is unrestricted is called a free head pile (Fig. 10.19b).

The mechanism of failure depends on the length to diameter or width ratio, soil type, and the fixity of the pile head. Free head piles tend to fail by rotation. Lateral loads and moments applied to a free head pile are initially resisted by the soil near ground level. For very small pile deflections, the soil behaves elastically and as the deflection increases the soil yields and then permanent soil displacement occurs. The soil resistance is shifted to the lower part of the pile as yielding progressively occurs from the top to the bottom of the pile. Fixed head piles tend to fail by translation. Piles in general are neither fixed head nor free head. They have undermined fixity somewhere between free head and fixed head conditions. You can view fixed head and free head as two limiting conditions in which piles in practice will respond somewhere within these limits.

Laterally loaded piles, particularly group piles, are difficult to analyze mainly because of the complexity of the soil–structure (pile) interaction. The displacements and rotations are in the directions of the resultant lateral load and resultant moment (Fig. 10.20). Outer piles in a group are subjected to uplift (pull) and compressive (push) forces while the piles in the center translate at the

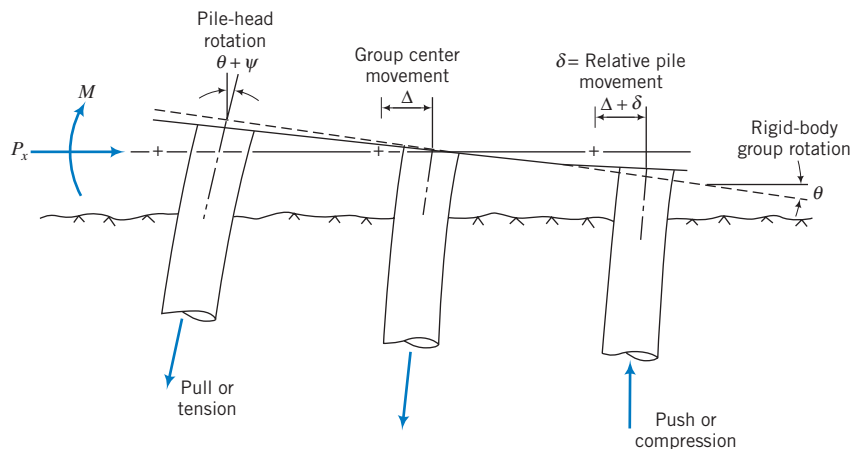


FIGURE 10.20 Group piles subjected to lateral loads.

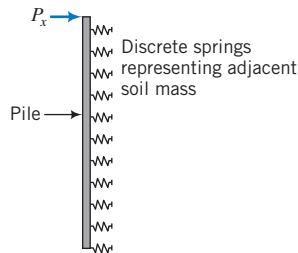


FIGURE 10.21 Simulation of adjacent soil mass as a set of discrete springs.

level of the superstructure connection. The response of a pile group to lateral loads and moments is influenced by

1. Geometry of the group
2. Pile–soil interaction
3. Stiffness or flexibility of the piles
4. Load conditions
5. Individual pile response
6. Pile group response resulting from individual pile responses

Two types of analyses are often used in design practice. One is the p - y (soil resistance–pile deflection) method (Reese, 1984, and others) that represents the soil as discrete springs (Fig. 10.21). The p - y relationships are obtained from results of high-quality, instrumented, laterally loaded test piles. General procedures have been developed to construct p - y curves for many soil types, lessening the need to conduct lateral load tests, at least in the preliminary stages of design. However, it is recommended that laterally loaded pile tests be conducted whenever such tests are feasible and economical. Computer programs (e.g., Com624 obtained from <http://www.fhwa.dot.gov/bridge/geosoft.htm#com624p> and various commercial derivatives of Com624) are available for routine use. The p - y method neglects soil continuity and soil shearing resistance.

The other method is a continuum analysis that represents the soil as a continuous medium with assumptions made on its stress–strain behavior or constitutive relationships (Fig. 10.22). The simplest stress–strain behavior is elastic described by Hooke’s law. The solution gives the load–deformation response of the pile for small strain levels (Poulos, 1971; Randolph, 1981). Soil yielding or pile yielding cannot be obtained from this analysis. To account for soil yielding, the soil can be assumed to be an elastoplastic material (Davies and Budhu, 1986; Budhu and Davies, 1987, 1988). A computer program for the elastoplastic (elastic–rigid plastic) solution is available on the CD (Chapter 10, APILES) accompanying this textbook. The use of this program and other programs based on the p - y method and the interpretation of the results for practice require significant experience.

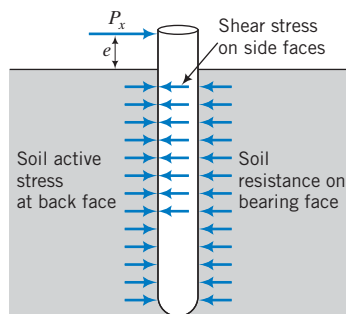


FIGURE 10.22 Soil stresses on a pile segment resulting from lateral loads treating soil as a continuum.

The essential points are

- 1. Lateral loads and moments cause piles to deflect laterally and the soil must provide reactions along the length of the pile to balance these applied loads and moments.*
- 2. The analysis of laterally loaded piles is a complex soil-structure interaction problem.*
- 3. Computer programs are available to analyze laterally loaded piles but the use and interpretation of the results from these programs require significant experience.*

EXAMPLE 10.11 Laterally Loaded Single Pile–Continuum Analysis

A 460-mm diameter timber pile of total length 10 m is embedded to a depth, L , of 9.14 m in a soft, normally consolidated, clay deposit of medium plasticity. The strength and elastic modulus of the soil varies linearly with depth. The working stress, σ_w , for timber of this quality is 11 MN/m^2 , and its Young's modulus of elasticity is 10.3 GN/m^2 . A lateral load is to be applied at a height, e , of 690 mm above ground level. Required are: (1) the working load; (2) the lateral deflection at the working load, and (3) the maximum bending moment at the working load. Assume the following. Adhesion factor, $\alpha_u = 1$, lateral earth pressure coefficient, $K_o = 1$, effective unit weight, $\gamma' = 9.4 \text{ kN/m}^3$; the slope of the linear variation of undrained shear strength with depth, $c = 2.7 \text{ kN/m}^3$; and the slope of the linear variation of elastic modulus with depth, $m = 2.7 \text{ MN/m}^3$, i.e., $m/c = 1,000$.

Strategy Use the program APILES located on the CD, Chapter 10.

Solution

Step 1: Enter input data interactively using the program APILES.

Step 2: Evaluate results.

The echoed input data and the results are given below.

TITLE: Example 10.11

Your input data is as follows

PILE DATA

Head type: Free head with eccentricity

Eccentricity = 0.69 m

Length = 9.14 m

Diameter = 0.46 m

Young's Modulus = .10E+08 kPa

Working stress = .11E+05 kPa

SOIL DATA

Soil type: Soft clay

Effective unit weight of soil = 9.4000 kN/m³

Earth pressure coefficient = 1.00

Adhesion factor = 1.00

Young's modulus at surface	= .00E+00 kPa
Rate of increase of Young's modulus with depth	= .27E+04 kN/m ³
Undrained shear strength at surface	= .00 kPa
Rate of increase of undrained shear strength with depth	= .27E+01 kN/m ³

The working load is: 42.54 kN; pile head displacement = 31 mm; maximum bending moment – 95 kN.m.

The bending moments and rotations are computed at ground surface

load	disp	moment	rotation	max. B.M.	depth to max. B.M.
kN	mm	kNm	radians	kNm	m
4.50	1.82	3.10	.04	7.45	1.66
4.91	1.99	3.39	.04	8.14	1.66
6.65	2.74	4.59	.06	11.29	1.66
9.40	4.03	6.48	.08	16.51	1.74
12.96	5.85	8.94	.12	23.45	1.84
13.77	6.27	9.50	.13	25.06	1.85
13.88	6.32	9.57	.13	25.25	1.86
14.48	6.65	9.99	.13	26.42	1.86
18.42	8.88	12.71	.18	34.38	1.92
19.62	9.61	13.54	.19	36.98	1.94
20.79	10.35	14.34	.20	39.64	1.96
23.74	12.31	16.38	.24	46.31	1.99
28.67	15.82	19.78	.30	57.83	2.06
30.96	17.61	21.36	.34	63.71	2.10
31.31	17.90	21.61	.34	64.64	2.10
37.83	23.57	26.10	.44	81.49	2.20
39.94	25.62	27.56	.47	87.52	2.24
45.25	31.14	31.22	.57	102.82	2.34
46.58	32.56	32.14	.59	106.68	2.36
50.68	37.58	34.97	.67	119.38	2.45
55.77	44.17	38.48	.77	135.47	2.54

The load-displacement results are shown in Fig. E10.11.

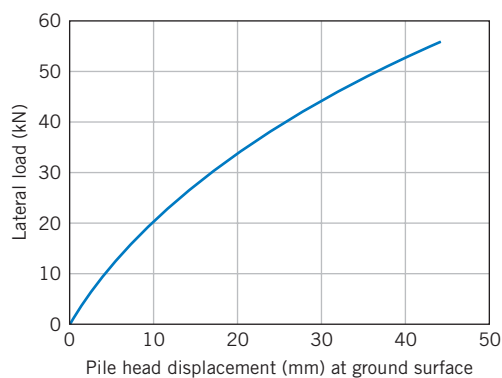


FIGURE E10.11

10.16 SUMMARY

Piles are used to support structural loads that cannot be supported on shallow foundations. The predominant types of pile material are steel, concrete, and timber. The selection of a particular type of pile depends on availability, environmental conditions, pile installation methods, and cost. Pile load capacity cannot be determined accurately because the method of installation invariably changes the soil properties near the pile. We do not know the extent of these changes. The equations for pile load capacities and settlement are, at best, estimates. Load capacities from pile load tests are preferred but these tests are expensive and may only be cost effective for large projects.

Practical Examples

EXAMPLE 10.12 *Pile Design for a Fish Port Facility*

A fish port facility is to be constructed near a waterfront area as shown in Figs. E10.12a,b. A soil investigation shows two predominant deposits. One is very soft, normally consolidated clay and the other is a stiff, overconsolidated clay. Soil data on the two deposits are shown in Fig. E10.12c. Determine the pile configuration (single or group piles), the pile length, and the expected settlement to support a design column load (working load) of 500 kN at A (Fig. E10.10). Timber piles of average diameter 0.38 m and average length 18 m are readily available. The elastic modulus of the pile is $E_p = 20,000$ MPa. The settlement should not exceed 0.5% of the pile diameter. The pile shafts within the tidal zone will be treated to prevent rot. From experience on this site, it is difficult to drive piles beyond 8 m in the stiff, overconsolidated clay. Driving tends to damage the pile head. You should allow for a 0.25 m cut out from the pile head.

Strategy The very soft silty clay is likely to cause downdrag on the pile. You should determine the single pile capacity assuming the full available length of the pile will be used. If a single pile is not capable of carrying the load, then you should design a pile group. Use an ESA when you are considering downdrag.

Solution 10.12

Step 1: Determine the geometric parameters, α , β , and N_q .

$$D = 0.38 \text{ m}; \quad \text{Perimeter} = \pi D = \pi \times 0.38 = 1.19 \text{ m};$$

$$A_b = \frac{\pi D^2}{4} = \frac{\pi \times 0.38^2}{4} = 0.113 \text{ m}^2$$

From borehole #1, the length of pile in soft clay: $L_1 = 7.4$ m

Length of pile in stiff clay: $L_2 = 18 - 7.4 - 3.5 - 0.25 = 6.85$ m.

The value 0.25 m is the cut-off length from the pile head and 3.5 m is the distance from the finished elevation to the surface of the soft soil.

$$\text{Soft clay, Table 10.3: } \beta = (1 - \sin 24^\circ) \tan 24^\circ (1)^{1/2} = 0.26$$

$$\text{Stiff clay, Table 10.3: } \beta = (1 - \sin 25^\circ) \tan 25^\circ (8)^{1/2} = 0.76$$

Assume $\psi_p = \pi/2$. Then

$$N_q = [\tan 25^\circ + \sqrt{1 + \tan^2(25^\circ)}]^2 \exp\left(2 \frac{\pi}{2} \tan 25^\circ\right) = 10.7$$

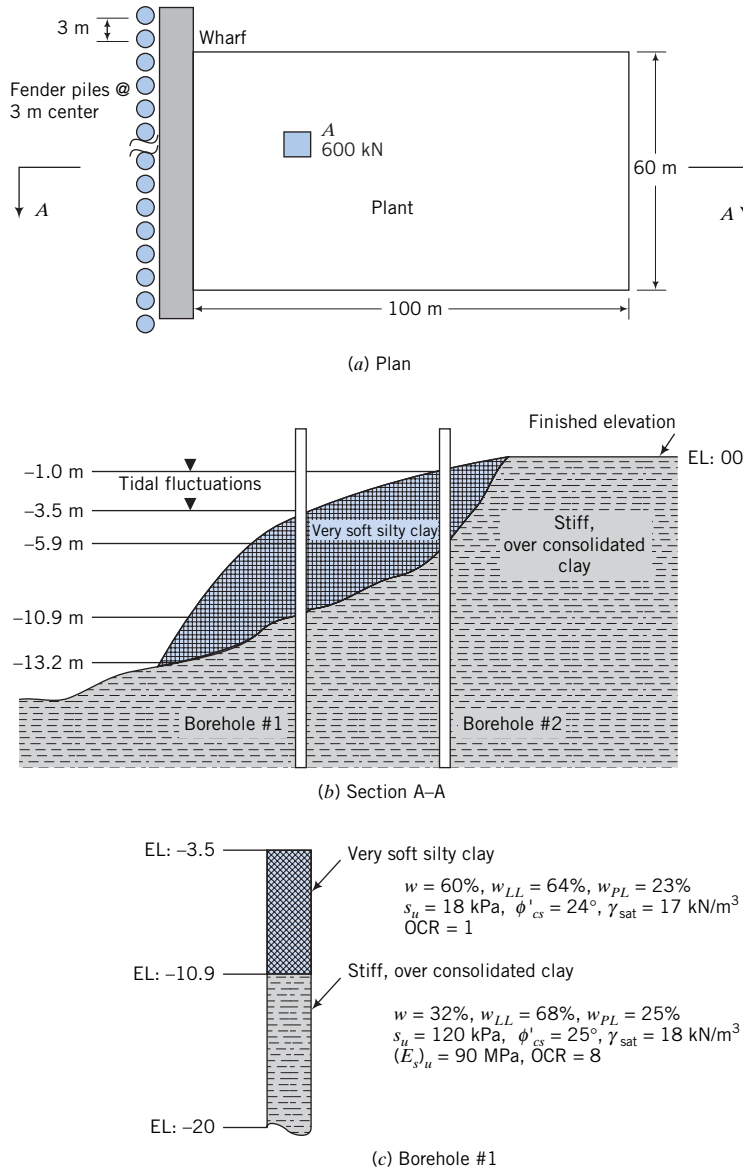


FIGURE E10.12

Step 2: Determine the negative skin friction for a single pile.

A conservative estimate is to assume negative skin friction over the whole length of the pile in the very soft clay.

$$\text{Soft clay: } \gamma' = 17 - 9.8 = 7.2 \text{ kN/m}^3$$

$$\text{Center of top half of soft clay: } \sigma'_z = 7.2 \times \frac{3.7}{2} = 13.3 \text{ kPa}$$

$$Q_{nf} = \beta \sigma'_z \times \text{Perimeter} \times L_1$$

$$Q_{nf} = 0.27 \times 13.3 \times 1.19 \times 7.4 = 32 \text{ kN}$$

Step 3: Determine the load capacity in the stiff clay for a single pile.

$$\text{Stiff clay: } \gamma' = 18 - 9.8 = 8.2 \text{ kN/m}^3$$

$$\text{Center of stiff clay: } \sigma'_z = 7.2 \times 7.4 + 8.2 \frac{6.85}{2} = 81.4 \text{ kPa}$$

$$\text{Base of stiff clay: } (\sigma'_z)_b = 7.2 \times 7.4 + 8.2 \times 6.85 = 109.5 \text{ kPa}$$

$$Q_f = \beta \sigma'_z \times \text{Perimeter} \times L_2 = 0.76 \times 81.4 \times 1.19 \times 68.5 = 504 \text{ kN}$$

$$Q_b = N_q (\sigma'_z)_b A_b = 10.7 \times 109.5 \times 0.113 = 132 \text{ kN}$$

Step 4: Determine the allowable load capacity of a single pile.

$$Q_a = \frac{Q_f + Q_b}{3} - Q_{nf} = \frac{504 + 132}{3} - 32 = 180 \text{ kN}$$

A single pile is inadequate for the load.

Step 5: Determine the block failure for the pile group.

Assume $\eta_e = 1$. Therefore,

$$\text{Number of piles required} = \frac{600}{180} = 3.3$$

Try 4 piles in a 2×2 matrix at a spacing of 1 m:

$$\frac{s}{D} = \frac{1}{0.38} = 2.6$$

$$B_g = L_g = s + D = 1 + 0.38 = 1.38 \text{ m;}$$

$$\text{Perimeter} = (1.38 + 1.38) \times 2 = 5.52 \text{ m; } A_b = 1.38^2 = 1.9 \text{ m}^2$$

$$\text{Soft clay: } Q_{nf} = 0.27 \times 13.3 \times 5.52 \times 7.4 = 147 \text{ kN}$$

$$\text{Stiff clay: } (Q_f)_g = 0.76 \times 81.4 \times 5.52 \times 6.85 = 2339 \text{ kN}$$

$$\text{Stiff clay: } (Q_b)_g = 10.7 \times 109.5 \times 1.9 = 2226 \text{ kN}$$

Step 6: Calculate the allowable load capacity for block failure mode.

$$(Q_a)_g = \frac{(Q_f)_g + (Q_b)_g}{3} - Q_{nf} = \frac{2339 + 2226}{3} - 147 = 1375 \text{ kN}$$

Step 7: Calculate the allowable load capacity for single pile failure mode.

$$n = 4 \text{ piles}$$

$$(Q_a)_g = nQ_a = 4 \times 180.6 = 722 \text{ kN}$$

Therefore, a 2×2 pile group is adequate. Single pile failure mode governs the design.

Step 8: Calculate the settlement.

Assume the full design load of $Q_a = 600 \text{ kN}$ will be carried by skin friction (floating pile) within the stiff clay.

$$\text{Load per pile } (Q_{af}) = \frac{600}{4} = 150 \text{ kN}$$

$$\frac{L}{D} = \frac{L_2}{D} = \frac{6.85}{0.38} = 18$$

Using Eq. (10.37) to calculate the elastic settlement, we get

$$I = 0.5 + \log(18) \approx 1.8$$

$$\rho_{es} = \frac{Q_{af}}{(E_{so})_u L_2} I = \frac{150}{90,000 \times 6.85} \times 1.8 = 0.43 \times 10^{-3} \text{ m} \approx 0.4 \text{ mm}$$

$$\rho_p = \frac{0.5Q_{af}L}{E_p A_p} = \frac{0.5 \times 150 \times 6.85}{20 \times 10^6 \times 0.113} = 227 \times 10^{-6} \text{ m} \approx 0.2 \text{ mm}$$

From Eq. (10.45): $R_s = n^\Phi$

$$\text{Assume } \Phi = 0.5: R_s = 4^{0.5} = 2$$

$$\text{Group piles: } (\rho_t)_g = R_s \rho_t + \rho_p = 2 \times 0.4 + 0.2 = 1.0 \text{ mm}$$

$$\frac{(\rho_r)_g}{D} = \frac{1.0}{380} = 2.6 \times 10^{-3} \approx 0.2\%$$

EXAMPLE 10.13 Design of Drilled Shafts

Computer Program Utility

Access Chapter 10 on the CD and click spreadsheet on the side bar for a spreadsheet to calculate the pile load capacity of straight drilled shafts.

A building is to be constructed at a site with a representative soil profile as shown in Fig. E10.13a. Nearby buildings of similar size are constructed on pile foundations. Previous records on driven piles near the site reveal significant heaving of prior installed piles during driving. In addition, pile-driving noise would be a problem with neighborhood groups. Consequently, drilled shafts should be considered. Design straight drilled shafts for the two column loads shown in Fig. E10.13b. FS desired = 2.5.

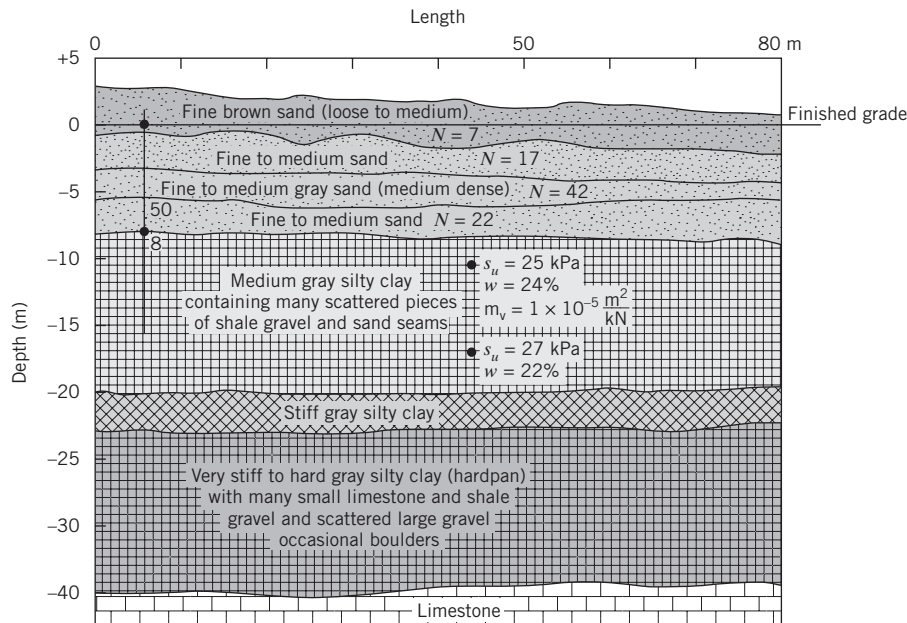


FIGURE E10.13a Soil profile along length of building.

Strategy Select drilled shaft diameters by adopting standard sizes used in the area. Study the soil profile and decide on which soil layer seems promising as an end-bearing layer. Now calculate shaft friction and end-bearing resistance for different depths of embedment. For a single shaft layout, you

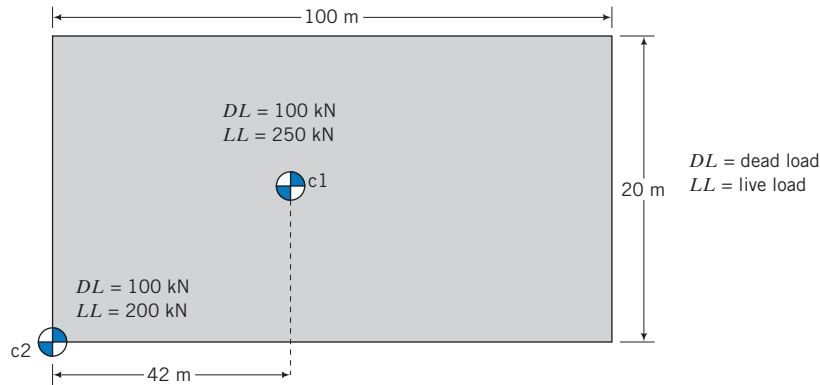


FIGURE E10.13b

need to check that the maximum diameter ($D_{\max}$) should be such that the column spacing exceeds $3D_{\max}$. If not, you will have to consider group action.

Solution 10.12

Step 1: Select drilled shaft diameters.

$$D_{\max} < \frac{\text{column spacing}}{3} = \frac{7}{3} = 2.33 \text{ m}$$

Assume $D = 0.6 \text{ m}$

Step 2: Select end bearing layer.

From inspection of the soil profile, we will try the medium gray silty clay layer as the end bearing layer. If the pile bearing capacity is inadequate, the stiff gray silty clay layer will then be considered.

Step 3: Calculate side friction and end-bearing capacity for different pile lengths.

For drilled shafts (straight shafts), neglect the skin friction of the top 1.5 m of soil below the finished grade and also the bottom length of one shaft diameter. No information on the unit weight of the sands is provided, so we use the correlations given in Table 7.4.

Skin friction

$$\text{Sand: } f_s = \beta \sigma'_z \quad (\text{Reese and O'Neill, 1988})$$

$$\text{Clay: } f_s = \alpha_u s_u$$

β and α_u are given in Tables 10.3 and 10.5.

End bearing

$$\text{Clay: } f_b = N_c(s_u) \leq 4.0 \text{ MPa; } N_c \text{ is given in Table 10.4}$$

Set up a spreadsheet to do the calculation. See spreadsheet drilledshafts.xls on the CD.

In the spreadsheet, the total, effective and porewater pressures for skin friction are calculated at the middle of each depth segment.

Step 4: Evaluate the calculated skin friction and end bearing and decide on pile length.

In the spreadsheet (drilledshafts.xls) on the CD, the friction on the top 1.5 m and on a length one diameter above the base is subtracted from the ultimate friction in columns Y and Z. The results show that TSA governs the design.

Pile c1

Applied vertical load = $Q_a = 350$ kN

Design load = $Q_{ult} = (FS) \times Q_a = 2.5 \times 350 = 875$ kN

Select pile diameter of 0.6 m.

Use $L = 10$ m, $Q_{ult} = 875$ kN (see Fig. E10.13c).

Pile c2

Applied vertical load = $Q_a = 300$ kN

Design load = $Q_{ult} = 2.5 \times 160 = 750$ kN

Use $L = 8.5$ m, $Q_{ult} = 803$ kN (see Fig. E10.13c).

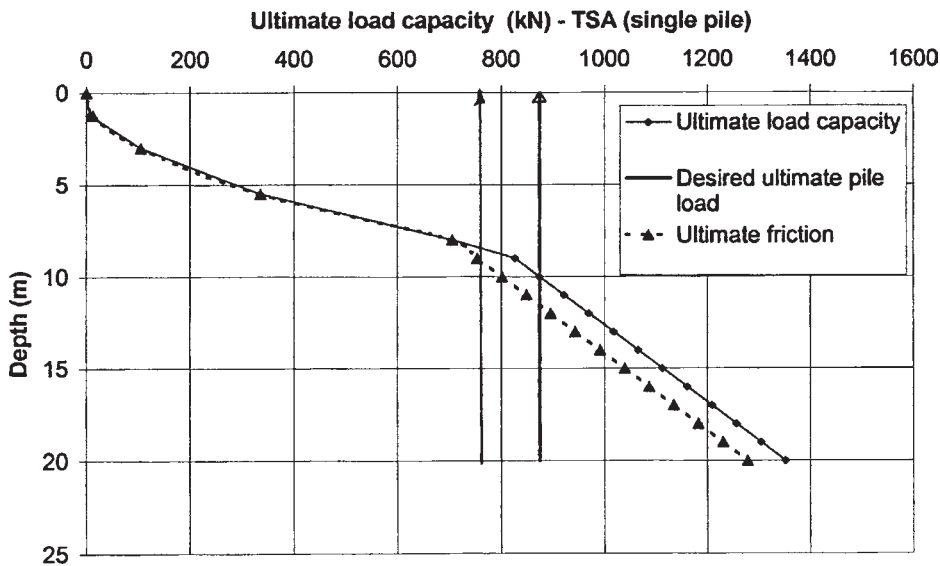
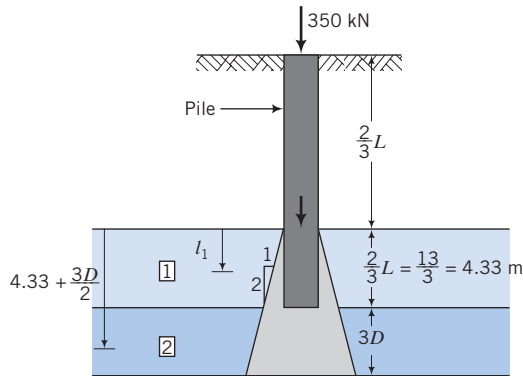


FIGURE E10.13c Ultimate load capacity variation with depth.

Step 5: Summarize design.

Drilled shaft	Diameter (m)	Embedded length (m)	$\frac{Q_f}{Q_{ult}}$
C1	0.6	10	$\frac{772.5}{875} \times 100 = 88\%$
C2	0.6	8.5	$\frac{701}{803} \times 100 = 87\%$

Most of the load is transferred as side friction. These piles are friction piles.
You can investigate alternative solutions using different shaft diameters.

EXERCISES

Problem Solving

- 10.1** Determine the length, L , required to support the load shown in Fig. P10.1. The diameter of the pile is 450 mm. A factor of safety of 3 is required.

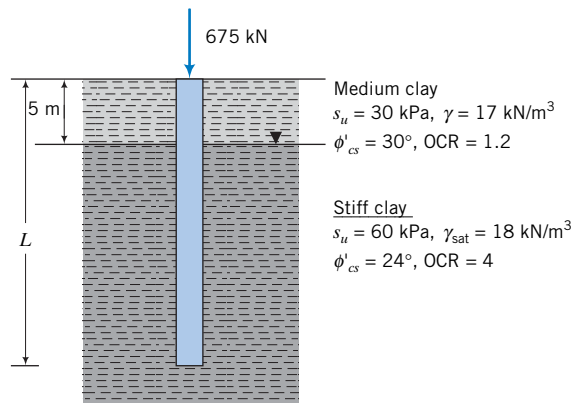


FIGURE P10.1

- 10.2** Determine the allowable load for a pile, 0.4 m in diameter, driven 20 m into the soil profile shown in Fig. P10.2. Groundwater is at 2 m below the surface but you can assume it will rise to the surface. A factor of safety of 3 is required.
- 10.3** A square precast concrete pile of sides 0.4 m is to be driven 12 m into the soil strata shown in Fig. P10.3. Estimate the allowable load capacity for a factor of safety of 3. Owing to changes in design requirements, the pile must support 20% more load. Determine the additional embedment depth required.
- 10.4** Estimate the allowable load capacity of a 0.5 m diameter pile embedded 17 m in the soil profile shown in Fig. P10.4. The factor of safety required is 3. The N values are blows/ft. Compare the load capacity for a driven pile and a drilled shaft.
- 10.5** The soil profile at a site for an offshore structure is shown in Fig. P10.5. Determine the allowable load for a driven pile with diameter 1.25 m and 50 m long. A factor of safety of 3 is required.

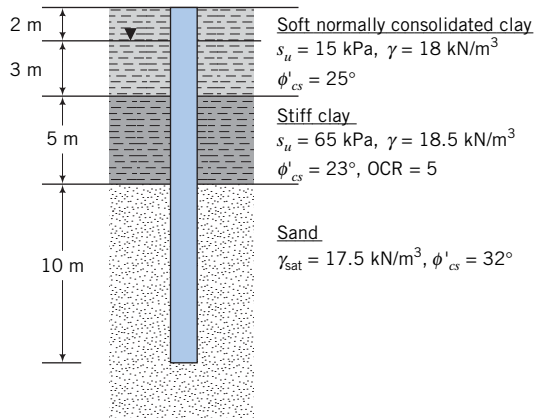


FIGURE P10.2

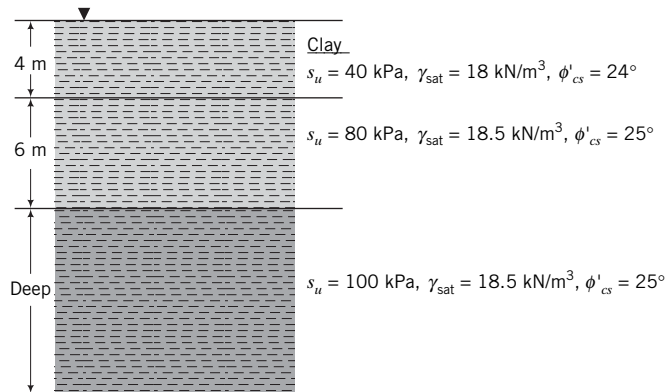


FIGURE P10.3

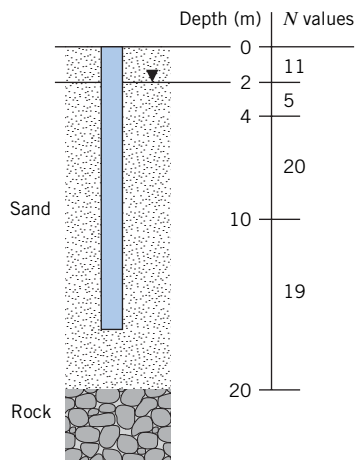


FIGURE P10.4

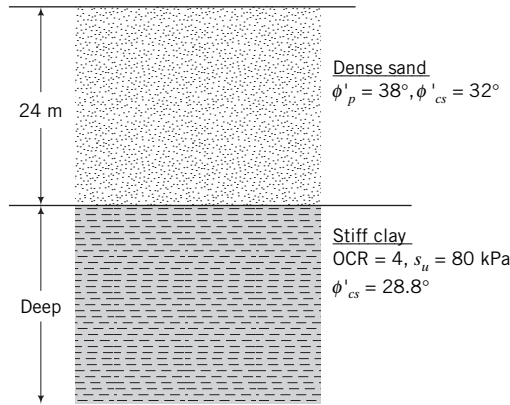


FIGURE P10.5

- 10.6** A soil profile consists of 3 m of a loose fill ($\phi'_{cs} = 15^\circ$, $\gamma_{sat} = 15 \text{ kN/m}^3$) over a stratified overconsolidated clay. The shear strength of the overconsolidated clay increases linearly with depth. At the top of the clay, $s_u = 90 \text{ kPa}$ and at a depth of 20 m (17 m into the clay), $s_u = 180 \text{ kPa}$. The critical state friction angle is $\phi'_{cs} = 28^\circ$, OCR = 8 and $\gamma_{sat} = 17 \text{ kN/m}^3$. Determine the ultimate load capacity of a drilled shaft (bored pile) 1 m in diameter embedded 18 m into the clay. Groundwater is at 8 m below the ground surface. You may assume that the soil above the groundwater level is saturated.
- 10.7** Estimate the allowable load capacity of a 3×3 pile group. Each pile has a diameter of 0.4 m and is driven 15 m into a soft clay (OCR = 1.2) whose undrained shear strength varies linearly with depth according to $s_u = 0.25 \sigma'_z$. The critical state friction angle is 30° and $\gamma_{sat} = 17.5 \text{ kN/m}^3$. The spacing of the piles is 1.5 m. Groundwater level is at ground surface. A factor of safety of 3 is required.

Practical

- 10.8** The soil profile and soil properties at a site are shown in the table below. A group of 12 concrete piles in a 3×4 matrix and of length 12 m is used to support a load. The pile diameter is 0.45 m and pile spacing is 1.5 m. Determine the allowable load capacity for a factor of safety of 3. Calculate the total settlement (elastic and consolidation) under the allowable load. Assume $E_p = 20 \times 10^6 \text{ kPa}$.

Depth (m)	Type of deposit	Soil test results
0–3	Sand	$\gamma = 17 \text{ kN/m}^3$, $\phi'_{cs} = 28^\circ$
3–groundwater level		$E'_{so} = 19 \text{ MN/m}^2$
3–6	Sand	$\gamma_{sat} = 17.5 \text{ kN/m}^3$, $\phi'_{cs} = 30^\circ$ $E'_{so} = 18 \text{ MN/m}^2$
6–15	Clay	$\gamma_{sat} = 18.5 \text{ kN/m}^3$, $\phi'_{cs} = 27^\circ$ $s_u = 30 \text{ kPa}$ $C_c = 0.4$, $C_r = 0.06$, OCR = 1.5 $E'_{so} = 30 \text{ MN/m}^2$, $\nu' = 0.3$
15–17	Soft clay	$\gamma_{sat} = 18 \text{ kN/m}^3$, $\phi'_{cs} = 24^\circ$ $s_u = 20 \text{ kPa}$ $C_c = 0.8$, OCR = 1.0 $E'_{so} = 10 \text{ MN/m}^2$, $\nu' = 0.3$
>17	Rock	

- 10.9** The results of field and laboratory tests at a site are shown in the table below. The soil is heavily overconsolidated ($\text{OCR} = 8$) over a depth of 30 m. Estimate the allowable load capacity for a steel cylindrical pile of diameter 1.5 m, length 15 m, wall thickness 65 mm driven with a driving shoe (displacement pile). Use a factor of safety of 1.5, $\phi'_{cs} = 32^\circ$ and α_u based on API (1984) (see Table 10.3).

Depth (m)	0	3	5	10	15	20	22	25	32	35	40
σ'_z (kPa)	0	42	58	137	198	368	297	342	431	460	500
s_u (kPa)	0	750	750	550	423	580	610	457	322	325	310

- 10.10** A pile group consisting of ten 450 mm diameter drilled shafts support a bridge abutment as shown in Fig. P10.10. The pile spacing in both directions is 1.5 m. The soil profile consists of a fine sand 2 m thick ($\gamma = 19 \text{ kN/m}^3$) above a medium sand ($\gamma_{\text{sat}} = 20 \text{ kN/m}^3$). Ground water is located at 2 m below ground surface. SPT results are shown in the table below. Determine the length of the pile group for $\text{FS} = 2$. Assume the SPT value is constant after 15 m.

Depth (m)	1	2	3	5	7	9	10	12	15
N (blows/ft)	5	5	7	12	16	18	19	25	38

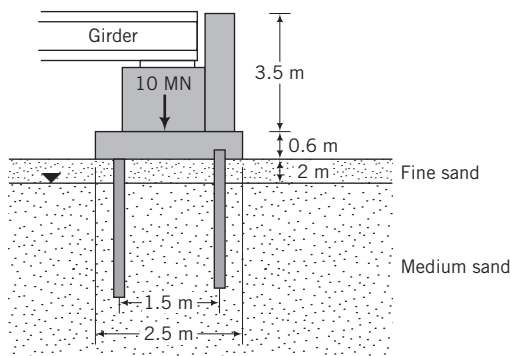


FIGURE P10.10

CHAPTER 11

TWO-DIMENSIONAL FLOW OF WATER THROUGH SOILS

11.0 INTRODUCTION

Many catastrophic failures in geotechnical engineering result from instability of soil masses due to groundwater flow. Lives are lost, infrastructures are damaged or destroyed, and major economic losses occur. In this chapter, you will study the basic principles of two-dimensional flow of water through soils. The topics covered here will help you to avoid pitfalls in the analyses and design of geotechnical systems where groundwater flow can lead to instability. The emphasis in this chapter is on gaining an understanding of the forces that provoke failures resulting from groundwater flow. You will learn methods to calculate flow, porewater pressure distribution, uplift forces, and seepage stresses for a few simple geotechnical systems.

When you complete this chapter, you should be able to:

- Calculate flow under and within earth structures.
- Calculate seepage stresses, porewater pressure distribution, uplift forces, hydraulic gradients, and the critical hydraulic gradient.
- Determine the stability of simple geotechnical systems subjected to two-dimensional flow of water.

You will use the following principles learned from previous chapters and your courses in mechanics:

- Statics
- Hydraulic gradient, flow of water through soils (Chapter 4)
- Effective stresses and seepage (Chapter 5)

Sample Practical Situation A deep excavation is required for the construction of a building. The soil is a silty sand with groundwater level just below ground level. The excavation cannot be made unless the sides are supported. You, a geotechnical engineer, are required to design the retaining structure for the excavation and to recommend a scheme to keep the inside of the excavation dry. Figure 11.1 shows the collapse of a sewer and a supported excavation by seepage forces. You should try to prevent such a collapse.

11.1 DEFINITIONS OF KEY TERMS

Equipotential line is a line representing constant head.

Flow line is the flow path of a particle of water.



FIGURE 11.1 Damage of a braced excavation by seepage forces. (Courtesy of George Tamaro, Mueser Rutledge Consulting Engineers.)

Flownet is a graphical representation of a flow field.

Seepage stress is the stress (similar to frictional stress in pipes) imposed on a soil as water flows through it.

Static liquefaction is the behavior of a soil as a viscous fluid when seepage reduces the effective stress to zero.

11.2 QUESTIONS TO GUIDE YOUR READING

1. What is the governing equation for two-dimensional flow and what are the methods adopted for its solution for practical problems?
2. What are flow lines and equipotential lines?
3. What is a flownet?
4. How do I draw a flownet?
5. What are the practical uses of a flownet?
6. What is the critical hydraulic gradient?
7. How do I calculate the porewater pressure distribution near a retaining structure and under a dam?
8. What are uplift pressures and how can I calculate them?
9. What do the terms static liquefaction, heaving, quicksand, boiling, and piping mean?
10. What are the forces that lead to instability due to two-dimensional flow?
11. How does seepage affect the stability of an earth retaining structure?

11.3 TWO-DIMENSIONAL FLOW OF WATER THROUGH POROUS MEDIA



Interactive Concept Learning and Self-Assessment

Access Chapter 11, Section 11.3 on the CD and learn about the two-dimensional flow of water through soils via animations. Take Quiz 11.3 to test your understanding.

The flow of water through soils is described by Laplace's equation. Flow of water through soils is analogous to steady state heat flow and flow of current in homogeneous conductors. The popular form of Laplace's equation for two-dimensional flow of water through soils is

$$k_x \frac{\partial^2 H}{\partial x^2} + k_z \frac{\partial^2 H}{\partial z^2} = 0 \quad (11.1)$$

where H is the total head and k_x and k_z are the hydraulic conductivities in the X and Z directions. Laplace's equation expresses the condition that the changes of hydraulic gradient in one direction are balanced by changes in the other directions.

The assumptions in Laplace's equation are:

- Darcy's law is valid.
- The soil is homogeneous and saturated.
- The soil and water are incompressible.
- No volume change occurs.

If the soil were an isotropic material then $k_x = k_z$ and Laplace's equation becomes

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial z^2} = 0 \quad (11.2)$$

The solution of any differential equation requires knowledge of the boundary conditions. The boundary conditions for most "real" structures are complex, so we cannot obtain an analytical solution or closed form solution for these structures. We have to resort to approximate solutions, which we can obtain using numerical methods such as finite difference, finite element, and boundary element. We can also use physical models to attempt to replicate the flow through the real structure.

In this chapter, we are going to consider two solution techniques for Laplace's equation. One is an approximate method called flownet sketching; the other is the finite difference technique, which you have encountered in Chapter 6. The flownet sketching technique is simple and flexible and conveys a picture of the flow regime. It is the method of choice among geotechnical engineers. But before we delve into these solution techniques, we will establish some key conditions that are needed to understand two-dimensional flow.

The solution of Eq. (11.1) depends only on the values of the total head within the flow field in the XZ plane. Let us introduce a velocity potential (ξ), which describes the variation of total head in a soil mass as

$$\xi = kH \quad (11.3)$$

where k is a generic hydraulic conductivity. The velocities of flow in the X and Z directions are

$$v_x = k_x \frac{\partial H}{\partial x} = \frac{\partial \xi}{\partial x} \quad (11.4)$$

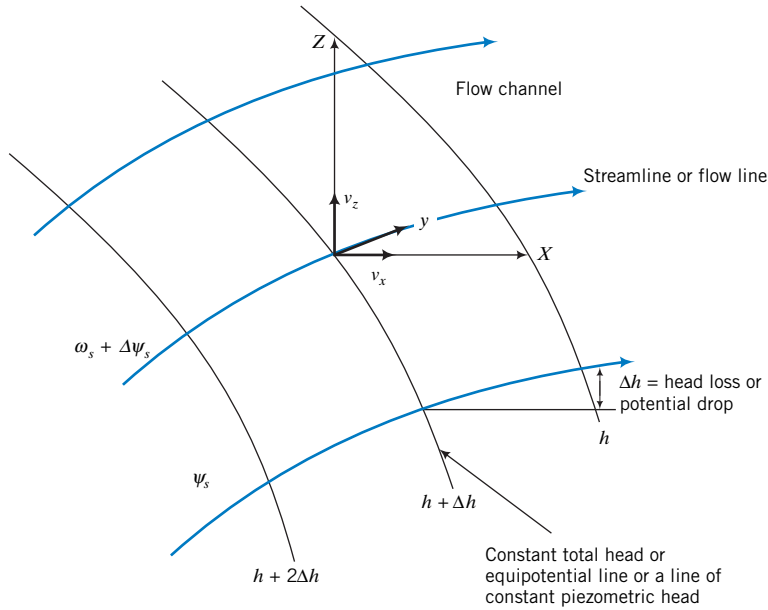


FIGURE 11.2 Illustration of flow terms.

$$v_z = k_z \frac{\partial H}{\partial z} = \frac{\partial \xi}{\partial z} \quad (11.5)$$

The inference from Eqs. (11.4) and (11.5) is that the velocity of flow (v) is normal to lines of constant total head as illustrated in Fig. 11.2. The direction of v is in the direction of decreasing total head. The head difference between two equipotential lines is called a potential drop or head loss.

If lines are drawn that are tangent to the velocity of flow at every point in the flow field in the XZ plane, we will get a series of lines that are normal to the equipotential lines. These tangential lines are called streamlines or flow lines (Fig. 11.2). A flow line represents the flow path that a particle of water is expected to take in steady state flow. A family of streamlines is called a stream function, $\psi_s(x, z)$.

The components of velocity in the X and Z directions in terms of the stream function are

$$v_x = \frac{\partial \psi_s}{\partial z} \quad (11.6)$$

$$v_z = \frac{\partial \psi_s}{\partial x} \quad (11.7)$$

Since flow lines are normal to equipotential lines, there can be no flow across flow lines. The rate of flow between any two flow lines is constant. The area between two flow lines is called a flow channel (Fig. 11.2). Therefore, the rate of flow is constant in a flow channel.

The essential points are:

1. Streamlines or flow lines represent flow paths of particles of water.
2. The area between two flow lines is called a flow channel.
3. The rate of flow in a flow channel is constant.

4. *Flow cannot occur across flow lines.*
5. *The velocity of flow is normal to the equipotential line.*
6. *Flow lines and equipotential lines are orthogonal (perpendicular) to each other.*
7. *The difference in head between two equipotential lines is called the potential drop or head loss.*

What's next . . . The flow conditions we established in the previous section allow us to use a graphical method to find solutions to two-dimensional flow problems. In the next section, we will describe flownet sketching and provide guidance in interpreting a flownet to determine flow through soils, the distribution of porewater pressures, and the hydraulic gradients.

11.4 FLOWNET SKETCHING



Interactive Concept Learning, Computer Program Utility, and Self-Assessment

Access Chapter 11, Section 11.4 on the CD to learn about sketching flownets; calculate porewater pressure and seepage forces through interactive animation. Take Quiz 11.4 to test your understanding.

Click on “2D Flow” on the sidebar to run a computer program that interactively plots flownets for retaining walls and dams. Input different boundary conditions and geometry to explore changes in the flownet. For example, you can drag a sheet pile up or down or position it at different points below a dam and explore how the flownet changes.

11.4.1 Criteria for Sketching Flownets

A flownet is a graphical representation of a flow field that satisfies Laplace's equation and comprises a family of flow lines and equipotential lines.

A flownet must meet the following criteria:

1. The boundary conditions must be satisfied.
2. Flow lines must intersect equipotential lines at right angles.
3. The area between flow lines and equipotential lines must be curvilinear squares. A curvilinear square has the property that an inscribed circle can be drawn to touch each side of the square and continuous bisection results, in the limit, in a point.
4. The quantity of flow through each flow channel is constant.
5. The head loss between each consecutive equipotential line is constant.
6. A flow line cannot intersect another flow line.
7. An equipotential line cannot intersect another equipotential line.

An infinite number of flow lines and equipotential lines can be drawn to satisfy Laplace's equation. However, only a few are required to obtain an accurate solution. The procedure for constructing a flownet is described next.

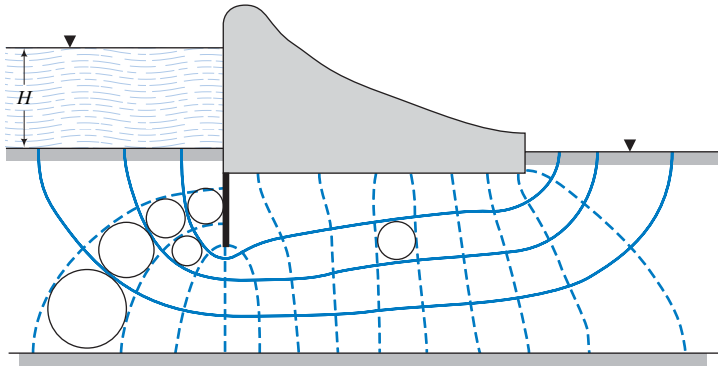


FIGURE 11.4 Flownet under a dam with a cutoff curtain (sheet pile) on the upstream end.

11.4.3 Anisotropic Soil

Equation (11.1) is Laplace's two-dimensional equation for anisotropic soils (the hydraulic conductivities in the X and Z directions are different). Let us manipulate this equation to transform it into a form for which we can use a procedure similar to the one for isotropic soils to draw and interpret flownets. Put $C = \sqrt{k_z/k_x}$ and $x_1 = Cx$. Then

$$\frac{\partial x_1}{\partial x} = C$$

$$\frac{\partial H}{\partial x} = \frac{\partial H}{\partial x_1} \frac{\partial x_1}{\partial x} = C \frac{\partial H}{\partial x_1}$$

and

$$\frac{\partial^2 H}{\partial x^2} = C^2 \frac{\partial^2 H}{\partial x_1^2}$$

Therefore, we can write Eq. (11.1) as

$$C^2 \frac{\partial^2 H}{\partial x_1^2} + C^2 \frac{\partial^2 H}{\partial z^2} = 0$$

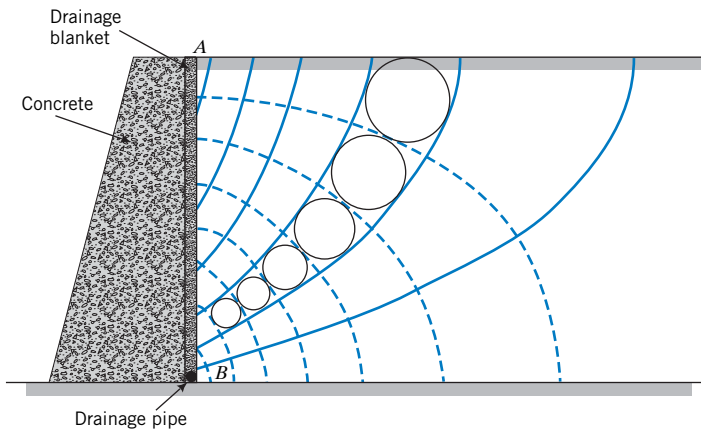


FIGURE 11.5 Flownet in the backfill of a retaining wall with a vertical drainage blanket.

which simplifies to

$$\frac{\partial^2 H}{\partial x_1^2} + \frac{\partial^2 H}{\partial z^2} = 0 \quad (11.8)$$

Equation (11.8) indicates that for anisotropic soils we can use the procedure for flownet sketching described for isotropic soils by scaling the x distance by $\sqrt{k_z/k_x}$. That is, you must draw the structure and flow domain by multiplying the horizontal distances by $\sqrt{k_z/k_x}$.

11.5 INTERPRETATION OF FLOWNET



Concept Learning and Self-Assessment

Access Chapter 11, Section 11.5 on the CD to review how to interpret flownets and take Quiz 11.5 to assess your understanding.

11.5.1 Flow Rate

Let the total head loss across the flow domain be ΔH , that is, the difference between upstream and downstream water level elevation. Then the head loss (Δh) between each consecutive pair of equipotential lines is

$$\Delta h = \frac{\Delta H}{N_d} \quad (11.9)$$

where N_d is the number of equipotential drops, that is, the number of equipotential lines minus one. In Fig. 11.3, $\Delta H = 8$ m and $N_d = 18$. From Darcy's law, the flow through each flow channel for an isotropic soil is

$$\Delta q = AKi = (b \times 1)k \frac{\Delta h}{L} = k \Delta h \frac{b}{L} = k \frac{\Delta H}{N_d} \frac{b}{L} \quad (11.10)$$

where b and L are defined as shown in Fig. 11.3. By construction, $b/L = 1$, and therefore the total flow is

$$q = k \sum_{i=1}^{N_f} \left(\frac{\Delta H}{N_d} \right)_i = k \Delta H \frac{N_f}{N_d} \quad (11.11)$$

where N_f is the number of flow channels (number of flow lines minus one); in Fig. 11.3, $N_f = 9$. The ratio N_f/N_d is called the shape factor. Finer discretization of the flownet by drawing more flow lines and equipotential lines does not significantly change the shape factor. Both N_f and N_d can be fractional. In the case of anisotropic soils, the quantity of flow is

$$q = \Delta H \frac{N_f}{N_d} \sqrt{k_x k_z} \quad (11.12)$$

11.5.2 Hydraulic Gradient

You can find the hydraulic gradient over each square by dividing the head loss by the length, L , of the cell; that is,

$$i = \frac{\Delta h}{L} \quad (11.13)$$

You should notice from Fig. 11.3 that L is not constant. Therefore, the hydraulic gradient is not constant. The maximum hydraulic gradient occurs where L is a minimum; that is,

$$i_{\max} = \frac{\Delta h}{L_{\min}} \quad (11.14)$$

where $L_{\min}$ is the minimum length of the cells within the flow domain. Usually, $L_{\min}$ occurs at exit points or around corners (e.g., point A in Fig. 11.3), and it is at these points that we can get the maximum hydraulic gradient.

11.5.3 Static Liquefaction, Heaving, Boiling, and Piping

Let us consider an element, B , of soil at a depth z near the downstream end of the sheet pile wall structure shown in Fig. 11.3. Flow over this element is upward. Therefore, the vertical effective stress is

$$\sigma'_z = \gamma'z - i\gamma_w z \quad (11.15)$$

If the effective stress becomes zero, the soil loses its intergranular frictional strength and behaves like a viscous fluid. The soil state at which the effective stress is zero is called static liquefaction. Various other names such as boiling, quicksand, piping, and heaving are used to describe specific events connected to the static liquefaction state. Boiling occurs when the upward seepage force exceeds the downward force of the soil. Piping refers to the subsurface “pipe-shaped” erosion that initiates near the toe of dams and similar structures. High localized hydraulic gradient statically liquefies the soil, which progresses to the water surface in the form of a pipe, and water then rushes beneath the structure through the pipe, leading to instability and failure. Quicksand is the existence of a mass of sand in a state of static liquefaction. Heaving occurs when seepage forces push the bottom of an excavation upward. A structure founded on a soil that statically liquefies will collapse. Liquefaction can also be produced by dynamic events such as earthquakes.

11.5.4 Critical Hydraulic Gradient

We can determine the hydraulic gradient that brings a soil mass (essentially, coarse-grained soils) to static liquefaction. Solving for i in Eq. (11.15) when $\sigma'_z = 0$, we get

$$i = i_{\text{cr}} = \frac{\gamma'}{\gamma_w} = \left(\frac{G_s - 1}{1 + e} \right) \frac{\gamma_w}{\gamma_w} = \frac{G_s - 1}{1 + e} \quad (11.16)$$

where i_{cr} is called the critical hydraulic gradient, G_s is specific gravity, and e is the void ratio. Since G_s is constant, the critical hydraulic gradient is solely a function of the void ratio of the soil. In designing structures that are subjected to steady state seepage, it is absolutely essential to ensure that the critical hydraulic gradient cannot develop.

11.5.5 Porewater Pressure Distribution

The porewater pressure at any point j is calculated as follows:

1. Select a datum. Let us choose the downstream water level as the datum (Fig. 11.3).
2. Determine the total head at j : $H_j = \Delta H - (N_d)_j \Delta h$, where $(N_d)_j$ is the number of equipotential drops at point j ; $(N_d)_j$ can be fractional. For example, at B , $H_B = \Delta H - 16.5 \Delta h$.
3. Subtract the elevation head at point j from the total head H_j to get the pressure head. For point B (Fig. 11.3), the elevation head h_z is $-z$ (point B is below the datum). The pressure head is then

$$(h_p)_j = \Delta H - (N_d)_j \Delta h - h_z \quad (11.17)$$

For point B , $(h_p)_B = \Delta H - 16.5 \Delta h - (-z) = \Delta H - 16.5 \Delta h + z$.

4. The porewater pressure is

$$u_j = (h_p)_j \gamma_w \quad (11.18)$$

11.5.6 Uplift Forces

Lateral and uplift forces due to groundwater flow can adversely affect the stability of structures such as dams and weirs. The uplift force per unit length (length is normal to the XZ plane) is found by calculating the porewater pressure at discrete points along the base (in the X direction, Fig. 11.4) and then finding the area under the porewater pressure distribution diagram, that is,

$$P_w = \sum_{j=1}^n u_j \Delta x_j \quad (11.19)$$

where P_w is the uplift force per unit length, u_j is the average porewater pressure over an interval Δx_j , and n is the number of intervals. It is convenient to use Simpson's rule to calculate P_w :

$$P_w = \frac{\Delta x}{3} \left(u_1 + u_n + 2 \sum_{\substack{i=3 \\ \text{odd}}}^n u_i + 4 \sum_{\substack{i=2 \\ \text{even}}}^n u_i \right) \quad (11.20)$$

EXAMPLE 11.1 Critical Hydraulic Gradient in an Excavation

An excavation is proposed for a site consisting of a homogeneous, isotropic layer of silty clay, 12.24 m thick, above a deep deposit of sand. The groundwater is 2 m below ground level (see Fig. E11.1). The void ratio of the silty clay is 0.62 and its specific gravity is 2.7. What is the limiting depth of the excavation to avoid heaving?

Strategy Heaving will occur if $i > i_{cr}$. Find the critical hydraulic gradient from the void ratio given and then find the depth at which this hydraulic gradient is reached.

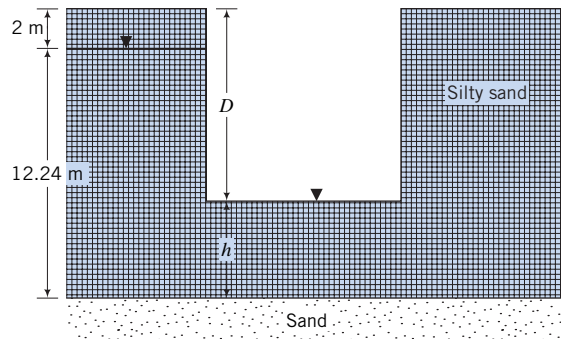


FIGURE E11.1

Solution 11.1

Step 1: Calculate i_{cr} .

$$i_{cr} = \frac{G_s - 1}{1 + e} = \frac{2.7 - 1}{1 + 0.62} = 1.05$$

Step 2: Determine D .

$$\text{Total head difference : } \Delta H = H - h = 12.24 - h$$

$$\text{Average hydraulic gradient : } i = \frac{\Delta H}{h}$$

Putting $i = i_{cr}$, we get

$$1.05 = \frac{12.24 - h}{h}$$

Solving for h , we get $h = 5.97$ m.

$$D = (12.24 + 2) - h = 14.24 - 5.97 = 8.27 \text{ m}$$

EXAMPLE 11.2 Flownet Sketching and Interpretation for a Dam**Computer Program Utility**

Access Chapter 11 on the CD, click Spreadsheet on the sidebar menu to view the solution of this problem using a spreadsheet. You can change input values and study how the flownet solution changes. You can also use the “2D Flow” program on the sidebar.

A dam, shown in Fig. E11.2a, retains 10 m of water. A sheet pile wall (cutoff curtain) on the upstream side, which is used to reduce seepage under the dam, penetrates 7 m into a 20.3 m thick silty sand stratum. Below the silty sand is a thick deposit of clay. The average hydraulic conductivity of the silty sand is 2.0×10^{-4} cm/s. Assume that the silty sand is homogeneous and isotropic.

- Draw the flownet under the dam.
- Calculate the flow, q .
- Calculate and draw the porewater pressure distribution at the base of the dam.
- Determine the uplift force.
- Determine and draw the porewater pressure distribution on the upstream and downstream faces of the sheet pile wall.

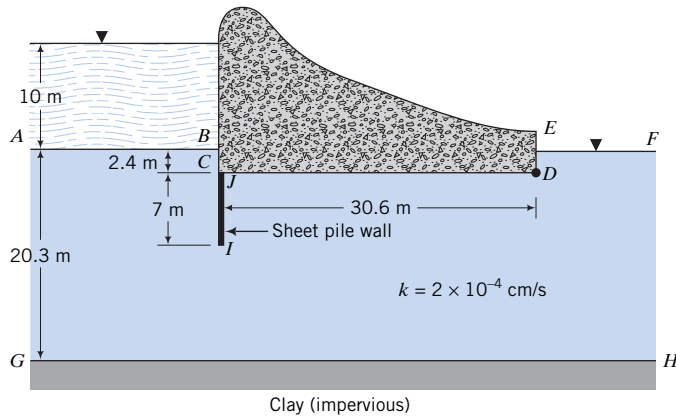


FIGURE E11.2a

- (f) Determine the resultant lateral force on the sheet pile wall due to the porewater.
- (g) Determine the maximum hydraulic gradient.
- (h) Will piping occur if the void ratio of the silty sand is 0.8?
- (i) What is the effect of reducing the depth of penetration of the sheet pile wall?

Strategy Follow the procedures described in Section 11.4 to draw the flownet and calculate the required parameters.

Solution 11.2

Step 1: Draw the dam to scale.

See Fig. E11.2b.

Step 2: Identify the impermeable and permeable boundaries.

With reference to Fig. E11.2a, AB and EF are permeable boundaries and are therefore equipotential lines. $BCIJDE$ and GH are impermeable boundaries and are therefore flow lines.

Step 3: Sketch the flownet.

Draw about three flow lines and then draw a suitable number of equipotential lines. Remember that flow lines are orthogonal to equipotential lines and the area between two consecutive flow lines and two consecutive equipotential lines is approximately a square. Use a circle template to assist you in estimating the square. Adjust/add/subtract flow lines and equipotential lines until you are satisfied that the flownet consists essentially of curvilinear squares. See sketch of flownet in Fig. E11.2b.

Step 4: Calculate the flow.

Select the downstream end, EF , as the datum.

$$\Delta H = 10 \text{ m}$$

$$N_d = 14, \quad N_f = 4$$

$$\text{Equation (11.11): } q = k \Delta H \frac{N_f}{N_d} = 2 \times 10^{-4} \times (10 \times 10^2) \times \frac{4}{14} = 0.057 \text{ cm}^3/\text{s}$$

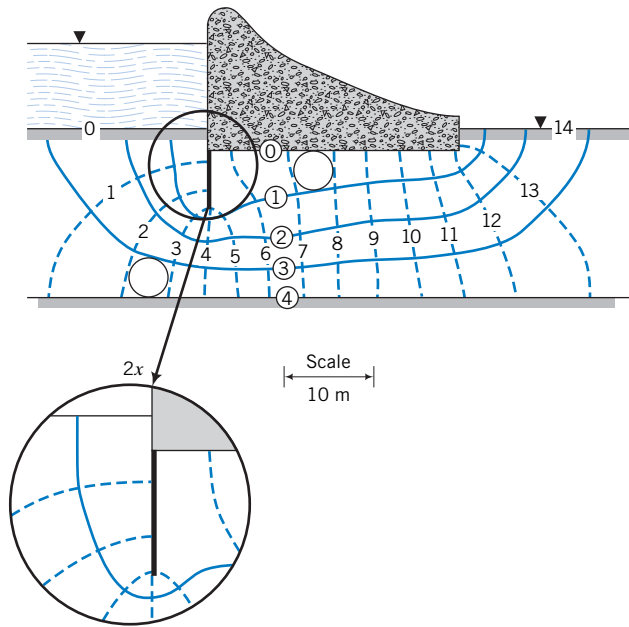


FIGURE E11.2b

Step 5: Determine the porewater pressure under the base of the dam.

Divide the base into a convenient number of equal intervals. Let us use 10 intervals; that is

$$\Delta x = \frac{30.6}{10} = 3.06 \text{ m}$$

Determine the porewater pressure at each nodal point. Use a table for convenience or, better yet, use a spreadsheet.

$$\Delta h = \frac{\Delta H}{N_d} = \frac{10}{14} = 0.714 \text{ m}$$

The calculation in the table below was done using a spreadsheet program.

Flow under a dam : $\Delta h = 0.714 \text{ m}$

Parameters	Under base of dam										
$x \text{ (m)}$	0	3.06	6.12	9.18	12.24	15.3	18.36	21.42	24.48	27.54	30.6
$N_d \text{ (m)}$	5.60	5.80	6.20	6.90	7.40	8.00	8.80	9.40	10.30	11.10	12.50
$N_d \Delta h \text{ (m)}$	4.00	4.14	4.43	4.93	5.28	5.71	6.28	6.71	7.35	7.93	8.93
$h_z \text{ (m)}$	-2.40	-2.40	-2.40	-2.40	-2.40	-2.40	-2.40	-2.40	-2.40	-2.40	-2.40
$h_p \text{ (m)} = \Delta H - N_d \Delta h - h_z$	8.40	8.26	7.97	7.47	7.12	6.69	6.12	5.69	5.05	4.47	3.48
$u \text{ (kPa)} = h_p \gamma_w$	82.3	80.9	78.1	73.2	69.7	65.5	59.9	55.7	49.4	43.9	34.1

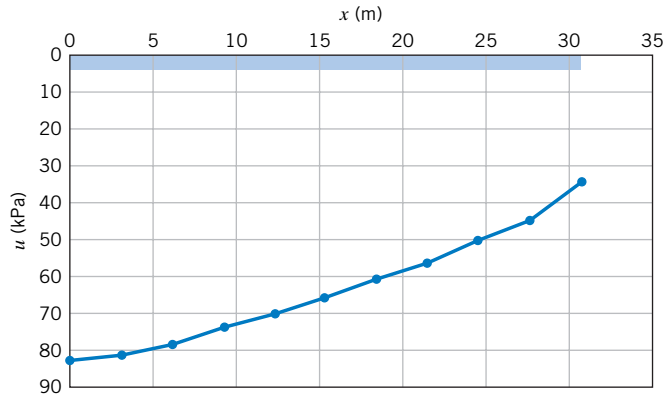


FIGURE E11.2c

Plot the porewater pressure distribution. See Fig. E11.2c.

Step 6: Calculate the uplift force and its location.

Using Simpson's rule [Eq. (11.20)], we find

$$P_w = \frac{3.06}{3} [82.3 + 34.1 + 2(78.1 + 69.7 + 59.9 + 49.4) + 4(80.9 + 73.2 + 65.5 + 55.7 + 43.9)]$$

$$= 1946.4 \text{ kN/m}$$

Step 7: Determine the porewater pressure distribution on the sheet pile wall. Divide the front face of the wall into six intervals of $7/6 = 1.17 \text{ m}$ and the back face into one interval. Six intervals were chosen because it is convenient for the scaling using the scale that was used to draw the flownet. The greater the intervals, the greater the accuracy. Only one interval is used for the back face of the wall because there are no equipotential lines that meet there. Use a spreadsheet to compute the porewater pressure distribution and the hydrostatic forces. The distributions of porewater pressure at the front and back of the wall are shown in Figs. E11.2d,e. Use Simpson's rule to calculate the hydrostatic force on the front face of the wall. The porewater pressure distribution at the back face is a trapezoid and the area is readily calculated.

Parameters	Front of wall							Back of wall	
$x \text{ (m)}$	0	1.17	2.33	3.50	4.67	5.83	7.00	7.00	0.00
$N_d \text{ (m)}$	0.70	1.00	1.30	1.60	1.90	2.40	3.00	5.00	5.60
$N_d \Delta h \text{ (m)}$	0.50	0.71	0.93	1.14	1.36	1.71	2.14	3.57	4.00
$h_z \text{ (m)}$	-2.40	-3.57	-4.73	-5.90	-7.07	-8.23	-9.40	-9.40	-2.40
$h_p \text{ (m)} = \Delta H - N_d \Delta h - h_z$	11.90	12.85	13.81	14.76	15.71	16.52	17.26	15.83	8.40
$u \text{ (kPa)} = h_p \gamma_w$	116.6	126.0	135.3	144.6	154.0	161.9	169.1	155.1	82.3
	Front	Back	Difference						
$P_w \text{ (kN/m)}$	1011.7	830.9	180.8						

Step 8: Determine the maximum hydraulic gradient.

The smallest value of L occurs at the exit. By measurement, $L_{\min} = 2 \text{ m}$.

$$i_{\max} = \frac{\Delta h}{L_{\min}} = \frac{0.714}{2} = 0.36$$

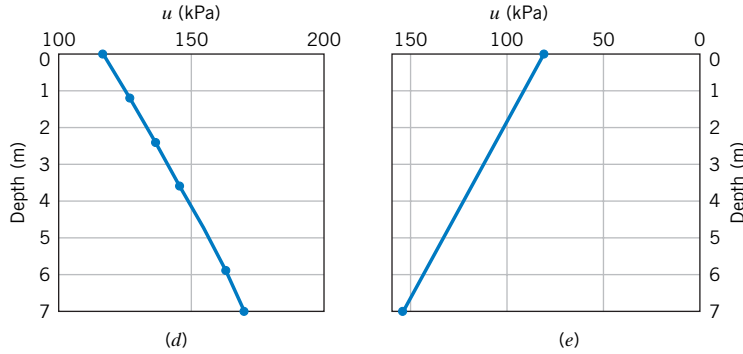


FIGURE E11.2d-e Porewater pressure distribution (d) at front of wall and (e) at back of wall.

Step 9: Determine if piping would occur.

$$\text{Equation (11.16): } i_{cr} = \frac{G_s - 1}{1 + e} = \frac{2.7 - 1}{1 + 0.8} = 0.89$$

Since $i_{max} < i_{cr}$, piping will not occur.

$$\text{Factor of safety against piping: } \frac{0.89}{0.36} = 2.5$$

Step 10: State the effect of reducing the depth of penetration of the sheet pile wall.

If the depth is reduced, the value of Δh increases and i_{max} is likely to increase. ■

11.6 FINITE DIFFERENCE SOLUTION FOR TWO-DIMENSIONAL FLOW



Computer Program Utility

Access Chapter 11 and click on Spreadsheet on the sidebar to run a finite difference program to solve the two-dimensional flow equation. The spreadsheet is based on Example 11.3, but you can modify a copy of the worksheet to solve or to explore the solutions of other problems.

In Chapter 6, we used the finite difference technique to solve the governing one-dimensional partial differential equation to determine the spatial variation of excess porewater pressure. We will do the same to solve Laplace's equation to determine two-dimensional confined flow through soils. Let us consider a grid of a flow domain as shown in Fig. 11.6, where (i, j) is a nodal point.

Using Taylor's theorem, we have

$$k_x \frac{\partial^2 H}{\partial x^2} + k_z \frac{\partial^2 H}{\partial z^2} = \frac{k_x}{\Delta x^2} (h_{i+1,j} + h_{i-1,j} - 2h_{i,j}) + \frac{k_z}{\Delta z^2} (h_{i,j+1} + h_{i,j-1} - 2h_{i,j}) = 0 \quad (11.21)$$

Let $\alpha = k_x/k_z$ and $\Delta x = \Delta z$ (i.e., we subdivide the flow domain into a square grid). Then solving for $h_{i,j}$ from Eq. (11.21) gives

$$h_{i,j} = \frac{1}{2(1 + \alpha)} (\alpha h_{i+1,j} + \alpha h_{i-1,j} + h_{i,j+1} + h_{i,j-1}) \quad (11.22)$$

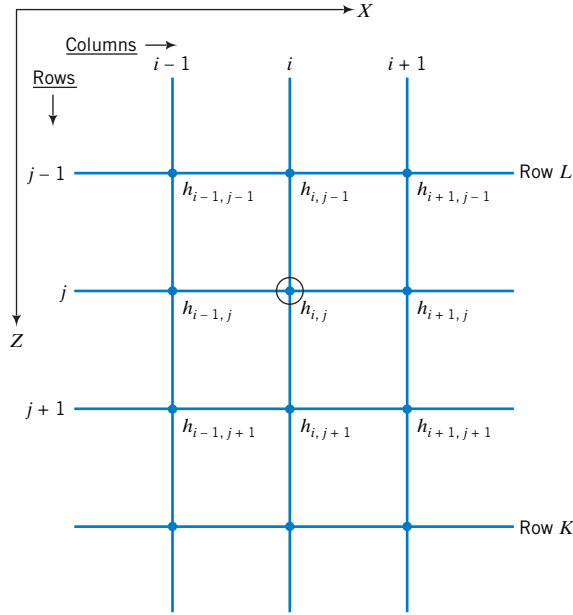


FIGURE 11.6 A partial grid of the flow domain.

For isotropic conditions, $\alpha = 1$ ($k_x = k_z$) and Eq. (11.22) becomes

$$h_{i,j} = \frac{1}{4}(h_{i+1,j} + h_{i-1,j} + h_{i,j+1} + h_{i,j-1}) \quad (11.23)$$

Since we are considering confined flow, one or more of the boundaries would be impermeable. Flow cannot cross impermeable boundaries and, therefore, for a horizontal impermeable surface,

$$\frac{\partial h}{\partial x} = 0 \quad (11.24)$$

The finite difference form of Eq. (11.24) is

$$\frac{\partial h}{\partial x} = \frac{1}{2\Delta x}(h_{i,j+1} - h_{i,j-1}) = 0 \quad (11.25)$$

Therefore, $h_{i,j+1} = h_{i,j-1}$ and, by substitution in Eq. (11.22), we get

$$h_{i,j} = \frac{1}{4}(h_{i+1,j} + h_{i-1,j} + 2h_{i,j-1}) \quad (11.26)$$

Various types of geometry of impermeable boundaries are encountered in practice, three of which are shown in Fig. 11.7. For Figs. 11.7a,b the finite difference equation is

$$h_{i,j} = \frac{1}{2}(h_{i+1,j} + h_{i,j-1}) \quad (11.27)$$

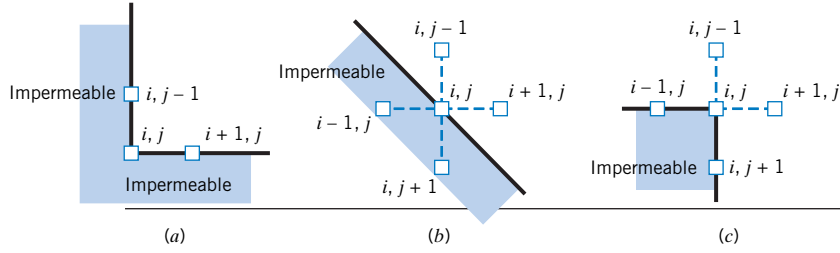


FIGURE 11.7 Three types of boundary encountered in practice.

and, for Fig. 11.7c,

$$h_{i,j} = \frac{1}{3}(h_{i,j-1} + h_{i+1,j} + h_{i,j+1}) + \frac{1}{2}h_{i-1,j} + \frac{1}{2}h_{i,j+1} \quad (11.28)$$

The porewater pressure at any node ($u_{i,j}$) is

$$u_{i,j} = \gamma_w(h_{i,j} - z_{i,j}) \quad (11.29)$$

where $z_{i,j}$ is the elevation head.

Contours of potential heads can be drawn from the discrete values of $h_{i,j}$. The finite difference equations for flow lines are analogous to the potential lines; that is, ψ_s replaces h in the above equations and the boundary conditions for ψ_s are specified rather than for h .

The horizontal velocity of flow at any node ($v_{i,j}$) is given by Darcy's law:

$$v_{i,j} = k_x i_{i,j}$$

where $i_{i,j}$ is the hydraulic gradient expressed as

$$i_{i,j} = \frac{\frac{1}{2}(h_{i+1,j} - h_{i-1,j})}{\Delta x} \quad (11.30)$$

Therefore,

$$v_{i,j} = \frac{k_x}{2\Delta x}(h_{i+1,j} - h_{i-1,j}) \quad (11.31)$$

The flow rate, q , is obtained by considering a vertical plane across the flow domain. Let L be the top row and K be the bottom row of a vertical plane defined by column i (Fig. 11.6). Then the expression for q is

$$q = \frac{k_x}{4} \left(h_{i+1,L} - h_{i-1,L} + 2 \sum_{j=L+1}^{K-1} (h_{i+1,j} - h_{i-1,j}) + h_{i+1,K} - h_{i-1,K} \right) \quad (11.32)$$

The procedure to determine the distribution of potential head, flow, and porewater pressure using the finite difference method is as follows:

1. Divide the flow domain into a square grid. Remember from Chapter 6 that finer grids give more accurate solutions than coarser grids, but are more tedious to construct and require more computational time. If the problem is symmetrical, you only need to consider one-half of the flow domain. For example, the sheet pile wall shown in Fig. 11.8 is symmetrical about the wall and only the left half may be considered. The total flow domain should have a width of at least four times the thickness of the soil layer. For example, if D is the thickness of the soil layer (Fig. 11.8), then the minimum width of the left half of the flow domain is $2D$.

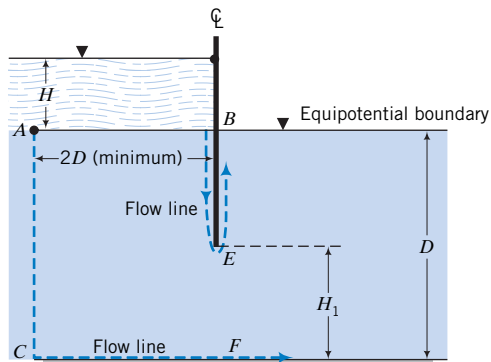


FIGURE 11.8 A sheet pile wall.

2. Identify boundary conditions, for example, impermeable boundaries (flow lines) and permeable boundaries (equipotential lines).
3. Determine the heads at the permeable or equipotential boundaries. For example, the heads along the equipotential boundary AB (Fig. 11.8) is ΔH . Therefore, all the nodes along this boundary will have a constant head of ΔH . Because of symmetry, the head along nodes directly under the sheet pile wall (EF) is $\Delta H/2$.
4. Apply the known heads to corresponding nodes and assume reasonable initial values for the interior nodes. You can use linear interpolation for the potential heads of the interior nodes.
5. Apply Eq. (11.23), if the soil is isotropic, to each node except (a) at impermeable boundaries, where you should use Eq. (11.26), (b) at corners, where you should use Eqs. (11.27) and (11.28) for the corners shown in Figs. 11.8a–c, and (c) at nodes where the heads are known.
6. Repeat item 5 until the new value at a node differs from the old value by a small numerical tolerance, for example, 0.001 m.
7. Arbitrarily select a sequential set of nodes along a column of nodes and calculate the flow, q , using Eq. (11.32). It is best to calculate $q' = q$ for a unit permeability value to avoid too many decimal points in the calculations.
8. Repeat items 1 to 6, to find the flow distribution by replacing heads by flow q' . For example, the flow rate calculated in item 7 is applied to all nodes along AC and CF (Fig. 11.8). The flow rate at nodes along BE is zero.
9. Calculate the porewater pressure distribution by using Eq. (11.29).

A spreadsheet program (see Spreadsheet on sidebar of Chapter 11 on the CD) can be prepared to automatically carry out the above procedure. However, you should carry out “hand” calculations at selected nodes to verify that the spreadsheet values are correct. The computer program “2D FLOW” on the sidebar is a finite difference program that you can use to draw flownets and explore the effects of some parameters such as sheet pile depth.

EXAMPLE 11.3 *Flownet Using Finite Difference Method*

Computer Program Utility

Access Chapter 11 and click on Spreadsheet on the sidebar to run a finite difference program to solve the two-dimensional flow equation. The spreadsheet is based on Example 11.3, but you can modify a copy of the worksheet to solve or to explore the solutions of other problems. You can also use the Flow Net program on the sidebar.

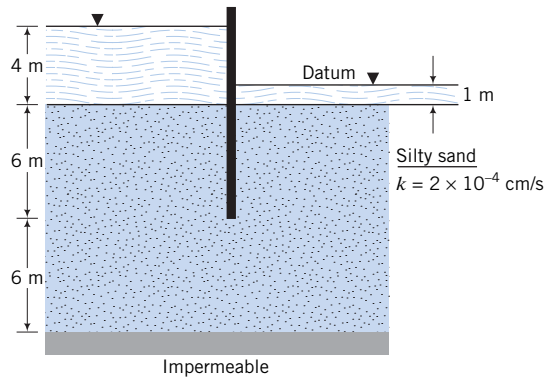


FIGURE E11.3a

Determine the flow under the sheet pile wall (Fig. E11.3a) and the porewater pressure distribution using the finite difference method.

Strategy Use a spreadsheet and follow the procedures in Section 11.6.

Solution 11.3

The finite difference solution is shown in Table E11.3 and in Fig. 11.3c.

Step 1: Divide the flow domain into a grid.

See Fig. E11.3b.

Since the problem is symmetrical, perform calculations for only one-half of the domain. Use the left half of the flow domain of width $2D = 2 \times 12 = 24$ m. Use a grid $2\text{ m} \times 2\text{ m}$.

Step 2: Identify the boundary conditions.

Permeable boundaries: AB and CD are equipotential lines.

Impermeable boundaries: BC , AE , and DE , are flow lines.

Step 3: Determine the heads at equipotential boundaries.

Along AB , the head difference is $4 - 1 = 3$ m.

Along CD , the head difference is $3/2 = 1.5$ m.

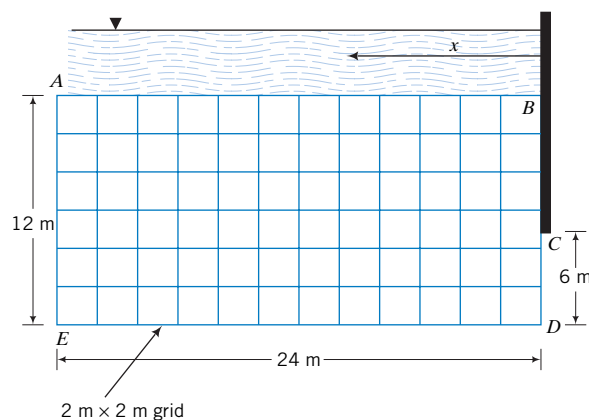


FIGURE E11.3b

TABLE E11.3[illegible]

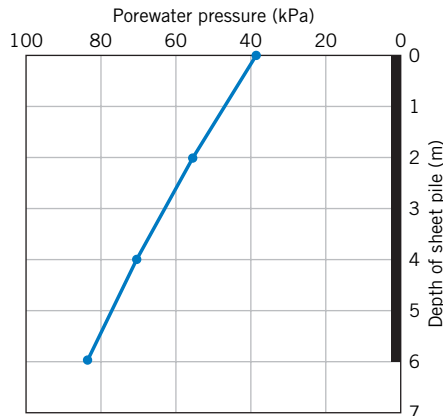


FIGURE E11.3c Porewater pressure distribution.

Step 4: Insert the heads at the nodes.

Set up the initial parameters in column *B*, rows 3 to 9. Note that cell *B5* is cell *B3* – cell *B2*. In cells *B12* to *N12* (corresponding to the equipotential boundary *AB*), copy cell *B5*. In cells *N15* to *N18*, insert cell *B5*/2. Arbitrarily insert values in all other cells from *B13* to *M18*, *N13*, and *N14*.

Step 5: Apply the appropriate equations.

On the impermeable boundaries—*B13* to *B18* (corresponding to *AE*), *C18* to *C22* (corresponding to *ED*), and *N13* to *N14* (corresponding to *BC*)—apply Eq. (11.26). You should note that some nodes (e.g., *B18*) are common. Apply Eq. (11.22) to all other cells except cells with known heads.

Step 6: Carry out the iterations.

In Excel, go to Tools → Options → Calculation. Select the following:

- (i) Automatic.
- (ii) Iteration, insert 100 in Maximum Iterations and 0.001 in Maximum change.
- (iii) Under Workbook Options, select Update remote reference, save external link values, accept labels in formulas. You can then click on Calculate Now (F9) or Calc.sheet.

Step 7: Calculate q .

Use Eq. (11.32) to calculate q' for a unit value of permeability. In the spreadsheet for this example, q' is calculated in cell *C20* as

$$\{(BL3 - BL5) + NL3 - NL5 + 2\} * (SUM(C13 : M13) - 2 * SUM(C15 : M15)) / 4$$

The actual value of q is

$$\begin{aligned} q &= kq' = 1.62 \times 2 \times 10^{-4} \times 10^{-2} \\ &= 3.2 \times 10^{-6} \text{ m/s} \quad (10^{-2} \text{ is used to convert cm/s to m/s}) \end{aligned}$$

Step 8: Calculate the flow for each cell (flow lines).

In cells *B36* to *B42* (corresponding to *AE*) and *C42* to *N32* (corresponding to *ED*), copy q' . The flow at the downstream end (cells *N36* to *N39*) is zero. Apply Eq. (11.26) to cells *C36* to *M41* and *N40* to *N41*. Apply Eq. (11.23) to all other cells except the cell with known values of q' . Carry out the reiterations.

Step 9: Calculate the porewater pressures.

From the potential heads, you can calculate the porewater pressure using Eq. (11.29). A plot of the porewater pressure distribution is shown in Fig. E11.3c. ■

11.7 FLOW THROUGH EARTH DAMS



Computer Program Utility

Access Chapter 11 on the CD and click Phreatic on the sidebar to run an interactive program to calculate and plot the phreatic surface for flow through an earth dam.

Flow through earth dams is an important design consideration. We need to ensure that the porewater pressure at the downstream end of the dam will not lead to instability and the exit hydraulic gradient does not lead to piping. The major exercise is to find the top flow line called the phreatic surface (Fig. 11.9). The pressure head on the phreatic surface is zero.

Casagrande (1937) showed that the phreatic surface can be approximated by a parabola with corrections at the points of entry and exit. The focus of the parabola is at the toe of the dam, point F (Fig. 11.9). The assumed parabola representing the uncorrected phreatic surface is called the basic parabola. Recall from your geometry course that the basic property of a parabola is that every point on it is equidistant from its focus and a line called the directrix. To draw the basic parabola, we must know point A , the focus F , and f (one-half the distance from the focus to the directrix). Casagrande (1937) recommended that point C be located at a distance $0.3AB$, where AB is the horizontal projection of the upstream slope at the water surface. From the basic property of a parabola, we get

$$2f = \sqrt{b^2 + H^2} - b \quad (11.33)$$

The equation to construct the basic parabola is

$$\sqrt{x^2 + z^2} = x + 2f$$

Solving for z , we obtain

$$z^2 = 4f(f + x) \quad (11.34)$$

or

$$z = 2\sqrt{f(f + x)} \quad (11.35)$$

Since H and b are known from the geometry of the dam, the basic parabola can be constructed. We now have to make some corrections at the upstream entry point and the downstream exit point.

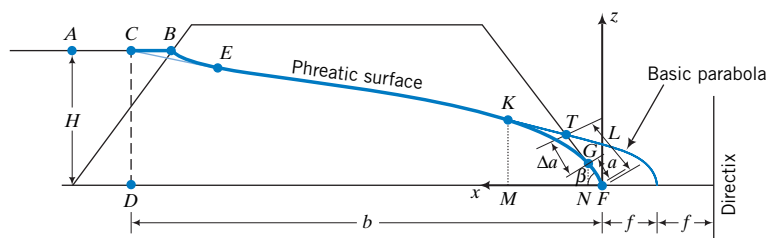


FIGURE 11.9 Phreatic surface within an earth dam.

The upstream end is corrected by sketching a transition curve (BE) that blends smoothly with the basic parabola. The correction for the downstream end depends on the angle β and the type of discharge face. Casagrande (1937) determined the length of the discharge face, a , for a homogeneous earth dam with no drainage blanket at the discharge face and $\beta \leq 30^\circ$ as follows. He assumed that Dupuit's assumption, which states that the hydraulic gradient is equal to the slope, dz/dx , of the phreatic surface, is valid. If we consider two vertical sections—one is KM of height z , and the other is GN of height $a \sin \beta$ —then the flow rate across KM is

$$q_{KM} = Aki = (z \times 1)k \frac{dz}{dx} \quad (11.36)$$

and across GN is

$$q_{GN} = Aki = (a \sin \beta \times 1)k \frac{dz}{dx} = (a \sin \beta)k \tan \beta \quad (11.37)$$

From the continuity condition at sections KM and GN , $q_{KM} = q_{GN}$; that is,

$$zk \frac{dz}{dx} = (a \sin \beta)k \tan \beta \quad (11.38)$$

which simplifies to

$$z \frac{dz}{dx} = a \sin \beta \tan \beta \quad (11.39)$$

We now integrate Eq. (11.39) within the limits $x_1 = a \cos \beta$ and $x_2 = b$, $z_1 = a \sin \beta$ and $z_2 = H$.

$$\int_{a \sin \beta}^H z dz = a \sin \beta \tan \beta \int_{a \cos \beta}^b dx$$

$$\therefore H^2 - a^2 \sin^2 \beta = a \sin \beta \tan \beta (b - a \cos \beta)$$

Simplification leads to

$$a = \frac{b}{\cos \beta} - \sqrt{\frac{b^2}{\cos^2 \beta} - \frac{H^2}{\sin^2 \beta}} = \frac{b}{\cos \beta} - \cos \beta \sqrt{b^2 - H^2 \cot^2 \beta} \quad (11.40)$$

Casagrande (1937) produced a chart relating $\Delta a/L$ (see Fig. 11.9 for definitions of Δa and L) with different values of β , as shown in Fig. 11.10.

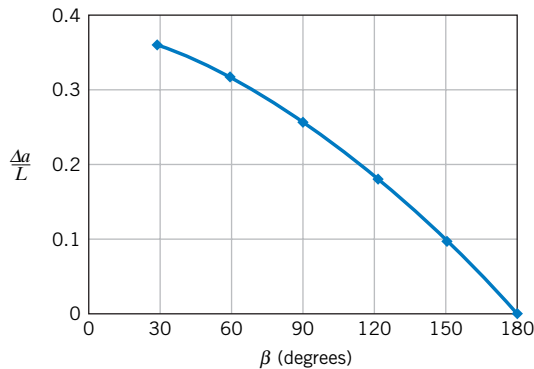


FIGURE 11.10 Correction factor for downstream face.

The flow through the dam is obtained by substituting Eq. (11.40) into Eq. (11.37), giving

$$\begin{aligned} q &= k \sin \beta \tan \beta \left(\frac{b}{\cos \beta} - \cos \beta \sqrt{b^2 - H^2 \cot^2 \beta} \right) \\ &= k \sin^2 \beta \left(\frac{b}{\cos^2 \beta} - \sqrt{b^2 - H^2 \cot^2 \beta} \right) \end{aligned} \quad (11.41)$$

Because the exit hydraulic gradient is often large, drainage blankets are used at the downstream end of dams to avoid piping. Figure 11.11 shows a horizontal drainage blanket at the toe of an earth dam. Seepage is controlled by the gradation of the coarse-grained soils used for the drainage blanket. The phreatic surface for dams with drainage blankets is forced to intersect the drainage blanket and does not intersect the downstream face of the dam. Therefore, no correction to the basic parabola is required on the downstream end of the dam.

The flow through the dam is

$$q = Aki = Ak \frac{dz}{dx}$$

where dz/dx is the slope of the basic parabola and the area $A = FJ \times 1$ (Fig. 11.11). From the geometry of the basic parabola, $FJ = 2f$ and the slope of the basic parabola at J is, from Eq. (11.34).

$$\frac{dz}{dx} = \frac{2f}{z} = \frac{2f}{2f} = 1$$

Therefore, the flow through a dam with a horizontal drainage blanket is

$$q = 2f \times k \times 1 = 2fk \quad (11.42)$$

The procedure to draw a phreatic surface within an earth dam, with reference to Fig. 11.9, is as follows:

1. Draw the structure to scale.
2. Locate a point A at the intersection of a vertical line from the bottom of the upstream face and the water surface, and a point B where the waterline intersects the upstream face.

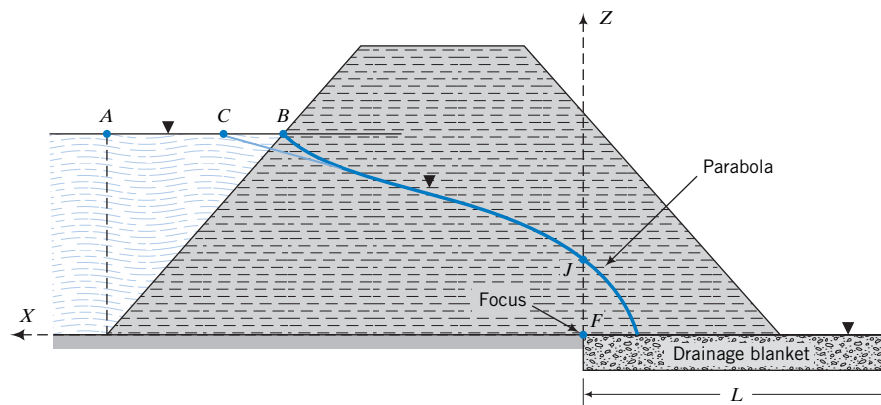


FIGURE 11.11 A horizontal drainage blanket at the toe of an earth dam.

3. Locate point C , such that $BC = 0.3AB$.
4. Project a vertical line from C to intersect the base of the dam at D .
5. Locate the focus of the basic parabola. The focus is located conveniently at the toe of the dam.
6. Calculate the focal distance, $f = (\sqrt{b^2 + H^2} - b)/2$, where b is the distance FD and H is the height of water on the upstream face.
7. Construct the basic parabola from $z = 2\sqrt{f(f+x)}$.
8. Sketch in a transition section BE .
9. Calculate the length of the discharge face, a , using

$$a = \frac{b}{\cos \beta} - \cos \beta \sqrt{b^2 - H^2 \cot^2 \beta}; \quad \beta \leq 30^\circ.$$

For $\beta > 30^\circ$, use Fig. 11.10 and (a) measure the distance TF , where T is the intersection of the basic parabola with the downstream face; (b) for the known angle β , read the corresponding factor $\Delta a/L$ from the chart; and (c) find the distance $a = TF(1 - \Delta a/L)$.

10. Measure the distance a from the toe of the dam along the downstream face to point G .
11. Sketch in a transition section, GK .
12. Calculate the flow using $q = ak \sin \beta \tan \beta$, where k is the hydraulic conductivity. If the downstream slope has a horizontal drainage blanket as shown in Fig. 11.11, the flow is calculated using $q = 2fk$.

What's next ... Coarse-grained soils are used as filters or drainage blankets to control seepage. Next, we present some simple guidelines for selecting soils for filters and the applications of geotextiles for soil filtration.

11.8 SOIL FILTRATION

The diameter D_{10} (Chapter 3) is called the effective size of the soil and was established by Allen Hazen (1893) in connection with his work on soil filters. The higher the D_{10} value, the coarser the soil and the better the drainage characteristics. The diameter of the finer particle sizes, in particular D_{15} , has been used to develop criteria for soil filters. Terzaghi and Peck (1948), for example, proposed the following set of criteria for an effective soil filter.

$$\frac{D_{15(F)}}{D_{15(BS)}} < 4 \quad (\text{to prevent the filter soil from being washed out}) \quad (11.43)$$

and

$$\frac{D_{15(F)}}{D_{15(BS)}} > 4 \quad (\text{to ensure a high rate of flow of water}) \quad (11.44)$$

where F denotes filter and BS is the base soil.

Most filters are multilayers of sand and gravel. These filters are difficult to construct and their efficiency is easily compromised by even a small amount of fines. Geotextiles—a permeable, polymeric material—have replaced soil filters. Geotextiles allow the passage of water but not fines. Consequently, the geotextile must be properly sized to prevent clogging. Construction is simplified and cost-effective compared with soil filters. Two examples of the applications of geotextiles in soil filtration are shown in Fig. 11.12.

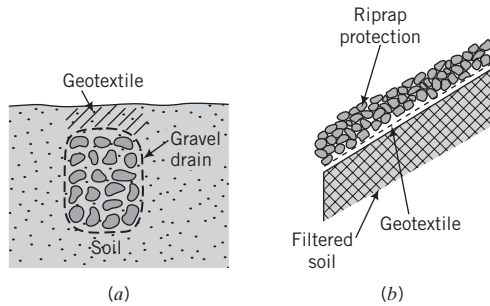


FIGURE 11.12 (a) A geotextile-wrapped gravel drain; (b) Geotextile used is filtration of riprap slope.

11.9 SUMMARY

The flow of water through soils is a very important consideration in the analysis, design, and construction of many civil engineering systems. The governing equation for flow of water through soils is Laplace's equation. In this chapter, we examined two types of solution for Laplace's equation for two-dimensional flow. One is a graphical technique, called flownet sketching, which consists of a network of flow and equipotential lines. The network (flownet) consists of curvilinear squares in which flow lines and equipotential lines are orthogonal to each other. From the flownet, we can calculate the flow rate, the distribution of heads, porewater pressures, seepage forces, and the maximum hydraulic gradient. The other type of solution is based on the finite difference method and requires, in most cases, the use of a spreadsheet or a computer program. Instability of structures embedded in soils could occur if the maximum hydraulic gradient exceeds the critical hydraulic gradient.

Practical Example

EXAMPLE 11.4 Determining Flow and Piping for an Excavation

A bridge pier is to be constructed in a riverbed by constructing a cofferdam as shown in Fig. E11.4a. A cofferdam is a temporary enclosure consisting of long, slender elements of steel, concrete, or timber members to support the sides of the enclosure. After constructing the cofferdam, the water within it will be pumped out. Determine (a) the flow rate using $k = 1 \times 10^{-4}$ cm/s and (b) the factor of safety against piping. The void ratio of the sand is 0.59.

There was a long delay before construction began and a 100 mm layer of silty clay with $k = 1 \times 10^{-6}$ cm was deposited at the site. What effect would this silty clay layer have on the factor of safety against piping?

Strategy The key is to draw a flownet and determine whether the maximum hydraulic gradient is less than the critical hydraulic gradient. The presence of the silty clay would result in significant head loss within it, and consequently the factor of safety against piping is likely to increase. Since the cofferdam is symmetrical about a vertical plane, you only need to draw the flownet for one-half of the cofferdam.

Solution 11.4

Step 1: Draw the cofferdam to scale and sketch the flownet.

See Fig. E11.4b.

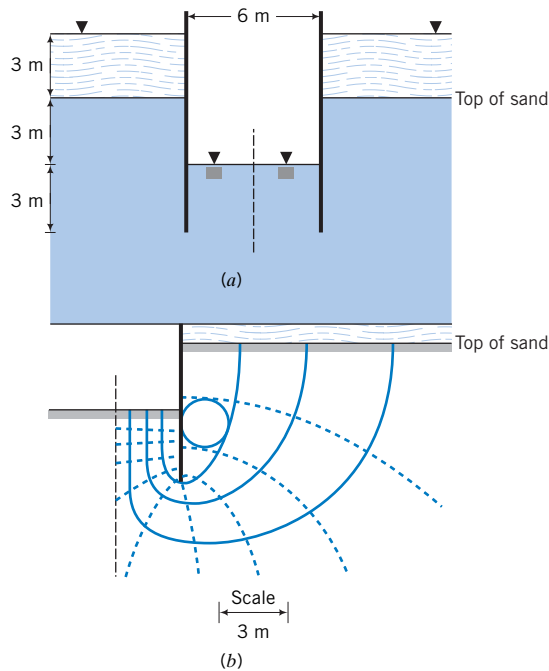


FIGURE E11.4a-b

Step 2: Determine the flow.

$$\Delta h = 6 \text{ m}; \quad N_f = 4, \quad N_d = 10$$

$$q = 2k\Delta H \frac{N_f}{N_d} = 2 \times 1 \times 10^{-4} \times 10^{-2} \times 6 \times \frac{4}{10} = 4.8 \times 10^{-6} \text{ m}^2/\text{s}$$

(Note: The factor 2 is needed because you have to consider both halves of the structure; the factor 10^{-2} is used to convert cm/s to m/s.)

Step 3: Determine the maximum hydraulic gradient.

$$L_{\min} \approx 0.3 \text{ m} \quad (\text{this is an average value of the flow length at the exit of the sheet pile})$$

$$i_{\max} = \frac{\Delta h}{L_{\min}} = \frac{\Delta H}{N_d L_{\min}} = \frac{6}{10 \times 0.3} = 2$$

Step 4: Calculate the critical hydraulic gradient.

$$i_{\text{cr}} = \frac{G_s - 1}{1 + e} = \frac{2.7 - 1}{1 + 0.59} = 1.07$$

Since $i_{\max} > i_{\text{cr}}$, piping is likely to occur; the factor of safety is $1.07/2 \approx 0.5$.

Step 5: Determine the effects of the silty clay layer.

Consider the one-dimensional flow in the flow domain, as shown in Fig. E11.4c. The head loss through 9 m of sand (6 m outside and 3 m inside of excavation) is 6 m in the absence of the silt layer. Let us find the new head loss in the sand due to the presence of the silt layer.

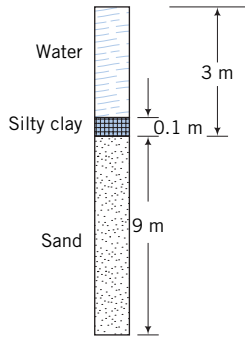


FIGURE E11.4c

From Darcy's law and the continuity condition: $k_{\text{sand}} i_{\text{sand}} = k_{\text{silt}} i_{\text{silt}}$

$$\text{or } k_{\text{sand}} \frac{\Delta h_{\text{sand}}}{L_{\text{sand}}} = k_{\text{silt}} \frac{\Delta h_{\text{silt}}}{L_{\text{silt}}}$$

Therefore,

$$\Delta h_{\text{sand}} = \frac{1 \times 10^{-6} \times 9 \Delta h_{\text{silt}}}{1 \times 10^{-4} \times 0.1} = 0.9 \Delta h_{\text{silt}}$$

But $\Delta h_{\text{sand}} + \Delta h_{\text{silt}} = 6 \text{ m}$; that is, $0.9 \Delta h_{\text{silt}} + \Delta h_{\text{silt}} = 6 \text{ m}$ and $\Delta h_{\text{silt}} = 3.16 \text{ m}$. Therefore,

$$\Delta h_{\text{sand}} = 6 - 3.16 = 2.84 \text{ m}$$

Thus, the maximum hydraulic gradient would be reduced. ■

EXERCISES

Theory

11.1 Derive a relationship between the critical hydraulic gradient, i_{cr} , and porosity, n .

Problem Solving

11.2 A sheet pile wall supporting 6 m of water is shown in Fig. P11.2.

- Draw the flownet.
- Determine the flow rate if $k = 0.0019 \text{ cm/s}$.
- Determine the porewater pressure distributions on the upstream and downstream faces of the wall.
- Would piping occur if $e = 0.55$?

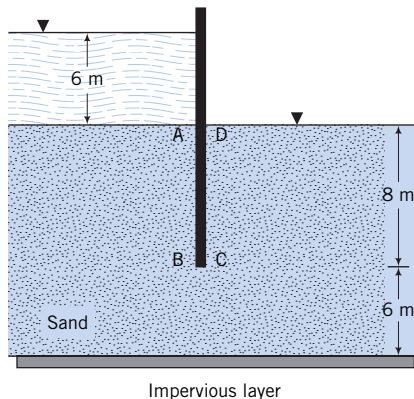


FIGURE P11.2

- 11.3** Repeat Exercise 11.2 using the finite difference method.
- 11.4** The sheet pile wall supporting 6 m of water has a clay (almost impervious) blanket of 3 m on the downstream side, as shown in Fig. P11.4.
- Draw the flownet.
 - Determine the flow rate if $k = 0.0019 \text{ cm/s}$.
 - Determine the porewater pressure distributions on the upstream and downstream faces of the wall.
 - Would piping occur if $e = 0.55$?

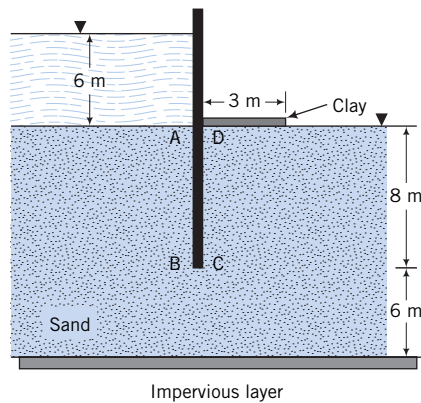


FIGURE P11.4

- 11.5** A section of a dam constructed from a clay is shown in Fig. P11.5. The dam is supported on 10 m of sandy clay with $k_x = 0.000012 \text{ cm/s}$ and $k_z = 0.00002 \text{ cm/s}$. Below the sandy clay is a thick layer of impervious clay.
- Draw the flownet under the dam.
 - Determine the porewater pressure distribution at the base of the dam.
 - Calculate the resultant uplift force and its location from the upstream face of the dam.

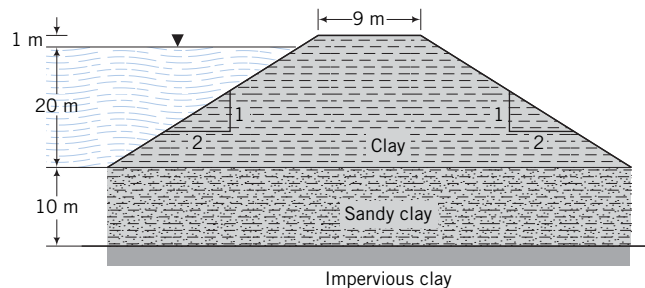


FIGURE P11.5

- 11.6** Draw the phreatic surface for the earth dam shown in Fig. P11.5. Determine the flow rate within the dam, if $k = k_x = 1 \times 10^{-6} \text{ cm/s}$.
- 11.7** Draw the phreatic surfaces and determine the flow rates with and without the drainage blanket for the dam shown in Fig. P11.7. The hydraulic conductivity of the clay is $k = k_x = 0.0000012 \text{ cm/s}$.

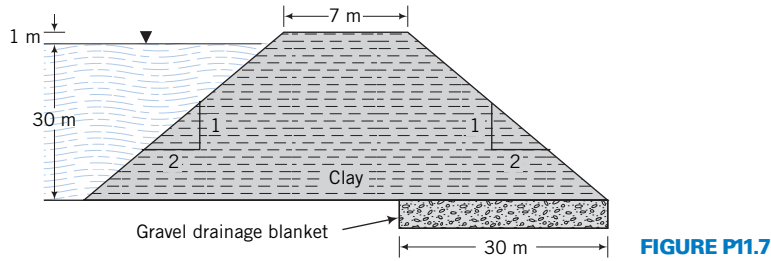


FIGURE P11.7

Practical

- 11.8** Borings at a site for a road pavement show a water-bearing stratum of coarse-grained soil below an 8 m thick deposit of a mixture of sand, silt, and clay. The water-bearing stratum below the mixture of sand, silt, and clay will create an artesian condition. The excess head is 1 m. One preliminary design proposal is to insert two trenches, as shown in Fig. P11.8, to reduce the excess head. Draw the flownet and determine the flow rate into each of the trenches, if $k = k_z = 1 \times 10^{-5}$ cm/s.

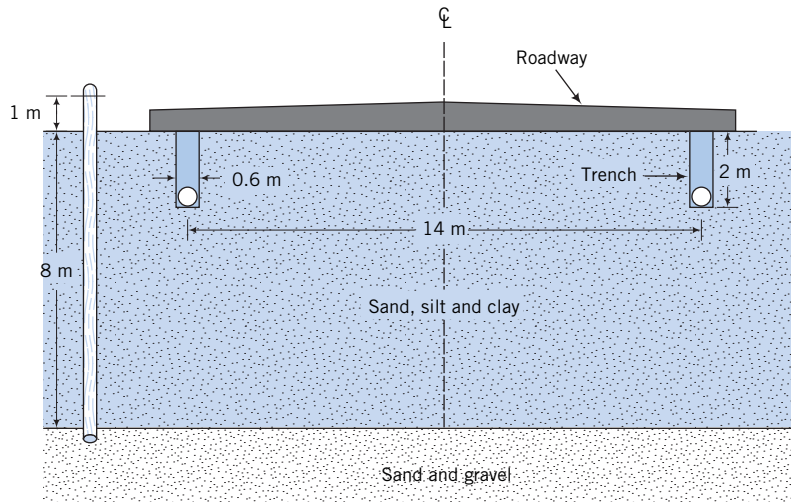


FIGURE P11.8

- 11.9** A retaining wall has a vertical drainage blanket (Fig. P11.9). After a heavy rainfall, a steady state seepage condition occurs. Draw the flownet and determine the porewater pressure distribution acting on a potential failure plane AB . The hydraulic conductivity is 1.8×10^{-4} cm/s.

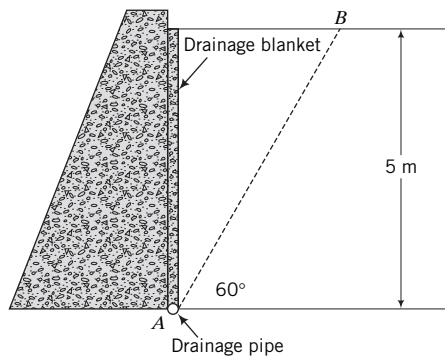


FIGURE P11.9

CHAPTER 12

STABILITY OF EARTH RETAINING STRUCTURES

ABET	ES	ED
	90	10

12.0 INTRODUCTION

Earth retaining structures are ubiquitous in the man-made environment. These structures have the distinction of being the first to be analyzed using mechanics (remember we mentioned Coulomb's analysis of the lateral earth pressure on the fortresses protected by soil in Chapter 1). In this chapter, we will analyze some typical earth retaining structures to determine their stability. The emphases will be on gaining an understanding of the forces that provoke failures and methods of analysis of simple earth retaining structures.

You should recall that stability refers to a condition in which a geotechnical system will not fail or collapse under any conceivable loading (static and dynamic loads, fluid pressure, seepage forces). Stability is synonymous with ultimate limit state but serviceability limit state is also important. In many circumstances, the serviceability limit state is the deciding design limit state. The methods of analyses that we are going to discuss do not consider the serviceability limit state. The analyses involved to determine the serviceability limit state are beyond the scope of this book.

When you complete this chapter, you should be able to:

- Understand and determine lateral earth pressures.
- Understand the forces that lead to instability of earth retaining structures.
- Determine the stability of simple earth retaining structures.

You will use the following principles learned from previous chapters and your courses in mechanics:

- Static equilibrium
- Effective stresses and seepage (Chapter 5)
- Mohr's circle (Chapter 5)
- Shear strength (Chapter 7)
- Two-dimensional flow of water through soils (Chapter 11)

Sample Practical Situation A retaining wall is required around a lake. You, a geotechnical engineer, are required to design the retaining wall. An example of a retaining structure in a waterfront area is shown in Fig. 12.1

12.1 DEFINITIONS OF KEY TERMS

Active earth pressure coefficient (K_a) is the ratio between the lateral and vertical principal effective stresses when an earth retaining structure moves away (by a small amount) from the retained soil.



FIGURE 12.1 A flexible retaining wall under construction.

Passive earth pressure coefficient (K_p) is the ratio between the lateral and vertical principal effective stresses when an earth retaining structure is forced against a soil mass.

Gravity retaining wall is a massive concrete wall relying on its mass to resist the lateral forces from the retained soil mass.

Flexible retaining wall or a sheet pile wall is a long slender wall relying on passive resistance and anchors or props for its stability.

Mechanical stabilized earth is a gravity type retaining wall in which the soil is reinforced by thin reinforcing elements (steel, fabric, fibers, etc.).

12.2 QUESTIONS TO GUIDE YOUR READING

1. What is meant by the stability of earth retaining structures?
2. What are the factors that lead to instability?
3. What are the main assumptions in the theory of lateral earth pressures?
4. When shall I use either Rankine's theory or Coulomb's theory?
5. Does Coulomb's theory give an upper bound or a lower bound solution?
6. What is the effect of wall friction on the shape of slip planes?
7. What are the differences among gravity wall, a cantilever wall, a cantilever sheet pile wall, and an anchored sheet pile wall?
8. How do I analyze a retaining wall to check that it is stable?
9. How deep can I make a vertical cut without wall supports?
10. What are mechanically stabilized earth walls?

12.3 BASIC CONCEPTS ON LATERAL EARTH PRESSURES



Interactive Concept Learning and Self-Assessment

Access Chapter 12, Section 12.3 on the CD to learn the basic concepts on lateral earth pressure through interactive visualizations. Take Quiz 12.3 to assess your understanding.

We have discussed the lateral earth pressure at rest and the lateral increases in stresses on a semi-infinite, isotropic, homogeneous, elastic soil mass from surface loading in Chapter 5. We are now going to consider the lateral earth pressures on a vertical wall that retains a soil mass. We will deal with two theories: one proposed by Coulomb (1776) and the other by Rankine (1857). First, we will develop a basic understanding of lateral earth pressures using a generic ϕ' and make the following assumptions:

1. The earth retaining wall is vertical.
2. The interface between the wall and soil is frictionless.
3. The soil surface is horizontal and no shear stress acts on horizontal and vertical boundaries.
4. The wall is rigid and extends to an infinite depth in a dry, homogeneous, isotropic soil mass.
5. The soil is loose and initially in an at-rest state.

Consider the wall as shown in Fig. 12.2. If the wall remains rigid and no movement (not even an infinitesimal movement) occurs, then the vertical and horizontal effective stresses at rest on elements A, at the back wall, and B, at the front wall (Fig. 12.2), are

$$\begin{aligned}\sigma'_z &= \sigma'_1 = \gamma'z \\ \sigma'_x &= \sigma'_3 = K_o \sigma'_1 = K_o \gamma'z\end{aligned}$$

where K_o is the lateral earth pressure at rest [Eq. (5.50)]. Mohr's circle for the at-rest state is shown by circle ① in Fig. 12.3.

Let us now assume a rotation about the bottom of the wall sufficient to produce slip planes in the soil mass behind and in front of the wall (Fig. 12.4). The rotation required, and consequently the lateral strains, to produce slip planes in front of the wall is much larger than that required for the back of the wall, as shown in Fig. 12.5. The soil mass at the back of the wall is assisting in producing failure while the

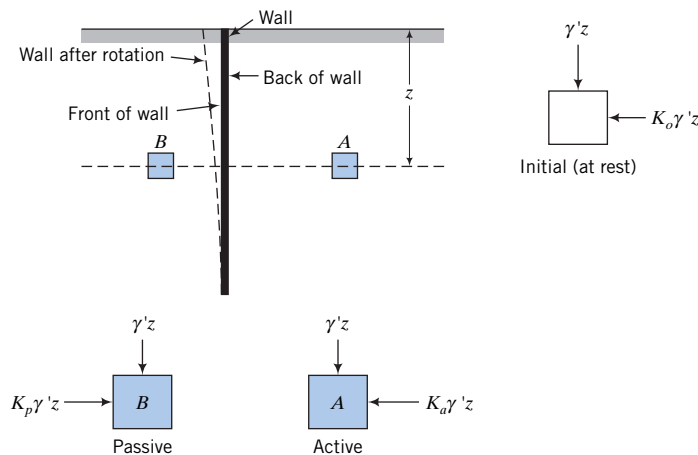


FIGURE 12.2 Stresses on soil elements in front and behind a retaining wall.

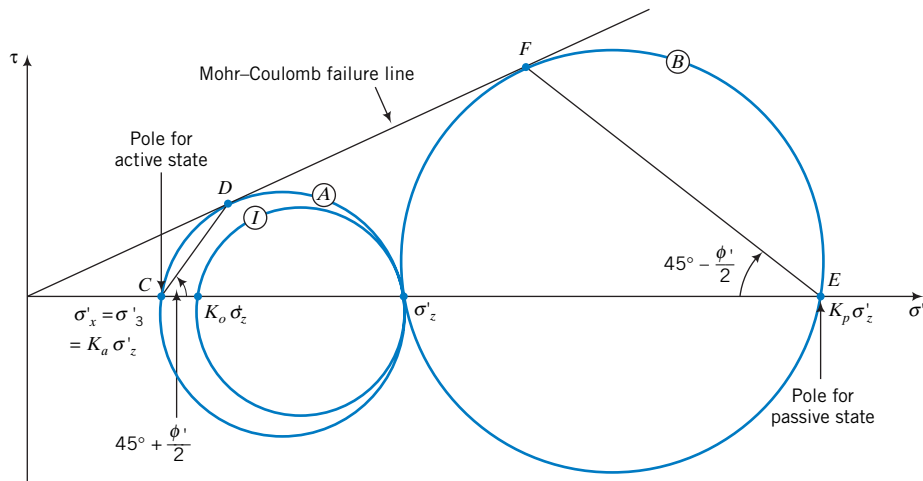


FIGURE 12.3 Mohr's circles at rest, active and passive states.

soil mass at the front of the wall is resisting failure. In the latter, you have to rotate the wall against the soil to produce failure.

What happens to the lateral effective stresses on elements A and B (Fig. 12.2) when the wall is rotated? The vertical stress will not change on either element but the lateral effective stress on element A will be reduced while that for element B will be increased. We can now plot two additional Mohr's circles: one to represent the stress state of element A (circle (A), Fig. 12.3) and the other to represent the stress state of element B (circle (B), Fig. 12.3). Both circles are drawn such that the decrease (element A) or increase (element B) in lateral effective stress is sufficient to bring the soil to the Mohr-Coulomb failure state. For element B to reach the failure state, the lateral effective stress must be greater than the vertical effective stress, as shown in Fig. 12.3.

The ratio of lateral principal effective stress to vertical principal effective stress is given by Eq. (7.26), which, for circle (A), is

$$\frac{(\sigma'_3)_f}{(\sigma'_1)_f} = \frac{(\sigma'_x)_f}{(\sigma'_z)_f} = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \tan^2 \left(45^\circ - \frac{\phi'}{2} \right) = K_a \quad (12.1)$$

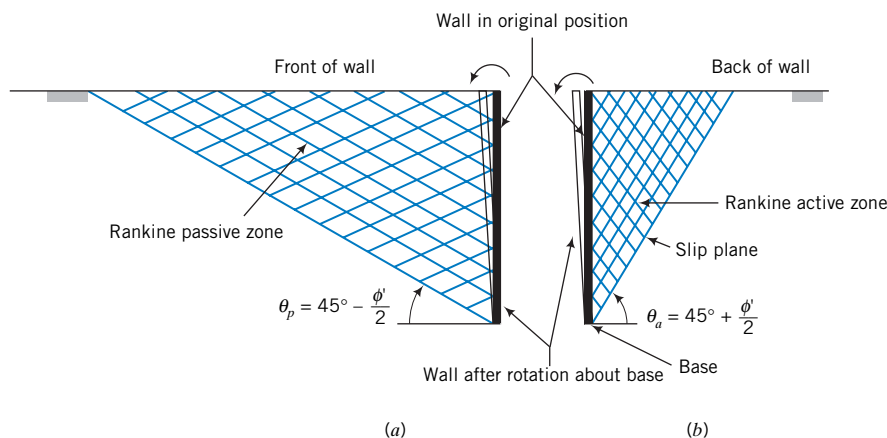


FIGURE 12.4 Slip planes within a soil mass near a retaining wall.

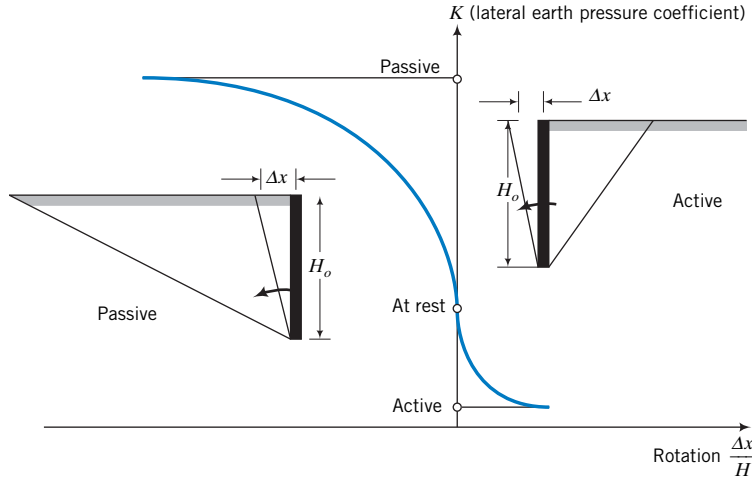


FIGURE 12.5 Rotation required to mobilize active and passive resistance.

where K_a is called the active lateral earth pressure coefficient. Similarly, for circle ③,

$$\frac{(\sigma'_1)_f}{(\sigma'_3)_f} = \frac{(\sigma'_x)_f}{(\sigma'_z)_f} = \frac{1 + \sin \phi'}{1 - \sin \phi'} = \tan^2 \left(45^\circ + \frac{\phi'}{2} \right) = K_p \quad (12.2)$$

where K_p is the passive earth pressure coefficient. Therefore,

$$K_p = \frac{1}{K_a}$$

If, for example, $\phi' = 30^\circ$, then $K_a = \frac{1}{3}$ and $K_p = 3$.

The stress states of soil elements A and B are called the Rankine active state and the Rankine passive state, respectively (named after the original developer of this theory, Rankine, 1857). Each of these Rankine states is associated with a family of slip planes. For the Rankine active state, the slip planes are oriented at

$$\theta_a = 45^\circ + \frac{\phi'}{2} \quad (12.3)$$

to the horizontal, as illustrated in Fig. 12.4b and proved in Chapter 7. For the Rankine passive state, the slip planes are oriented at

$$\theta_p = 45^\circ - \frac{\phi'}{2} \quad (12.4)$$

to the horizontal, as illustrated in Fig. 12.4a.

The lateral earth pressure for the Rankine active state is

$$(\sigma'_x)_a = K_a \sigma'_z = K_a \gamma' z \quad (12.5)$$

and for the Rankine passive state it is

$$(\sigma'_x)_p = K_p \sigma'_z = K_p \gamma' z \quad (12.6)$$

Equations (12.5) and (12.6) indicate that, for a homogeneous soil layer, the lateral earth pressure varies linearly with depth, as shown in Figs. 12.6a,b. The lateral active and passive coefficients are applied

only to effective stresses. For soils above the ground water level $\gamma' = \gamma$ while for soils below the groundwater level $\gamma' = (\gamma_{\text{sat}} - \gamma_w)$.

The lateral earth force is the area of the lateral stress diagram, which, for the Rankine active state, is

$$P_a = \int_0^{H_o} K_a \gamma' z = \frac{1}{2} K_a \gamma' H_o^2 \quad (12.7)$$

and, for the Rankine passive state, is

$$P_p = \int_0^{H_o} K_p \gamma' z = \frac{1}{2} K_p \gamma' H_o^2 \quad (12.8)$$

These lateral forces, P_a and P_p , are located at the centroid of the lateral earth pressure distribution diagram. In this case, the centroid is at $H_o/3$ from the base.

If groundwater is present, you need to add the hydrostatic pressure (porewater pressure) to the lateral earth pressure. For example, if the groundwater level is at distance h_w from the base of the wall (Fig. 12.6c), the hydrostatic pressure is

$$u = \gamma_w h_w \quad (12.9)$$

and the hydrostatic force is

$$P_w = \frac{1}{2} \gamma_w h_w^2 \quad (12.10)$$

Surface stresses also impose lateral earth pressures on retaining walls. A uniform surface stress, q_s , will transmit a uniform active lateral earth pressure of $K_a q_s$ (Fig. 12.6d) and a uniform passive lateral earth pressure of $K_p q_s$. The active and passive lateral earth pressures due to the soil, groundwater, and the uniform surface stresses are then

$$(\sigma_x)_a = K_a \sigma'_z + K_a q_s + (u)_a \quad (12.11)$$

and

$$(\sigma_x)_p = K_p \sigma'_z + K_p q_s + (u)_p \quad (12.12)$$

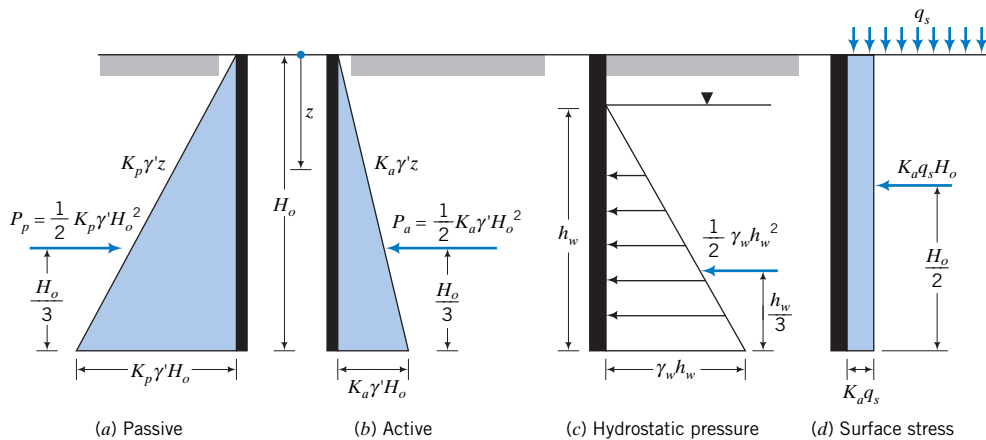


FIGURE 12.6 Variation of active and passive lateral earth pressures, hydrostatic pressure, and a uniform surface stress with depth.

where u is the hydrostatic pressure and the subscripts a and p denote active and passive states, respectively.

The lateral forces from a line load and a strip load are given in Chapter 5. For other types of surface loads you can consult Poulos and Davis (1974).

The essential points are:

1. *The lateral earth pressures on retaining walls are related directly to the vertical effective stress through two coefficients. One is the active earth pressure coefficient,*

$$K_a = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \tan^2 \left(45^\circ - \frac{\phi'}{2} \right)$$

and the other is the passive earth pressure coefficient,

$$K_p = \frac{1 + \sin \phi'}{1 - \sin \phi'} = \tan^2 \left(45^\circ + \frac{\phi'}{2} \right) = \frac{1}{K_a}$$

2. *Substantially more movement is required to mobilize the full passive earth pressure than the full active earth pressure.*
3. *A family of slip planes occurs in the Rankine active and passive states. In the active state, the slip planes are oriented at $45^\circ + \phi'/2$ to the horizontal, while for the passive case they are oriented at $45^\circ - \phi'/2$ to the horizontal.*
4. *The lateral earth pressure coefficients, developed so far, are only valid for a smooth, vertical wall supporting a homogeneous soil mass with a horizontal surface.*
5. *The lateral earth pressure coefficients must be applied only to effective stresses.*

EXAMPLE 12.1 Lateral Earth Pressure and Force

Determine the active lateral earth pressure on the frictionless wall shown in Fig. E12.1a. Calculate the resultant force and its location from the base of the wall. Neglect seepage effects.

Strategy The lateral earth pressure coefficients can only be applied to the effective stresses. You need to calculate the vertical effective stress, apply K_a , and then add the porewater pressure.

Solution 12.1

Step 1: Calculate K_a .

$$K_a = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \frac{1 - \sin(30^\circ)}{1 + \sin(30^\circ)} = \frac{1}{3}$$

$$\text{or } \tan^2 \left(45^\circ - \frac{\phi'}{2} \right) = \tan^2 \left(45^\circ - \frac{30^\circ}{2} \right) = \frac{1}{3}$$

Step 2: Calculate the vertical effective stress.

$$\text{At the surface : } \sigma'_z = 0, \quad u = 0$$

$$\text{At the base : } \sigma'_z = \gamma' H_o = (20 - 9.8) \times 5 = 51 \text{ kPa}$$

$$u = \gamma_w H_o = 9.8 \times 5 = 49 \text{ kPa}$$

Step 3: Calculate the lateral effective stress.

$$(\sigma'_x)_a = K_a \sigma'_z = \frac{1}{3} \times 51 = 17 \text{ kPa}$$

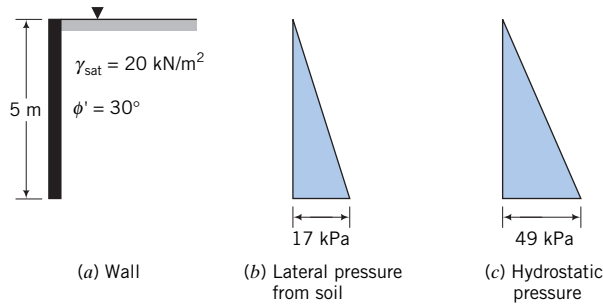


FIGURE E12.1

Step 4: Sketch the lateral earth pressure distributions.

See Figs. E12.1b, c.

Step 5: Calculate the lateral force.

$$P_a = P_s + P_w$$

where P_s is the lateral force due to the soil solids and P_w is the lateral force due to the porewater.

$$P_a = \frac{1}{2}(\sigma'_x)_a H_o + \frac{1}{2}uH_o = \left(\frac{1}{2} \times 17 \times 5\right) + \left(\frac{1}{2} \times 49 \times 5\right) = 165 \text{ kN}$$

Step 6: Determine the location of the resultant.

Since both the lateral earth pressure and the porewater pressure distributions are triangular over the whole depth, the resultant is at the centroid of the triangle, that is, $\bar{z} = H_o/3 = 5/3 = 1.67 \text{ m}$ from the base of the wall. ■

EXAMPLE 12.2 Lateral Earth Pressure in Layered Soils

For the frictionless wall shown Fig. E12.2a, determine the following:

- The active lateral earth pressure distribution with depth.
- The passive lateral earth pressure distribution with depth.
- The magnitudes and locations of the active and passive forces.
- The resultant force and its location.
- The ratio of passive moment to active moment.

Strategy There are two layers. It is best to treat each layer separately. Neither K_a nor K_p should be applied to the porewater pressure. You do not need to calculate K_p for the top soil layer. Since the water level on both sides of the wall is the same, the resultant hydrostatic force is zero. However, you are asked to determine the forces on each side of the wall; therefore, you have to consider the hydrostatic force. A table is helpful to solve this type of problem.

Solution 12.2

Step 1: Calculate the active lateral earth pressure coefficients.

$$\text{Top layer (0–2 m): } K_a = \tan^2\left(45^\circ - \frac{\phi'}{2}\right) = \tan^2\left(45^\circ - \frac{25^\circ}{2}\right) = 0.41$$

$$\text{Bottom layer (2–6 m): } K_a = \tan^2\left(45^\circ - \frac{\phi'}{2}\right) = \tan^2\left(45^\circ - \frac{30^\circ}{2}\right) = \frac{1}{3}; \quad K_p = \frac{1}{K_a} = 3$$

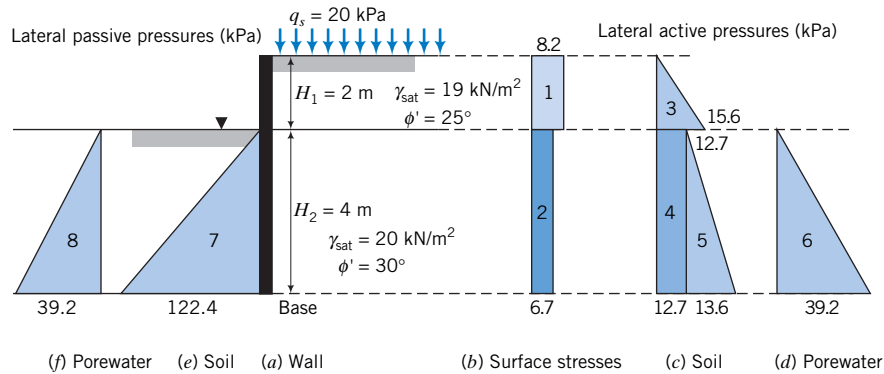


FIGURE E12.2

Step 2: Calculate the active and passive lateral earth pressures.

Use a table as shown below to do the calculations or use a spreadsheet.

Active	Depth ^a (m)	u (kPa)	σ_z (kPa)	$\sigma'_z = \sigma_z - u$	$(\sigma'_x)_a = K_a \sigma'_z$ (kPa)
Surcharge	0	0	20	20	$0.41 \times 20 = 8.2$
	2–6	0	20	20	$\frac{1}{3} \times 20 = 6.7$
Soil	0	0	0	0	0
	2 ⁻	0	$\gamma_1 H_1 = 19 \times 2 = 38$	38	$0.41 \times 38 = 15.6$
	2 ⁺	0	$\gamma_1 H_1 = 19 \times 2 = 38$	38	$\frac{1}{3} \times 38 = 12.7$
	6	$\gamma_w H_2 = 9.8 \times 4 = 39.2$	$\gamma_1 H_1 + \gamma_2 H_2 = 19 \times 2 + 20 \times 4 = 118$	78.8	$\frac{1}{3} \times 78.8 = 26.3$
Passive	Depth (m)	u (kPa)	σ_z (kPa)	$\sigma'_z = \sigma_z - u$	$(\sigma'_x)_p = K_p \sigma'_z$
Soil	0	0	0	0	0
	4	$\gamma_w H_2 = 9.8 \times 4 = 39.2$	$\gamma_2 H_2 = 20 \times 4 = 80$	40.8	$3 \times 40.8 = 122.4$

^aThe – and + superscripts indicate that you are calculating the stress just above (–) and just below (+) 2 m.

See Figs. E12.2b–e for the pressure distributions.

Step 3: Calculate the hydrostatic force.

$$P_w = \frac{1}{2} \gamma_w H_2^2 = \frac{1}{2} \times 9.8 \times 4^2 = 78.4 \text{ kN}$$

Step 4: Calculate the resultant lateral forces and their locations.

See the table below for calculations. Active moments are assumed to be negative.

Active Area	Depth (m)	Force (kN)	Moment arm from base (m)	Moment (kN · m)
1	0–2	$8.2 \times 2 = 16.4$	$4 + 1 = 5 = 4.42$	–82.0
2	2–6	$6.7 \times 4 = 26.8$	$\frac{4}{2} = 2$	–53.6
3	0–2	$\frac{1}{2} \times 15.6 \times 2 = 15.6$	$\frac{2}{3} + 4 = 4.67$	–72.9
4	2–6	$12.7 \times 4 = 50.8$	$\frac{4}{2} = 2$	–101.6
5	2–6	$\frac{1}{2} \times 13.6 \times 4 = 27.2$	$\frac{4}{3} = 1.33$	–36.3
6	2–6	78.4	$\frac{4}{3} = 1.33$	–104.5
(water)				

Σ Active lateral forces = 215.2

Σ Active moments = –450.9

(Continued)

(Continued)

Passive Area	Depth (m)	Force (kN)	Moment arm from base (m)	Moment (kN · m)
7	2–6	$\frac{1}{2} \times 122.4 \times 4 = 244.8$	$\frac{4}{3}$	326.4
8 (water)	2–6	<u>78.4</u>	$\frac{5.4}{3}$	<u>104.5</u>
Σ Passive forces = 323.2			Σ Passive moments = 430.9	

Location of resultant active lateral earth force:

$$\bar{z}_a = \frac{\Sigma \text{ Moments}}{\Sigma \text{ Active lateral forces}} = \frac{450.9}{215.2} = 2.09 \text{ m}$$

Location of passive lateral force: $\bar{z}_p = \frac{4}{3} = 1.33 \text{ m}$

Step 5: Calculate the resultant lateral force.

$$R_x = P_p - P_a = 323.2 - 215.2 = 108 \text{ kN/m}$$

Step 6: Calculate the ratio of moments.

$$\text{Ratio of moments} = \frac{\Sigma \text{ Passive moments}}{\Sigma \text{ Active moments}} = \frac{430.9}{450.9} = 0.96$$

Since the active moment is greater than the passive moment, the wall will rotate. ■

What's next... The pioneer of earth pressure theory is Coulomb (1776). We are going to introduce his ideas in the next section.

12.4 COULOMB'S EARTH PRESSURE THEORY



Interactive Concept Learning, Computer Program Utility, and Self-Assessment

Access Chapter 12, Section 12.4 on the CD to learn about Coulomb's earth pressure theory. Take Quiz 12.4 to test your understanding. Click on K_a and K_p coeff on the sidebar to run a computer program that will give the values of the active and passive coefficients for Coulomb and Rankine (Sections 12.3 and 12.5) methods.

Coulomb (1776) proposed that a condition of limit equilibrium exists through which a soil mass behind a vertical retaining wall will slip along a plane inclined an angle θ to the horizontal. He then determined the slip plane by searching for the plane on which the maximum thrust acts. We begin consideration of Coulomb's theory by reminding you of the basic tenets of limit equilibrium. The essential steps in the limit equilibrium method are (1) selection of a plausible failure mechanism, (2) determination of the forces acting on the failure surface, and (3) use of equilibrium equations to determine the maximum thrust.

Let us consider a vertical, frictionless wall of height H_o , supporting a soil mass with a horizontal surface (Fig. 12.7a). We are going to assume a dry, homogeneous soil mass and postulate that slip occurs on a plane inclined an angle θ to the horizontal. Since the soil is dry, $\gamma' = \gamma$. We can draw the free-body diagram as shown in Fig. 10.7b and solve for P_a using statics as follows:

$$\Sigma F_x = P_a + T \cos \theta - N \sin \theta = 0$$

$$\Sigma F_z = W - T \sin \theta - N \cos \theta = 0$$

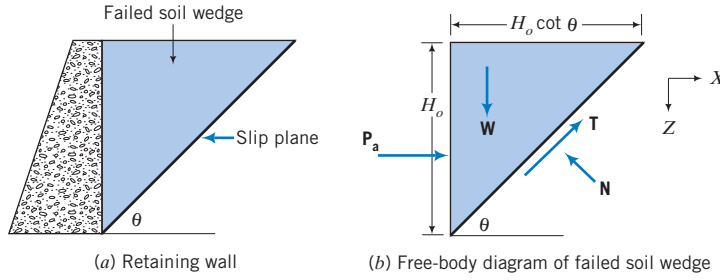


FIGURE 12.7 Coulomb failure wedge.

The weight of the sliding mass of soil is

$$W = \frac{1}{2} \gamma H_o^2 \cot \theta$$

At limit equilibrium,

$$T = N \tan \phi'$$

Solving for P_a we get

$$P_a = \frac{1}{2} \gamma H_o^2 \cot \theta \tan(\theta - \phi') \quad (12.13)$$

To find the maximum thrust and the inclination of the slip plane, we use calculus to differentiate Eq. (12.13) with respect to θ :

$$\frac{\partial P_a}{\partial \theta} = \frac{1}{2} \gamma H_o^2 [\cot \theta \sec^2(\theta - \phi') - \csc^2 \theta \tan(\theta - \phi')] = 0$$

which leads to

$$\theta = \theta_{cr} = 45^\circ + \frac{\phi'}{2} \quad (12.14)$$

Substituting this value of θ into Eq. (12.13) we get

$$P_a = \frac{1}{2} \gamma H_o^2 \tan^2 \left(45^\circ - \frac{\phi'}{2} \right) = \frac{1}{2} K_a \gamma H_o^2 \quad (12.15)$$

This is the same result obtained earlier from considering Mohr's circle.

The solution from a limit equilibrium method is analogous to an upper bound solution because it gives a solution that is usually greater than the true solution. The reason for this is that a more efficient failure mechanism may be possible than the one we postulated. For example, rather than a planar slip surface we could have postulated a circular slip surface or some other geometric form, and we could have obtained a maximum horizontal force lower than for the planar slip surface.

For the Rankine active and passive states, we considered the stress states and obtained the distribution of lateral stresses on the wall. At no point in the soil mass did the stress state exceed the failure stress state and static equilibrium is satisfied. The solution for the lateral forces obtained using the Rankine active and passive states is analogous to a lower bound solution—the solution obtained is usually lower than the true solution because a more efficient distribution of stress could exist. If the lower bound solution and the upper bound solution are in agreement, we have a true solution, as is the case here.

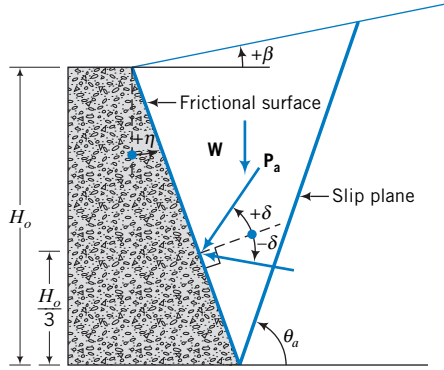


FIGURE 12.8 Retaining wall with sloping back, wall friction, and sloping soil surface for use with Coulomb's method for active condition.

Poncelet (1840), using Coulomb's limit equilibrium approach, obtained expressions for K_a and K_p for cases where wall friction (δ) is present, the wall face is inclined at an angle η to the vertical, and the backfill is sloping at an angle β . With reference to Fig. 12.8, K_{aC} and K_{pC} are

$$K_{aC} = \frac{\cos^2(\phi' - \eta)}{\cos^2 \eta \cos(\eta + \delta) \left[1 + \left\{ \frac{\sin(\phi' + \delta) \sin(\phi' - \beta)}{\cos(\eta + \delta) \cos(\eta - \beta)} \right\}^{1/2} \right]^2} \quad (12.16)$$

$$K_{pC} = \frac{\cos^2(\phi' + \eta)}{\cos^2 \eta \cos(\eta - \delta) \left[1 - \left\{ \frac{\sin(\phi' + \delta) \sin(\phi' + \beta)}{\cos(\eta - \delta) \cos(\eta - \beta)} \right\}^{1/2} \right]^2} \quad (12.17)$$

The subscript C denotes Coulomb. You should note that $K_{pC} \neq 1/K_{aC}$. Recall that the lateral earth pressure coefficients are applied to the effective not total vertical stress. In the absence of groundwater, the total and effective vertical stresses are equal.

The inclination of the slip plane to the horizontal is

$$\tan \theta = \left[\frac{(\sin \phi' \cos \delta)^{1/2}}{\cos \phi' \{\sin(\phi' + \delta)\}^{1/2}} \right] \pm \tan \phi' \quad (12.18)$$

where the positive sign refers to the active state (θ_a) and the negative sign refers to the passive state (θ_p).

Wall friction causes the slip planes in both the active and passive states to be curved. The curvature in the active case is small in comparison to the passive case. The implication of the curved slip surface is that the values of K_{aC} and K_{pC} from Eqs. (12.16) and (12.17) are not accurate. In particular, the passive earth pressures are overestimated using Eq. (12.17). For the active state, the error is small and can be neglected. The error for the passive state is small if $\delta < \phi'/3$. In practice, δ is generally greater than $\phi'/3$.

Several investigators have attempted to find K_a and K_p using nonplanar slip surfaces. For example, Caquot and Kerisel (1948) used logarithm spiral slip surfaces while Packshaw (1969) used circular failure surfaces. The Caquot and Kerisel (1948) values of K_p are generally used in practice. Table 12.1 lists factors that can be applied to K_{pC} to correct it for a logarithm spiral slip surface.

The lateral forces are inclined at δ to the normal on the sloping wall face. The sign conventions for δ and β are shown in Fig. 12.8. You must use the appropriate sign in determining K_{aC} and K_{pC} . The direction of the frictional force on the wall depends on whether the wall moves relative to the soil or the soil moves relative to the wall. In general, the active wedge moves downward relative to the wall and the passive wedge moves upward relative to the wall. The frictional forces developed are shown in Fig. 12.9.

TABLE 12.1 Correction Factors to Be Applied to K_{pC} to Approximate a Logarithm Spiral Slip Surface for a Backfill with a Horizontal Surface and Sloping Wall Face

ϕ'	δ/ϕ'							
	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1	0.0
15	0.96	0.93	0.91	0.88	0.85	0.83	0.80	0.78
20	0.94	0.90	0.86	0.82	0.79	0.75	0.72	0.68
25	0.91	0.86	0.81	0.76	0.71	0.67	0.62	0.57
30	0.88	0.81	0.75	0.69	0.63	0.57	0.52	0.47
35	0.84	0.75	0.67	0.60	0.54	0.48	0.42	0.36
40	0.78	0.68	0.59	0.51	0.44	0.38	0.32	0.26

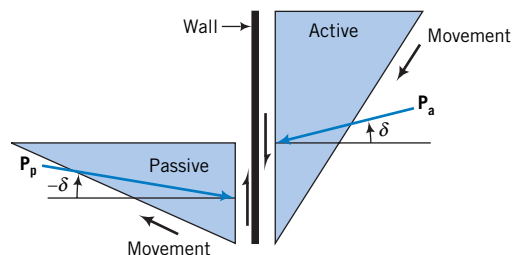
The sense of the inclination of active lateral force is then positive while the sense of the inclination of passive lateral force is negative. Load bearing walls with large vertical loads tend to move downward relative to the soil. In such a situation the frictional force on both sides of the wall will be downward and the lateral earth pressures are increased on both sides of the wall. In practice, these increases in pressure are ignored and the sense of the active and lateral forces are taken as shown in Fig. 12.9.

The horizontal components of the lateral forces are $P_{ax} = P_a \cos \delta$ and $P_{px} = P_p \cos \delta$, and the vertical components are $P_{az} = P_a \sin \delta$ and $P_{pz} = P_p \sin \delta$. The point of application of these forces is $H_o/3$ from the base of the wall (Fig. 12.8). Typical values of δ for the interfaces of coarse-grained soils and concrete or steel walls range from $\frac{1}{2}\phi'$ to ϕ' .

During compaction of the backfill soil, additional lateral stresses are imposed by compaction equipment. For walls with sloping backfill, heavy compaction equipment is generally used. The wall pressures from using heavy compaction equipment can be in excess of the active earth pressures. You should account for the lateral stresses in designing retaining walls. You may refer to Ingold (1979), who used elastic theory to estimate the lateral stresses imposed by construction equipment. Some practicing geotechnical engineers prefer to account for the additional compaction stresses by assuming that the resultant active lateral earth force acts at $0.4H_o$ or $0.5H_o$ rather than $\frac{1}{3}H_o$ from the base of the wall. Alternatively, you can multiply the active earth pressures by a factor ($\simeq 1.20$) to account for compaction stresses.

The essential points are:

1. *Coulomb's analysis of the lateral forces on a retaining structure is based on limit equilibrium.*
2. *Wall friction causes the slip planes to curve, which leads to an over-estimation of the passive earth pressure using Coulomb's analysis.*
3. *For calculation of the lateral earth pressure coefficients you can use Eqs. (12.16) and (12.17), and correct K_{pC} using the factors listed in Table 12.1.*
4. *The active and passive forces are inclined at an angle δ from the normal to the wall face.*

**FIGURE 12.9** Directions of active and passive forces when wall friction is present.

What's next... Coulomb's analysis is based on a postulated failure mechanism and the stresses within the soil mass are not considered. Rankine (1857) proposed an analysis based on the stress state of the soil. You have already encountered his approach in Section 12.3. Next, we introduce Rankine's solution for a wall with a sloping backfill and a sloping wall face.

12.5 RANKINE'S LATERAL EARTH PRESSURE FOR A SLOPING BACKFILL AND A SLOPING WALL FACE



Computer Program Utility

Access Chapter 12 on the CD and click on K_a and K_p coeff on the sidebar to run a computer program that will give the values of the active and passive coefficients for Rankine and Coulomb methods. Take Quiz 12.5 to test your understanding.

Rankine (1857) established the principle of stress states or stress field in solving stability problems in soil mechanics. We have used Rankine's method in developing the lateral earth pressures for a vertical, frictionless wall supporting a dry, homogeneous soil with a horizontal surface. Rankine (1857) derived expressions for K_a and K_p for a soil mass with a sloping surface that was later extended to include a sloping wall face by Chu (1991). You can refer to Rankine's paper and Chu's paper for the mathematical details.

With reference to Fig. 12.10, the lateral earth pressure coefficients according to Rankine's analysis are

$$K_{aR} = \frac{\cos(\beta - \eta) \sqrt{1 + \sin^2 \phi' - 2 \sin \phi' \cos \theta_a}}{\cos^2 \eta (\cos \beta + \sqrt{\sin^2 \phi' - \sin^2 \beta})} \quad (12.19)$$

and

$$K_{pR} = \frac{\cos(\beta - \eta) \sqrt{1 + \sin^2 \phi' + 2 \sin \phi' \cos \theta_p}}{\cos^2 \eta (\cos \beta - \sqrt{\sin^2 \phi' - \sin^2 \beta})} \quad (12.20)$$

where the subscript R denotes Rankine, and

$$\theta_a = \frac{\pi}{4} + \frac{\phi'}{2} + \frac{\beta}{2} - \frac{1}{2} \sin^{-1} \left(\frac{\sin \beta}{\sin \phi'} \right) \quad (12.21)$$

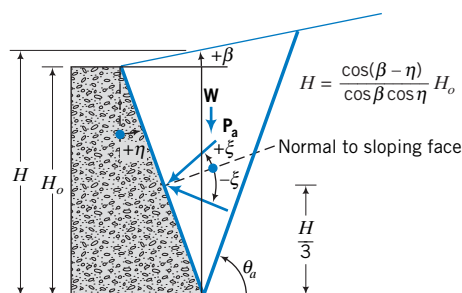


FIGURE 12.10 Retaining wall with sloping soil surface, frictionless soil-wall interface, and sloping back for use with Rankine's method.

$$\theta_p = \frac{\pi}{4} - \frac{\phi'}{2} + \frac{\beta}{2} + \frac{1}{2} \sin^{-1} \left(\frac{\sin \beta}{\sin \phi'} \right) \quad (12.22)$$

are the inclinations of the slip planes to the horizontal. The sign conventions for η and β are shown in Fig. 12.10; anticlockwise rotation is positive.

The active and passive lateral earth forces are

$$P_a = \frac{1}{2} K_{aR} \gamma' H_o^2 \quad \text{and} \quad P_p = \frac{1}{2} K_{pR} \gamma' H_o^2$$

These forces are inclined at

$$\xi_a = \tan^{-1} \left(\frac{\sin \phi' \sin \theta_a}{1 - \sin \phi' \cos \theta_a} \right) \quad (12.23)$$

$$\xi_p = \tan^{-1} \left(\frac{\sin \phi' \sin \theta_p}{1 + \sin \phi' \cos \theta_p} \right) \quad (10.24)$$

to the normal of the wall face. The angles ξ_a and ξ_p (reminder: the subscripts a and p denote active and passive) are not interface friction values.

In the case of a wall with a vertical face, $\eta = 0$, Eqs. (12.19) and (12.20) reduce to

$$K_{aR} = \frac{1}{K_{pR}} = \cos \beta \left(\frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi'}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi'}} \right) \quad (10.25)$$

and the active and passive lateral earth forces act in a direction parallel to the soil surface, that is, they are inclined at an angle β to the horizontal. Thus, $P_{ax} = P_a \cos \beta$ and $P_{az} = P_a \sin \beta$.

The essential points are:

1. Rankine used stress states of a soil mass to determine the lateral earth pressures on a frictionless wall.
2. The active and passive lateral earth forces are inclined at ξ_a and ξ_p , respectively, from the normal to the wall face. If the wall face is vertical, the active and passive lateral earth forces are parallel to the soil surface.

What's next... We considered the lateral earth pressures for a dry soil mass, which is analogous to an effective stress analysis. Next, we will consider total stress analysis.

12.6 LATERAL EARTH PRESSURES FOR A TOTAL STRESS ANALYSIS



Interactive Concept Learning and Self-Assessment

Access Chapter 12, Section 12.6 on the CD to learn about lateral earth pressure for a total stress analysis. Learn how tension cracks are formed and their effects on the lateral earth pressures. Take Quiz 12.6 to test your understanding.

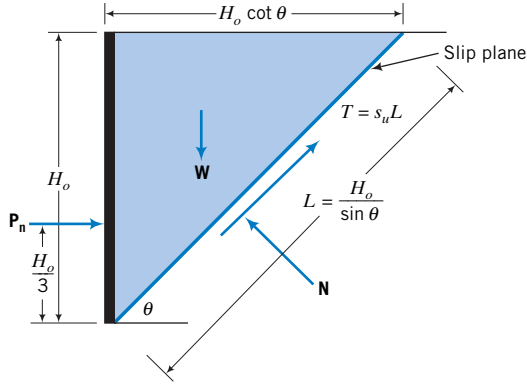


FIGURE 12.11 Forces on a retaining wall for a total stress analysis.

Figure 12.11 shows a smooth, vertical wall supporting a homogeneous soil mass under undrained conditions. Using the limit equilibrium method, we will assume, for the active state, that a slip plane is formed at an angle θ to the horizontal. The forces on the soil wedge are shown in Fig. 12.11.

Using static equilibrium, we obtain the sum of the forces along the slip plane:

$$P_a \cos \theta + T - W \sin \theta = 0 \quad (12.26)$$

But $T = s_u L = s_u (H_o / \sin \theta)$ and $W = \frac{1}{2} \gamma H_o^2 \cot \theta$. Equation (12.26) then yields

$$P_a = \frac{1}{2} \gamma H_o^2 - \frac{s_u H_o}{\sin \theta \cos \theta} = \frac{1}{2} \gamma H_o^2 - \frac{2s_u H_o}{\sin 2\theta}$$

To find the maximum active lateral earth force, we differentiate P_a with respect to θ and set the result equal to zero, giving

$$\frac{\partial P_a}{\partial \theta} = 4s_u H_o \cot 2\theta \csc 2\theta = 0$$

The solution is $\theta = \theta_a = 45^\circ$.

By substituting $\theta = 45^\circ$ into the above equation for P_a , we get the maximum active lateral earth force as

$$P_a = \frac{1}{2} \gamma H_o^2 - 2s_u H_o \quad (12.27)$$

If we assume a uniform distribution of stresses on the slip plane, then the active lateral stress is

$$(\sigma_x)_a = \gamma z - 2s_u \quad (12.28)$$

Let us examine Eq. (12.28). If $(\sigma_x)_a = 0$, for example, when you make an excavation, then solving for z in Eq. (12.28) gives

$$z = z_{cr} = \frac{2s_u}{\gamma} \quad (12.29)$$

Depth z_{cr} is the depth at which tension cracks would extend into the soil (Fig. 12.12). If the tension crack is filled with water, the critical depth can extend to

$$z'_{cr} = \frac{2s_u}{\gamma'} \quad (12.30)$$

In addition, the soil in the vicinity of the crack is softened and a hydrostatic pressure, $\gamma_w z'_{cr}$, is imposed on the wall. Often, the critical depth of water-filled tension cracks in overconsolidated clays is greater

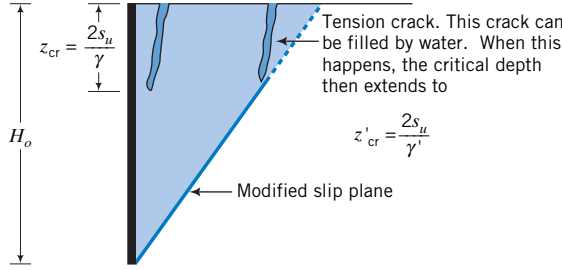


FIGURE 12.12 Tension cracks behind a retaining wall.

than the wall height. For example, if $s_u = 80 \text{ kPa}$, $\gamma = \gamma_{\text{sat}} = 18 \text{ kN/m}^3$, then $z'_{\text{cr}} = 19.5 \text{ m}$. A wall height equivalent to the depth of the tension crack of 19.5 m is substantial. As a comparison, the average height of the Great Wall of China is about 12 m. When tension cracks occur they modify the slip plane, as shown in Fig. 12.12; no shearing resistance is available over the length of the slip plane above the depth of the tension cracks.

For an unsupported excavation, the active lateral force is also zero. From Eq. (12.27), we get

$$\frac{1}{2} \gamma H_o^2 - 2s_u H_o = 0$$

and, solving for H_o , we obtain

$$H_o = H_{\text{cr}} = \frac{4s_u}{\gamma} \quad (12.31)$$

If the excavation is filled with water, then

$$H'_{\text{cr}} = \frac{4s_u}{\gamma'} \quad (12.32)$$

We have two possible unsupported depths as given by Eqs. (12.29) and (12.31). The correct solution lies somewhere between these critical depths. In design practice, a value of

$$\frac{3.6s_u}{\gamma} \leq H_{\text{cr}} \leq \frac{3.8s_u}{\gamma} \quad (12.33)$$

is used for unsupported excavation in fine-grained soils. If the excavation is filled with water,

$$\frac{3.6s_u}{\gamma'} \leq H'_{\text{cr}} \leq \frac{3.8s_u}{\gamma'} \quad (12.34)$$

The passive lateral earth force for a total stress analysis, following a procedure similar to that for the active state above, can be written as

$$P_p = \frac{1}{2} \gamma H_o^2 + 2s_u H_o \quad (12.35)$$

and the passive lateral pressure is

$$(\sigma_x)_p = \gamma z + 2s_u \quad (12.36)$$

We can write Eqs. (12.28) and (12.36) using apparent active and passive lateral earth pressures for the undrained condition as

$$(\sigma_x)_a = \sigma_z - K_{au}s_u \quad (12.37)$$

$$(\sigma_x)_p = \sigma_z + K_{pu}s_u \quad (12.38)$$

where K_{au} and K_{pu} are the undrained active and passive lateral earth pressure coefficients. In our case, for a smooth wall supporting a soil mass with a horizontal surface, $K_{au} = K_{pu} = 2$.

Walls that are embedded in fine-grained soils may be subjected to an adhesive stress (s_w) at the wall face. The adhesive stress is analogous to a wall–soil interface friction for an effective stress analysis. The undrained lateral earth pressure coefficients are modified to account for adhesive stress as

$$K_{au} = K_{pu} = 2\sqrt{1 + \frac{s_w}{s_u}} \quad (12.39)$$

The essential points are:

1. *Lateral earth pressures for a total stress analysis are found using apparent lateral earth pressure coefficients K_{au} and K_{pu} . These coefficients are applied to the undrained shear strength. For smooth, vertical walls, $K_{au} = K_{pu} = 2$.*
2. *Tension cracks of theoretical depth $2s_u/\gamma$, or $2s_u/\gamma'$ if water fills the tension cracks, are usually formed in fine-grained soils and they modify the slip plane. If water fills the cracks, it softens the soil and a hydrostatic stress is imposed on the wall over the depth of the tension crack. You must pay particular attention to the possibility of the formation of tension cracks, and especially so if these cracks can be filled with water, because they can initiate failure of a retaining structure.*
3. *The theoretical maximum depth of an unsupported vertical cut in fine-grained soils is $H_{cr} = 4s_u/\gamma$ or, if the cut is filled with water, $H'_{cr} = 4s_u/\gamma'$.*

What's next. . . You were introduced to Coulomb's and Rankine's analyses of the active and passive lateral earth pressures. Concerns were raised regarding the accuracy of, in particular, K_{pC} because wall friction causes the failure surface to diverge from a plane surface. Several investigators have proposed values of K_p , assuming curved failure surfaces. The question that arises is: "What values of K_a , K_p , ϕ' , and s_u should be used in the analyses of earth retaining structures?" Next, we will attempt to address this question. You are forewarned that the answer will not be definite. A geotechnical engineer usually has his/her preferences based on his/her experience with a particular method.

12.7 APPLICATION OF LATERAL EARTH PRESSURES TO RETAINING WALLS

Field and laboratory tests have not confirmed the Coulomb and Rankine theories. In particular, field and laboratory test results showed that both theories overestimate the passive lateral earth pressures. Values of K_p obtained by Caquot and Kerisel (1948) lower the passive lateral earth pressures but they are still higher than experimental results. Other theories, for example, plasticity theory (Rosenfarb and Chen, 1972), have been proposed, but these theories are beyond the scope of this book and they too do not significantly change the Coulomb and Rankine passive lateral earth pressures for practical ranges of friction angle and wall geometry.

Rankine's theory was developed based on a semi-infinite "loose granular" soil mass for which the soil movement is uniform. Retaining walls do not support a semi-infinite mass but a soil mass of fixed depth. Strains, in general, are not uniform in the soil mass unless the wall rotates about its base to induce a state of plastic equilibrium.

The strains required to achieve the passive state are much larger than for the active case (Fig. 12.5). For sands, a decrease in lateral earth pressure of 40% of the at-rest lateral earth pressure can be sufficient to reach an active state, but an increase of several hundred percent in lateral earth pressure over the at-rest lateral earth pressure is required to bring the soil to a passive state. Because of the large strains that occur to achieve the passive state, it is customary to apply a factor of safety to the passive lateral earth pressure.

We have assumed a generic friction angle for the soil mass. Backfills are usually coarse-grained soils compacted to greater than 95% Proctor dry unit weight. If samples of the backfill were to be tested in shear tests in the laboratory at the desired degree of compaction, the samples may show peak shear stresses resulting in ϕ'_p . If you use ϕ'_p to estimate the passive lateral earth pressure using either the Coulomb or Rankine method, you are likely to overestimate it because the shear strains required to develop the passive lateral earth pressure are much greater than those required to mobilize ϕ'_p in the lab. For granular materials, ϕ'_p is mobilized at shear strains $\approx 2\%$. The use of ϕ'_p in the Rankine or Coulomb equations is one reason for the disagreement between the predicted passive lateral earth pressures and experimental results.

Bolton and Powrie (1988) demonstrated that a wall rotation of $0.005H_o$ induces tensile and compressive strains of ± 0.005 corresponding to an average shear strain, $\gamma \approx 1\%$. Tolerable wall rotations are generally less than $0.005H_o$. Consequently, ϕ'_p (which requires $\gamma \approx 2\%$ to be mobilized for granular soils) would not be mobilized.

Large shear strains ($\gamma > 10\%$) are required to mobilize ϕ'_{cs} . For a backfill consisting of loose, coarse-grained soils, the displacement of the wall required to mobilize ϕ'_{cs} is intolerable in practice. You should then use conservative values of ϕ' in design. The maximum ϕ'_{design} should be ϕ'_{cs} . In practice, factors of safety are applied to allow for uncertainties of loads and soil properties.

Sometimes, it is desirable that the wall remains rigid because of nearby structures that are sensitive to lateral displacements. In this case, you should use K_o as the lateral earth pressure coefficient.

Wall friction causes the active lateral earth pressures to decrease and the passive lateral earth pressures to increase. For active lateral earth pressures, the Coulomb equation is sufficiently accurate for practice, but for passive lateral earth pressures you should use the Caquot and Kerisel (1948) values or similar values (e.g., Packshaw, 1969).

The total stress analysis should only be used in temporary works. But even for these works, you should be cautious in relying on the undrained shear strength. A total stress analysis should be used in conjunction with an effective stress analysis for retaining structures supporting fine-grained soils.

When a wet clay soil is excavated, negative excess porewater pressures develop and give the soil a greater undrained strength than prior to excavation. You should not rely on this gain in strength because the excess porewater pressures will dissipate with time. On inserting a wall, the soil at the wall-soil interface is remolded and you should use conservative values of wall adhesion. The maximum values of wall adhesion (Padfield and Mair, 1984) should be the lesser of

$$\text{Active state : } s_w = 0.5s_u \quad \text{or} \quad s_w \leq 50 \text{ kPa} \quad (12.40)$$

$$\text{Passive state : } s_w = 0.5s_u \quad \text{or} \quad s_w \leq 25 \text{ kPa} \quad (12.41)$$

The interface friction between the wall face and the soil depends on the type of backfill used and construction methods. If the surface texture of the wall is rougher than D_{50} of the backfill, the strength characteristics of backfill would control the interface friction. In such a case, the interface friction angle can be taken as equivalent to ϕ'_{cs} . If the wall surface is smooth compared with D_{50} of the backfill, the interface friction value can be assumed, in the absence of field measurements, to be between $\frac{2}{3}\phi'_{cs}$ and $\frac{1}{2}\phi'_{cs}$.

During the life of a retaining wall, a gap may open between the wall and the backfill due to wall movement, deficient construction method, and unanticipated environmental conditions (drought, rainfall, floods, etc.). You need to be cautious in using adhesion and interface friction because these may become nonexistent during the life of a wall. More importantly, water can seep through the gap and subject the wall to hydrostatic pressures. When a gap could develop, use hydrostatic pressure rather than the lateral soil pressure.

A layer of coarse-grained soil is often used in construction to rest the base of gravity retaining walls (see Section 12.8) founded on clays. The interface friction angle for sliding would then be the

lesser of the interface friction between the layer of coarse-grained soil and the wall base, and the interface friction between the layer of coarse-grained soil and the clay.

In summary, you should use:

1. $\phi'_{\text{design}} \leq \phi'_{\text{cs}}$ because:
 - (a) Uncertainties of loads and soil properties exist.
 - (b) Tolerable rotations of walls ($< 0.005 H_0$) would not normally mobilize peak shear strength or critical state shear strength.
 - (c) Prior to wall completion, shear bands (thin zones of soil that reached critical state) in the compacted backfill may develop.
 - (d) Quality of construction methods and construction loading, environmental conditions (floods, heavy rainfall, etc.), human and animal interventions (excavation at toe, dumping of materials on top of wall, burrowing of holes, etc.) cannot be estimated (at least, accurately) in advance.
 - (e) You should design the wall so that the backfill soil behaves in a ductile manner. Using ϕ'_{cs} or lower values would allow the soil to respond in a ductile manner.
2. Total stress analysis in conjunction with an effective stress analysis for fine-grained soils.
3. Conservative values for wall friction ($\delta \approx \frac{1}{2} \phi'_{\text{cs}}$) and wall adhesion ($s_w = 0.5s_u$ but ≤ 50 kPa for active state and $s_w \leq 25$ kPa for passive case).
4. High-quality drainage system to drain water from the backfill and away from the wall.

What's next... In the next three sections, we will analyze retaining walls to determine their stability. We will consider an ESA (effective stress analysis) and a TSA (total stress analysis). We begin by considering the possible failure modes.

12.8 TYPES OF RETAINING WALLS AND MODES OF FAILURE



Concept Learning and Self Assessment

Access Chapter 12, Section 12.8 on the CD to learn how retaining walls are likely to behave and fail. Take Quiz 12.8 to test your understanding.

There are two general classes of retaining walls. One class is rigid and consists of concrete walls relying on gravity for stability (Fig. 12.13). These are called cast-in-place (CIP) gravity and semi-gravity walls. The other class is flexible and consists of long slender members of either steel or concrete or wood and relies on passive soil resistance and anchors for stability (Fig. 12.14).

There are four modes of failures for rigid retaining walls—translational failure, rotation and bearing capacity failure, deep-seated failure, and structural failure (Fig. 12.15). Flexible walls, also called sheet pile walls, failed either by deep-seated failure, rotation at the base, rotation about the anchor or prop, failure of the anchor, bending of the wall, or seepage-induced failure (Fig. 12.16).

Seepage-induced failure is avoided in rigid retaining walls by providing adequate drainage systems, two of which are depicted in Fig. 12.17. The design of drainage systems is beyond the scope of this book. Flow nets discussed in Chapter 11 are used in designing drainage systems.

Flexible retaining walls are often used in waterfront structures and as temporary supports for excavations. Seepage forces are generally present and must be considered in evaluating the stability of these walls.

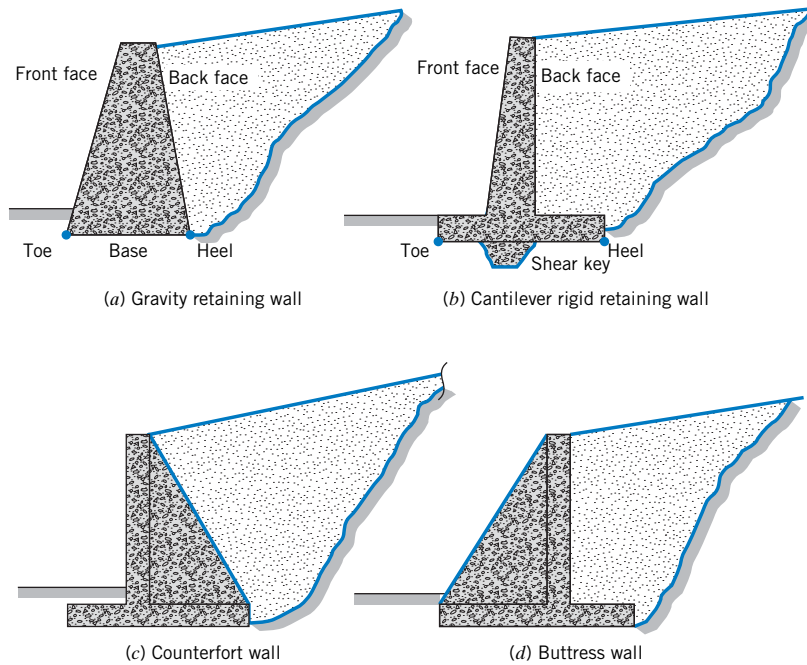


FIGURE 12.13 Types of rigid retaining walls.

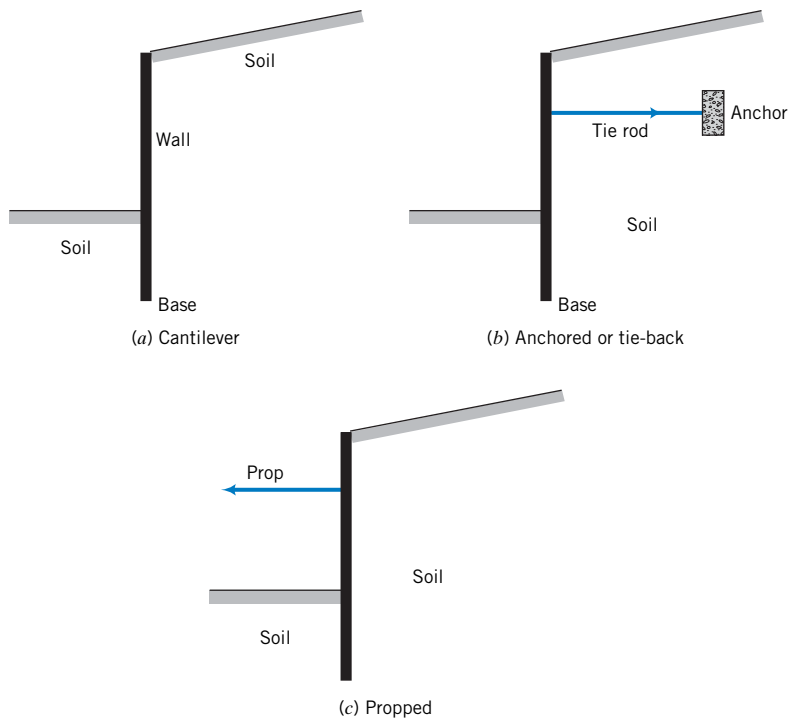


FIGURE 12.14 Types of flexible retaining walls.

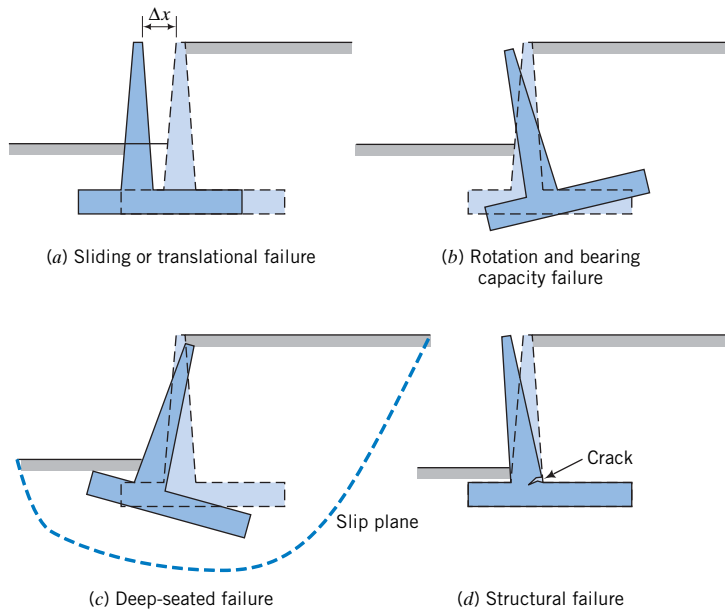


FIGURE 12.15 Failure modes for rigid retaining walls (the dotted lines show the original position of the wall).

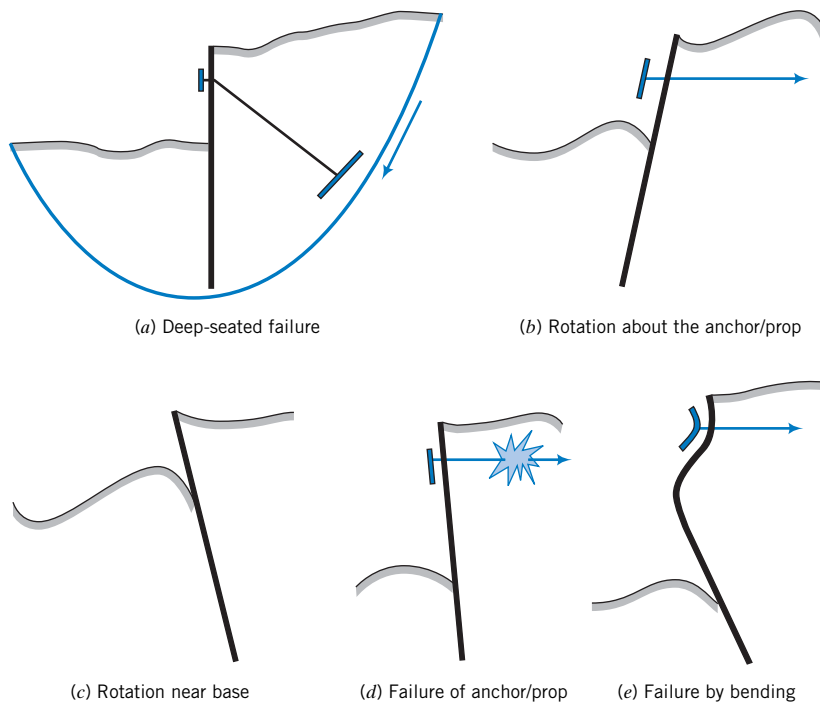


FIGURE 12.16 Failure modes for flexible retaining walls.

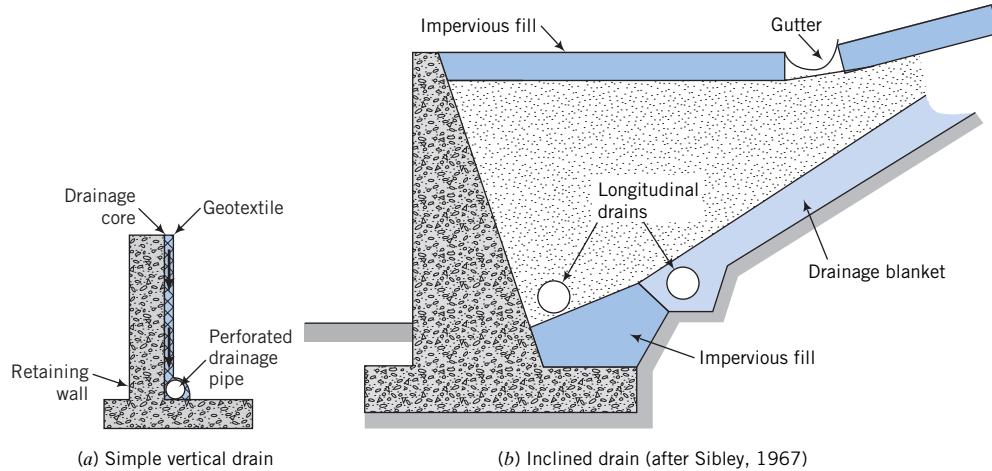


FIGURE 12.17 Two types of drainage system behind rigid retaining walls.

12.9 STABILITY OF RIGID RETAINING WALLS



Concept Learning

Access Chapter 12, Section 12.9 on the CD to learn about rigid retaining walls. Take Quiz 12.9 to test your understanding.

CIP gravity retaining walls (Fig. 12.13a) are massive concrete walls. Their stability depends mainly on the self-weight of the walls. Cantilever walls (Fig. 12.13b)—CIP semi-gravity walls—utilize the backfill to help mobilize stability and are generally more economical than CIP gravity retaining walls. A rigid retaining wall must have an adequate factor of safety to prevent excessive translation, rotation, bearing capacity failure, deep-seated failure, and seepage-induced instability.

12.9.1 Translation

A rigid retaining wall must have adequate resistance against translation. That is, the sliding resistance of the base of the wall must be greater than the resultant lateral force pushing against the wall. The factor of safety against translation, $(FS)_T$, is

$$(FS)_T = \frac{T}{P_{ax}}; \quad (FS)_T \geq 1.5 \quad (12.42)$$

where T is the sliding resistance at the base and P_{ax} is the lateral force pushing against the wall. The sliding resistance is $T = R_z \tan \phi'_b$ for an ESA, and $T = s_w B$ for a TSA (if the base rests directly on fine-grained soils). R_z is the resultant vertical force, ϕ'_b is the interfacial friction angle between the base of the wall and the soil

$$\phi'_b \approx \frac{1}{2} \phi'_{cs} \text{ to } \frac{2}{3} \phi'_{cs}$$

and B is the projected horizontal width of the base. Typical sets of forces acting on gravity and cantilever rigid retaining walls are shown in Fig. 12.18.

Using statics, we obtain, for an ESA,

$$(FS)_T = \frac{[(W_w + W_s + P_{az}) \cos \theta_b - P_{ax} \sin \theta_b] \tan \phi'_b}{P_{ax} \cos \theta_b + (W_w + W_s + P_{az}) \sin \theta_b} \quad (12.43)$$

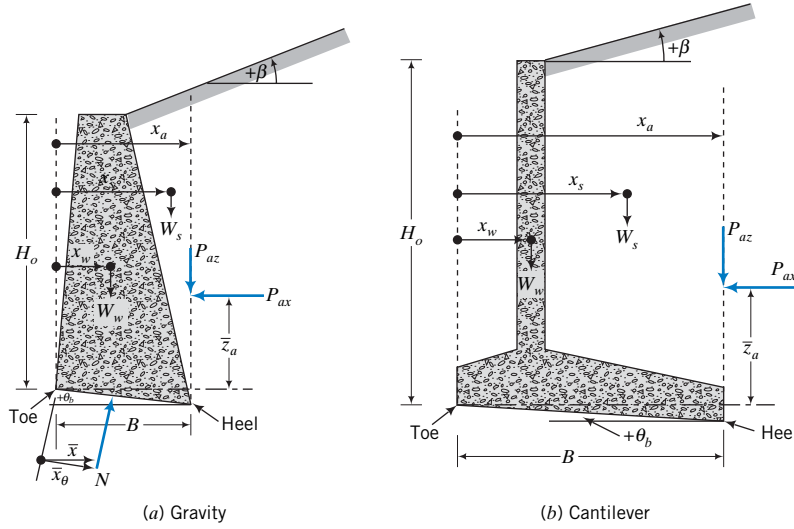


FIGURE 12.18 Forces on rigid retaining walls.

where W_w is the weight of the wall, W_s is the weight of the soil wedge, P_{az} and P_{ax} are the vertical and horizontal components of the active lateral force, and θ_b is the inclination of the base to the horizontal (θ_b is positive if the inclination is counterclockwise, as shown in Fig. 12.18). If $\theta_b = 0$ (base is horizontal), then

$$(FS)_T = \frac{(W_w + W_s + P_{az}) \tan \phi'_b}{P_{ax}} \quad (12.44)$$

For a TSA,

$$(FS)_T = \frac{s_w B / \cos \theta_b}{P_{ax} \cos \theta_b + (W_w + W_s + P_{ax}) \sin \theta_b} \quad (12.45)$$

Where s_w is the adhesive stress (see Eqs. 12.40 and 12.41)

If $\theta_b = 0$, then

$$(FS)_T = \frac{s_w B}{P_{ax}} \quad (12.46)$$

The embedment of rigid retaining walls is generally small and the passive lateral force is not taken into account. If base resistance is inadequate, the width B of the wall can be increased. For cantilever walls, a shear key (Fig. 12.13b) can be constructed to provide additional base resistance against sliding.

12.9.2 Rotation

A rigid retaining wall must have adequate resistance against rotation. The rotation of the wall about its toe is satisfied if the resultant vertical force lies within the middle third of the base. Taking moments about the toe of the base, the resultant vertical force at the base is located at

$$\bar{x}_\theta = \frac{W_w x_w + W_s x_s + P_{az} x_a - P_{ax} \bar{z}_a}{(W_w + W_s + P_{az}) \cos \theta_b - P_{ax} \sin \theta_b} \quad (12.47)$$

where $\bar{z}_a$ is the location of the active lateral earth force from the toe. The wall is safe against rotation if $B/3 \leq \bar{x} \leq 2B/3$; that is, $e = |(B/2 - \bar{x})| \leq B/6$, where e is the eccentricity of the resultant vertical load and $\bar{x} = \bar{x}_\theta \cos \theta_b$.

If $\theta_b = 0$, then

$$\bar{x} = \frac{W_w x_w + W_s x_s + P_{az} x_a - P_{ax} \bar{z}_a}{W_w + W_s + P_{az}} \quad (12.48)$$

12.9.3 Bearing Capacity

A rigid retaining wall must have a sufficient margin of safety against soil bearing capacity failure. The maximum pressure imposed on the soil at the base of the wall must not exceed the allowable soil bearing capacity; that is,

$$\sigma_{\max} \leq q_a \quad (12.49)$$

where $\sigma_{\max}$ is the maximum vertical stress imposed and q_a is the allowable soil bearing capacity.

12.9.4 Deep-Seated Failure

A rigid retaining wall must not fail by deep-seated failure, where by a slip surface encompasses the wall and the soil adjacent to it. In Chapter 13, we will discuss deep-seated failure.

12.9.5 Seepage

A rigid retaining wall must have adequate protection from groundwater seepage. The porewater pressures and the maximum hydraulic gradient developed under seepage must not cause any of the four stability criteria stated above to be violated and static liquefaction must not occur, that is, $i_{\max} < i_{cr}$. Usually,

$$i_{\max} \leq i_{cr}/(\text{FS})_s$$

where $(\text{FS})_s$ is a factor of safety for seepage and is conventionally greater than 3. To avoid seepage-related failures, adequate drainage should be installed in the backfill to dissipate excess porewater pressures quickly. Coarse-grained soils are preferable for the backfill because of their superior drainage characteristics compared with fine-grained soils.

12.9.6 Procedures to Analyze Gravity Retaining Walls

The essential steps in determining the stability of rigid retaining walls are as follows:

- 1. Calculate the active lateral earth force and its components. If the wall is smooth, use Rankine's equations because they are simpler than Coulomb's equations to calculate the active lateral earth pressure coefficient.*
- 2. Determine the weight of the wall and soil above the base.*
- 3. Use Eq. (12.43) or Eq. (12.44) to find $(\text{FS})_T$.*
- 4. Use Eq. (12.47) or Eq. (12.48) to determine the location of the resultant vertical force, R_z , from the toe of the wall.*

5. Check that the eccentricity is less than $B/6$. If it is, then the wall is unlikely to fail by rotation.
6. Determine the maximum soil pressure from $\sigma_{\max} = (R_z/A) (1 + 6e/B)$.
7. Calculate the ultimate bearing capacity q_u , described in Chapter 9. In most cases, R_z would be eccentric.
8. Calculate the factor of safety against bearing capacity failure: $(FS)_B = q_u/\sigma_{\max}$.

EXAMPLE 12.3 Mass Gravity Wall

Computer Program Utility

Access Chapter 12 on the CD and click Spreadsheet on the sidebar to view examples of retaining wall analysis using a spreadsheet. You can change input values and explore how the stability of the walls changes.

A gravity retaining wall, shown in Fig. E12.3a, is required to retain 5 m of soil. The backfill is a coarse-grained soil with $\gamma_{\text{sat}} = 18 \text{ kN/m}^3$, $\phi'_{\text{cs}} = 30^\circ$. The existing soil (below the base) has the following properties: $\gamma_{\text{set}} = 20 \text{ kN/m}^3$, $\phi'_p = 36^\circ$. The wall is embedded 1 m into the existing soil and a drainage system is provided as shown. The groundwater level is 4.5 m below the base of the wall. Determine the stability of the wall for the following conditions: Assume $\delta = 20^\circ$.

- (a) Wall friction is zero.
- (b) Wall friction is 20° .
- (c) The drainage system becomes clogged during several days of a rainstorm and the groundwater rises to the surface. Neglect seepage forces.

The unit weight of concrete is $\gamma_c = 24 \text{ kN/m}^3$.

Strategy For zero wall friction, you can use Rankine's method. But for wall friction, you should use Coulomb's method. The passive resistance is normally neglected in rigid retaining walls. Since only active lateral forces are considered, K_a from the Rankine and Coulomb methods should be accurate enough. Since groundwater is below the base, $\gamma' = \gamma_{\text{sat}}$ over the wall depth.

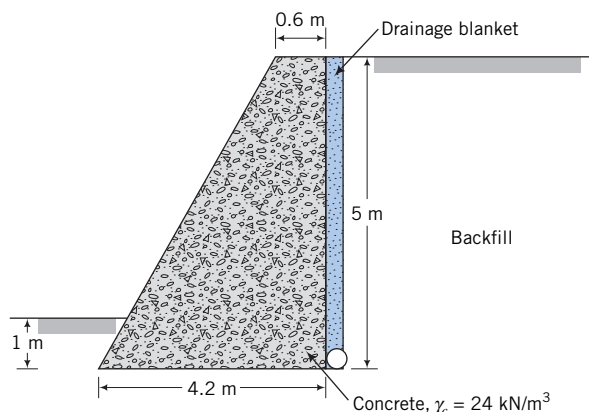


FIGURE E12.3a

Solution 12.3**Step 1:** Determine K_a .

$$\text{Rankine : } \delta = 0, \quad K_{aR} = \tan^2 \left(45^\circ - \frac{\phi_{cs}}{2} \right) = \tan^2 \left(45^\circ - \frac{30^\circ}{2} \right) = \frac{1}{3}$$

$$\text{Coulomb : } \delta = 20^\circ, \quad \phi' = \phi'_{cs}, \quad \beta = 0, \quad \eta = 0; \quad \text{and from Eq. (12.16)}$$

$$K_{aC} = \frac{\cos^2(30^\circ - 0^\circ)}{\cos^2 0^\circ \cos(0^\circ + 20^\circ) \left[1 + \left\{ \frac{\sin(30^\circ + 20^\circ) \sin(30^\circ - 0^\circ)}{\cos(0^\circ + 20^\circ) \cos(0^\circ - 0^\circ)} \right\}^{1/2} \right]^2} = 0.3$$

Step 2: Determine the lateral forces.

Air forces are per unit length of wall.

$$\text{Rankine : } P_{aR} = \frac{1}{2} K_{aR} \gamma_{\text{sat}} H_o^2 = \frac{1}{2} \times \frac{1}{3} \times 18 \times 5^2 = 75 \text{ kN}$$

 P_{aR} acts horizontally because the ground surface is horizontal.

$$\text{Coulomb : } P_{aC} = \frac{1}{2} K_{aC} \gamma_{\text{sat}} H_o^2 = \frac{1}{2} \times 0.3 \times 18 \times 5^2 = 67.5 \text{ kN}$$

 P_{aC} acts at an angle $\delta = 20^\circ$ to the horizontal (see Fig. E12.3b).

$$\text{Horizontal component of } P_{aC}: (P_{ax})_C = P_{aC} \cos \delta = 67.5 \cos 20^\circ = 63.4 \text{ kN}$$

$$\text{Vertical component of } P_{aC}: (P_{az})_C = P_{aC} \sin \delta = 67.5 \sin 20^\circ = 23.1 \text{ kN}$$

Step 3: Determine wall stability.

Consider a unit length of wall.

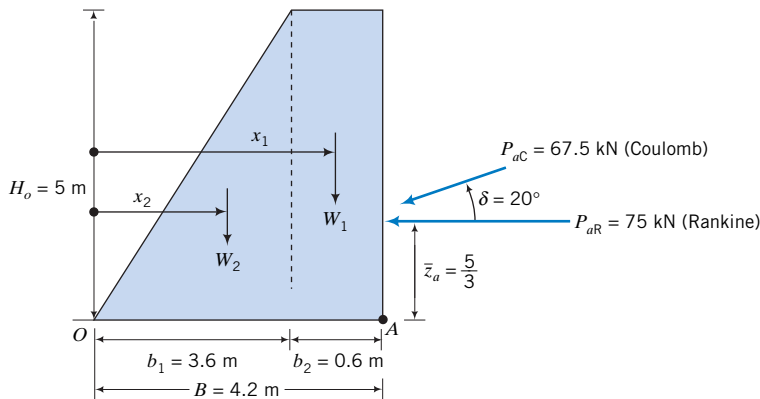
$$W_1 = b_2 H_o \gamma_c = 5 \times 0.6 \times 24 = 72 \text{ kN}$$

$$W_2 = \frac{1}{2} b_1 H_o \gamma_c = \frac{1}{2} \times 3.6 \times 5 \times 24 = 216 \text{ kN}$$

$$W = W_1 + W_2 = 72 + 216 = 288 \text{ kN}$$

or

$$W = \frac{1}{2} (B + b_2) H_o \gamma_c = \frac{1}{2} (4.2 + 0.6) \times 5 \times 24 = 288 \text{ kN}$$

**FIGURE E12.3b**

Calculate the location of the resultant from O (Fig. E12.3b).

$$\text{Rankine : } M_O = W_1x_1 + W_2x_2 - P_{aR}\bar{z}_a = 72(3.6 + 0.3) + 216 \times \left(\frac{2}{3} \times 3.6\right) - 75 \times \frac{5}{3} = 674.2 \text{ kN} \cdot \text{m}$$

$$R_z = W = 288 \text{ kN}$$

$$\bar{x} = \frac{M_O}{R_z} = \frac{674.2}{288} = 2.34 \text{ m}$$

$$\text{Coulomb : } M_O = W_1x_1 + W_2x_2 + (P_{az})_C \times B - (P_{ax})_C \times \bar{z}_a \\ = 72(3.6 + 0.3) + 216 \times \left(\frac{2}{3} \times 3.6\right) + 23.1 \times 4.2 - 63.4 \times \frac{5}{3} = 790.6 \text{ kN}$$

$$R_z = W + (P_{az})_C = 288 + 23.1 = 311.1 \text{ kN}$$

$$\bar{x} = \frac{M_O}{R_z} = \frac{790.6}{311.1} = 2.54 \text{ m}$$

Base resistance: $T = R_z \tan \phi'_b$, where R_z is the resultant vertical force

Assume $\phi'_b = \frac{2}{3}\phi' = \frac{2}{3} \times 36^\circ = 24^\circ$

$$\text{Rankine : } T = 288 \times \tan 24^\circ = 128.2 \text{ kN}$$

$$\text{Coulomb : } T = (288 + 23.1) \times \tan 24^\circ = 138.5 \text{ kN}$$

$$\text{Rankine : } (\text{FS})_T = \frac{T}{P_{aR}} = \frac{128.2}{75} = 1.7 > 1.5; \text{ therefore satisfactory}$$

$$\text{Coulomb : } (\text{FS})_T = \frac{T}{(P_{ax})_C} = \frac{138.5}{63.4} = 2.2 > 1.5; \text{ therefore satisfactory}$$

With wall friction, the factor of safety against translation is greater than without wall friction.

Determine Rotational Stability

$$\text{Rankine : } e = \left| \frac{B}{2} - \bar{x} \right| = \left| \frac{4.2}{2} - 2.34 \right| = 0.24 \text{ m}$$

$$\text{Coulomb : } e = \left| \frac{B}{2} - \bar{x} \right| = \left| \frac{4.2}{2} - 2.54 \right| = 0.44 \text{ m}$$

$$\frac{B}{6} = \frac{4.2}{6} = 0.7 > e$$

The resultant vertical forces for both Rankine and Coulomb method lie within the middle one-third of the base and, therefore, overturning is unlikely to occur

Determining Factor of Safety Against Bearing Capacity Failure Since the resultant vertical force is located within the middle one-third, tension will not develop in the soil.

$$\sigma_{\max} = \frac{R_z}{A} \left(1 + \frac{6e}{B} \right)$$

$$\text{Rankine : } (\sigma_{\max})_R = \frac{288}{4.2 \times 1} \left(1 + \frac{6 \times 0.24}{4.2} \right) = 92.1 \text{ kPa}$$

$$\text{Coulomb : } (\sigma_{\max})_C = \frac{311.1}{4.2 \times 1} \left(1 + \frac{6 \times 0.44}{4.2} \right) = 120.6 \text{ kPa}$$

The maximum stress occurs at A (Fig. E12.3b) for both the Rankine and Coulomb methods. The base of the wall can be taken as a strip, surface foundation, that is, $B/L \rightarrow 0$, and $D_f = 0$. The groundwater level is below $B = 4.2$ m from the base, so groundwater would have no effect on the bearing capacity.

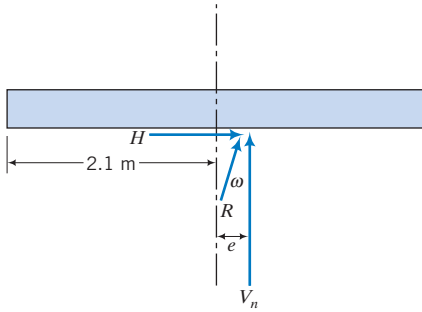


FIGURE E12.3c

The resultant force, R , is eccentric and inclined to the vertical (Fig. E12.3c). Therefore, you should use the bearing capacity equation for inclined load with the width modified for eccentricity.

Rankine: $H = P_{aR} = 75 \text{ kN}$; $V_n = R_z = 288 \text{ kN}$, $\frac{H}{V_n} = \frac{75}{288} = 0.26$; $\omega = \tan^{-1} \frac{H}{V_n} = \tan^{-1}(0.26) = 14.6^\circ$

$\frac{B'}{L'} = 0$, for a strip footing; $B' = B - 2e = 4.2 - 2 \times 0.24 = 3.72 \text{ m}$

$n = \left(2 + \frac{B'}{L'}\right) / \left(1 + \frac{B'}{L'}\right) = 2$; $i_\gamma = \left(1 - \frac{H}{V_n}\right)^{n+1} = (1 - 0.26)^{2+1} = 0.41$

$N_\gamma = 0.1054 \exp(9.6\phi'_p) = 0.1054 \exp\left(9.6 \times 3.7 \times \frac{\pi}{80}\right) = 43.9$

$q_u = 0.5\gamma B' N_\gamma i_\gamma = 0.5 \times 3.73 \times 43.9 \times 0.41 = 653 \text{ kPa}$

$(FS)_B = \frac{q_u}{(\sigma_{\max})_R} = \frac{653}{92.1} = 7.1 > 3$; okay

Coulomb: $H = (P_{ax})_C = 63.4 \text{ kN}$; $V_n = R_z = 311.1 \text{ kN}$, $\frac{H}{V_n} = \frac{63.4}{311.1} = 0.20$;

$\omega = \tan^{-1} \frac{H}{V_n} = \tan^{-1}(0.2) = 11.3^\circ$

$\frac{B'}{L'} = 0$, for a strip footing; $B' = B - 2e = 4.2 - 2 \times 0.44 = 3.32 \text{ m}$

$n = \left(2 + \frac{B'}{L'}\right) / \left(1 + \frac{B'}{L'}\right) = 2$; $i_\gamma = \left(1 - \frac{H}{V_n}\right)^{n+1} = (1 - 0.20)^{2+1} = 0.51$

$N_\gamma = 0.1054 \exp(9.6\phi'_p) = 0.1054 \exp\left(9.6 \times 3.7 \times \frac{\pi}{180}\right) = 43.9$

$q_u = 0.5\gamma B' N_\gamma i_\gamma = 0.5 \times 20 \times 3.32 \times 43.9 \times 0.51 = 743 \text{ kPa}$

$(FS)_B = \frac{q_u}{(\sigma_{\max})_C} = \frac{743}{120.6} = 6.2 > 3$; okay

Step 4: Determine the effects of water from the rainstorm.

Using Rankine's method (zero wall friction)

$$P_{aR} = \frac{1}{2} K_a \gamma' H_o^2 + \frac{1}{2} \gamma_w H_o^2 = \frac{1}{2} \times \frac{1}{3} \times (18 - 9.8) \times 5^2 + \frac{1}{2} \times 9.8 \times 5^2 = 34.2 + 122.5 = 156.7 \text{ kN}$$

Location of resultant from O

$$\begin{aligned} M_O &= W_1 x_1 + W_2 x_2 - P_{aR} \bar{z}_a = 72(3.6 + 0.3) + 216 \times \left(\frac{2}{3} \times 3.6\right) \\ &\quad - 156.7 \times \frac{5}{3} = 538 \text{ kN} \cdot \text{m} \\ \bar{x} &= \frac{538}{288} = 1.87 \text{ m} \end{aligned}$$

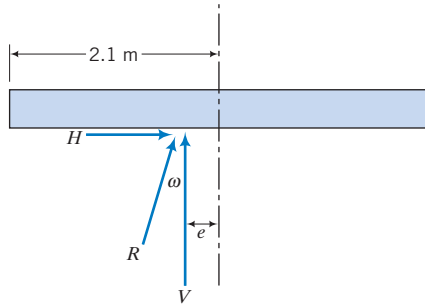


FIGURE E12.3d

Translation

$$(\text{FS})_T = \frac{97.5}{156.7} = 0.62 < 1$$

The wall will fail by translation.

Rotation

$$e = \left| \frac{B}{2} - \bar{x} \right| = \left| \frac{4.2}{2} - 1.87 \right| = 0.23 < \frac{B}{6} \left(= \frac{4.2}{6} = 0.7 \right)$$

The wall is unlikely to fail by rotation.

Bearing capacity

$$\sigma_{\max} = \frac{288}{4.2 \times 1} \left(1 + \frac{6 \times 0.23}{4.2} \right) = 91.1 \text{ Pa}$$

The maximum stress now occurs at O rather than at A .

The eccentric of the resultant force, R , is now on the opposite side of the centroid (Fig. E12.3d).

$$H = P_{aR} = 156.7 \text{ kN}; \quad V_n = R_z = 288 \text{ kN}, \quad \frac{H}{V_n} = \frac{156.7}{288} = 0.54; \quad \omega = \tan^{-1} \frac{H}{V_n} = \tan^{-1}(0.54) = 28.4^\circ$$

$$B' = B - 2e = 4.2 - 2 \times 0.23 = 3.74 \text{ m}$$

$$n = \left(2 + \frac{B'}{L'} \right) / \left(1 + \frac{B'}{L'} \right) = 2; \quad i_\gamma = \left(1 - \frac{H}{V_n} \right)^{n+1} = (1 - 0.54)^{2+1} = 0.1$$

$$q_u = 0.5\gamma' B' N_\gamma i_\gamma = 0.5 \times (20.9.8) \times 3.74 \times 43.9 \times 0.1 = 83.7 \text{ kPa}$$

$$(\text{FS})_B = \frac{q_u}{(\sigma_{\max})_R} = \frac{83.7}{91.1} = 0.9 < 3; \quad \text{not okay}$$

The wall will fail by bearing capacity failure. ■

EXAMPLE 12.4 Cantilever Gravity Wall

Determine the stability of the cantilever, gravity retaining wall shown in Fig. E12.4a. The existing soil is a clay and the backfill is a coarse-grained soil. The base of the wall will rest on a 50 mm thick, compacted layer of the backfill. The interface friction between the base and the compacted layer of backfill is 25° . Groundwater level is 8 m below the base.

Strategy You should use Coulomb's method to determine the lateral earth pressure because of the presence of wall friction. The height of the wall for calculating the lateral earth pressure is the vertical height from the base of the wall to the soil surface. You should neglect the passive resistance of the 1.0 m of soil behind the wall.

Soil mass

All forces are per meter length of wall.

$$\text{Lateral force from soil mass: } P_{aC} = \frac{1}{2} K_{aC} \gamma_{\text{sat}} H_o^2 = \frac{1}{2} \times 0.41 \times 18 \times 7.42^2 = 203.2 \text{ kN}$$

$$\text{Horizontal component: } F_{ax} = P_{aC} \cos \delta = 203.2 \cos 15^\circ = 196.3 \text{ kN}$$

$$\text{Vertical component: } F_{az} = P_{aC} \sin \delta = 203.2 \sin 15^\circ = 52.6 \text{ kN}$$

Surcharge

$$F_x = K_a q_s H_o \cos \delta = 0.41 \times 20 \times 7.42 \times \cos 15^\circ = 58.8 \text{ kN}$$

$$F_z = K_a q_s H_o \sin \delta = 0.41 \times 20 \times 7.42 \times \sin 15^\circ = 15.7 \text{ kN}$$

Resultant force components

$$P_{ax} = F_{ax} + F_x = 196.3 + 58.8 = 255.1 \text{ kN}$$

$$P_{az} = F_{az} + F_z = 52.6 + 15.7 = 68.3 \text{ kN}$$

Step 2: Determine the resultant vertical force per unit length and its location.

A table is useful to keep the calculation tidy and easy to check.

Part	Force (kN/m)	Moment arm from toe (m)	Moment (+kN · m)
1	$0.5 \times 0.42 \times 3 \times 18 = 11.3$	3.80	42.9
2	$3 \times 6.1 \times 18 = 329.4$	3.30	1087.0
3	$0.4 \times 6.1 \times 23.5 = 57.3$	1.60	91.7
4	$0.5 \times 0.36 \times 6.1 \times 23.5 = 25.8$	1.28	33.0
5	$0.9 \times 4.8 \times 23.5 = 101.5$	2.40	243.6
6	$3 \times 20 = 60.00$	3.3	198.00
	585.3		1696.2 (+)
P_{az}	68.3	4.8	327.9(+)
	$R_z = 653.6$		$\Sigma \text{ Moments} = 2024.1(+)$
P_{ax}	255.1	2.75	701.5(-)
			$\Sigma M_o = 1322.6(+)$

The location of the resultant horizontal component of force from the toe is

$$\bar{z} = \frac{F_{ax} \frac{H_o}{3} + F_x \frac{H_o}{2}}{F_{ax} + F_x} = \frac{196.3 \times \frac{7.42}{3} + 58.8 \times \frac{7.42}{2}}{196.3 + 58.8} = 2.75 \text{ m}$$

The location of the resultant vertical component of force from the toe is

$$\bar{x} = \frac{\Sigma M_o}{R_z} = \frac{1322.6}{653.6} = 2.02$$

Step 3: Determine the eccentricity.

$$e = \frac{B}{2} - \bar{x} = \frac{4.8}{2} - 2.02 = 0.38 \text{ m}$$

Step 4: Determine the stability.

Rotation

$$\frac{B}{6} = \frac{4.8}{6} = 0.8 \text{ m} > e (= 0.38 \text{ m}); \text{ therefore rotation is satisfactory}$$

Translation

$$T = R_z \tan \phi'_b = 656 \times \tan 25^\circ = 305.9 \text{ kN/m}$$

$$(\text{FS})_T = \frac{T}{P_{ax}} = 1.2 < 1.5; \quad \text{therefore, translation is not satisfactory}$$

In design, you can consider placing a key at the base to increase the factor of safety against translation.

Bearing capacity

$$\sigma_{\max} = \frac{656}{4.8 \times 1} \left(1 + \frac{6 \times 0.38}{4.8} \right) = 201.5 \text{ kPa}$$

$$B' = B - 2e = 4.8 - 2 \times 0.38 = 4.06 \text{ m}$$

$$H = P_{ax} = 255.1 \text{ kN}; \quad V_n = R_z = 653.6 \text{ kN}, \quad \frac{H}{V_n} = \frac{255.1}{653.6} = 0.39;$$

$$\omega = \tan^{-1} \frac{H}{V_n} = \tan^{-1}(0.39) = 21.3^\circ$$

$$n = \left(2 + \frac{B'}{L'} \right) / \left(1 + \frac{B'}{L'} \right) = 2; \quad i_\gamma = \left(1 - \frac{H}{V_n} \right)^{n+1} = (1 - 0.39)^{2+1} = 0.23$$

$$N_\gamma = 0.1054 \exp(9.6\phi'_p) = 0.1054 \exp\left(9.6 \times 35 \times \frac{\pi}{180}\right) = 37.2$$

$$q_u = 0.5\gamma B' N_\gamma i_\gamma = 0.5 \times 19 \times 4.06 \times 37.2 \times 0.23 = 330 \text{ kPa}$$

$$(\text{FS})_B = \frac{q_u}{(\sigma_{\max})} = \frac{330}{201.5} = 1.6 < 3$$

Therefore, bearing capacity is not satisfactory.

What's next... In the next section, we will study how to determine the stability of flexible retaining walls.

12.10 STABILITY OF FLEXIBLE RETAINING WALLS



Concept Learning and Self-Assessment

Access Chapter 12, Section 12.10 on the CD to learn about flexible retaining walls. Take Quiz 12.10 to assess your understanding.

Sheet pile walls are flexible and are constructed using steel or thin concrete slabs or wood. Two types of sheet pile walls are common. One is a cantilever wall commonly used to support soils to a height of less than 3 m (Fig. 12.14). The other is an anchored or propped sheet pile wall (Fig. 12.14) commonly used to support deep excavations and as waterfront retaining structures. Cantilever sheet pile walls rely on the passive soil resistance for their stability, while anchored sheet pile walls rely on a combination of anchors and passive soil resistance for their stability. The stability of sheet pile walls must satisfy all the criteria for rigid retaining walls described in Section 12.9 except bearing capacity. However, because sheet pile walls are used in situations where seepage may occur, it is necessary to pay particular attention to seepage-related instabilities.

12.10.1 Analysis of Sheet Pile Walls in Uniform Soils

In analyzing sheet pile walls, we are attempting to determine the depth of embedment, d , for stability. The analysis is not exact and various simplifications are made. The key static equilibrium condition is moment equilibrium. Once we determine d , the next step is to determine the size of the wall. This is

done by calculating the maximum bending moment and then determining the section modulus by dividing the maximum bending moment by the allowable bending stress of the material constituting the sheet pile, for example, steel, concrete, or wood.

An effective stress analysis is generally used to analyze sheet pile walls and as such we must evaluate the porewater pressure distribution and seepage pressures. We can use flow net sketching or numerical methods to determine the porewater pressure distribution and seepage pressures. However, approximate methods are often used in practice. If the groundwater level on both sides of a sheet pile wall is the same, then the resultant porewater pressures and seepage pressures are zero (Fig. 12.19a). You can then neglect the effects of groundwater in determining the stability of sheet pile walls. However, you must use effective stresses in your calculations of the lateral earth forces.

The approximate distribution of porewater pressures in front of and behind sheet pile walls for conditions in which the water tables are different is obtained by assuming a steady state seepage condition and uniform distribution of the total head. Approximate resultant porewater pressure distributions for some common conditions (Padfield and Mair, 1984) are shown in Fig. 12.19.

The maximum porewater pressures (u_B), maximum porewater forces (P_w) and their locations ($\bar{z}_w$), and the seepage force per unit volume (j_s) are as follows:

Case (a)—Fig. 12.19a

Resultant porewater pressure is zero and the seepage force is zero.

Case (b)—Fig. 12.19b

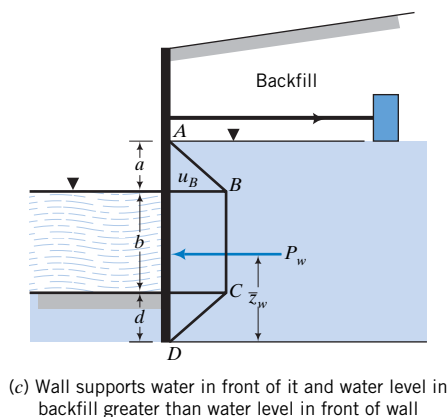
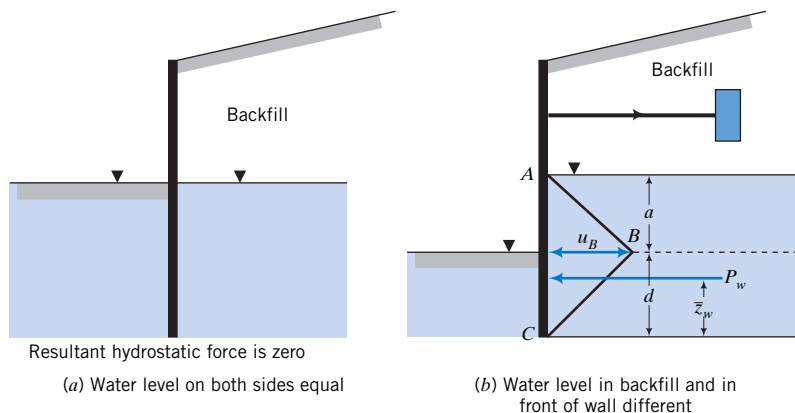


FIGURE 12.19 Approximate resultant porewater pressure distribution behind flexible retaining walls.

$$u_B = \frac{2ad}{a+2d} \gamma_w \quad (12.50)$$

$$P_w = \frac{ad(a+d)}{a+2d} \gamma_w \quad (12.51)$$

$$\bar{z}_w = \frac{a+2d}{3} \quad (10.52)$$

$$j_s = \frac{a}{a+2d} \gamma_w \quad (10.53)$$

Case (c)—Fig. 12.19c

$$u_B = u_C = \frac{a(b+2d)}{a+b+2d} \gamma_w \quad (12.54)$$

$$P_w = \frac{1}{2} \left[\frac{a(b+2d)(a+2b+d)}{a+b+2d} \right] \gamma_w \quad (12.55)$$

$$\bar{z}_w = \frac{a^2 + 3a(b+d) + 3b(b+2d) + 2d^2}{3(a+2b+d)} \quad (12.56)$$

$$j_s = \frac{a}{a+b+2d} \gamma_w \quad (12.57)$$

Recall that j_s is the seepage pressure per unit volume and the resultant effective stress is increased when seepage is downward (behind the wall) and is decreased when seepage is upward (in front of the wall) as discussed in Chapter 5.

12.10.2 Analysis of Sheet Pile Walls in Mixed Soils

Sheet pile walls may penetrate different soil types. For example, Fig. 12.20 shows a sheet pile wall that supports a coarse-grained soil but is embedded in a fine-grained soil. In this case, you should consider a mixed analysis. For short-term condition, an effective stress analysis can be used for the coarse-grained

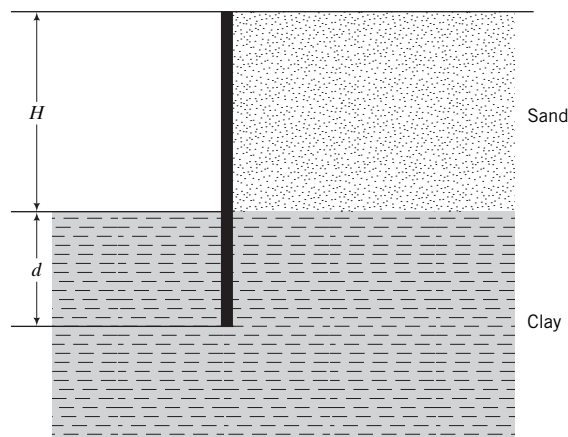


FIGURE 12.20 Sheet pile wall penetrating different soils.

soil but a total stress analysis should be used for the fine-grained soil. For long-term conditions, an effective stress analysis should be carried out for both soil types.

12.10.3 Consideration of Tension Cracks in Fine-Grained Soils

If a sheet pile wall supports fine-grained soils, you should consider the formation of tension cracks. The theoretical depth of a tension crack is

$$z_{cr} = \frac{2s_u}{\gamma} \quad \text{or} \quad \frac{2s_u}{\gamma'} \quad (12.58)$$

The latter is applicable when the tension cracks are filled with water. The depth of a tension crack is sometimes greater than the wall height. In this situation, you can assume a minimum active lateral effective pressure of $5z$ (kPa) as suggested by Padfield and Mair (1984), where z is the depth measured from the top of the wall. When water fills the tension cracks, you should apply the full hydrostatic pressure to the wall over a depth equivalent to the depth of the tension crack or the wall height, whichever is smaller. In fine-grained soils, loss of moisture at the wall-soil interface can cause the soil to shrink, creating a gap. This gap can be filled with water. In this case, you should apply the full hydrostatic pressure over the depth of the wall.

12.10.4 Methods of Analyses

Several methods have been proposed to determine the stability of sheet pile walls. These methods differ in the way the lateral stresses are distributed on the wall and the way the factor of safety is applied in solving for the embedment depth. We will discuss three methods in this book. In the first method, called the factored moment method (FMM), you would determine an embedment depth to satisfy moment equilibrium by applying a factor of safety (FS)_p on the passive resistance. The factor of safety, (FS)_p, is usually between 1.5 and 2.0.

In the second method, called the factored strength method (FSM), reduction factors are applied to the shear strength parameters. These reduction factors are called mobilization factors because they are intended to limit the shear strength parameters to values that are expected to be mobilized by the design loads. A mobilization factor, F_ϕ , is applied to the friction angle, ϕ'_{cs} , and a mobilization factor, F_u , is applied to s_u . The application of these mobilization factors results in a higher active pressure and a lower passive pressure than the unfactored soil strength parameters. The design parameters are

$$\phi'_{\text{design}} = \frac{\phi'_{cs}}{F_\phi} \quad (12.59)$$

and

$$(s_u)_{\text{design}} = \frac{s_u}{F_u} \quad (12.60)$$

where

$$F_\phi = 1.2 \text{ to } 1.5 \quad \text{and} \quad F_u = 1.5 \text{ to } 2$$

The results from the FSM are sensitive to F_ϕ and F_u .

The third method, called the net passive pressure method (NPPM), utilizes a net available passive resistance (Burland et al., 1981). A vertical line is drawn from the active pressure at the

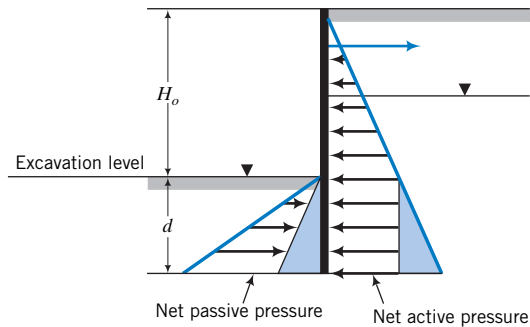


FIGURE 12.21 Net pressures for the NPPM.

excavation level to the base of the wall (Fig. 12.21). The shaded region of pressure on the active side (Fig. 12.21) is subtracted from the passive pressure to give the net passive pressure shown by the area hatched by arrows. The factor of safety for the NPPM is

$$(FS)_r = \frac{\Sigma \text{ Moments of net available passive resistance}}{\Sigma \text{ Moments of lateral forces causing rotation}};$$

$$(FS)_r = 1.5 \text{ to } 2 \text{ with } 2 \text{ most often used}$$

The flexibility of sheet pile walls leads to lateral pressure distributions that do not correspond to the Rankine active and passive states. Expected lateral pressure distributions on a “stiff” flexible wall and a less “stiff” flexible wall are shown in Fig. 12.22. The net effect of the distribution of the lateral pressures as illustrated in Fig. 12.22 is a reduction in maximum bending moment from that calculated using Rankine active and passive states.

Rowe (1957) developed a method, based on laboratory tests, to reduce the calculated maximum bending moment to account for the effects of wall flexibility on the bending moment. Rowe’s moment reduction is applicable when a factor of safety has been applied on the passive resistance as in the FMM. There is some debate on the applicability of Rowe’s method. Some engineers prefer to calculate

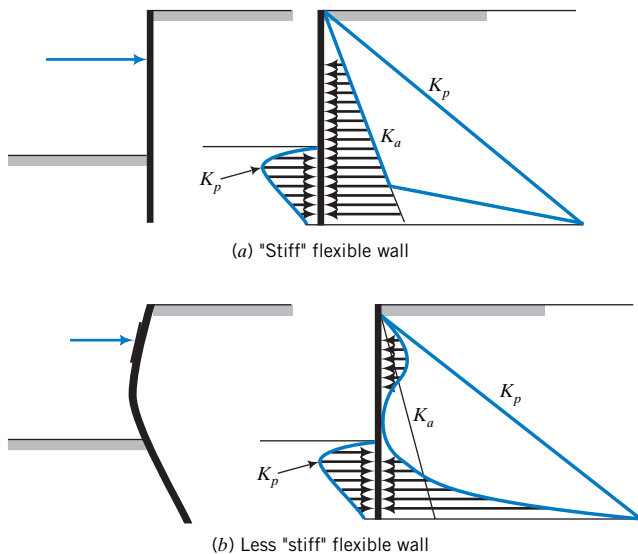


FIGURE 12.22 Lateral pressure distributions expected on a “stiffer” flexible wall and less “stiff” flexible wall. (After Padfield and Mair, 1984.)

the maximum bending moment at limit equilibrium ($(FS)_p = F_\phi = (FS)_r = F_u = 1$) and use it as the design moment. This is the preferred method in this book.

To account for soil-wall interface friction, you need to use K_{aC} and K_{pC} . However, the active and passive coefficients derived by Caquot and Kerisel (1948) are regarded as more accurate than those of Coulomb. For a flexible retaining wall, only the horizontal components of the lateral forces are important. In Appendix D, the horizontal components of the active and passive coefficients of Caquot and Kerisel (1948) as tabulated by Kerisel and Absi (1990) are plotted for some typical backfill slopes and soil-wall interface friction angles. We will use the values of the lateral earth pressure coefficients in Appendix D in some of the example problems in this chapter.

12.10.5 Analysis of Cantilever Sheet Pile Walls

Cantilever sheet pile walls are analyzed by assuming that rotation occurs at some point, O , just above the base of the wall (Fig. 12.23). The consequence of assuming rotation above the base is that, below the point of rotation, the lateral pressure is passive behind the wall and active in front of the wall (Fig. 12.23b). To simplify the analysis, a force R (Fig. 12.23c) is used at the point of rotation to approximate the net passive resistance below it (the point of rotation). By taking moments about O , the unknown force R is eliminated and we then obtain one equation with one unknown, that is, the unknown depth, d_o . To account for this simplification, the depth d_o is increased by 20% to 30% to give the design embedment depth, d .

The general procedure for determining d for stability and to determine the wall size is as follows:

1. Arbitrarily select a point O at a distance d_o from the excavation level.
2. Calculate the active and passive earth pressures using the FMM or FSM or NPPM.
3. Calculate the net porewater pressure (u) distribution and the seepage force per unit volume (j_s). The effective unit weight is increased by j_s in the active zone and is decreased by j_s in the passive zone.
4. Determine the unknown depth d_o by summing moments about O .
5. Calculate d by increasing d_o by 20% or 30% to account for simplifications made in the analysis. The depth of penetration d is therefore $1.2d_o$ or $1.3d_o$.
6. Calculate R by summing forces horizontally over the depth $(H_o + d)$.
7. Calculate the net passive resistance, $(P_p)_{\text{net}}$, over the distance, $d - d_o$, below O .
8. Check that R is less than $(P_p)_{\text{net}}$. If not, extend the depth of embedment and recalculate R .
9. Calculate the maximum bending moment (M_{max}) over the depth $(H_o + d_o)$ using unfactored passive resistance (FMM), unfactored strength values (FSM), and $(FS)_r = 1$ (NPPM).

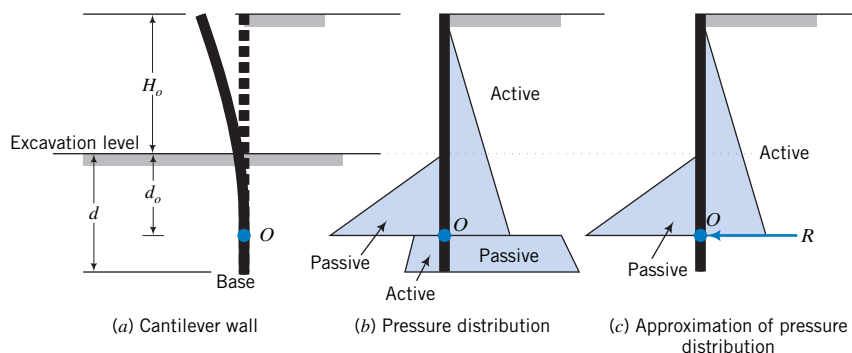


FIGURE 12.23 Approximation of pressure distributions in the analysis of cantilever flexible retaining walls (Padfield and Mair, 1984).

10. Determine the section modulus, $S_x = M_{\max}/f_a$, where $M_{\max}$ is the maximum bending moment and f_a is the allowable bending stress of the wall material.

12.10.6 Anchored Sheet Pile Walls

There are two methods used to analyze anchored sheet pile walls. One is the free earth method, the other is the fixed earth method. We will be discussing the free earth method, because it is frequently used in design practice.

In the free earth method, it is assumed that (1) the depth of embedment of the wall is insufficient to provide fixity at the bottom end of the wall and (2) rotation takes place about the point of attachment of the anchor, O (Fig. 12.24a). The expected bending moment diagram, based on the above assumptions, is depicted in Fig. 12.24b.

The procedure for analyzing an anchored sheet pile wall is as follows:

1. Assume a depth of embedment, d .
2. Calculate the active and passive pressures using the FMM or FSM or NPPM.
3. Calculate the net porewater pressure (u) distribution and the seepage force per unit volume (j_s). The effective unit weight is increased by j_s in the active zone and is decreased by j_s in the passive zone.
4. Determine d by taking moments about the point of attachment of the anchor, O . Usually, you will get a cubic equation, which you can solve by iteration or by using a goal seek option in a spreadsheet program or by using a polynomial function on a calculator.
5. Recalculate d using unfactored passive resistance (FMM), unfactored strength (FSM), and $(FS)_r = 1$ (NPPM). Use this recalculated depth to determine the anchor force and the maximum bending moment.
6. Determine the anchor force per unit length of wall, T_a , by summing forces in the horizontal direction. The anchor force, T_a , is multiplied by a factor of safety $(FS)_a$, usually 2.
7. Determine the location of the anchor plates or deadmen. Let d_z be the depth of the bottom of the anchor plate from the ground surface (Fig. 12.25). The force mobilized by the anchor plate must balance the design anchor force, that is,

$$\frac{1}{2} \gamma' d_z^2 (K_p - K_a) = T_a \times (FS)_a$$

Solving for d_z , we get

$$d_z = \sqrt{\frac{2T_a \times (FS)_a}{\gamma' (K_p - K_a)}} \quad (12.61)$$

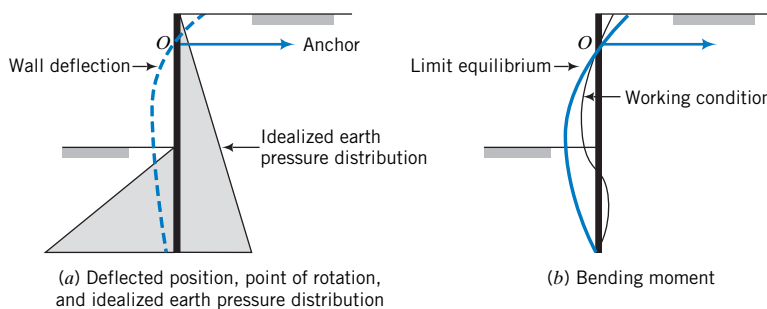


FIGURE 12.24 Free earth conditions for anchored retaining walls (Padfield and Mair, 1984).

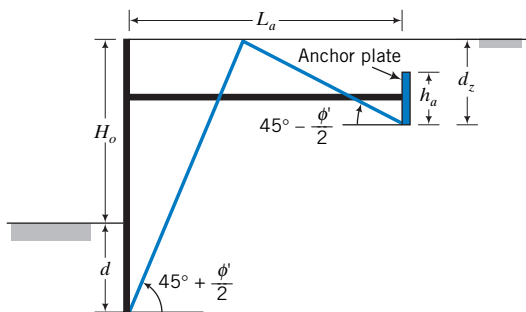


FIGURE 12.25 Location of anchor plates.

A passive wedge develops in front of the anchor plate and an active wedge develops behind the retaining wall. The anchor plate must be located outside the active slip plane. The minimum anchor length (L_a) of the anchor rod, with reference to Fig. 12.25, is

$$L_a = (H_o + d) \tan(45^\circ - \phi'/2) + d_z \tan(45^\circ + \phi'/2) \quad (12.62)$$

8. Calculate the spacing of the anchors. Let s be the longitudinal spacing of the anchors and h_a be the height of the anchor plate. If $h_a \geq d_z/2$, the passive resistance of the anchor plate is assumed to be developed over the full depth d_z . From static equilibrium of forces in the horizontal direction, we obtain

$$s = \frac{\gamma' d_z^2 L_a}{2 T_a (\text{FS})_a} (K_p - K_a) \quad (12.63)$$

9. Calculate the maximum bending moment ($M_{\max}$) using the embedment depth at limit equilibrium (unfactored passive resistance, unfactored strength values, or $(\text{FS})_r = 1$).
10. Determine the section modulus, $S_x = M_{\max}/f_a$, where f_a is the allowable bending stress of the wall material.

EXAMPLE 12.5 Cantilever Flexible Wall

Computer Program Utility

Access Chapter 12 on the CD and click Spreadsheet on the sidebar to view examples of retaining wall analysis using a spreadsheet. You can change input values and explore how the stability of the walls changes.

Determine the depth of embedment required for stability of the cantilever sheet pile wall shown in Fig. E12.5a. Compare the results of the three methods—FMM, FSM, and NPPM—using $(\text{FS})_p = 2.0$, $F_\phi = 1.25$, and $(\text{FS})_r = 1.5$. Calculate the maximum bending moment for each of these

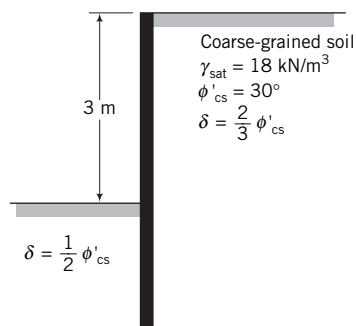


FIGURE E12.5a

methods. Groundwater is below the base of the wall. The tolerable rotation is $0.005 H_o$ and less, where H_o is the wall height.

Strategy You should use the Kerisel and Absi (1990) passive pressures (see Appendix D) and either the Kerisel and Absi (1990) or Coulomb active pressures. The key is to determine the lateral forces and then find d_o (an arbitrarily selected embedment depth at which rotation is presumed to occur) using moment equilibrium. Since you have to find the depth for unfactored values to calculate the maximum bending moment, you should determine K_{ax} and K_{px} for factored and unfactored values at the very beginning of your solution.

Solution 12.5

Step 1: Calculate K_{ax} and K_{px} .

$$\text{For FSM: } \phi'_{\text{design}} = \frac{\phi'_{\text{cs}}}{F_\phi} = \frac{30}{1.25} = 24^\circ$$

$$\text{For FMM and NPPM: } \phi'_{\text{design}} = \phi'_{\text{cs}}$$

Use the Kerisel and Absi (1990) K_{ax} and K_{px} (Appendix D).

$$\text{FSM: } K_{ax} = 0.36 \quad (\phi'_{\text{design}} = 24^\circ, \delta = \frac{2}{3}\phi'_{\text{design}}, \beta/\phi'_{\text{design}} = 0)$$

$$K_{px} = 3.3 \quad (\phi'_{\text{design}} = 24^\circ, \delta = \frac{1}{2}\phi'_{\text{design}}, \beta/\phi'_{\text{design}} = 0)$$

$$\text{FMM and NPPM: } K_{ax} = 0.28 \quad (\phi'_{\text{cs}} = 30^\circ, \delta = \frac{2}{3}\phi'_{\text{cs}}, \beta/\phi'_{\text{cs}} = 0)$$

$$K_{px} = 4.6 \quad (\phi'_{\text{cs}} = 30^\circ, \delta = \frac{1}{2}\phi'_{\text{design}}, \beta/\phi'_{\text{cs}} = 0)$$

Step 2: Determine the lateral earth pressure distributions.

The lateral pressure distributions for the FMM and FSM have the same shape but different magnitudes because of the different lateral earth pressure coefficients (Fig. E12.5b). The lateral pressure distribution for the NPPM is shown in Fig. E12.5c. Since groundwater is not within the depth of the retaining wall, $\gamma' = \gamma_{\text{sat}} = \gamma$.

With reference to Fig. E12.5b:

Active case

$$P_{ax} = \frac{1}{2} K_{ax} \gamma (H_o + d_o)^2 = \frac{1}{2} \times K_{ax} \times 18(3 + d_o)^2 = 9K_{ax}(3 + d_o)^2$$

$$\bar{z}_a = \frac{H_o + d_o}{3} = \frac{3 + d_o}{3}$$

$$(M_O)_a = P_{ax} \bar{z}_a = 9K_{ax}(3 + d_o)^2 \left(\frac{3 + d_o}{3} \right) = 3K_{ax}(3 + d_o)^2$$

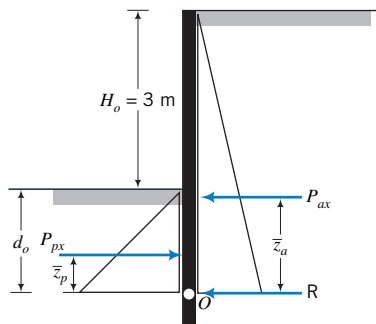


FIGURE E12.5b

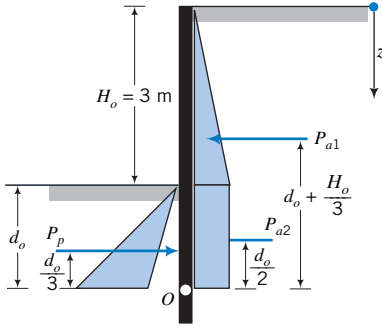


FIGURE E12.5c

Passive case

$$P_{px} = \frac{1}{2} K_{px} \gamma d_o^2 = \frac{1}{2} \times K_{px} \times 18 \times d_o^2 = 9K_{px} d_o^2$$

$$\bar{z}_p = \frac{d_o}{3}$$

$$(M_o)_p = P_{px} \bar{z}_p = 9K_{px} d_o^2 \times \frac{d_o}{3} = 3K_{px} d_o^3$$

Step 3: Find d_o .

All forces are calculated per meter length of wall.

FMM

The passive pressure is factored by $(FS)_p$.

$$K_{ax} = 0.28 \quad \text{and} \quad K_{px} = 4.6$$

$$(M_o)_a = 3 \times 0.28(3 + d_o)^3 = 0.84(3 + d_o)^3$$

$$(M_o)_p = \frac{3 \times 4.6 d_o^3}{(FS)_p} = \frac{3 \times 4.6 d_o^3}{2} = 6.9 d_o^3$$

$$\text{For equilibrium: } (M_o)_a = (M_o)_p$$

$$\therefore 0.84(d_o^3 + 9d_o^2 + 27d_o + 27) = 6.9d_o^3$$

which simplifies to

$$7.21d_o^3 - 9d_o^2 - 27d_o - 27 = 0$$

By trial and error or by using the polynomial function on a calculator, $d_o = 2.95$ m.

FSM

$$K_{ax} = 0.36 \quad \text{and} \quad K_{px} = 3.3$$

$$(M_o)_a = 3 \times 0.36(3 + d_o)^3 = 1.08(3 + d_o)^3$$

$$(M_o)_p = 3 \times 3.3 d_o^3 = 9.9 d_o^3$$

$$\text{For equilibrium: } (M_o)_a = (M_o)_p$$

$$1.08(d_o^3 + 9d_o^2 + 27d_o + 27) = 9.9 d_o^3$$

which simplifies to

$$8.17d_o^3 - 9d_o^2 - 27d_o - 27 = 0$$

By trial and error or by using the polynomial function on a calculator, $d_o = 2.75$ m.

NPPM

$$K_{ax} = 0.28 \quad \text{and} \quad K_{px} = 4.6$$

The pressure diagram for the NPPM is shown in Fig. E12.5c.

$$P_{a1} = \frac{1}{2} \times K_{ax} \gamma H_o^2 = \frac{1}{2} \times 0.28 \times 18 \times 3^2 = 22.7 \text{ kN}$$

$$P_{a2} = K_{ax} \gamma H_o d_o = 0.28 \times 18 \times 3 \times d_o = 15.1 d_o$$

$$P_p = \frac{1}{2} (K_{px} - K_{ax}) \gamma d_o^2 = \frac{1}{2} (4.5 - 0.28) \times 18 d_o^2 = 38.9 d_o^2$$

$$(M_o)_a = P_{a1} \left(d_o + \frac{H_o}{3} \right) + P_{a2} \frac{d_o}{2} = 7.6 d_o^2 + 22.7 d_o + 22.7$$

$$(M_o)_p = P_p \frac{d_o}{3} = 13 d_o^3$$

$$(FS)_r = \frac{(M_o)_p}{(M_o)_a}$$

For $(FS)_r = 1.5$,

$$1.5(7.6 d_o^2 + 22.7 d_o + 22.7) = 13 d_o^3$$

Rearranging, we get

$$7.67 d_o^3 - 7.6 d_o^2 - 22.7 d_o - 22.7 = 0$$

By trial and error or by using the polynomial function on a calculator, $d_o = 2.41 \text{ m}$.

Step 4: Calculate the design depth.

$$\text{FMM: } d = 1.2 d_o = 1.2 \times 2.95 = 3.54 \text{ m}$$

$$\text{FSM: } d = 1.2 d_o = 1.2 \times 2.75 = 3.3 \text{ m}$$

$$\text{NPPM: } d = 1.2 d_o = 1.2 \times 2.41 = 2.89 \text{ m}$$

Step 5: Determine R .

$$R = P_{px} - P_{ax}$$

$$\text{FMM: } d_o = 2.95 \text{ m; } R = 9 \times 4.5 \times 2.95^2 - 9 \times 0.28 \times (3 + 2.95)^2 = 271.1 \text{ kN/m}$$

$$\text{FSM: } d_o = 2.75 \text{ m; } R = 9 \times 3.3 \times 2.75^2 - 9 \times 0.36 \times (3 + 2.75)^2 = 109.4 \text{ kN/m}$$

$$\begin{aligned} \text{NPPM: } d_o = 2.41 \text{ m; } R &= 9 \times (4.6 - 0.28) \times 2.41^2 \\ &\quad - (9 \times 0.28 \times 3^2 + 0.28 \times 18 \times 3 \times 2.41) = 166.7 \text{ kN/m} \end{aligned}$$

To calculate the net resistance below the assumed point of rotation, O , calculate the average passive pressure at the back of the wall and the average active pressure in front of the wall. Remember that below the point of rotation, passive pressure acts at the back of the wall and active pressure acts at the front of the wall. The mid-depth between d_o and $1.2 d_o$ is $1.1 d_o$.

FMM

$$\begin{aligned} \text{Average passive lateral pressure} &= K_{px} \gamma (H_o + 1.1 d_o) = 4.6 \times 18 \\ &\quad \times (3 + 1.1 \times 2.95) = 517.1 \text{ kPa} \end{aligned}$$

$$\begin{aligned} \text{Average active lateral pressure} &= K_{ax} \gamma \times 1.1 d_o = 0.28 \times 18 \times 1.1 \times 2.95 \\ &= 16.4 \text{ kPa} \end{aligned}$$

$$\text{Net lateral pressure} = 517.1 - 16.4 = 500.7 \text{ kPa}$$

$$\text{Net force} = 500.7 \times 0.2 d_o = 295.4 \text{ kN} > R (= 271.1 \text{ kN});$$

therefore, depth of penetration is satisfactory

FSM

$$\begin{aligned}\text{Average passive lateral pressure} &= K_{px}\gamma(H_o + 1.1d_o) = 3.3 \times 18 \\ &\quad \times (3 + 1.1 \times 2.75) = 357.9 \text{ kPa}\end{aligned}$$

$$\begin{aligned}\text{Average active lateral pressure} &= K_{ax}\gamma \times 1.1d_o = 0.36 \times 18 \times 1.1 \times 2.75 \\ &= 19.6 \text{ kPa}\end{aligned}$$

$$\text{Net lateral pressure} = 357.9 - 19.6 = 338.3 \text{ kPa}$$

$$\text{Net force} = 338.3 \times 0.2d_o = 186.1 \text{ kN} > R (= 109.4 \text{ kN});$$

therefore, depth of penetration is satisfactory

NPPM

$$\begin{aligned}\text{Average passive lateral pressure} &= K_{px}\gamma(H_o + 1.1d_o) = 4.6 \times 18 \\ &\quad \times (3 + 1.1 \times 2.41) = 467.9 \text{ kPa}\end{aligned}$$

$$\begin{aligned}\text{Average active lateral pressure} &= K_{ax}\gamma \times 1.1d_o = 0.28 \times 18 \times 1.1 \times 2.41 \\ &= 13.4 \text{ kPa}\end{aligned}$$

$$\text{Net lateral pressure} = 467.9 - 13.4 = 454.5 \text{ kPa}$$

$$\text{Net force} = 454.5 \times 0.2d_o = 219.1 \text{ kN} > R (= 166.7 \text{ kN});$$

therefore, depth of penetration is satisfactory

Step 6: Determine the maximum bending moment.

$$\text{Maximum bending moment for (FS)}_p = F_\phi = 1.$$

Let z be the location of the point of maximum bending moment (point of zero shear) such that $z > H_o$.

$$\begin{aligned}M_z &= \frac{1}{2}K_{ax}\gamma z^2 \times \frac{z}{3} - \frac{1}{2}K_{px}\gamma(z - H_o)^2 \times \frac{(z - H_o)}{3} \\ &= \frac{1}{2} \times 4.6 \times 18 \times \frac{(z - 3)^3}{3} \\ &= 0.84z^3 - 13.8(z - 3)^3\end{aligned}$$

To find z at which the bending moment is maximum, we need to differentiate the above equation with respect to z and set the result equal to zero.

$$\frac{dM_z}{dz} = 0 = 2.52z^2 - 41.4(z - 3)^2 = 38.9z^2 - 248.4z + 372.6 = 0$$

Solving for z , we get $z = 3.68 \text{ m}$ or 2.53 m . The correct answer is 3.68 m since zero shear cannot occur above the excavation level in this problem (positive shear in the active zone only gets reduced below the excavation level).

$$M_z = 0.84 \times 3.68^3 - 13.8(3.68 - 3)^3 = 36 \text{ kN} \cdot \text{m}$$

For this problem, it is easy to use calculus to determine the maximum bending moment. For most problems, you will have to find the shear force distribution with depth, identify or calculate the point of zero shear, and then calculate the maximum bending moment. ■

EXAMPLE 12.6 Anchored Flexible Wall

Determine the embedment depth and the anchor force of the tied-back wall shown in Fig. E12.6a using the FSM.

Strategy You should use the Kerisel and Absi (1990) passive pressures (see Appendix D) and either the Kerisel and Absi (1990) or Coulomb active pressures. Groundwater level on both sides of the wall is the same, so seepage will not occur.

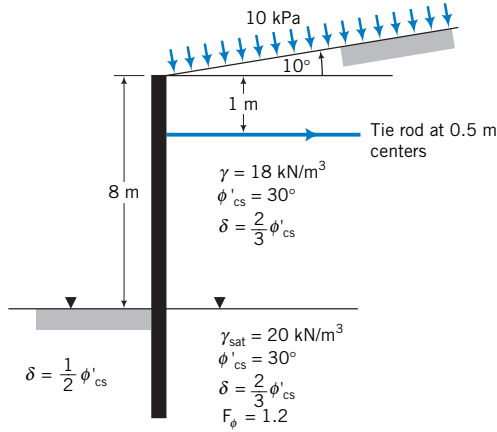


FIGURE E12.6a

Solution 12.6**Step 1:** Determine K_{ax} and K_{px} .

$$\phi'_{\text{design}} = \frac{\phi'_{cs}}{F_\phi} = \frac{30}{1.2} = 25^\circ; \quad \frac{\beta}{\phi'_{\text{design}}} = \frac{10}{25} = 0.4$$

From Appendix D,

$$K_{ax} = 0.42 \quad (\phi'_{\text{design}} = 25^\circ, \beta/\phi'_{\text{design}} = 0.4, \delta = \frac{2}{3}\phi'_{\text{design}})$$

$$K_{px} = 3.4 \quad (\phi'_{\text{design}} = 25^\circ, \beta/\phi'_{\text{design}} = 0, \delta = \frac{1}{2}\phi'_{\text{design}})$$

For unfactored strength values,

$$K_{ax} = 0.31 \quad (\phi'_{cs} = 30^\circ, \beta/\phi'_{cs} = 10/30 = \frac{1}{3}, \delta = \frac{2}{3}\phi'_{cs})$$

$$K_{px} = 4.6 \quad (\phi'_{cs} = 30^\circ, \beta/\phi'_{cs} = 0, \delta = \frac{1}{2}\phi'_{cs})$$

Step 2: Determine the lateral forces and moments.

Use a table to facilitate ease of computation and checking. (See Fig. E12.6b.) Below the groundwater level, $\gamma' = 20 - 9.8 = 10.2 \text{ kN/m}^3$.

Part	Horizontal force (kN)	Moment arm from anchor (m)	Moment (kN · m) + ↻
1	$K_{ax} q_s (H_o + d) =$ $0.42 \times 10 \times (8 + d) = 4.2d + 33.6$	$\left(\frac{H_o + d}{2}\right) - h =$ $\left(\frac{8 + d}{2}\right) - 1 = 3 + \frac{d}{2}$	$-(2.1d^2 + 29.4d + 100.8)$
2	$\frac{1}{2} K_{ax} \gamma H_o^2 =$ $\frac{1}{2} \times 0.42 \times 18 \times 8^2 = 241.9$	$\frac{2}{3} H_o - h =$ $\frac{2}{3} \times 8 - 1 = 4.33$	-1047.4
3	$K_{ax} \gamma H_o d =$ $0.42 \times 18 \times 8 \times d = 60.5d$	$H_o - h + \frac{d}{2} = 7 + \frac{d}{2}$	$-(30.3d^2 + 423.5d)$
4	$\frac{1}{2} K_{ax} \gamma' d^2 =$ $\frac{1}{2} \times 0.42 \times 10.2 \times d^2 = 2.1d^2$ $\sum 2.1d^2 + 64.7d + 275.5$	$H_o - h + \frac{2}{3}d = 7 + \frac{2}{3}d$	$-(1.4d^3 + 14.7d^2)$
5	$\frac{1}{2} K_{px} \gamma' d^2 =$ $\frac{1}{2} \times 3.4 \times 10.2 \times d^2 = 17.3d^2$	$H_o - h + \frac{2}{3}d = 7 + \frac{2}{3}d$	$11.5d^3 + 121.1d^2$
			$\sum M = -(10.1d^3 + 74d^2 + 452.9d^2 + 1148.2)$

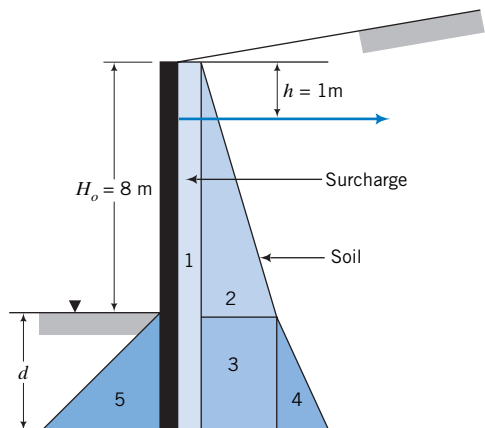


FIGURE E12.6b

Step 3: Determine d .

Equate the sum of moments to zero (simplify equation by dividing by the coefficient of d^3).

$$d^3 + 7.3d^2 - 44.8d - 113.7 = 0$$

By trial and error or by using the polynomial function on a calculator, $d = 5.38$ m.

Step 4: Determine d for the unfactored strength values.

To calculate the new depth of penetration for unfactored strength values, use proportionality, for example,

Active moment = Active moment for factored strength values

$$\begin{aligned} & \times \frac{\text{Unfactored } K_{ax}}{\text{Factored } K_{ax}} \\ (M_o)_A &= -(1.4d^3 + 47.1d^2 + 452.9d + 1148.2) \times \frac{0.31}{0.42} \\ &= -(d^3 + 34.8d^2 + 334.3d + 847.5) \end{aligned}$$

$$\text{Passive moment: } (M_o)_p = (11.5d^3 + 121.1d^2) \times \frac{4.6}{3.4} = 15.5d^3 + 163.9d^2$$

$$\text{Sum of moments: } (M_o)_p + (M_o)_a = 14.5d^3 + 129.1d^2 - 334.3d - 847.5$$

Solving, we get $d = 3.38$ m

Step 5: Determine the anchor force for $d = 3.38$ m.

$$\Sigma \text{ Active forces} = (2.1d^2 + 64.7d + 275.5) \times \frac{0.31}{0.42} = 381.9 \text{ kN}$$

$$\Sigma \text{ Passive forces} = 17.3d^2 \times \frac{4.6}{3.4} = 265.8 \text{ kN}$$

$$T_a = 381.9 - 265.8 = 116.1 \text{ kN}$$

$$(T_a)_{\text{design}} = (\text{FS})T_a = 2 \times 116.1 = 232.2 \text{ kN}$$

EXAMPLE 12.7 Anchored Flexible Wall

Determine the embedment depth and the design anchor force required for stability of the sheet pile wall shown in Fig. E12.7a using the NPPM.

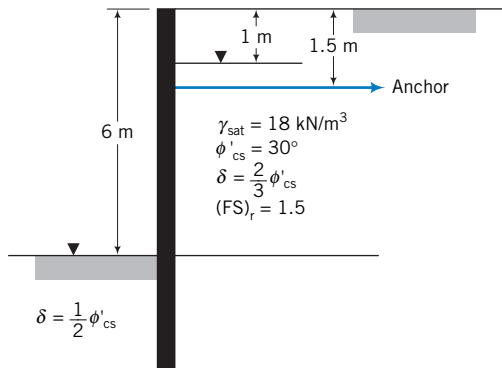


FIGURE E12.7a

Strategy In the NPPM, you must use the unfactored strength values to calculate K_{ax} and K_{px} and then determine the net active and net passive lateral pressures. To calculate d , you have to do iterations. A simple approach to solve for d is to set up the forces and moments in terms of the unknown d and then assume values of d until you find a d value that gives the required factor of safety $((\text{FS})_r \cong 1.5)$. A spreadsheet program or a programmable calculator is very helpful in solving this type of problem. In Excel, for example, you can use the Goal Seek function to find d . It is quite easy to make errors in calculations, so you should recheck your work and you must conduct a “hand” check when using outputs from computer programs. Since the groundwater levels are different in front and behind the wall, you need to consider seepage assuming a steady state seepage condition. To calculate the anchor force, you have to find d for $(\text{FS})_r = 1$ and then multiply the anchor force by 2 (factor of safety).

Solution 12.7

Step 1: Determine K_{ax} and K_{px} .

From Appendix D,

$$K_{ax} = 0.28 \quad (\phi'_{\text{cs}} = 30^\circ, \beta/\phi'_{\text{cs}} = 0, \delta = \frac{2}{3}\phi'_{\text{cs}})$$

$$K_{px} = 4.6 \quad (\phi'_{\text{cs}} = 30^\circ, \beta/\phi'_{\text{cs}} = 0, \delta = \frac{1}{2}\phi'_{\text{cs}})$$

Step 2: Determine the net lateral pressures. Make a table to do the calculations and draw a diagram of the lateral earth pressure distribution.

See Fig. E12.7b.

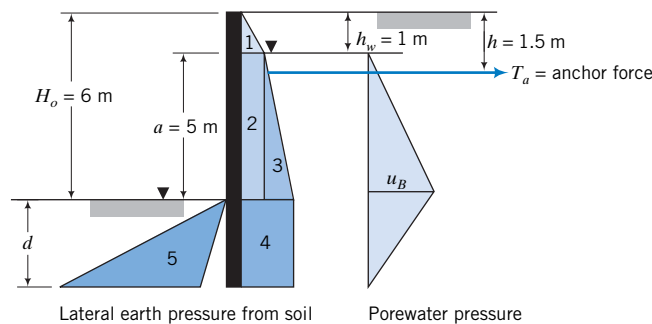


FIGURE E12.7b

Below groundwater level

$$\begin{aligned}\text{Average seepage force/unit volume: } j_s &= \left(\frac{a}{a+2d} \right) \gamma_w \\ &= \left(\frac{5}{5+2d} \right) 9.8 \text{ kN/m}^3\end{aligned}$$

$$\text{Active zone: } \gamma' = \gamma_{\text{sat}} - \gamma_w + j_s = 18 - 9.8 + j_s = 8.2 + j_s \text{ kN/m}^3$$

$$\text{Passive zone: } \gamma' = \gamma_{\text{sat}} - \gamma_w - j_s = 18 - 9.8 - j_s = 8.2 - j_s \text{ kN/m}^3$$

Depth (m)	Active pressure (kPa)	Passive pressure (kPa)
0	0	0
1	$K_a \gamma h_w = 0.28 \times 18 \times 1 = 5$	0
6	$K_a \gamma h_w + K_a (\gamma' + j_s) (H_o - h_w)$ $= 5.0 + 0.28 \times (8.2 + j_s) \times 5 = 16.5 + 1.4j_s$	0
6 + d	$16.8 + 1.4j_s$	$K_p (8.2 - j_s) d - K_a (8.2 + j_s) d$ $= 4.6(8.2 - j_s) d - 0.28(8.2 + j_s) d$ $= (35.4 - 4.88j_s) d$
Water	$u_B = \frac{2(ad)}{a+2d} \gamma_w = \frac{98.1d}{5+2d}$	

Step 3: Calculate the lateral forces, the moment, and $(FS)_r$.

All forces and moments are per meter length of wall. The moment is the sum of moments about the anchor position and R_x is the resultant active lateral force. In the first column under moment, a value of d is guessed and $(FS)_r$ is calculated. In the second column under moment, the value of $d = 5.75$ m was obtained using a spreadsheet program (the actual value obtained from the spreadsheet program is $d = 5.73$ m for $(FS)_r = 1.5$). If you use a spreadsheet program, the first column under moment is not needed. In Excel, you use Tools→Goal Seek to find d to satisfy the desired value of $(FS)_r$.

Part	Forces (kN)	Moment arm (m)	Moment (kN · m)	
			$d = 7$ m	$d = 5.75$ m
1	$0.5 \times 5.0 \times 1 = 2.5$	$h - h_w + \frac{h_w}{3} =$ $0.5 + 0.33 = 0.83$	-2.1	-2.2
2	$5.0 \times 5 = 25$	$\frac{a}{2} - (h - h_w) = 2.5 - 0.5 = 2$	-50.0	-50.0
3	$0.5 \times (11.5 + 1.4j_s) \times 5$ $= 28.8 + 3.8j_s$	$\frac{2a}{3} - (h - h_w) = \frac{10}{3} - 0.5$ $= 2.83$	-109.2	-113.4
4	$(16.5 + 1.4j_s) \times d$ $= (16.5 + 1.4j_s) d$ Water: $\frac{ad(a+d)\gamma_w}{a+2d} = \frac{49d(5+d)}{5+2d}$	$H_o - h - \frac{d}{2} = 4.5 + \frac{d}{2}$ $H_o + d - h - z_w$ $= 4.5 + d - \frac{5+2d}{3} = \frac{8.5+d}{3}$	-1126.2	-876.0
			-1119.3	-871.9
	$R_x = 56.3 + 3.8j_s$ $+(16.5 + 1.4j_s)d + \frac{49d(5+d)}{5+2d}$	$\sum M_d$	-2402.6	-1909.2

(continued)

(Continued)

Part	Forces (kN)	Moment arm (m)	$\sum M_r = \frac{\text{Moment (kN} \cdot \text{m)}}{d = 7 \text{ m} \quad d = 5.75 \text{ m}}$	
5	$0.5 \times (35.4 - 4.88j_s)d^2$ $= (17.7 - 2.4j_s)d^2$	$H_o - h + \frac{2d}{3} = 4.5 + \frac{2d}{3}$	5179.8	2912.8
$(FS)_r = \frac{\sum M_r}{\sum M_d}$			2.16	1.53

Step 4: Calculate the anchor forces for $(FS)_r = 1$.For $(FS)_r = 1$, $d = 4.62$ m. Substituting $d = 4.62$ m, we get**Active zone**

$$j_s = \left(\frac{5}{5 + 2 \times 4.62} \right) 9.8 = 3.44 \text{ kN/m}^3$$

$$R_x = 56.3 + (3.8 \times 3.27) + (16.5 + 1.4 \times 3.44)4.62 + \frac{49 \times 4.62(5 + 4.62)}{5 + 2 \times 4.62}$$

$$= 320.1 \text{ kN}$$

Passive

$$\text{Passive lateral force} = (17.7 - 2.4 \times 3.44)4.62^2 = 201.6 \text{ kN}$$

$$T_a = \text{Active lateral force} - \text{Passive lateral force} = 320.1 - 201.6$$

$$= 118.5 \text{ kN}$$

$$\text{Design anchor force} = 2T_a = 237 \text{ kN}$$

What's next... Sometimes, it is not possible to use an anchored sheet pile wall for an excavation. For example, an existing building near a proposed excavation can preclude the use of anchors. You may have to brace your sheet piles within the excavation using struts. The analysis of braced excavation (also called cofferdam) is presented in the next section.

12.11 BRACED EXCAVATION

Braced excavations consist of sheet piles driven into the soil to form the sides of an excavation (Fig. 12.26a) such as in the construction of bridge piers and abutments. As excavation proceeds within the area enclosed by the sheet piles, struts are added to keep the sheet piles in place.

The top struts are installed followed by others at lower depths. The wall displacements before the top struts are installed are usually very small but get larger as the excavation gets deeper. The largest wall displacement occurs at the base of the excavation (Fig. 12.26a). Wall displacements are inconsistent with all the earth pressure theories.

The critical design elements in a braced excavation are the loads on the struts, which are usually different because of different lateral loads at different depths, the time between excavations, and the installation procedure. Failure of a single strut can be catastrophic because it can lead to the collapse of the whole system. The analysis for the forces and deflection in braced

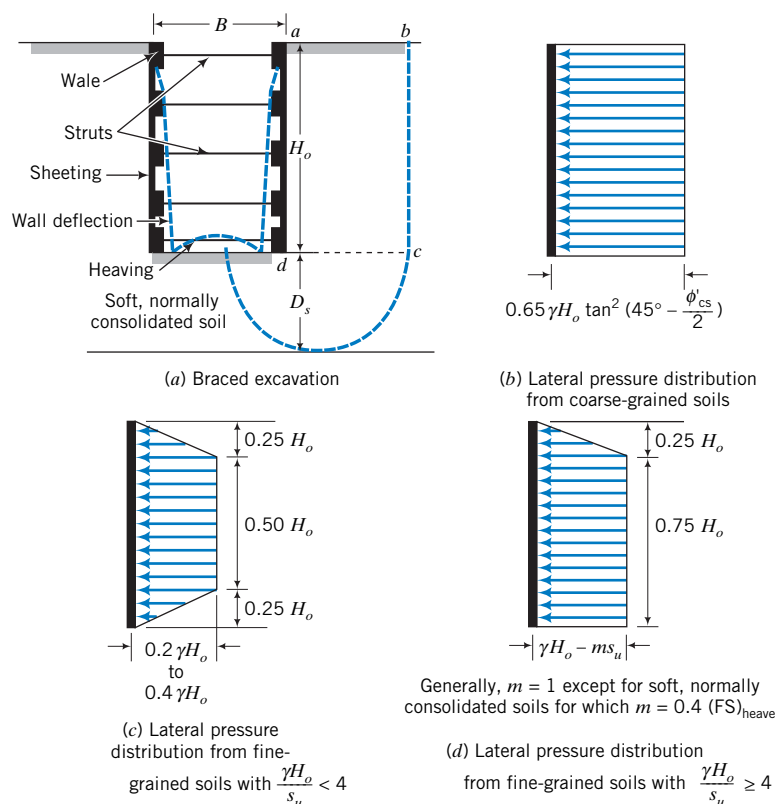


FIGURE 12.26 Braced excavations.

excavation should ideally consider the construction sequence, and numerical methods such as the finite element method are preferred. Semi-empirical methods are often used for shallow braced excavations and in the preliminary design of deep braced excavations. The finite element method is beyond the scope of this book. We will only discuss a semi-empirical method.

Lateral stress distributions for use in the semi-empirical method are approximations from field measurements of strut loads in different types of soil. The lateral stress distributions used for coarse-grained and fine-grained soils are shown in Figs. 12.26b–d. These lateral stress distributions are not real but average approximate stress distributions. The lateral stress distribution for coarse-grained soils (Fig. 12.26b) was extrapolated from strut loads measured for dense sand adjacent to the excavation. The appropriate value of friction angle is ϕ'_p but because we cannot rely on dilation, the design friction angle should be ϕ'_{cs} . For fine-grained soils, a total stress analysis is used and the lateral stress distribution depends on the stability number, $\gamma H_o/s_u$ (Peck, 1969). If the stability number is less than 4, the stress state of the soil adjacent to the excavation can be assumed to be elastic and the recommended lateral stress distribution is depicted in Fig. 12.26c. However, if the stability number is greater than or equal to 4, the stress state of the soil adjacent to the bottom of the excavation is expected to be plastic and the recommended lateral stress distribution is depicted in Fig. 12.26d.

If the soil below the base of the excavation is a soft, normally consolidated soil, it is possible that heaving can occur. The column of soil, $abcd$ (Fig. 12.26a), above the base acts as a surcharge on the soil below the excavation level. This surcharge load may exceed the bearing capacity of the soft, normally consolidated soil, resulting in heaving. Bjerrum and Eide (1956) suggested that the excavation could be

viewed as a footing of width B and embedment depth H_o . They showed that the factor of safety against bottom heave is

$$(FS)_{\text{heave}} = N_c \frac{s_u}{\gamma H_o + q_s} \quad (12.64)$$

where N_c is a bearing capacity coefficient given by Skempton (1951). The coefficient N_c can be approximated, for practical purposes, by

$$N_c = 6 \left(1 + 0.2 \frac{H_o}{B} \right) \quad \text{for} \quad \frac{H_o}{B} \leq 2.5 \quad \text{and} \quad N_c = 9I_c \quad \text{for} \quad \frac{H_o}{B} > 2.5, I_c = 0.84 + 0.16 \frac{B}{L} \quad (12.65)$$

where H_o/B is the depth to width ratio.

If $(FS)_{\text{heave}} < 1.5$, the sheeting should be extended below the base of the excavation for stability. The value of the coefficient m (Fig. 12.26d) is usually 1 except when the soil below the excavation is a deep, soft, normally consolidated soil, in which case $m \approx 0.4(FS)_{\text{heave}}$.

The strut loads at each level are found by assuming hinged connections of the struts to the sheet piles. A free-body diagram is drawn for each level and the forces imposed on the struts are determined using static equilibrium. Displacements of the walls are an important design consideration as adjacent structures may be affected. The method discussed above does not consider displacements. Analyses using numerical methods (e.g., finite element method) are better suited for the overall analysis of braced excavation.

The procedure for analysis of braced excavation is as follows:

1. Check the stability against bottom heave using Eq. (12.64). If $(FS)_{\text{heave}} < 1.5$, the walls should be extended below the base.
2. Determine the lateral stress on the walls for your soil type (Figs. 12.26b-d).
3. Treat the connections of the wall (sheet pile) to the struts as hinges.
4. Draw a free-body diagram at each level of the excavation.
5. Solve for the forces in the struts using the static equilibrium equations on each free-body diagram.

EXAMPLE 12.8 Braced Excavation

Determine the forces on the struts for the braced excavation in soft, normally consolidated clay as shown in Fig. E12.8a.

Strategy You need to determine the approximate lateral stress distribution by calculating $\gamma H_o/s_u$. To find the forces on the struts, draw free-body diagrams—one at each level—and use statics.

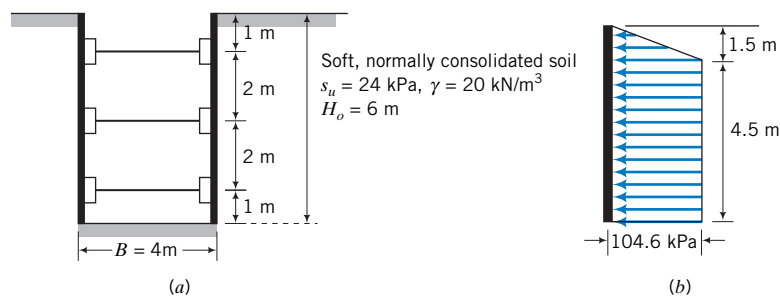


FIGURE E12.8a,b

Solution 12.8**Step 1:** Check for stability against bottom heave.

$$\frac{H_o}{B} = \frac{6}{4} = 1.5, \quad N_c = 6 \left(1 + 0.2 \frac{H_o}{B} \right) = 6(1 + 2 \times 0.15) = 7.6,$$

$$q_s = 0$$

$$(\text{FS})_{\text{heave}} = N_c \frac{s_u}{\gamma H_o + q_s} = 7.6 \frac{24}{20 \times 6} = 1.52 > 1.5;$$

therefore, excavation is safe against bottom heave

Step 2: Determine the lateral pressure diagram.

$$\frac{\gamma H_o}{s_u} = \frac{20 \times 6}{24} = 5; \quad \text{use Fig. 12.26d}$$

$$m = 0.4(\text{FS})_{\text{heave}} = 0.4 \times 1.6 = 0.64$$

The maximum lateral pressure is

$$\gamma H_o - m s_u = 20 \times 6 - 0.64 \times 24 = 104.6 \text{ kPa}$$

Step 3: Draw the pressure diagram.

See Fig. E12.8b.

Step 4: Calculate the forces on the struts at each level.

All loads are per meter length of wall.

Level 1 (Fig. E12.8c)

$$\Sigma M_{B_1} = 0 = 2.0A$$

$$- \left[\frac{1}{2} \times 104.6 \times 1.5 \times \left(\frac{1.5}{3} + 1.5 \right) + 1.5 \times 104.6 \times 0.75 \right]$$

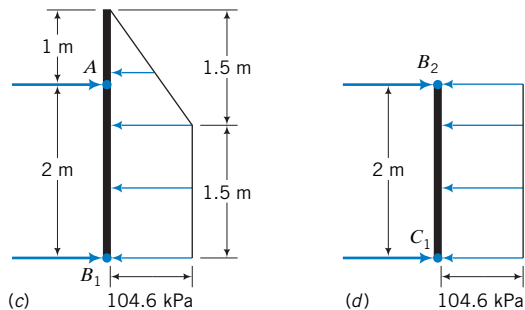
$$\therefore A = 137.3 \text{ kN/m}$$

$$\Sigma F_x = 0: \quad A + B_1 = \frac{1}{2} \times 104.6 \times 1.5 + 1.5 \times 104.6 = 235.4 \text{ kN/m}$$

$$\therefore B_1 = 235.4 - 137.3 = 98.1 \text{ kN/m}$$

Level 2 (Fig. E12.8d)

$$B_2 = C_1 = \frac{104.6 \times 2}{2} = 104.6 \text{ kN/m}$$

**FIGURE E12.8c,d**

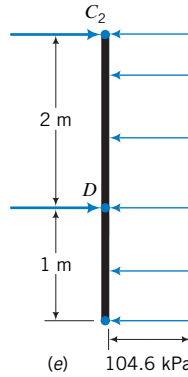


FIGURE E12.8e

Level 3 (Fig. E12.8e)

$$\Sigma F_x = 0: \quad C_2 = 104.6 \times 1 = 104.6 \text{ kN/m}$$

Step 5: Calculate the forces on each strut.

$$A = 137.3 \text{ kN/m}$$

$$B = B_1 + B_2 = 98.1 + 104.6 = 202.7 \text{ kN/m}$$

$$C = C_1 + C_2 = 104.6 + 104.6 = 209.2 \text{ kN/m}$$

What's next... Soils reinforced by metal strips or geotextiles have become popular earth retaining structures because they are generally more economical than gravity retaining walls. These walls are called mechanical stabilized earth (MSE) walls. A brief introduction to MSE walls is presented next.

12.12 MECHANICAL STABILIZED EARTH WALLS



Concept Learning and Self-Assessment

Access Chapter 12, Section 12.12 on the CD to learn about mechanized stabilized earth walls. Take Quiz 12.12 to test your understanding.

Mechanical stabilized earth (MSE) walls (Fig. 12.27a) are used for a variety of retaining structures. Metal strips (Fig. 12.27b), geotextiles (Fig. 12.27c), or geogrids (Fig. 12.27d), reinforce the soil mass. Other names used are geosynthetics and geocomposites. Geotextile is any permeable textile material used in geotechnical applications. Geosynthetics are polymeric, planar material (polypropylene, polyethylene, polyester, polyamide, and nylon). Geocomposite is a product made from a combination of geosynthetics. A geogrid is a polymeric product formed by joining intersecting ribs.

Geotextiles are manufactured from filaments and yarns that are combined to form planar products called fabrics. In geotechnical applications, nonwoven and woven fabrics are popular. Nonwoven geotextiles are manufactured from continuous filaments formed by extruding polymers through spinnerets or staple fibers (2 cm to 10 cm of filaments). Woven geotextiles are made from yarns knitted using various weave patterns such as plain, twill, satin, or various combinations of these.

The American Society for Test and Materials (ASTM) has a number of standard tests to determine the physical and mechanical properties of geotextiles. For retaining walls, the key

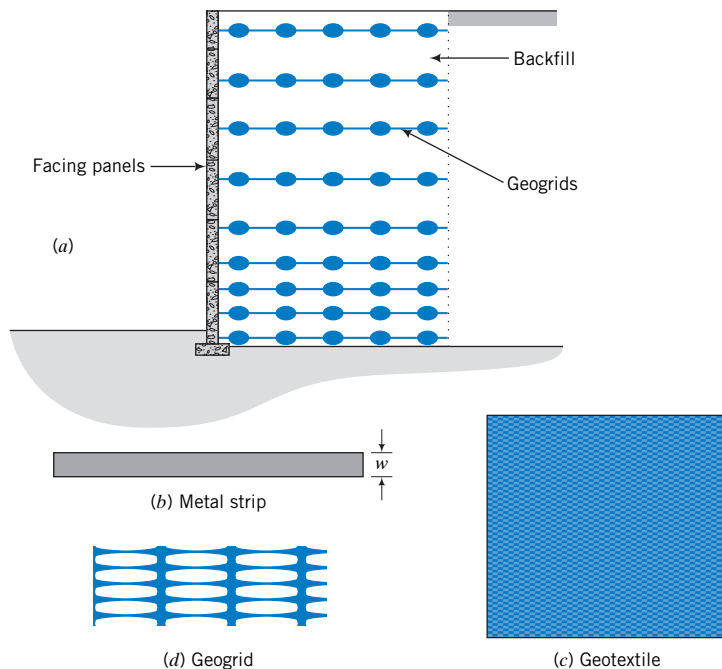


FIGURE 12.27 (a) A geogrid reinforced wall, (b) metal strip, (c) geotextile, and (d) geogrid.

properties are tensile strength, creep, and durability. The ASTM tests for tensile strength and creep are

1. Grab tensile strength test (ASTM D4632). This test gives the ultimate tensile load (N) of the geotextile.
2. Wide width tensile test (ASTM D4595). This test gives the load per unit width (N/m) of geotextile. You can also obtain the wide width modulus from this test.
3. Tensile creep test (ASTM D5262). This test gives the percentage of the ultimate tensile load at which the rate of creep strain approaches zero with time.

Geotextiles suffer strength reduction from ultraviolet light degradation. Consequently, during construction it is essential not to expose the geotextile unduly to sunlight. Specifications normally limit the exposure time to less than 14 days.

MSE walls are generally more economical than gravity walls. The basic mechanics of MSE walls is described in the next section.

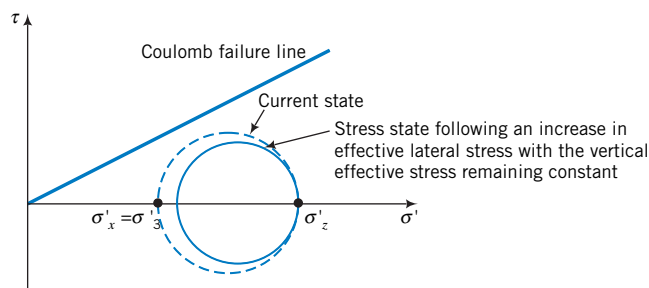


FIGURE 12.28 Effects of increasing the lateral soil resistance by reinforcement.

12.12.1 Basic Concepts

You should recall from Chapter 7 that if a load is applied to a soil mass under axisymmetric undrained conditions, the lateral strain (ε_3) is one-half the axial strain (ε_1) as expressed by Eq. (7.48). If the undrained restriction is lifted, then you can expect lateral strains greater than one-half the vertical strains. If we were to install strips of metal in the lateral directions of the soil mass, then the friction at the interfaces of the metal strips and the soil would restrain lateral displacements. The net effect is the imposition of a lateral resistance on the soil mass that causes Mohr's circle to move away from the failure line (Fig. 12.28). The lateral force imposed on the soil depends on the interface friction value between the reinforcing element and the soil mass and the vertical effective stress. For a constant interface friction value, the lateral frictional force would increase with depth. The reinforcing material will fail if the lateral stress exceeds its tensile strength.

The **essential point** in MSE walls is that the reinforcement serves as an internal lateral confinement that allows the soil to mobilize more shearing resistance.

12.12.2 Stability of Mechanical Stabilized Earth Walls

There are two sets of stability criteria to be satisfied for MSE walls. One is the internal stability; the other is the external stability. The external stability of a MSE wall is determined by analogy to a gravity retaining wall with a vertical face, FG , as illustrated in Fig. 12.29. The internal stability depends on the tensile strength of the reinforcing material and the slip at the interface of the reinforcing material and the soil. Tensile failure of the reinforcing material at any depth leads to progressive collapse of the wall, while slip at the interface of the reinforcing material and the soil mass leads to redistribution of stresses and progressive deformation of the wall.

Two methods of analysis are used to determine the internal stability. One method is based on an analogy with anchored flexible retaining walls and is generally used for reinforcing material with high extensibility, for example, polymeric such as geotextiles and geogrids. The Rankine active earth pressure theory is used with the active slip plane inclined at $\theta_a = 45^\circ + \phi'_{cs}/2$ to the horizontal, as shown in Fig. 12.29. The frictional resistance develops over an effective length, L_e , outside the active slip or failure zone. At a depth z , the frictional resistance developed on both surfaces of the reinforcing material (Fig. 12.29) is

$$P_r = 2wL_e(\sigma'_z + q_s) \tan \phi_i \quad (12.66)$$

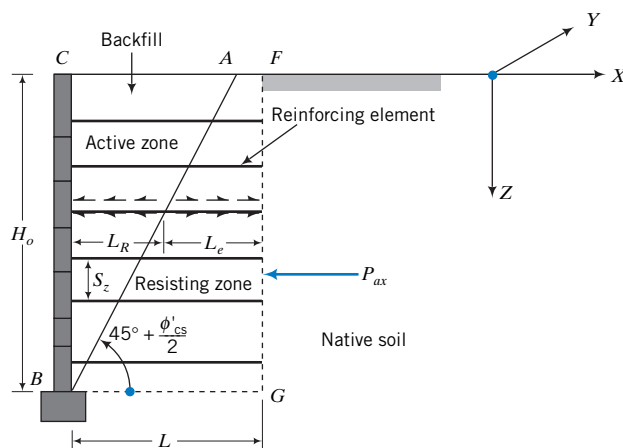


FIGURE 12.29 MSE wall using Rankine's method.

where w is the width of the reinforcing material, σ'_z is the vertical effective stress, q_s is the surcharge, and ϕ_i is the friction angle at the soil-reinforcement interface. Consider a layer of reinforcement at a depth z . The tensile force is

$$T = K_{aR}(\sigma'_z + q_s)S_zS_y \quad (12.67)$$

where S_z and S_y are the spacing in the Z and Y directions and T is the tensile force per unit length of wall. For geotextiles or geogrids, you would normally consider one unit length of wall and one unit width, so $S_y = 1$ and $w = 1$.

By setting $T = P_r$, we can find the effective length of the reinforcement required for limit equilibrium (factor of safety = 1). To find the design effective length, a factor of safety (FS)_{*i*} is applied on the tensile force, T , and by solving for L_e from Eqs. (12.66) and (12.67), we get

$$L_e = \frac{K_{aR}(\sigma'_z + q_s)S_zS_y(\text{FS})_i}{2w(\sigma'_z + q_s)\tan\phi_i} = \frac{K_{aR}S_zS_y(\text{FS})_i}{2w\tan\phi_i} \quad (12.68)$$

where (FS)_{*i*} ranges from 1.3 to 1.5.

The total length of reinforcement is

$$L = L_e + L_R \quad (12.69)$$

where L_R is the length of reinforcement within the active failure zone.

Because L_R is zero at the base of the wall, the calculated length of reinforcement at the base is often the shortest. This calculated length, while adequate for internal stability, is often inadequate for translation or bearing capacity (external stability). You should check whether the calculated length of reinforcement at the base is adequate for translation or bearing capacity as follows:

1. Calculate the maximum lateral active earth force, P_{ax} .

$$P_{ax} = \frac{1}{2}K_{aC}\gamma H_o^2 \cos\delta + K_{aC}q_s H_o \cos\delta = H_o K_{aC} \cos\delta (0.5\gamma H_o + q_s)$$

where K_{aC} is the active lateral pressure coefficient using Coulomb's method with wall friction. The friction angle to use is $\phi' = (\phi'_{cs})_{\text{backfill}}$ and the wall friction is $\delta \approx \frac{2}{3}(\phi'_{cs})_{\text{native soil}}$ to $(\phi'_{cs})_{\text{native soil}}$. Coulomb's method is used because the interface between the reinforced backfill soil and the native soil (FG, Fig. 12.29) is frictional. If the interface friction is neglected, then you can use K_{aR} instead of K_{aC} .

2. Calculate the length of reinforcement required for translation. For short-term loading in clays, the base resistance is $T = L_b s_w$, where L_b is the length of reinforcement at the base and s_w is the adhesion stress. By analogy with sheet pile retaining walls, we will use $s_w = 0.5s_u$ (maximum $s_w = 50$ kPa). For external stability against translation $T = P_{ax}(\text{FS})_T$, where (FS)_{*T*} is a factor of safety against translation; usual range 1.5–3.0. The required length at the base against translation for short-term loading in clays is then

$$L_b = \frac{P_{ax}(\text{FS})_T}{s_w} \quad (12.70)$$

For long-term loading, $T = \sum_{i=1}^n W_i \tan\phi'_b$, where W_i is the weight of soil layer i , n is the number of layers, and ϕ'_b is the effective interfacial friction angle between the reinforcement and the soil at the base. Assuming a uniform soil unit weight throughout the height of the wall, then

$$T = \gamma H_o L_b \tan\phi'_b$$

The length of reinforcement at the base required to prevent translation under long-term loading is

$$L_b = \frac{P_{ax}(FS)_T}{\gamma H_o \tan \phi'_b} = \frac{(K_{ac})_x H_o (0.5\gamma H_o + q_s)}{\gamma H_o \tan \phi'_b} = \frac{(K_{ac})_x (0.5H_o + q_s/\gamma)(FS)_T}{\tan \phi'_b} \quad (12.71)$$

where $(K_{ac})_x$ is the horizontal component of Coulomb's active lateral earth pressure coefficient. You should use the larger value of L_b obtained from Eqs. (12.70) and (12.71).

The procedure for analysis of a reinforced soil wall using polymeric materials is as follows:

1. Calculate the allowable tensile strength per unit width of the reinforcing polymeric material.

$$T_{all} = T_{ult} \frac{1}{(FS)_{ID} \times (FS)_{CR} \times (FS)_{CD} \times (FS)_{BD}} \quad (12.72)$$

where T_{all} is the allowable tensile strength, T_{ult} is the ultimate tensile strength, FS is a factor of safety, and the subscripts have the following meaning:

ID—installation damage

CR—creep

CD—chemical degradation

BD—biological degradation

Typical values of the various factors of safety are shown in Table 12.2.

2. Calculate the vertical spacing at different wall heights.

$$s_z = \frac{T_{all}}{K_{aR}(\sigma'_z + q_s)(FS)_{sp}} \quad (12.73)$$

where $(FS)_{sp}$ is a factor of safety between 1.3 and 1.5. It is customary to calculate the minimum vertical spacing using $\sigma'_z = \gamma' H_o$ and then check the vertical spacing required at $H_o/3$ and $2H_o/3$. For practical reasons, the vertical spacing either is kept at a constant value or varies by not more than three different values along the height.

3. Determine the length of reinforcement required at the base for external stability from Eqs. (12.70) and (12.71).
4. Determine the total length of reinforcement at different levels.

$$L = L_e + L_R$$

where

$$L_R = (H_o - z) \tan \left(45^\circ - \frac{\phi'_{cs}}{2} \right) \quad (12.74)$$

TABLE 12.2 Typical Ranges of Factor of Safety

Factor of safety	Range
$(FS)_{ID}$	1.1 to 2.0
$(FS)_{CR}$	2.0 to 4.0
$(FS)_{CD}$	1.0 to 1.5
$(FS)_{BD}$	1.0 to 1.3

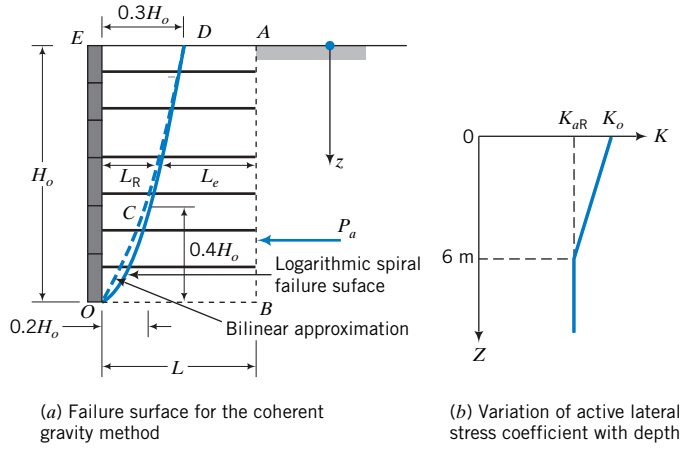


FIGURE 12.30 Coherent gravity method.

5. Determine the external stability (translation and bearing capacity). Remember that translation is already satisfied from item 3 above. Overturning is not crucial in MSE walls because these walls are flexible and cannot develop moment. However, you can verify that overturning is satisfied if $e < L_b/6$, where e is the eccentricity of the vertical forces.

The other method is the coherent gravity method (Juran and Schlosser, 1978) and is applicable to low extensible reinforcing materials such as metal strips. Failure is assumed to occur progressively along a path defined by the maximum tensile strains at each level of the reinforcing material. The failure surface is a logarithm spiral that is approximated to a bilinear surface to simplify calculations (Fig. 12.30a). The lateral active pressure coefficient is assumed to vary linearly from K_o at the top of the wall to K_{aR} at a depth of 6 m and below (Fig. 12.30b).

The variation of lateral stress coefficient with depth (Fig. 12.30b) is

$$K = K_{aR} \frac{z}{6} + K_o \left(1 - \frac{z}{6}\right) \quad \text{for } z \leq 6 \text{ m} \quad (12.75)$$

$$K = K_{aR} \quad \text{for } z > 6 \text{ m} \quad (12.76)$$

The length of reinforcement within the failure zone (Fig. 12.30a) is

$$L_R = 0.2H_o + \left(0.1H_o - \frac{z}{6}\right) \quad \text{for } z \leq 0.6H_o \quad (12.77)$$

$$L_R = \frac{1}{2}H_o \left(1 - \frac{z}{H_o}\right) \quad \text{for } z > 0.6H_o \quad (12.78)$$

The procedure for analyzing a MSE wall using low extensible materials is as follows:

1. Select the spacing of the reinforcement in the Z and Y directions and the width of the reinforcement. Use a manufacturer's catalog to provide information on standard sizes.
2. Calculate the required maximum thickness of the reinforcement.

$$t_r = \frac{K_{aR}(\gamma H_o + q_s)S_z S_y (FS)_{tr}}{w f_y} \quad (12.79)$$

where K_{aR} is the Rankine active earth pressure coefficient, f_y is the yield stress of the reinforcement, w is the width of the reinforcement, and $(FS)_{rr}$ is a factor of safety, usually 3. The design thickness is the calculated thickness plus a corrosion thickness expected for the design life of the wall.

3. Determine the length of reinforcement required at the base from Eqs. (12.70) and (12.71).
4. Determine the total length of reinforcement at each level.

$$L = L_e + L_R$$

where L_R is given by Eqs. (12.77) and (12.78) and L_e is determined from Eq. (12.68) by using the appropriate value of the active lateral earth pressure coefficient from Eqs. (12.75) and (12.76).

5. Determine the external stability by assuming the wall is a gravity wall.

EXAMPLE 12.9 Geotextile MSE Wall



Computer Program Utility

Access Chapter 12 on the CD and click Spreadsheet on the sidebar to view examples of MSE retaining wall analysis using a spreadsheet. You can change input values and explore how the stability of the wall changes.

Design a 4 m mechanical stabilized earth wall using a geotextile as the reinforcement. The backfill would be a compacted, coarse-grained soil with $\phi'_{cs} = 30^\circ$ and $\gamma_{sat} = 18 \text{ kN/m}^3$. The surcharge is 15 kPa. The geotextile selected has an ultimate wide-width tensile strength of 58.5 kN/m and the soil-geotextile interface friction value is 20° . The native soil is a clay with parameters $\gamma_{sat} = 18.5 \text{ kN/m}^3$, $\phi'_{cs} = 32^\circ$, $\sigma'_b = 20^\circ$, and $s_u = 60 \text{ kPa}$.

Strategy Follow the procedure for mechanical stabilized earth walls using a polymeric product.

Solution 12.9

Step 1: Calculate the allowable tensile strength of the geotextile.

From Table 12.2, use $(FS)_{ID} = 1.5$, $(FS)_{CR} = 2$, $(FS)_{CD} = 1.3$, and $(FS)_{BD} = 1.3$.

$$T_{all} = \frac{58.5}{1.5 \times 2 \times 1.3 \times 1.3} = 11.5 \text{ N/mm} = 11.5 \text{ kN/m}$$

Step 2: Calculate the vertical spacing.

$$K_{aR} = \tan^2(45 - \phi'_{cs}/2) = \tan^2(45 - 30/2) = \frac{1}{3}$$

$$\text{Lateral stress due to surcharge : } K_a q_s = \frac{1}{3} \times 15 = 5 \text{ kPa}$$

$$\sigma_x = K_{aR} \sigma'_z + K_{aR} q_s = \frac{1}{3} \gamma' z + 5 = \frac{1}{3} \times 18 \times z + 5 = 6z + 5$$

$$(\sigma_x)_{\max} = 6 \times H_o + 5 = 6 \times 4 + 5 = 29 \text{ kPa}$$

From Eq. (12.69) with $(FS)_{sp} = 1.3$, we get

$$(S_z)_{\min} = \frac{T_{all}}{K_{aR}(\sigma'_z + q_s)(FS)_{sp}} = \frac{11.5}{29 \times 1.3} = 0.305 \text{ m} = 305 \text{ mm}$$

Check spacing requirement at midheight ($z = 2$):

$$\sigma_x = 6 \times 2 + 5 = 17 \text{ kPa}$$

$$S_z = \frac{11.5}{17 \times 1.3} = 0.520 \text{ m} = 520 \text{ mm}$$

You should try to minimize the number of layers and keep the spacing to easily measurable values. Use $S_z = 250 \text{ mm}$ for the bottom half of the wall and 500 mm for the top half of the wall.

Step 3: Determine the length of reinforcement required at the base for translation. From Eq. (12.16),

$$K_{aC} = 0.3, \quad \delta = 20^\circ, \quad \text{and} \quad (K_{aC})_x = K_{aC} \cos \delta = 0.3 \times \cos 20 = 0.28$$

$$P_{ax} = \frac{1}{2} (K_{aC})_x \gamma H_o^2 + (K_{aC})_x q_s H_o = \frac{1}{2} \times 0.28 \times 1.8 \times 4^2 + 0.28 \times 15 \times 4$$

$$= 57.1 \text{ kN/m}$$

$$s_w = 0.5s_u = 0.5 \times 60 = 30 \text{ kPa} < 50 \text{ kPa}; \quad \text{therefore use } s_w = 30 \text{ kPa}$$

$$\text{Equation (12.70): } L_b = \frac{P_{ax}(\text{FS})_T}{s_w} = \frac{57.1 \times 1.5}{30} = 2.9 \text{ m}$$

$$\text{Equation (12.71): } L_b = \frac{(K_{aC})_x (0.5H_o + q_s/\gamma') (\text{FS})_T}{\tan \phi'_b}$$

$$= \frac{0.28(0.5 \times 4 + 15/18)1.5}{\tan(\frac{2}{3} \times 28)} = 3.5 \text{ m}$$

Use $L_b = 3.5 \text{ m}$.

Step 4: Determine the total length of reinforcement at each level for internal stability.

Use a table to determine the total length as shown below.

Use $(\text{FS})_t = 1.3$.

$z \text{ (m)}$	$S_z \text{ (m)}$	$L_R \text{ (m)} = \frac{(H_o - z)}{\tan(45^\circ - \phi'_{cs}/2)}$	$L_e \text{ (m)} = \frac{K_{aR} S_z (\text{FS})_t}{2 \tan \phi'_i}$	$L \text{ (m)}$
0.50	0.50	2.02	0.3	2.3
1.00	0.50	1.73	0.3	2.0
1.50	0.50	1.44	0.3	1.7
2.00	0.50	1.15	0.3	1.5
2.25	0.25	1.01	0.15	1.2
2.50	0.25	0.87	0.15	1.0
2.75	0.25	0.72	0.15	0.9
3.00	0.25	0.58	0.15	0.7
3.25	0.25	0.43	0.15	0.6
3.50	0.25	0.29	0.15	0.4
3.75	0.25	0.14	0.15	0.3
4.00	0.25	0.00	0.15	0.2

Since L_b is greater than the total length required for internal stability at each level, use $L = L_b = 3.5 \text{ m}$. The total length can be reduced toward the top of the wall but for construction purposes, it is best to use, in most cases, a single length of polymeric product.

Step 5: Check external stability.

Stability against translation is already satisfied in Step 3.

Check the bearing capacity.

$$(\sigma_z)_{\max} = \gamma H_o = 18 \times 4 = 72 \text{ kPa}$$

TSA

$$q_u = 5.14s_u i_c; \quad i_c = 1 \quad \text{for strip footing}$$

$$q_u = 5.14 \times 60 \times 1 = 308.4 \text{ kPa}$$

$$(\text{FS})_B = \frac{q_{\text{ult}}}{(\sigma_x)_{\text{max}}} = \frac{308.4}{72} = 4.3 > 3; \quad \text{okay}$$

ESA

Use rough footing because a layer of geotextile is placed at the interface of the wall and foundation.

$$H = P_{ax} = 57.1 \text{ kN}; \quad V_n = \gamma H_o B = 18 \times 4 \times 3.5 = 252 \text{ kN},$$

$$\frac{H}{V_n} = \frac{57.1}{252} = 0.23; \quad \omega = \tan^{-1} \frac{H}{V_n} = \tan^{-1}(0.23) = 13^\circ$$

$$n = \left(2 + \frac{B'}{L'}\right) / \left(1 + \frac{B'}{L'}\right) = 2; \quad i_\gamma = \left(1 - \frac{H}{V_n}\right)^{n+1} = (1 - 0.23)^{2+1} = 0.46$$

$$N_\gamma = 0.1054 \exp(9.6\phi'_p) = 0.1054 \exp\left(9.6 \times 32 \times \frac{\pi}{180}\right) = 22.5$$

$$q_u = 0.5\gamma B N_\gamma i_\gamma = 0.5 \times 18.5 \times 3.5 \times 22.5 \times 0.46 = 335 \text{ kPa}$$

$$(\text{FS})_B = \frac{q_u}{(\sigma_{\text{max}})} = \frac{335}{72} = 4.6 > 3; \quad \text{okay}$$

EXAMPLE 12.10 Metal Ties MSE Wall

Design a mechanical stabilized wall 6 m high using metal ties. The backfill is a coarse-grained soil with $\gamma_{\text{sat}} = 16.5 \text{ kN/m}^3$ and $\phi'_{\text{cs}} = 32^\circ$. Galvanized steel ties are available with a yield strength $f_y = 2.5 \times 10^5 \text{ kPa}$ and a rate of corrosion of 0.025 mm/year. A factor of safety of 3 is desired for a design life of 50 years. The soil–tie interface friction value is 21° . The foundation soil is coarse-grained with parameters $\gamma_{\text{sat}} = 18 \text{ kN/m}^3$, $\phi'_p = 35^\circ$, and $\phi'_b = 20^\circ$. The surcharge is 15 kPa.

Strategy You have to guess the spacing of the ties. You can obtain standard widths and properties of ties from manufacturers' catalogs. Follow the procedure for the coherent gravity method.

Solution 12.10

Step 1: Assume spacing and width of ties.

$$\text{Assume } S_z = 0.5 \text{ m}, \quad S_y = 1 \text{ m}, \quad \text{and } w = 75 \text{ mm}$$

Step 2: Calculate required thickness of reinforcement.

$$K_o = 1 - \sin \phi'_{\text{cs}} = 1 - \sin 32^\circ = 0.47$$

$$K_{aR} = \tan^2(45 - \phi'_{\text{cs}}/2) = \tan^2(45 - 32/2) = 0.31$$

$$t_r = \frac{K_{aR}(\gamma' H_o + q_s) S_z S_y (\text{FS})_{tr}}{w f_y} = \frac{0.31(16.5 \times 6 + 15) \times 0.5 \times 1 \times 3}{0.075 \times 2.5 \times 10^5}$$

$$= 283 \times 10^{-5} \text{ m} = 2.8 \text{ mm}$$

$$t_{\text{corrosion}} = \text{Annual corrosion rate} \times \text{design life} = 0.025 \times 50 = 1.25 \text{ mm}$$

$$t_{\text{design}} = \text{Calculated thickness} + \text{Corrosion thickness} = 2.8 + 1.25 = 4.05 \text{ mm}$$

You will select a tie thickness from standard sizes closest to 4.05 mm. Use $t = 5 \text{ mm}$.

Step 3: Determine the length of reinforcement required at the base.

$$\text{From Eq. (12.16): } K_{aC} = 0.28 \quad (\phi'_{cs} = 32^\circ, \delta = 28^\circ)$$

$$(K_{aC})_x = K_{aC} \cos \delta = 0.28 \cos 20 = 0.26$$

$$\text{From Eq. (12.71): } L_b = L_b = \frac{0.26(0.5 \times 6 + 15/16.5)1.5}{\tan 20^\circ} = 4.2 \text{ m}$$

For internal stability, the effective length at the wall base is

$$\text{Equation (12.68): } L_e = \frac{0.31 \times 0.5 \times 1 \times 1.3}{2 \times 0.075 \times \tan 21^\circ} = 3.5 \text{ m} < 4.2 \text{ m}$$

Step 4: Determine the total length of reinforcement.

Use a table as shown below. $(FS)_t = 1.3$.

z (m)	S_z (m)	K Eq. (12.75)	L_e (m) Eq. (12.68)	L_r (m) Eq. (12.77)	L (m)	Recommended L (m)
0.50	0.50	0.46	5.15	1.72	6.9	7.0
1.00	0.50	0.44	5.00	1.63	6.6	7.0
1.50	0.50	0.43	4.85	1.55	6.4	6.5
2.00	0.50	0.42	4.69	1.47	6.2	6.5
2.50	0.50	0.40	4.54	1.38	5.9	6.5
3.00	0.50	0.39	4.39	1.30	5.7	6.5
3.50	0.50	0.38	4.23	1.22	5.5	6.5
4.00	0.50	0.36	4.08	1.00	5.1	6.5
4.50	0.50	0.35	3.93	0.75	4.7	5.0
5.00	0.50	0.33	3.77	0.50	4.3	5.0
5.50	0.50	0.32	3.62	0.25	3.9	5.0
6.00	0.50	0.31	3.47	0.00	3.5	5.0

Step 5: Check for external stability.

Translation is satisfied because L used at the base is greater than L_b .

Check bearing capacity.

$$(\sigma_z)_{\max} = \gamma H_o = 16.5 \times 6 = 99 \text{ kPa}$$

$$H = 0.5\gamma K_{(aC)x} H_o^2 + K_{(aC)x} H_o q_s = 0.5 \times 16.5 \times 0.26 \times 6^2 + 0.26 \times 6 \times 15 = 100.6 \text{ kN}$$

$$V_n = \gamma H_o B = 16.5 \times 6 \times 4.2 = 415.8 \text{ kN}, \quad \frac{H}{V_n} = \frac{100.6}{415.8} = 0.24; \quad \omega = \tan^{-1} \frac{H}{V_n} = \tan^{-1}(0.24) = 13.5^\circ$$

$$n = \left(2 + \frac{B'}{L'}\right) / \left(1 + \frac{B'}{L'}\right) = 2; \quad i_\gamma = \left(1 + \frac{H}{V_n}\right) = (1 - 0.24)^{2+1} = 0.44$$

$$N_\gamma = 0.1054 \exp(9.6\phi'_p) = 0.1054 \exp\left(9.6 \times 35 \times \frac{\pi}{180}\right) = 37.2$$

$$q_u = 0.5\gamma B N_\gamma i_\gamma = 0.5 \times 18 \times 4.2 \times 37.2 \times 0.44 = 618.7 \text{ kPa}$$

$$(FS)_B = \frac{q_u}{(\sigma_{\max})} = \frac{618.7}{99} = 6.2 > 3; \quad \text{okay}$$

12.13 OTHER TYPES OF RETAINING WALLS

There are a variety of retaining walls that are now used in geotechnical and other applications (e.g., landscaping). Some of these are listed below.



FIGURE 12.31 Gabion basket wall.

12.13.1 Modular Gravity Walls

Modular gravity walls consist of a structural container filled with materials of various types. These walls are used in projects such as highways, dikes, and channels that conventionally utilize cast-in-place concrete gravity walls. The design method is discussed in Section 12.9. Examples of modular gravity walls are

- *Gabion baskets.* Gabion basket walls consist of prefabricated steel wire or polypropylene or polyethylene or nylon baskets filled with rocks and stacked horizontally and vertically (Fig. 12.31). Basket size varies but is normally 2 m in length $\times$ 1 m wide $\times$ 1 m high. Long baskets exceed 2 m in length are reinforced with diaphragms to strengthen the baskets for ease of construction. Gabion baskets are suitable for gravity walls where rock fill material is readily available. Common applications are for erosion protection of banks of rivers and streams, and for retaining walls along roads and highways, especially in rugged terrain.
- *Bin walls.* Bin-walls are gravity walls in which earth fill is placed in a bin made from metal (steel) or timber or concrete (Fig. 12.32). These walls require a small amount of settlement of the vertical corner members. If the foundation is rigid, a compressible cushion of approximately 200 mm of loose fill is placed under the grade plates or concrete base.
- *Precast modular concrete walls.* These walls are constructed by stacking precast blocks made from concrete and other materials. There is a large variety of blocks; the major differences being the material used to make the blocks, size, and interlocking mechanism.

12.13.2 In Situ Reinforced Walls

These walls utilize reinforcing elements to form a composite with the soil. The reinforcing elements are nails or small-diameter cast-in-place concrete piles or small-diameter steel pipe piles. One popular in situ reinforced wall is soil nail walls. Closely spaced nails are installed by drilling inclined holes into the soil and grouting the hole. Shear stresses from the soil are transferred to the nails and are resisted by

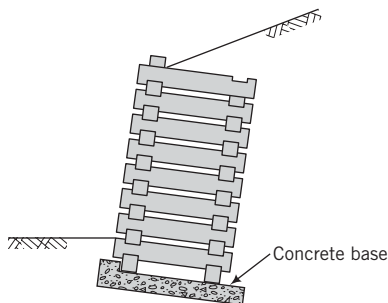


FIGURE 12.32 A crib wall.

tensile forces in the nails. The faces of soil nail walls are shotcrete or precast concrete panels or cast-in-place concrete.

12.13.3 Chemically Stabilized Earth Walls (CSE)

CSE walls are in situ soils mixed with chemical grouts such as lime or lime-cement mixtures to form columns of overlapping soils. Sometimes reinforcements are added to the soil-grout mixtures before they harden. CSE walls can retain soils up to great depths. They are also used in seepage control.

12.14 SUMMARY

We have considered lateral earth pressures and their applications to several types of earth retaining structures in this chapter. Two earth pressure theories are in general use: one developed by Coulomb and the other by Rankine. Coulomb's equations for the lateral earth pressure coefficients are based on limit equilibrium and include the effects of soil-wall friction, wall slope, and backfill slope. Rankine's equations are based on stress states of the backfill and do not account for soil-wall friction. The failure planes in the Coulomb and Rankine methods are planar surfaces. Soil-wall friction causes the failure plane to curve, resulting in higher active lateral earth pressure coefficients and lower passive earth pressure coefficients than those found using Coulomb's equations.

Because of the flexibility of some earth retaining structures and construction methods used in practice, the "real" lateral earth pressures are different from either the Coulomb or Rankine theories. It is suggested in this book that the appropriate value of friction angle to use in the analysis of earth retaining structures is ϕ'_{cs} . Three methods of analysis for flexible earth retaining walls were considered. The differences in the methods result mainly from how the lateral stresses are considered and how the factor of safety is applied.

Practical Examples

EXAMPLE 12.11 *Cantilever Gravity Walls for Material Storage*



Computer Program Utility

Access Chapter 12 on the CD and click on Spreadsheet on the sidebar to view examples of retaining wall analysis using a spreadsheet. You can change input values and explore how the stability of the walls changes.

Ore from a manufacturing plant is to be stored between two cantilever gravity retaining walls, as shown in Fig. E12.11a. A gantry crane will run on top of the walls to place the ore. The crane will apply a maximum vertical load of 24 kN and a horizontal load of ± 4.5 kN on each wall. The base of the ore pile is only permitted to come within 0.5 m of the top of the walls. The ore surface should be the maximum admissible slope. The walls will be restrained from spreading outward by steel ties or rods at 1 m centers anchored to the base slab. A 2 m layer of the ore spoils of similar characteristics to the ore to be stored would be compacted to support the base of the wall. Determine (a) the stability of the wall for the geometry shown in Fig. E12.11a and (b) the force in the tie rods, assuming they resist all the horizontal loads. If the tie rods were not present, would the walls be safe? Do you expect any alignment problems with the gantry crane?

Strategy The maximum slope angle would be the friction angle. Since the storage is symmetrical, each wall will carry identical loads. You will have to make an assumption regarding wall

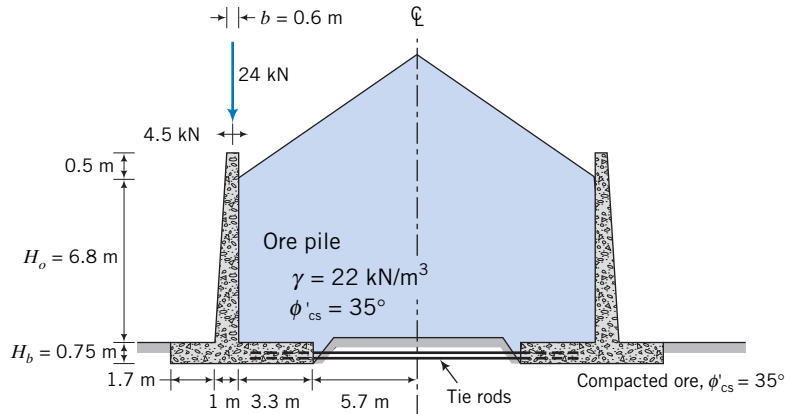


FIGURE E12.11a

friction. You can assume $\delta = \frac{1}{2}\phi'_{cs}$ or $\delta = \frac{2}{3}\phi'_{cs}$ and use Coulomb's method to determine the lateral forces.

Solution 12.11

Step 1: Determine the lateral forces.

The maximum admissible slope is $\beta = \phi'_{cs}$. Therefore, $\beta = 35^\circ$.

$$H_o = 0.75 + 6.8 + 3.3 \tan 35^\circ = 9.86 \text{ m}$$

$$\text{From Eq. (12.16): } K_a = 0.7 \text{ for } \beta = 35^\circ, \phi' = 35^\circ, \delta = \frac{1}{2}\phi'$$

$$P_a = \frac{1}{2} K_a \gamma H_o^2 = \frac{1}{2} \times 0.7 \times 22 \times 9.86^2 = 748.7 \text{ kN/m}$$

$$P_{ax} = P_a \cos \delta = 748.7 \times \cos 17.5^\circ = 714.0 \text{ kN/m}$$

$$P_{az} = P_a \sin \delta = 748.7 \times \sin 17.5^\circ = 225.1 \text{ kN/m}$$

Step 2: Calculate forces and moments. Draw a diagram (show only one-half because of symmetry, see Fig. E12.11b) and use a table to do the calculations.

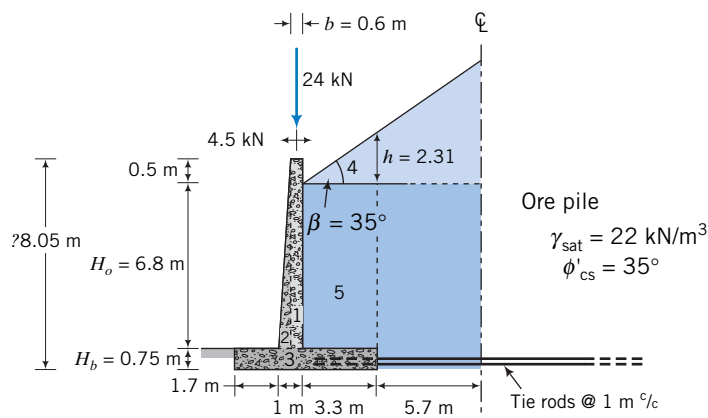


FIGURE E12.11b

Part	Force (kN/m)	Moment arm from 0 (m)	Moment (kN · m)
<i>Wall</i>			
<i>Vertical forces</i>			
1	$(6.8 + 0.5) \times 0.6 \times 24 = 105.1$	$1.7 + 0.4 + 0.6/2 = 2.4$	+252.2
2	$\frac{1}{2}(6.8 + 0.5) \times 0.4 \times 24 = 35$	$1.7 + \frac{2}{3} \times 0.4 = 1.97$	69.0
3	$0.75 \times 6 \times 24 = 108.0$	$6/2 = 3.00$	+324.0
<i>Soil</i>			
4	$\frac{1}{2} \times 3.3 \times 2.31 \times 22 = 83.9$	$1.7 + 1 + (\frac{2}{3} \times 3.3) = 4.9$	+411.1
5	$3.3 \times 6.8 \times 22 = 493.7$	$1.7 + 1 + 3.3/2 = 4.35$	+2147.6
P_{az}	225.1	6	+1350.6
<i>Gantry</i>			
P_z	24	2.4	+57.6
$R_z = \Sigma \text{ Vertical forces} = 1074.8$			
<i>Gantry</i>			
<i>Lateral forces</i>			
P_x	4.5	$0.75 + 6.8 + 0.5 = 8.05$	-36.2
<i>Soil</i>			
P_{ax}	714	$9.87/3 = 3.29$	-2349.1
$R_x = \Sigma \text{ Lateral forces} = 718.5$			
$M = \Sigma \text{ Moments} = 2226.8$			

Step 3: Determine stability.

Rotation

$$\bar{x} = \frac{M}{R_z} = \frac{2226.8}{1074.8} = 2.07 \text{ m}$$

$$e = \frac{B}{2} - \bar{x} = \frac{6}{2} - 2.07 = 0.93 \text{ m}$$

$$\frac{B}{6} = \frac{6}{6} = 1 \text{ m}; \quad e < \frac{B}{6}; \quad \text{therefore, wall is safe against rotation}$$

Sliding

$$T = \text{sliding resistance at base} = R_z \tan \phi'_b = 1074.8 \tan 20^\circ = 391.2 \text{ kN}$$

$$(\text{FS})_T = \frac{T}{R_x} = \frac{391.2}{718.5} = 0.54 < 1.5$$

$\therefore$ Sliding would occur if tie rods were not present.

Forces on tie rods

$$\text{For } (\text{FS})_T = 1.5: \quad T_a + T = 1.5R_x$$

where T_a is the force on the tie rods/unit length of wall.

$$T_a = 1.5 \times 718.5 - 391.2 = 686.6 \text{ kN/m}$$

Bearing capacity

All horizontal forces accommodated by tie rods.

$$B' = B - 2e = 6 - 2 \times 0.93 = 4.14 \text{ m}$$

$$V = 1074.8 \text{ kN}$$

$$N_\gamma = 0.1054 \exp(9.6\phi'_p) = 0.1054 \exp\left(9.6 \times 35 \times \frac{\pi}{180}\right) = 37.2$$

$$q_u = 0.5\gamma B' N_\gamma = 0.5 \times 18 \times 4.14 \times 37.2 = 1386.1 \text{ kPa}$$

$$\sigma_{\max} = \frac{\Sigma R_z}{B \times 1} \left(1 + \frac{6e}{B}\right) = \frac{1074.8}{6 \times 1} \left(1 + \frac{6 \times 0.93}{6}\right) = 345.7 \text{ kPa}$$

$$(\text{FS})_B = \frac{1386.1}{345.7} = 4 > 3; \quad \text{therefore, bearing capacity is satisfactory}$$

Summary of Results

1. The wall is unlikely to rotate significantly.
2. Without the tie rods, the wall will translate.
3. The design tie rod force is ≈ 687 kN/m.
4. The soil bearing capacity is adequate.
5. Assuming that the base slab is rigid and the loading is symmetrical, there should be no alignment problem.

EXAMPLE 12.12 Flexible Wall Near a Lake

A retaining wall is required near a man-made lake in a housing scheme development. The site is a swamp and the topsoil consisting mainly of organic material will be removed up to the elevation of a deep deposit of clay with a silt and sand mixture. The wall is expected to retain a sand backfill of height 6 m. It is anticipated that a rapid drawdown condition could occur and the lake emptied for a long period. A flexible retaining wall is proposed as shown in Fig. E12.12a. Determine the embedment depth and the force for an anchor spacing of 3 m. A surcharge of 10 kPa should be considered.

Strategy You can use either the FSM or NPPM or both. Because the clay layer has silt and sand, you should consider seepage forces for long-term conditions (assuming that the drawdown level would remain for some time).

Solution 12.12

Step 1: Determine the lateral earth pressure coefficients.

Use Appendix D to find the lateral earth pressure coefficients.

$$\text{Sand: } K_{ax} = 0.28 \quad (\phi'_{cs} = 30^\circ, \delta = \frac{2}{3}\phi'_{cs})$$

$$\text{Clay: } K_{ax} = 0.34 \quad (\phi'_{cs} = 27^\circ, \delta = \frac{2}{3}\phi'_{cs})$$

$$K_{px} = 3.8 \quad (\phi'_{cs} = 27^\circ, \delta = \frac{1}{2}\phi'_{cs})$$

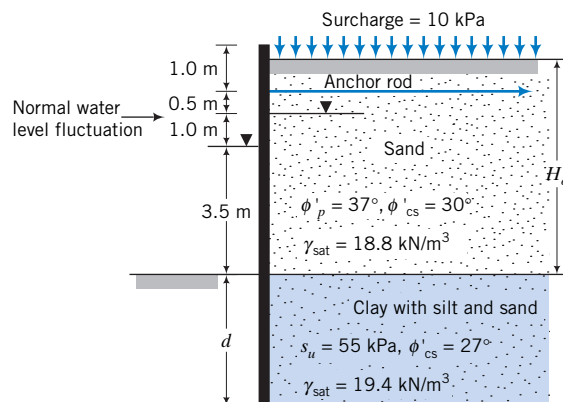


FIGURE E12.12a

Step 2: Determine the seepage forces and porewater pressures.

$$\text{Sand: } \gamma' = 18.8 - 9.8 = 9 \text{ kN/m}^3$$

$$\text{Clay: } \gamma' = 19.4 - 9.8 = 9.6 \text{ kN/m}^3$$

$$j_s = \frac{a}{a + 2d} \gamma_w = \left(\frac{3.5}{3.5 + 2d} \right) 9.8 = \frac{34.3}{3.5 + 2d} \text{ kN/m}^3$$

Active state

$$\text{Sand: } \gamma'_{ja} = \left(9 + \frac{34.3}{3.5 + 2d} \right) = \frac{65.8 + 18d}{3.5 + 2d}$$

$$\text{Clay: } \gamma'_{ja} = \left(9.6 + \frac{34.3}{3.5 + 2d} \right) = \frac{67.9 + 19.2d}{3.5 + 2d}$$

Passive state

$$\text{Clay: } \gamma'_{jp} = \left(9.6 - \frac{34.3}{3.5 + 2d} \right) = \frac{19.2d - 0.7}{3.5 + 2d}$$

$$u_B = \left(\frac{2ad}{a + 2d} \right) \gamma_w = \left(\frac{2 \times 3.5d}{3.5 + 2d} \right) 9.8 = \frac{68.6d}{3.5 + 2d}$$

$$P_w = \frac{1}{2} (a + d) u_B = \frac{1}{2} (3.5 + d) \frac{68.6d}{3.5 + 2d} = \left(\frac{120d + 34.3d^2}{3.5 + 2d} \right)$$

$$\bar{z}_w = \frac{a + 2d}{3} = \frac{3.5 + 2d}{3}$$

$$H_o - h_w = 6 - 2.5 = 3.5 \text{ m}; \quad H_o - h = 6 - 1 = 5 \text{ m};$$

$$H_w - h = 2.5 - 1.0 = 1.5 \text{ m}$$

Step 3: Carry out the calculations.

Draw a diagram of the pressure distributions. Use a table (see below) or a spreadsheet program to carry out the calculations. See Fig. E12.12b for the lateral pressure distributions.

Step 4: Calculate the anchor force.

$$T_a = 317.1 - 208.7 = 108.4 \text{ kN/m}$$

Assume a factor of safety of 2.

$$\text{Design anchor force} = T_a(\text{FS}) \times \text{Anchor spacing} = 108.4 \times 2 \times 3 = 650.4 \text{ kN}$$

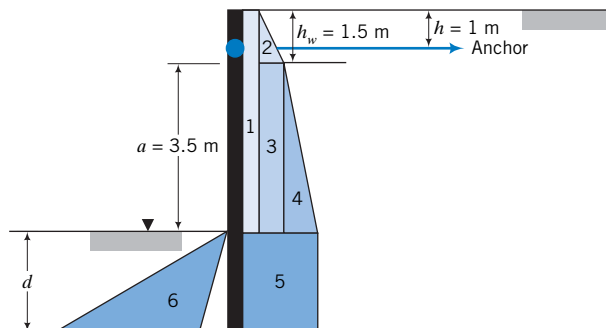


FIGURE E12.12b

Part	Pressure (kPa)	Force (kN/m)	Moment arm from 0	Moment = Force $\times$ Moment arm	D = 5.57 m	D = 4.39 m	
					Moment (kN $\cdot$ m/m)	Moment (kN $\cdot$ m/m)	Force (kN/m)
1	$p_{a1} = K_{ax} q_s$ $= 0.3 \times 10 = 3$	$p_{a1} H_0 = 3 \times 6 = 18$	$\frac{H_0}{2} - h = \frac{6}{2} - 1 = 2$	(-)33.6	-33.6	-33.6	16.8
2	$p_{a2} = K_{ax} \gamma h_w$ $= 0.3 \times 18.8 \times 2.5 = 14.1$	$\frac{1}{2} p_{a2} h_2 = \frac{1}{2} \times 14.1 \times 2.5 = 17.6$	$\frac{2}{3} h_w - 1 = \frac{2}{3} \times 2.5 - 1 = 0.67$	(-)11.0	-11.0	-11.0	16.5
3	$p_{a3} = p_{a2} = 14.1$	$p_{a3} a = 14.1 \times 3.5 = 49.4$	$\frac{a}{2} + (h_w - h) = \frac{3.5}{2} + 1.5 = 3.25$	(-)149.7	-149.7	-149.7	46.1
4	$p_{a4} = K_{ax} \gamma_{ja} a$ $= 0.3 \left(\frac{65.8 + 18d}{3.5 + 2d} \right) 3.5$ $= \frac{69.1 + 18.9d}{3.5 + 2d}$	$\frac{1}{2} p_{a4} a = \frac{1}{2} \left(\frac{69.1 + 18.9d}{3.5 + 2d} \right) \times 3.5$ $= \frac{120.9 + 33.1d}{3.5 + 2d}$	$\frac{2}{3} a + (h_w - h) = \frac{2}{3} \times 3.5 + 1.5 = 3.83$	(-) $\frac{463 + 126.8d}{3.5 + 2d}$	-74.6	-77.5	20.2
5	$p_{a5} = p_{a1} + p_{a2} + p_{a3} + p_{a4}$ $= 3 + 14.1 + 14.1 + \frac{69.1 + 18.9d}{3.5 + 2d}$ $= \frac{178.3 + 81.3d}{3.5 + 2d}$	$p_{a5} d = \left(\frac{129d + 53.1d^2}{3.5 + 2d} \right) d$	$\frac{d}{2} + (H_0 - h) = \frac{d}{2} + 5 = \frac{d + 10}{2}$	(-) $\left(\frac{26.5d^2 + 330d + 645}{3.5 + 2d} \right) d$	-1173.1	-869.2	120.8
Water	$u_b = \frac{8.6d}{3.5 + 2d}$	$P_w \frac{(120 + 34.3d)d}{3.5 + 2d}$	$(H_0 - h) + d - y_w = 5 + d - \frac{3.5 + 2d}{3}$ $= \frac{11.5 + d}{3}$	(-) $\left(\frac{11.4d^2 + 171.4d + 460}{3.5 + 2d} \right) d$	-673.0	-512.4	96.7
6	$p_{p6} = K_{px} \gamma_{jp} d - K_{ax} \gamma_{ja} d$ $= \left[3.8 \left(\frac{19.2d - 0.7}{3.5 + 2d} \right) - 0.34 \left(\frac{67.9 + 19d}{3.5 + 2d} \right) \right]$ $= \left(\frac{66.4d - 25.7}{3.5 + 2d} \right) d$	$\frac{1}{2} p_{p6} d = \frac{1}{2} \left(\frac{66.4d - 25.7}{3.5 + 2d} \right) d$ $= \left(\frac{33.2d - 12.9}{3.5 + 2d} \right) d^2$	$\frac{2}{3} d + (H_0 - h) = \frac{2}{3} d + 5 = \frac{2d + 15}{3}$	(+) $\frac{(22.1d^2 + 157.5d - 64.5)d^2}{3.5 + 2d}$	$\Sigma = -2114.9$ $+3172.9$	$\Sigma = -1653.4$ $+1653.9$	$\Sigma = 317.1$ 208.7

EXERCISES

Theory

- 12.1** Show, using Mohr's circle, that the depth of a tension crack is $z_{cr} = 2s_u/\gamma_{sat}$ for a saturated clay.
- 12.2** Show that a tension crack will not appear in a saturated clay if a surface stress of $q_s \geq 2s_u$ is present.
- 12.3** A trench is to be excavated in a clay soil with $\gamma_{sat} = 20 \text{ kN/m}^3$ and $s_u = 15 \text{ kPa}$.
- Determine the theoretical maximum depth of the trench without lateral support.
 - It was decided to use a bentonite slurry of unit weight γ_f to fill the trench as excavation proceeds. Show that the theoretical maximum depth of the trench is $H_{cr} = \frac{4s_u}{\gamma_{sat} - \gamma_f}$ where γ_{sat} is the saturated unit weight of the soil and s_u is the undrained shear strength.
 - How much deeper can the trench be excavated by using a bentonite slurry of $\gamma_f = 11 \text{ kN/m}^3$?

Problem Solving

- 12.4** Plot the variation of active and passive lateral pressures with depth for the soil profile shown in Fig. P12.4.

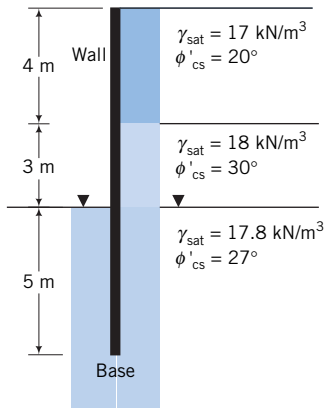


FIGURE P12.4

- 12.5** A retaining wall 8 m high supports a soil of saturated unit weight 18 kN/m^3 , $\phi'_{cs} = 30^\circ$. The backfill is subjected to a surcharge of 15 kPa . Calculate the active force on the wall if (a) the wall is smooth and (b) the wall is rough ($\delta = 20^\circ$). Groundwater is below the base of the wall.
- 12.6** A retaining wall 5 m high was designed to stabilize a slope of 15° . The back of the wall is inclined 10° to the vertical and may be assumed to be rough with $\delta = 20^\circ$. The soil parameters are $\phi'_{cs} = 30^\circ$ and $\gamma_{sat} = 17.5 \text{ kN/m}^3$. After a flood, the groundwater level, which is usually below the base of the wall, rose to the surface. Calculate the lateral force on the wall. Neglect seepage effects.
- 12.7** Figure P12.7 shows rigid walls of height H_o with different geometries. Sketch the distribution of lateral earth pressures on each wall; indicate the location and direction of the resultant lateral force. Show on your diagram what other forces act on the wall, for example, the weight of the wall

(W_w) and the weight of the soil (W_s). You should consider two cases of soil–wall friction: (a) $\delta > 0$ and (b) $\delta = 0$.

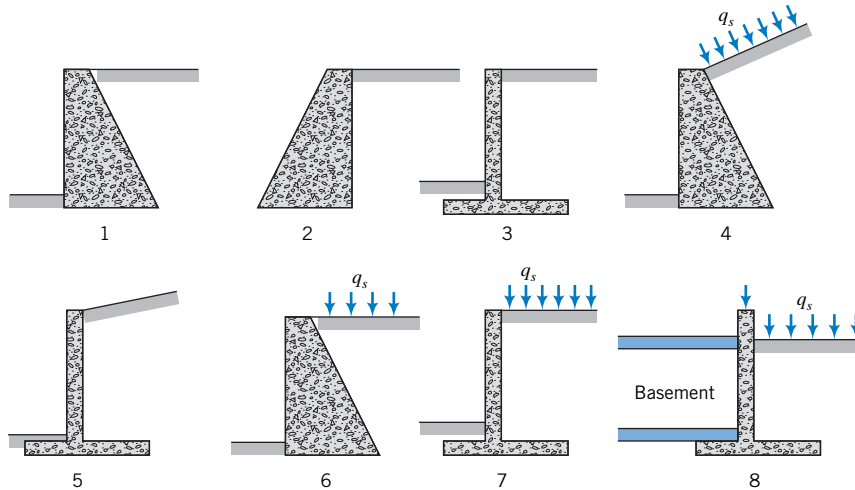


FIGURE P12.7

12.8 Which of the two walls in Fig. P12.8 gives the larger horizontal force? (Show your calculations.)

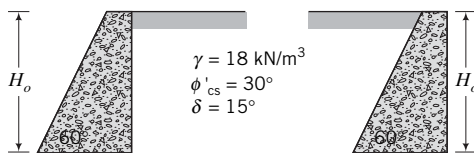


FIGURE P12.8

12.9 Determine the stability of the concrete gravity wall shown in Fig. P12.9.

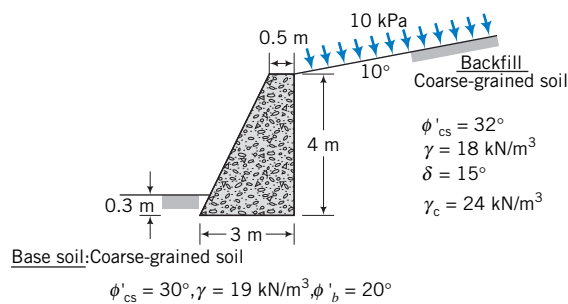


FIGURE P12.9

12.10 Determine the stability of the cantilever wall shown in Fig. P12.10 and sketch a drainage system to prevent buildup of porewater pressures behind the wall.

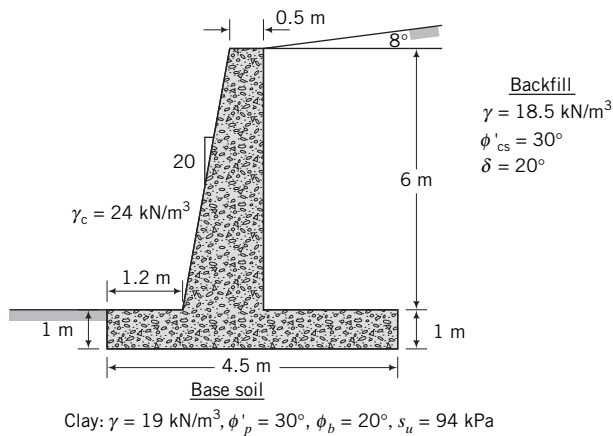


FIGURE P12.10

- 12.11** The drainage system of a cantilever wall shown in Fig. P12.11 became blocked after a heavy rainstorm and the groundwater level, which was originally below the base, rose to 1.5 m below the surface. Determine the stability of the wall before and after the rainfall. Neglect seepage effects.

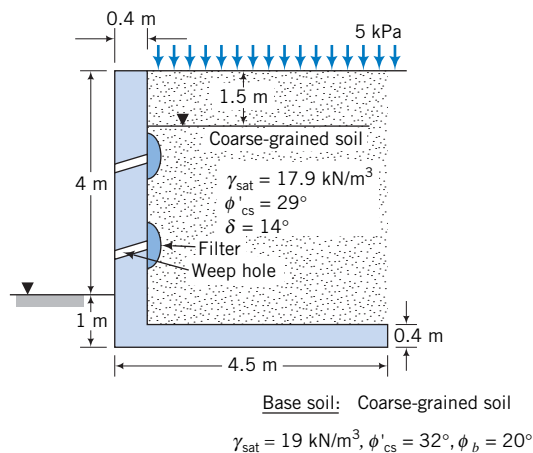


FIGURE P12.11

- 12.12** Determine the embedment depth, d , and maximum bending moment for the cantilever sheet pile wall shown in Fig. P12.12. Use the factored strength method (FSM) with $F_\phi = 1.25$.

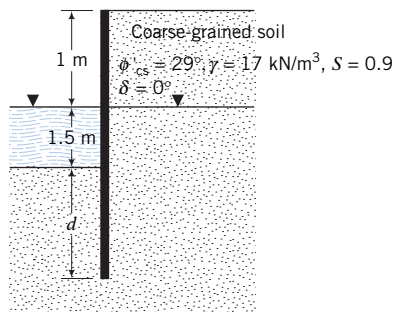


FIGURE P12.12

- 12.13** Determine the embedment depth, d , and maximum bending moment for the cantilever sheet pile wall shown in Fig. P12.13 for long-term conditions. Use the FSM with $F_\phi = 1.25$, and the NPPM with $(FS)_r = 1.5$. Compare the results.

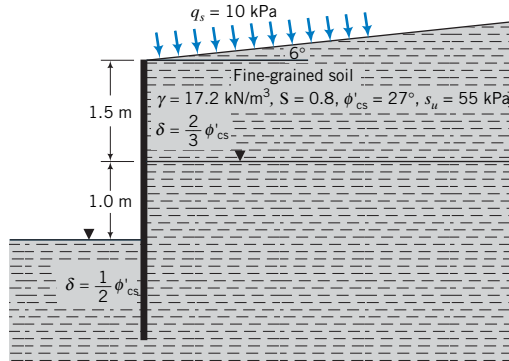


FIGURE P12.13

- 12.14** An anchored sheet pile wall is shown in Fig. P12.14. Determine the embedment depth, the maximum bending moment, and the force on the anchor per unit length of wall. Use either FSM ($F_\phi = 1.25$) or the NPPM with $(FS)_r = 1.5$. Assume the soil above the groundwater to be saturated.

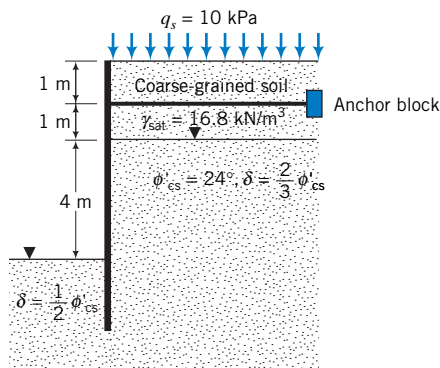


FIGURE P12.14

- 12.15** Determine the depth of embedment and the anchor force per unit length of wall for the retaining wall shown in Fig. P12.15 using the FMM. Assume the soil above the groundwater to be saturated.

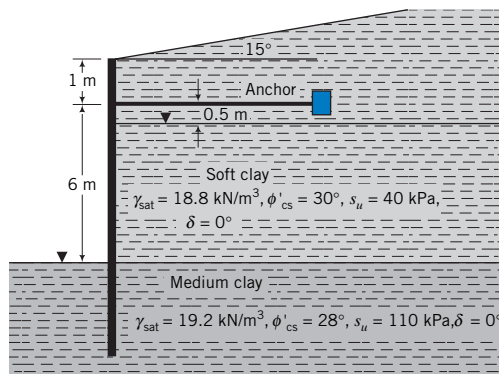


FIGURE P12.15

12.16 Calculate the strut loads per meter length for the braced excavation shown in Fig. P12.16.

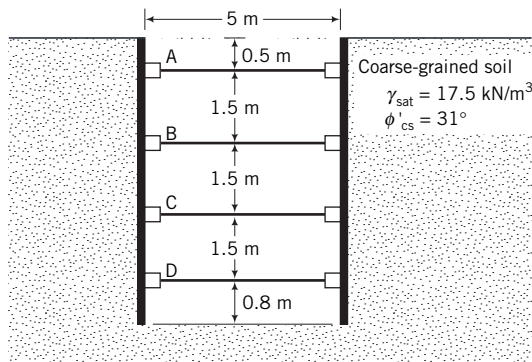


FIGURE P12.16

12.17 A braced excavation is required in a soft clay as shown in Fig. P12.17. Determine the load on the struts per meter length and the factor of safety against bottom heave.

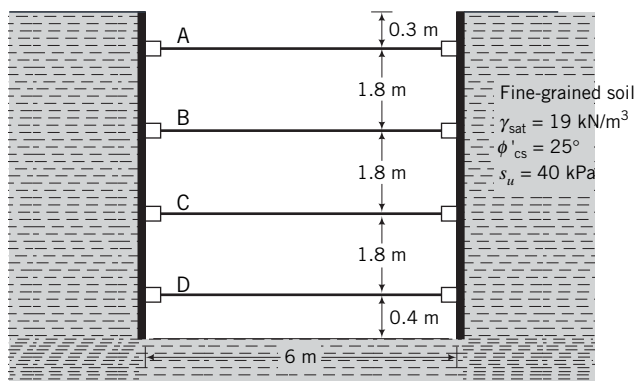


FIGURE P12.17

12.18 A 6 m high geotextile wall is required to support a coarse-grained backfill with $\gamma_{\text{sat}} = 17.5 \text{ kN/m}^3$ and $\phi'_{\text{cs}} = 29^\circ$. The foundation (base) soil is a clay with $\gamma_{\text{sat}} = 18 \text{ kN/m}^3$, $\phi_p = 28^\circ$, $s_u = 72 \text{ kPa}$, and $\phi'_b = 16^\circ$. The ultimate strength of the geotextile is 45 kN/m and the soil–geotextile interface friction angle is 20° . The permanent surcharge is 15 kPa . Determine the spacing and length of geotextile required for stability.

12.19 Redo Exercise 12.18 using galvanized steel ties 75 mm wide with a yield strength $2.5 \times 10^5 \text{ kPa}$, tie–soil interface friction of 20° , and rate of corrosion of 0.025 mm/yr . The design life is 50 years.

Practical

12.20 A section of an approach to a bridge is shown in Fig. P12.20. The sides of the approach are to be supported using MSE walls with $1 \text{ m} \times 1 \text{ m}$ facing panels. Steel ties of width 300 mm are readily available. Determine the length and thickness of the ties required for stability. The design life is 50 years and the rate of corrosion is 0.025 mm/yr . The yield strength is $2.5 \times 10^5 \text{ kPa}$.

- 12.23** A roadway is to be constructed near an existing slope. The engineer decided to use an MSE wall as shown in Fig. P12.23.

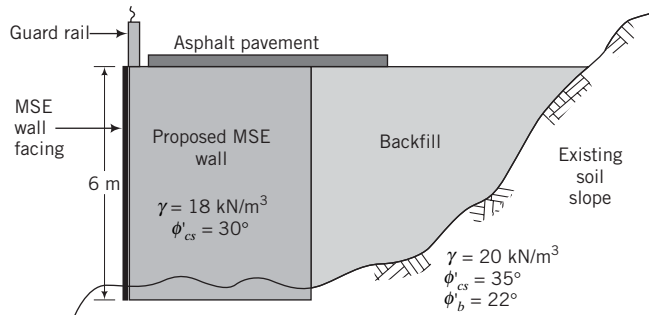


FIGURE P12.23

With a surcharge load of 15 kPa (Guard rail + pavement + construction), compare designs using:

- 4 mm thick, 50 mm wide galvanized steel ties; $f_y = 450 \text{ MPa}$, corrosion rate = 0.025 mm/yr, design life = 75 years, soil-tie interface friction = 20° . Factor of safety desired = 3.
 - Wide-width geotextile of ultimate yield strength 58.5 kN/m and soil-geotextile interface friction = 20° .
- 12.24** A cantilever retaining wall is required to support a cut near a residential area. A preliminary geometry of the wall is shown in Fig. P12.24. A masonry wall 1.2 m high will sit on top of the wall, centered. The masonry wall load (including construction loading) is 9 kN/m and the uniform loading from the homesite is 10 kPa.

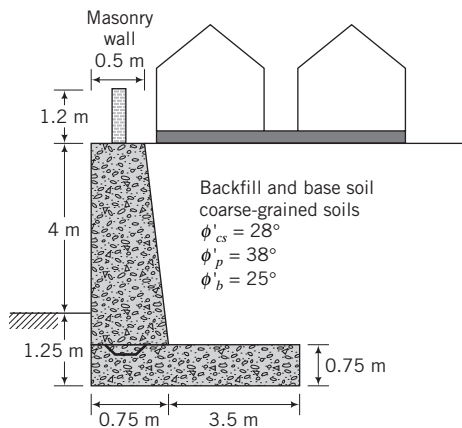


FIGURE P12.24

The wall rotation is limited to $0.0005 H_o$ (H_o is the height of the wall). Check the adequacy of the wall. Determine a suitable wall geometry if the given geometry is unsatisfactory. Sketch a drainage scheme for the wall. (*Hint:* The tolerable wall rotation is very small; it is less than the wall rotation to mobilize the active earth pressure.)

CHAPTER 13

SLOPE STABILITY

13.0 INTRODUCTION

Slopes in soils and rocks are ubiquitous in nature and in man-made structures. Highways, dams, levees, canals, and stockpiles are constructed by sloping the lateral faces of the soil because slopes are generally less expensive than constructing walls. Natural forces (wind, water, snow, etc.) change the topography on Earth and other planets, often creating unstable slopes. Failure of natural slopes (landslides) and man-made slopes have resulted in much death and destruction. Some failures are sudden and catastrophic; others are insidious. Some failures are widespread; others are localized.

Geotechnical engineers have to pay particular attention to geology, surface drainage, groundwater, and the shear strength of soils in assessing slope stability. However, we are handicapped by the geological variability of soils and methods of obtaining reliable values of shear strength. The analyses of slope stability are based on simplifying assumptions and the design of a stable slope relies heavily on experience and careful site investigation.

In this chapter, we will discuss a few simple methods of analysis from which you should be able to:

- Estimate the stability of slopes with simple geometry and geological features.
- Understand the forces and activities that provoke slope failures.
- Understand the effects of geology, seepage, and porewater pressures on the stability of slopes.

You would make use of the following:

- Shear strength of soils (Chapter 7)
- Effective stresses and seepage (Chapter 5)
- Flow through dams (Chapter 11)

Sample Practical Situation A reservoir is required to store water for domestic use. Several sites were investigated and the top choice is a site consisting of clay soils (clay is preferred because of its low permeability—it is practically impervious). The soils would be excavated, forming sloping sides. You are required to determine the maximum safe slope for the reservoir. Slope failures occur frequently, one of which is shown in Fig. 13.1. Your job is to prevent such failure.

13.1 DEFINITIONS OF KEY TERMS

Slip or failure zone is a thin zone of soil that reaches the critical state or residual state and results in movement of the upper soil mass.



FIGURE 13.1 Slope failure near a roadway. (Courtesy of Todd Mooney)

Slip plane or **failure plane** or **slip surface** or **failure surface** is the surface of sliding.

Sliding mass is the mass of soil within the slip plane and the ground surface.

Slope angle (α_s) is the angle of inclination of a slope to the horizontal. The slope angle is sometimes referred to as a ratio, for example, 2:1 (horizontal: vertical).

Porewater pressure ratio (r_u) is the ratio of porewater force on a slip surface to the total weight of the soil and any external loading.

13.2 QUESTIONS TO GUIDE YOUR READING

1. What types of slope failure are common in soils?
2. What factors provoke slope failures?
3. What is an infinite slope failure?
4. What methods of analysis are used to estimate the factor of safety of a slope?
5. What are the assumptions of the various methods of analysis?
6. How does seepage affect the stability of slopes?
7. What is the effect of rapid drawdown on slope stability?

13.3 SOME TYPES OF SLOPE FAILURE



Concept Learning and Self-Assessment

Access Chapter 13, Sections 13.3 and 13.4 on the CD for a summary of types and causes of slope failures. Take Quiz 13.3/4 to test your understanding.

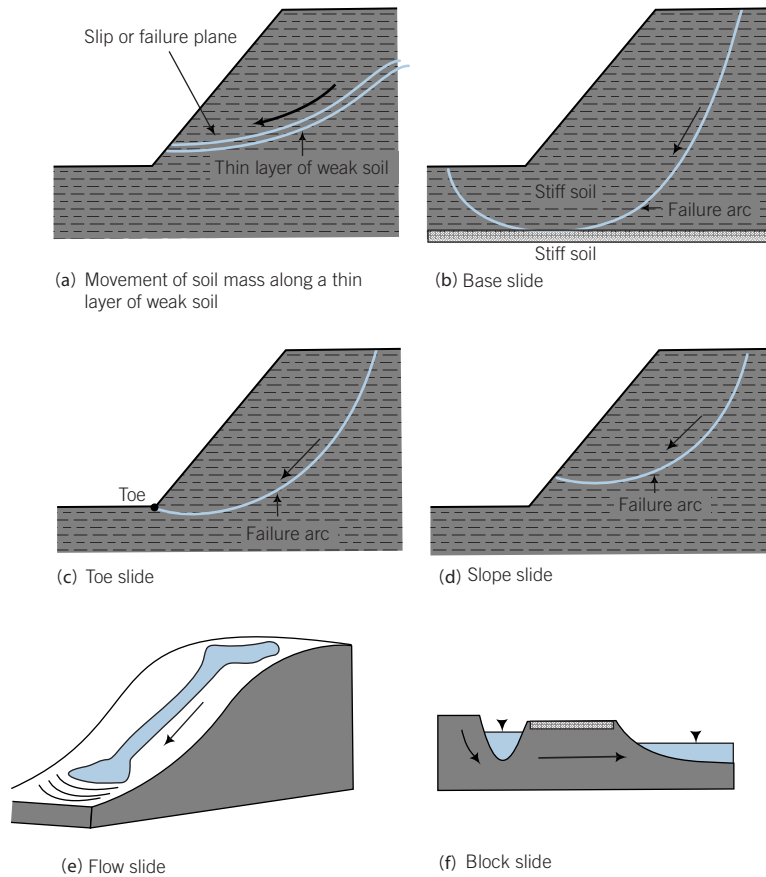


FIGURE 13.2 Some common types of slope failure.

Slope failures depend on the soil type, soil stratification, groundwater, seepage, and the slope geometry. We will introduce a few types of slope failure that are common in soils. Failure of a slope along a weak zone of soil is called a translational slide (Fig. 13.2a). The sliding mass can travel long distances before coming to rest. Translational slides are common in coarse-grained soils.

A common type of failure in homogeneous fine-grained soils is a rotational slide that has its point of rotation on an imaginary axis parallel to the slope. Three types of rotational failure often occur. One type, called a base slide, occurs by an arc engulfing the whole slope. A soft soil layer resting on a stiff layer of soil is prone to base failure (Fig. 13.2b). The second type of rotational failure is the toe slide, whereby the failure surface passes through the toe of the slope (Fig. 13.2c). The third type of rotational failure is the slope slide, whereby the failure surface passes through the slope (Fig. 13.2d).

A flow slide occurs when internal and external conditions force a soil to behave like a viscous fluid and flow down even shallow slopes, spreading out in several directions (Fig. 13.2e). The failure surface is ill defined in flow slides. Multiple failure surfaces usually occur and change continuously as flow proceeds. Flow slides can occur in dry and wet soils.

Block or wedge slides occur when a soil mass is shattered along joints, seams, fissures, and weak zones by forces emanating from adjacent soils. The shattered mass moves as blocks and wedges down the slope (Fig. 13.2f).

What's next . . . What causes the slope failures that we briefly described above? The causes are many and varied. In the next section, we will describe some common causes of slope failure.

13.4 SOME CAUSES OF SLOPE FAILURE

Slope failures are caused, in general, by natural forces, human misjudgment and activities, and burrowing animals. We will describe below some of the main factors that provoke slope failures.

13.4.1 Erosion

Water and wind continuously erode natural and man-made slopes. Erosion changes the geometry of the slope (Fig. 13.3a), ultimately resulting in slope failure or, more aptly, a landslide. River and streams continuously scour their banks undermining their natural or man-made slopes (Fig. 13.3b).

13.4.2 Rainfall

Long periods of rainfall saturate, soften, and erode soils. Water enters into existing cracks and may weaken underlying soil layers, leading to failure, for example, mud slides (Fig. 13.3c).

13.4.3 Earthquakes

Earthquakes induce dynamic forces (Fig. 13.3d), especially dynamic shear forces that reduce the shear strength and stiffness of the soil. Porewater pressures in saturated coarse-grained soils could rise to a value equal to the total mean stress and cause these soils to behave like viscous fluids—a phenomenon known as dynamic liquefaction. Structures founded on these soils would collapse; structures buried within them would rise. The quickness (a few seconds) in which the dynamic forces are induced prevents even coarse-grained soils from draining the excess porewater pressures. Thus, failure in a seismic event often occurs under undrained conditions.

13.4.4 Geological Features

Many failures commonly result from unidentified geological features. A thin seam of silt (a few millimeters thick) under a thick deposit of clay can easily be overlooked in drilling operations or one may be careless in assessing borehole logs only to find later that the presence of the silt caused a catastrophic failure. Sloping, stratified soils are prone to translational slide along a weak layer(s) (Fig. 13.3e). You must pay particular attention to geological features in assessing slope stability.

13.4.5 External Loading

Loads placed on the crest of a slope (the top of the slope) add to the gravitational load and may cause slope failure (Fig. 13.3f). A load placed at the toe, called a berm, will increase the stability of the slope. Berms are often used to remediate problem slopes.

13.4.6 Construction Activities

Construction activities near the toe of an existing slope can cause failure because lateral resistance is removed (Fig. 13.3g). We can conveniently divide slope failures due to construction activities into two cases. The first case is excavated slope and the second case is fill slope.

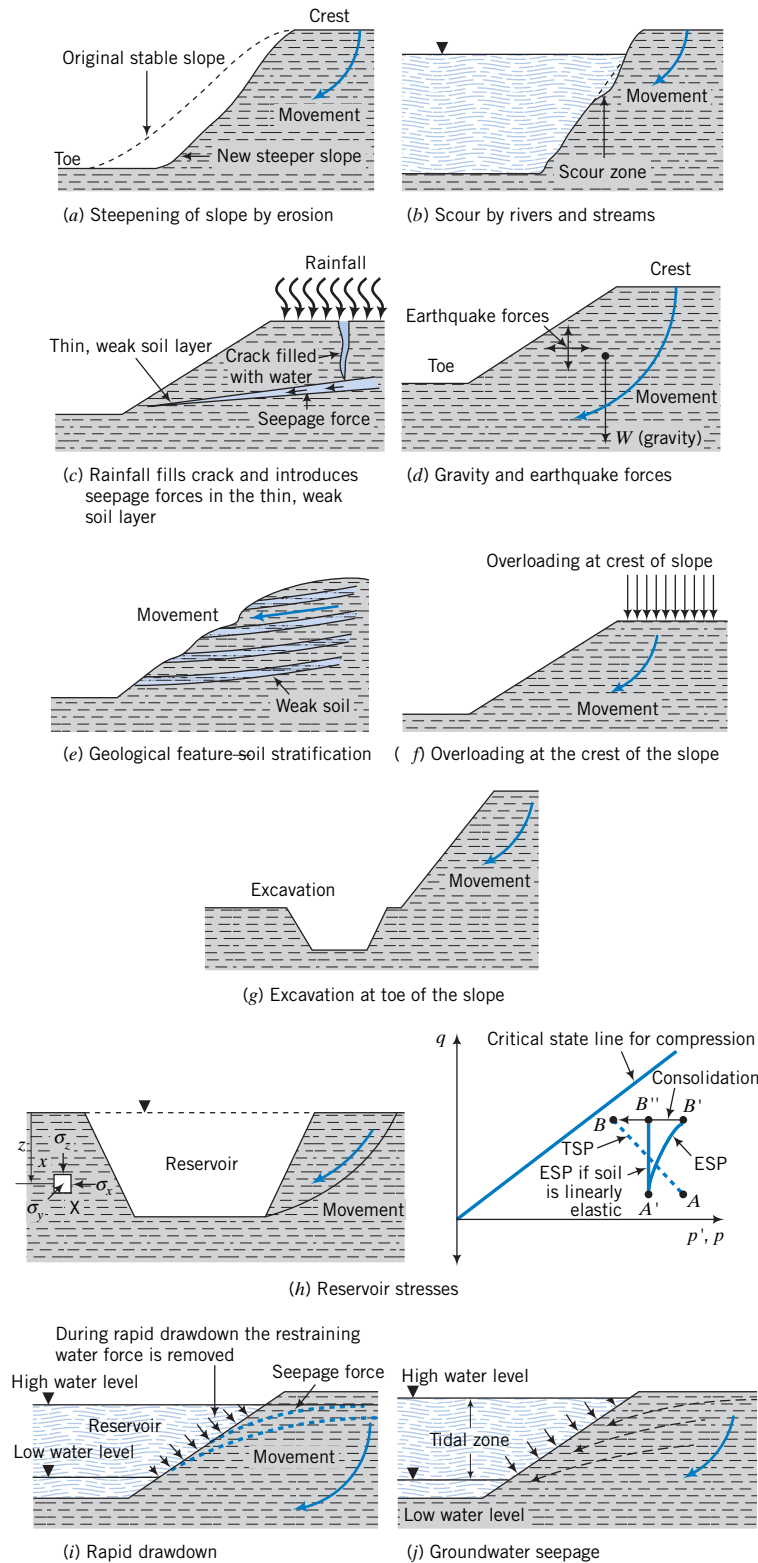


FIGURE 13.3 Some causes of slope failure.

13.4.6.1 Excavated Slopes When excavation occurs, the total stresses are reduced and negative porewater pressures are generated in the soil. With time the negative porewater pressures dissipate, causing a decrease in effective stresses and consequently lowering the shear strength of the soil. If slope failures were to occur, they would most likely take place after construction is completed.

We can use our knowledge of stress paths (Chapter 5) to provide insight on the possible effects of excavation on slope stability. Let us consider a construction activity involving excavation of a normally consolidated fine-grained soil to construct a reservoir (Fig. 13.3h). Let us consider an element of soil, X , at a depth z below the surface.

The initial vertical effective stress is $\sigma'_{z0} = \gamma'z$ and the lateral effective stresses are $\sigma'_{x0} = \sigma'_{y0} = K_0\sigma'_{z0}$. The initial porewater pressure is $u_0 = \gamma_w z$. The stress invariants are

$$p'_0 = \frac{\sigma'_{z0}}{3}(1 + 2K_0), \quad p_0 = \frac{\sigma'_{z0}}{3}(1 + 2K_0) + u_0, \quad \text{and} \quad q_0 = \sigma'_{z0}(1 - K_0)$$

In stress space $\{p'(p), q\}$, the initial total stresses are represented by point A and the initial effective stresses are represented by point A' . The excavation will cause a reduction in σ_x (i.e., $\Delta\sigma_x < 0$) but very little change in σ_z (i.e., $\Delta\sigma_z \cong 0$) and σ_y (i.e., $\Delta\sigma_y \cong 0$). The change in mean total stress is then $\Delta p = -\Delta\sigma_x/3$ and the change in deviatoric stress is $\Delta q = \Delta\sigma_x$. The total stress path (TSP) is depicted as AB in Fig. 13.3h. Although B is near the failure line, the soil is not about to fail because failure is dictated by effective not total stresses. In some cases, the slope fails because the undrained shear strength is exceeded. We will discuss slope failure under undrained condition later in this chapter.

If the soil were a linear, elastic material, the ESP would be $A'B''$ (recall that for elastic material $\Delta p' = 0$ under undrained condition). For an elastoplastic soil, the TSP will be nonlinear. Assuming our soil is elastoplastic, then $A'B'$ would represent our ESP. The ESP moves away from the failure line and the excess porewater pressure is negative. Therefore, failure is unlikely to occur during the excavation stage unless the undrained shear strength of the soil is exceeded.

After the excavation, the excess porewater pressure would dissipate with time. Since no further change in q occurs, the ESP must move from B' to B , that is, toward the failure line. The implication is that slope instability would most likely occur under drained condition (after the excavation). The illustration of the excavation process using stress paths further demonstrates the power of stress paths to provide an understanding of construction events in geotechnical engineering.

13.4.6.2 Fill Slopes Fill slopes are common in embankment construction. Fill (soil) is placed at the site and compacted to specifications, usually greater than 95% Proctor maximum dry unit weight. The soil is invariably unsaturated and negative porewater pressures develop. The soil on which the fill is placed, which we will call the foundation soil, may or may not be saturated. If the foundation soil is saturated, then positive porewater pressures will be generated from the weight of the fill and the compaction process. The effective stresses decrease and consequently the shear strength decreases. With time the positive porewater pressures dissipate, the effective stresses increase and so does the shear strength of the soil. Thus, slope failures in fill slopes are likely to occur during or immediately after construction.

13.4.7 Rapid Drawdown

Reservoirs can be subjected to rapid drawdown. In this case the lateral force provided by the water is removed and the excess porewater pressure does not have enough time to dissipate (Fig. 13.3i). The net effect is that the slope can fail under undrained conditions. If the water level in the reservoir remains at low levels and failure did not occur under undrained conditions, seepage of groundwater would occur and the additional seepage forces can provoke failure (Fig. 13.3j).

The essential points are:

1. *Geological features and environmental conditions (e.g., external loads and natural forces) are responsible for most slope failures.*
2. *The common modes of slope failure in soils are by translation, rotation, flow, and block movements.*

What's next . . . In the next section, we will study how to analyze a slope of infinite extent. We will make use of the limit equilibrium method of analysis and consider long-term and short-term conditions.

13.5 INFINITE SLOPES



Concept Learning and Self Assessment

Access Chapter 13, Section 13.5 on the CD to learn about infinite slope failures. Take Quiz 13.5 to test your understanding.

Infinite slopes have dimensions that extend over great distances. The assumption of an infinite slope simplifies stability calculations. Let us consider a clean, homogeneous coarse-grained soil of infinite slope, α_s . To use the limit equilibrium method, we must first speculate on a failure or slip mechanism. We will assume that slip would occur on a plane parallel to the slope. If we consider a slice of soil between the surface of the slope and the slip plane, we can draw a free-body diagram of the slice as shown in Fig. 13.4.

The forces acting on the slice per unit thickness are the weight $W_j = \gamma b_j z_j$, the shear forces X_j and X_{j+1} on the sides, the normal forces E_j and E_{j+1} on the sides, the normal force N_j on the slip plane, and the mobilized shear resistance of the soil, T_j , on the slip plane. We will assume that forces that provoke failure are positive. If seepage is present, a seepage force $J_s = i\gamma_w b_j z_j$ develops, where i is the hydraulic gradient. The sign of the seepage force depends on the seepage direction. In our case the seepage force is positive because seepage is in the direction of positive forces. For a uniform slope of infinite extent, $X_j = X_{j+1}$ and $E_j = E_{j+1}$.

To continue with the limit equilibrium method, we must now use the equilibrium equations to solve the problem. But before we do this, we will define the factor of safety of a slope. The factor of safety (FS) of a slope is defined as the ratio of the available shear strength of the soil, τ_f , to the minimum shear strength required to maintain stability, τ_m , that is,

$$\text{FS} = \frac{\tau_f}{\tau_m} \quad (13.1)$$

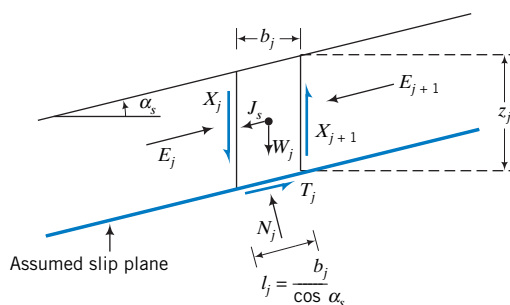


FIGURE 13.4 Forces on a slice of soil in an infinite slope.

The shear strength of soils is governed by the Mohr–Coulomb failure criterion (Chapter 7): that is, $\tau_f = \sigma'_n \tan \phi'$ for an effective stress analysis and $\tau_f = s_u$ for a total stress analysis. The factor of safety is then

$$\text{ESA : } \boxed{\text{FS} = \frac{\sigma'_n \tan \phi'}{\tau_m} = \frac{N' \tan \phi'}{T_m}} \quad (13.2)$$

$$\text{TSA : } \boxed{\text{FS} = \frac{s_u}{\tau_m}} \quad (13.3)$$

where N' is the normal effective force on the slip plane and T_m is the mobilized shear force.

Let us now use statics to solve for the factor of safety. First, we will consider a **slope without seepage** and groundwater below the slip plane. Because the groundwater is below the slip plane, the effective stress is equal to the total stress. From statics,

$$N'_j = W'_j \cos \alpha_s \quad \text{and} \quad T_j = W'_j \sin \alpha_s$$

From the definition of factor of safety for the ESA [Eq. (13.2)], we get

$$\therefore \text{FS} = \frac{N'_j \tan \phi'}{T_j} = \frac{W'_j \cos \alpha_s \tan \phi'}{W'_j \sin \alpha_s} = \frac{\tan \phi'}{\tan \alpha_s} \quad (13.4)$$

At limit equilibrium, $\text{FS} = 1$. Therefore,

$$\boxed{\alpha_s = \phi'} \quad (13.5)$$

The implication of Eq. (13.5) is that the maximum slope angle of a coarse-grained soil cannot exceed ϕ' .

We will now consider groundwater within the sliding mass and assume that **seepage is parallel to the slope**. The seepage force is

$$J_s = i \gamma_w b_j z_j$$

Since seepage is parallel to the slope, $i = \sin \alpha_s$. From statics,

$$N'_j = W'_j \cos \alpha_s = \gamma' b_j z_j \cos \alpha_s \quad (13.6)$$

and

$$\begin{aligned} T_j &= W'_j \sin \alpha_s + J_s = \gamma' b_j z_j \sin \alpha_s + \gamma_w b_j z_j \sin \alpha_s = (\gamma' + \gamma_w) b_j z_j \sin \alpha_s \\ &= (\gamma_{\text{sat}}) b_j z_j \sin \alpha_s \end{aligned}$$

From the definition of factor of safety [Eq. (13.2)], we get

$$\text{FS} = \frac{N'_j \tan \phi'}{T_j} = \frac{\gamma' b_j z_j \cos \alpha_s \tan \phi'}{\gamma_{\text{sat}} b_j z_j \sin \alpha_s} = \frac{\gamma'}{\gamma_{\text{sat}}} \frac{\tan \phi'}{\tan \alpha_s} \quad (13.7)$$

At limit equilibrium, $\text{FS} = 1$. Therefore,

$$\tan \alpha_s = \frac{\gamma'}{\gamma_{\text{sat}}} \tan \phi' \quad (13.8)$$

For most soils, $\gamma'/\gamma_{\text{sat}} \approx \frac{1}{2}$. Thus, seepage parallel to the slope reduces the limiting slope of a clean, coarse-grained soil by about one-half.

The shear stress on the slip plane for a TSA, which is applicable to the short-term slope stability in fine-grained soils, is

$$\tau_j = \frac{T_j}{l_j} = \frac{W_j \sin \alpha_s}{l_j} = \frac{W_j \sin \alpha_s \cos \alpha_s}{b_j} = \frac{\gamma b_j z_j}{b_j} \sin \alpha_s \cos \alpha_s = \gamma z_j \sin \alpha_s \cos \alpha_s \quad (13.9)$$

The factor of safety [Eq. (13.3)] is

$$FS = \frac{s_u}{\sum \tau_j} = \frac{s_u}{\gamma z \sin \alpha_s \cos \alpha_s} = \frac{2s_u}{\gamma z \sin(2\alpha_s)} \quad (13.10)$$

At limit equilibrium, $FS = 1$. Therefore,

$$\alpha_s = \frac{1}{2} \sin^{-1}(2s_u/\gamma z) \quad (13.11)$$

and

$$z = \frac{2s_u}{\gamma \sin(2\alpha_s)} \quad (13.12)$$

The critical value of z occurs when $\alpha_s = 45^\circ$, that is,

$$z = z_{cr} = 2s_u/\gamma \quad (13.13)$$

which is the depth of tension cracks. Therefore, the maximum slope of fine-grained soils under short-term loading, assuming an infinite slope failure mechanism, is 45° . For slopes greater than 45° and depths greater than $2s_u/\gamma$, the infinite slope failure mechanism is not tenable. The infinite slope failure mechanism is more relevant to coarse-grained soils than fine-grained soils because most slope failures observed in fine-grained soils are finite and rotational.

The essential points are:

1. *The maximum stable slope in a coarse-grained soil, in the absence of seepage, is equal to the friction angle.*
2. *The maximum stable slope in coarse-grained soils, in the presence of seepage parallel to the slope, is approximately one-half the friction angle.*
3. *The critical slope angle in fine-grained soils is 45° for an infinite slope failure mechanism.*
4. *The critical depth is the depth of tension cracks, that is, $2s_u/\gamma$.*

EXAMPLE 13.1 *Infinite Slope Stability Considering Seepage*

Dry sand is to be dumped from a truck on the side of a roadway. The properties of the sand are $\phi'_{cs} = 30^\circ$, $\gamma = 17 \text{ kN/m}^3$, and $\gamma_{sat} = 17.5 \text{ kN/m}^3$. Determine the maximum slope angle of the sand in (a) the dry state, (b) the saturated state, without seepage and (c) the saturated state if groundwater is present and seepage were to occur parallel to the slope. What is the safe slope in the dry state for a factor of safety of 1.25?

Strategy The solution to this problem is a straightforward application of Eqs. (13.5) and (13.8).

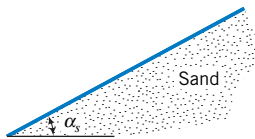


FIGURE E13.1

Solution 13.1

Step 1: Sketch a diagram.

See Fig. E13.1.

Step 2: Determine the maximum slope under the dry condition.

$$\text{Equation (13.5): } \alpha_s = \phi'_{cs} = 30^\circ$$

You should note that any small disturbance would cause this slope to fail since it is at limit equilibrium.

For $FS = 1.25$, the safe slope is

$$\alpha_s = \frac{\tan \phi'_{cs}}{FS} = \frac{\tan 30^\circ}{1.25} = 24.8^\circ$$

Step 3: Determine the maximum slope for the saturated condition.

We see that ϕ'_{cs} is not significantly affected by whether the soil is dry or wet. Therefore, $\alpha_s = 30^\circ$.

Step 4: Determine the maximum slope for seepage parallel to the slope.

$$\text{Equation (13.8): } \tan \alpha_s = \frac{\gamma'}{\gamma_{sat}} \tan \phi'_{cs}$$

$$\gamma' = \gamma_{sat} - \gamma_w = 17.5 - 9.8 = 7.7 \text{ kN/m}^3$$

$$\tan \alpha_s = \frac{\gamma'}{\gamma_{sat}} \tan \phi'_{cs} = \frac{7.7}{17.5} \tan 30^\circ = 0.25$$

$$\alpha_s = 14^\circ$$

EXAMPLE 13.2 *Infinite Slope Failure in Clay Soils*

A trench was cut in a clay slope (Fig. E13.2) to carry TV and telephone cables. When the trench reached a depth of 1.9 m, the excavation was stopped. On resumption the top portion of the clay suddenly failed when a depth of 2 m was excavated, engulfing the trench and injuring several workers. On investigating the failure, you observed a slip plane approximately parallel to the original slope. Determine the undrained shear strength of the clay.

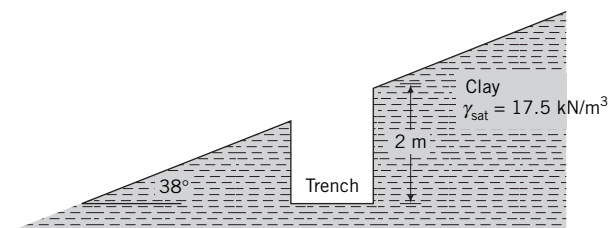


FIGURE E13.2

Strategy The failure observed can be analyzed as an infinite slope failure in a fine-grained soil.

Solution 13.2

Step 1: Determine s_u .

$$\text{From Eq. (13.10): } s_u = \frac{\gamma z (\sin 2\alpha_s)}{2} \text{FS}$$

Since the slope failed, FS = 1.

$$\therefore s_u = \frac{1.75 \times 2 \times \sin(2 \times 38^\circ)}{2} = 17 \text{ kPa}$$

What's next ...In the next section, we will discuss the methods of analyses used to evaluate the stability of slopes.

13.6 TWO-DIMENSIONAL SLOPE STABILITY ANALYSES

Slope stability can be analyzed using one or more of the following: the limit equilibrium method, limit analysis, finite difference method, and finite element method. Limit equilibrium is often the method of choice but the finite element method (FEM) or the finite difference method (FDM) is more flexible and general. You should recall (Chapters 9 and 12) that in the limit equilibrium method, a failure mechanism must be postulated and then the equilibrium equations are used to solve for the collapse load. Several failure mechanisms must be investigated and the minimum load required for collapse is taken as the collapse load. The limit equilibrium method normally gives an upper bound solution (answer higher than the “true” collapse load) because a more efficient mechanism of collapse is possible than those postulated. The limit analysis makes use of the stress–strain characteristics and a failure criterion for the soil. The solution from a limit analysis gives both an upper bound and a lower bound (answer lower than the “true” collapse load). The FEM or FDM requires the discretization of the soil domain, and makes use of the stress–strain characteristics of the soil and a failure criterion to identify soil regions that have reached the failure stress state. The FEM or FDM does not require speculation on a possible failure surface. We will concentrate on the limit equilibrium method in this book because of its simplicity. We will develop the analyses using a generic friction angle, ϕ' . Later, we will discuss the appropriate ϕ' to use. We will use an effective stress analysis (ESA) and a total stress analysis (TSA).

What's next ...Homogeneous fine-grained soils have been observed to fail through a rotational mechanism. Next, we will consider methods of analyses for two dimensional slope failure.

13.7 ROTATIONAL SLOPE FAILURES



Concept Learning, Computer Program Utility

Access Chapter 13, Section 13.7 on the CD to learn about rotational slope failures. Go to www.grow.arizona.edu and search for “slopes.” Run one of the slope stability programs.

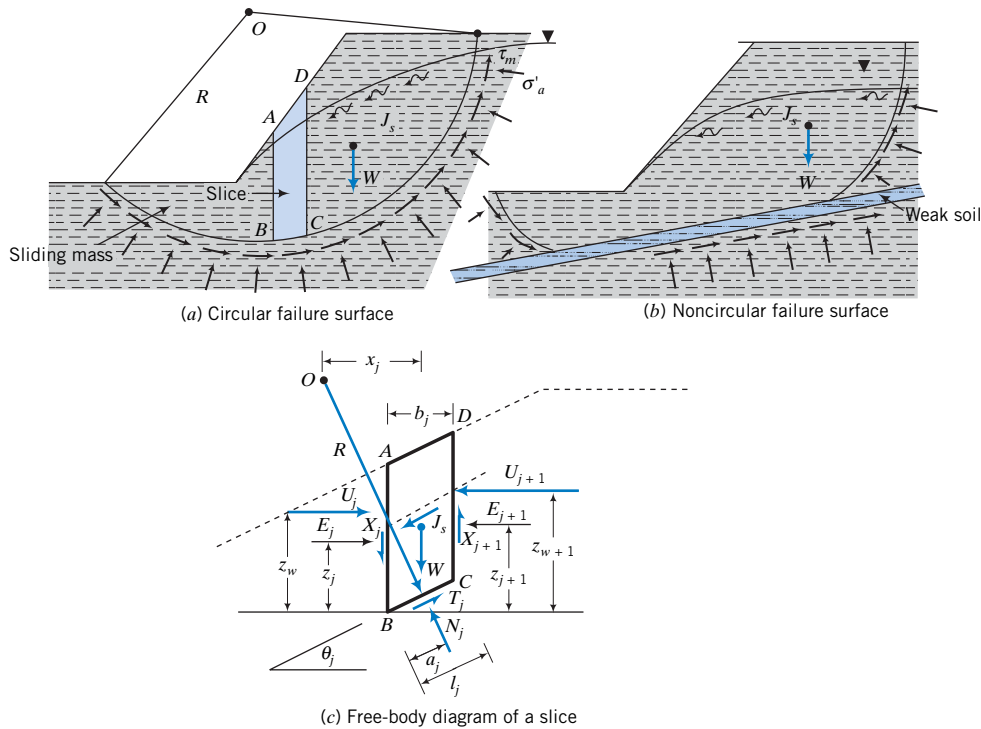


FIGURE 13.5 Circular and noncircular failure surfaces and the forces on a slice of soil.

We will continue to use the limit equilibrium method but instead of a planar slip surface of infinite extent we will assume circular (Fig. 13.5a) and noncircular (Fig. 13.5b) slip surfaces of finite extent. We will assume the presence of a phreatic surface within the sliding mass.

A free-body diagram of the postulated circular failure mechanism would show the weight of the soil within the sliding mass acting at the center of mass. If seepage is present, then the seepage forces, J_s , which may vary along the flow path, are present. The forces resisting outward movement of slope are the shearing forces mobilized by the soil along the slip surface.

We must now use statics to determine whether the disturbing forces and moments created by W and J_s exceed the resisting forces and moments due to the shearing forces mobilized by the soil. However, we have several problems in determining the forces and moments. Here is a list.

- It is cumbersome, if not difficult, to determine the location of the center of mass especially when we have layered soils and groundwater.
- The problem is statically indeterminate.
- We do not know how the mobilized shear strength, τ_m , of the soil varies along the slip surface.
- We do not know how the normal effective stress, σ'_n , varies along the slip surface.
- We do not know how the seepage forces vary within the soil mass and along the failure surface.
- Even the weight of sliding mass is difficult to calculate because of soil layering (different unit weights of the soils) and complex geometry of some slopes.

One approach to solve our problem is to divide the sliding mass into an arbitrary number of slices and then sum the forces and moments of each slice. Of course, the larger the number of slices, the better the accuracy of our answer. Dividing the area inside the sliding mass into slices presents new problems. We now have to account for the internal forces or interfacial forces between the slices.

Let us consider an arbitrary slice, $ABCD$ (Fig. 13.5a), and draw a free-body diagram of the forces acting on the slices as illustrated in Fig. 13.5c. The forces have the following meaning:

- W_j is the total weight of a slice including any external load.
- E_j is the interslice lateral effective force.
- $(J_s)_j$ is the seepage force on the slice.
- N_j is the normal force along the slip surface.
- T_j is the mobilized shear force along the slip surface.
- X_j is the interslice shear force.
- U_j is the force from the porewater pressure.
- z_j is the location of the interslice lateral effective force.
- z_w is the location of the porewater force.
- a_j is the location of the normal effective force along the slip surface.
- b_j is the width of the slice.
- l_j is the length of slip surface along the slice.
- θ_j is the inclination of the slip surface within the slice to the horizontal plane.

The side BC is assumed to be a straight line.

We now have to obtain the values of 13 parameters. We can find W_j , U_j , $(J_s)_j$, b_j , l_j , z_w , and θ_j from the geometry of the slice, the unit weights of the soils, and the location of the phreatic surface. We have six unknowns for each slice and only three equilibrium equations; our problem is then statically indeterminate. To solve our problem, we have to make assumptions regarding three of the unknown parameters. Several solution methods have evolved depending on the assumptions made about the unknown parameters and which equilibrium condition (force, moment, or both) is satisfied. Table 13.1 provides a summary of methods that have been proposed.

TABLE 13.1 Slope Analyses Methods Based on Limit Equilibrium

Method	Assumption	Failure surface	Equilibrium equation satisfied	Solution by
Swedish method (Fellenius, 1927)	Resultant of interslice force is zero; $J_s = 0$	Circular	Moment	Calculator
Bishop's simplified method (Bishop, 1955)	E_j and E_{j+1} are collinear $X_j - X_{j+1} = 0$; $J_s = 0$	Circular	Moment	Calculator
Bishop's method (Bishop, 1955)	E_j and E_{j+1} are collinear; $J_s = 0$	Circular	Moment	Calculator/computer
Morgenstern and Price (1965)	Relationship between E and X of the form $X = \lambda f(x)E$, $f(x)$ is a function ≈ 1 , λ is a scale factor, $J_s = 0$	Any shape	All	Computer
Spencer (1967)	Interslice forces are parallel; $J_s = 0$	Any shape	All	Computer
Bell's method (Bell, 1968)	Assumed normal stress distribution along failure surface; $J_s = 0$	Any shape	All	Computer
Janbu (1973)	$X_j - X_{j+1}$ replaced by a correction factor, f_0 ; $J_s = 0$	Noncircular	Horizontal forces	Calculator
Sarma (1976) ^a	Assumed distribution of vertical interslice forces; $J_s = 0$	Any shape	All	Computer

^aSarma's method includes seismic forces.

We will describe the methods developed by Bishop (1955) and by Janbu (1973) because they are popular methods and require only a calculator or a spreadsheet program. Computer programs are commercially available for all the methods listed in Table 13.1. Before you use these programs, you should understand the principles employed and the assumptions made in their development. The methods of Bishop (1955) and Janbu (1973) were developed by assuming that soil is a cohesive–frictional material. We will modify the derivation of the governing equations of these methods by considering soil as a dilatant–frictional material. We will also develop separate governing equations for an effective stress analysis and a total stress analysis. However, we will retain the names of original developers. For long-term or drained condition, we have to conduct an effective stress analysis. For short-term or undrained condition in fine-grained soils, we have to conduct a total stress analysis.

13.8 METHOD OF SLICES

13.8.1 Bishop's Method

Bishop (1955) assumed a circular slip surface as shown in Fig. 13.5a. Let us apply the equilibrium equations to the forces on the slice shown in Fig. 13.5c, assuming that E_j and E_{j+1} , U_j and U_{j+1} are collinear, N_j acts at the center of the arc length, that is, at $l_j/2$, and $(J_s)_j = 0$.

Summing forces vertically, we get

$$N_j \cos \theta_j + T_j \sin \theta_j - W_j - X_j + X_{j+1} = 0 \quad (13.14)$$

The force due to the porewater pressure (U_j) is $U_j = u_j l_j$. From the principle of effective stress,

$$N'_j = N_j - u_j l_j \quad (13.15)$$

Combining Eqs. (13.14) and (13.15), we get

$$N'_j \cos \theta_j = W_j + X_j - X_{j+1} - T_j \sin \theta_j - u_j l_j \cos \theta_j \quad (13.16)$$

For convenience, let us define the force due to the porewater as a function of W_j as

$$r_u = \frac{u_j b_j}{W_j} \quad (13.17)$$

where r_u is called the porewater pressure ratio. Substituting Eq. (13.17) into (13.16) yields

$$N'_j \cos \theta_j = W_j(1 - r_u) - T_j \sin \theta_j + (X_j - X_{j+1}) \quad (13.18)$$

Bishop (1955) considered only moment equilibrium such that, from Fig. 13.5c,

$$\Sigma W_j x_j = \Sigma T_j R \quad (13.19)$$

where x_j is the horizontal distance from the center of the slice to the center of the arc of radius R , and T_j is the mobilized shear force. Solving for T_j from Eq. (13.19) and noting that $x_j = R \sin \theta_j$, we get

$$\Sigma T_j = \Sigma \frac{W_j x_j}{R} = \Sigma W_j \sin \theta_j \quad (13.20)$$

Recall from Eq. (13.1) that the factor of safety is defined as

$$FS = \frac{\tau_f}{\tau_m} = \frac{(T_f)_j}{T_j} \quad (13.21)$$

where T_f is the soil shear force at failure. In developing the governing equation for FS, we will, firstly, consider effective stress and later total stress. For an ESA,

$$FS = \frac{N'_j \tan(\phi')_j}{T_j} \quad (13.22)$$

By rearranging Eq. (13.22), we get

$$T_j = \frac{N'_j \tan(\phi')_j}{FS} \quad (13.23)$$

Substituting Eq. (13.23) into (13.18) yields

$$N'_j \cos \theta_j = W_j(1 - r_u) - \frac{N'_j \tan(\phi')_j \sin \theta_j}{FS} + (X_j - X_{j+1}) \quad (13.24)$$

Solving for N'_j , we get

$$N'_j = \frac{W_j(1 - r_u) + (X_j - X_{j+1})}{\cos \theta_j + \frac{\tan(\phi')_j \sin \theta_j}{FS}} \quad (13.25)$$

Putting

$$m_j = \frac{1}{\cos \theta_j + \frac{\tan(\phi')_j \sin \theta_j}{FS}} \quad (13.26)$$

we can write N'_j as

$$N'_j = [W_j(1 - r_u) + (X_j - X_{j+1})] m_j \quad (13.27)$$

Substituting Eq. (13.22) into (13.20) gives

$$\Sigma \frac{N'_j \tan(\phi')_j}{FS} = \Sigma W_j \sin \theta_j \quad (13.28)$$

Combining Eqs. (13.27) and (13.28) yields

$$FS = \frac{\Sigma [W_j(1 - r_u) + (X_j - X_{j+1})] \tan(\phi')_j m_j}{\Sigma W_j \sin \theta_j} \quad (13.29)$$

Equation (13.29) is Bishop's equation for an ESA. Bishop (1955) showed that neglecting $(X_j - X_{j+1})$ resulted in about 1% of error. Therefore, neglecting $(X_j - X_{j+1})$, we get

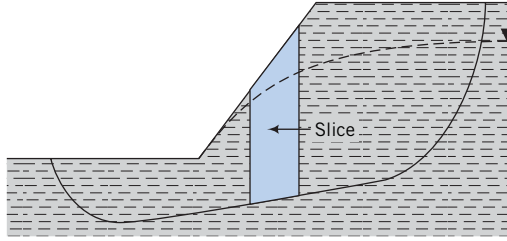
$$FS = \frac{\Sigma [W_j(1 - r_u)] \tan(\phi')_j m_j}{\Sigma W_j \sin \theta_j} \quad (13.30)$$

Equation (13.30) is Bishop's simplified equation for an ESA. If groundwater is below the slip surface, $r_u = 0$ and

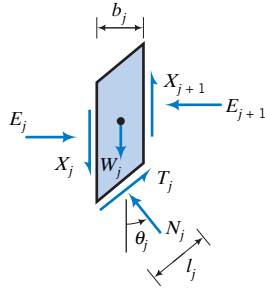
$$FS = \frac{\Sigma W_j \tan(\phi')_j m_j}{\Sigma W_j \sin \theta_j} \quad (13.31)$$

Let us now consider a TSA. The mobilized shear force on the slip surface is

$$T_j = \frac{(s_u)_j l_j}{FS} \quad (13.32)$$



(a) Noncircular slip surface



(b) Forces on a slice

FIGURE 13.6 Failure surface proposed by Janbu and forces on a slice of soil.

where $(s_u)_j$ is the undrained shear strength of the soil along the slip surface within the slice. Combining Eqs. (13.20) and (13.22) yields

$$FS = \frac{\sum (s_u)_j l_j}{\sum W_j \sin \theta_j} \quad (13.33)$$

Since $b_j = l_j \cos \theta_j$, Eq. (13.33) becomes

$$FS = \frac{\sum (s_u)_j \frac{b_j}{\cos \theta_j}}{\sum W_j \sin \theta_j} \quad (13.34)$$

13.8.2 Janbu's Method

Janbu (1973) assumed a noncircular slip surface (Fig. 13.6a). The forces acting on a slice are as shown in Fig. 13.6b. Janbu considered equilibrium of horizontal forces and assumed that $E_j - E_{j+1} = 0$.

The factor of safety, defined with respect to equilibrium of horizontal forces, is

$$FS = \frac{\sum \text{Resisting forces}}{\sum \text{Disturbing forces}} = \frac{\sum (T_f)_j \cos \theta_j}{\sum [W_j + (X_j - X_{j+1})] \tan \theta_j} \quad (13.35)$$

Noting that $(T_f)_j = T_j(FS) = N_j \tan(\phi')_j FS$, we can combine Eqs. (13.35) and (13.27) to yield, for an ESA,

$$FS = \frac{\sum [W_j(1 - r_u) + (X_j - X_{j+1})] m_j \tan \phi'_j \cos \theta_j}{\sum [W_j + (X_j - X_{j+1})] \tan \theta_j} \quad (13.36)$$

Janbu (1973) then replaced the interslice shear forces (X_j and X_{j+1}) by a correction factor f_o as shown in Fig. 13.7. The simplified form of Janbu's equation for an ESA is

$$FS = \frac{f_o \sum W_j (1 - r_u) m_j \tan \phi'_j \cos \theta_j}{\sum W_j \tan \theta_j} \quad (13.37)$$

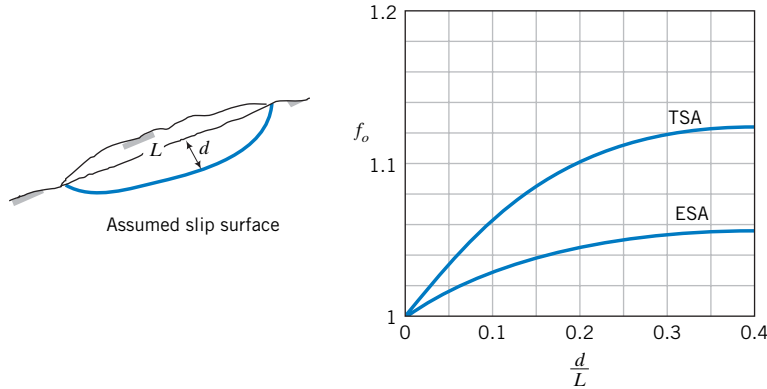


FIGURE 13.7 Correction factor for Janbu's method.

If the groundwater is below the slip surface, $r_u = 0$, and

$$FS = f_o \frac{\sum W_j m_j \tan \phi'_j \cos \theta_j}{\sum W_j \tan \theta_j} \quad (13.38)$$

For a TSA,

$$FS = \frac{\sum (s_u)_j b_j}{\sum [W_j + (X_j - X_{j+1})] \tan \theta_j} \quad (13.39)$$

Replacing the effects of $(X_i - X_{i+1})$ by a correction factor f_o (Fig. 13.7), we get

$$FS = f_o \frac{\sum (s_u)_j b_j}{\sum W_j \tan \theta_j} \quad (13.40)$$

The essential points are:

1. **Bishop (1955) assumed a circular slip plane and considered only moment equilibrium. He neglected seepage forces and assumed that the lateral normal forces are collinear. In Bishop's simplified method, the resultant interface shear is assumed to be zero.**
2. **Janbu (1973) assumed a noncircular failure surface and considered equilibrium of horizontal forces. He made similar assumptions to Bishop (1955), except that a correction factor is applied to replace the interface shear.**
3. **For slopes in fine-grained soils, you should conduct both an ESA and a TSA for long-term loading and short-term loading respectively. For slopes in coarse-grained soils, only an ESA is necessary for short-term and long-term loading provided the loading is static.**

13.9 APPLICATION OF THE METHOD OF SLICES

The shear strength parameters are of paramount importance in slope stability calculations. The soils at the slip surface are at or near the critical or the residual state. You should use $\phi' = \phi'_{cs}$ in all slope stability calculations except for fissured overconsolidated clays. Progressive failure usually occurs in fissured overconsolidated clays. The appropriate value of ϕ' to use is ϕ'_r —the residual friction angle. The measured undrained shear strength is often unreliable. You should use conservative values of s_u for a TSA.

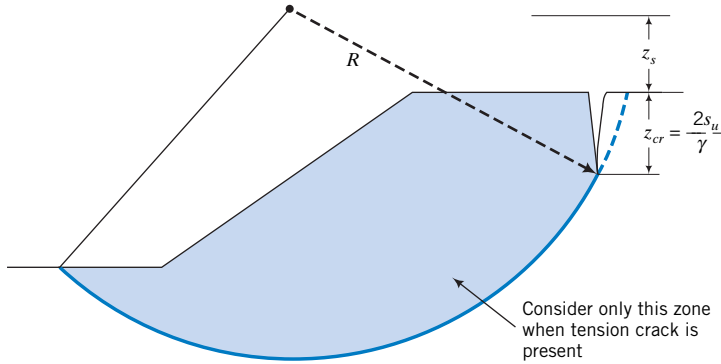


FIGURE 13.8 Effect of tension crack on the slip surface.

Tension cracks in fine-grained soils tend to develop on the crest and the face of slopes in fine-grained soils. There are three important effects of tension cracks. First, they modify the slip surface. The slip surface does not intersect the ground surface but stops at the base of the tension crack (Fig. 13.8). Recall from Chapter 12 that the depth of a tension crack is $z_{cr} = 2s_u/\gamma$.

Second, the tension crack may be filled with water. In this case, the critical depth is $z'_{cr} = 2s_u/\gamma'$ and a hydrostatic pressure is applied along the depth of the crack. The net effect is a reduction in the factor of safety because the disturbing moment is increased. The additional disturbing moment from the hydrostatic pressure is $\frac{1}{2}\gamma_w z_{cr}^2(z_s + \frac{2}{3}z_{cr})$, where z_s is the vertical distance from the top of the tension crack to the center of rotation. The factor of safety using Bishop's simplified method becomes

$$\text{ESA: } FS = \frac{\sum W_j(1 - r_u)(\tan \theta'_j)m_j}{\sum W_j \sin \theta_j + \frac{\frac{1}{2}\gamma_w z_{cr}^2(z_s + \frac{2}{3}z_{cr})}{R}} \quad (13.41)$$

$$\text{TSA: } FS = \frac{\sum (s_u)_j \frac{b_j}{\cos \theta_j}}{\sum W_j \sin \theta_j + \frac{\frac{1}{2}\gamma_w z_{cr}^2(z_s + \frac{2}{3}z_{cr})}{R}} \quad (13.42)$$

Third, the tension crack provides a channel for water to reach underlying soil layers. The water can introduce seepage forces and weaken these layers. The locations of the tension cracks and the critical slip plane are not sensitive to the location of the phreatic surface.

Dams and cuts supporting reservoirs can be subjected to rapid drawdown. Consider the earth dam shown in Fig. 13.9. When the reservoir is full, the groundwater level within the dam will equilibrate with the reservoir water level. If water is withdrawn rapidly, the water level in the reservoir will drop but very little change in the groundwater level in the dam will occur. In fine-grained soils, a few weeks of

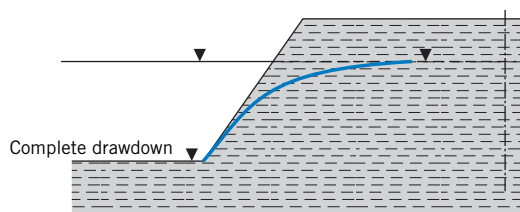


FIGURE 13.9 Drawdown in a reservoir.

drawdown can be rapid because of the low permeability of these soils. Because the restraining lateral force of the water in the reservoir is no longer present and the porewater pressure in the dam is high, the FS will be reduced. The worst case scenario is rapid, complete drawdown. If a partial drawdown occurs and is maintained, then the phreatic surface will keep changing and seepage forces (resulting from porewater pressure gradients) are present in addition to the porewater pressures.

13.10 PROCEDURE FOR THE METHOD OF SLICES



Computer Program Utility

Access Chapter 13 on the CD and click on Spreadsheet on the sidebar to view a spreadsheet program on Jaubou's and Bishop's methods. You can use this spreadsheet or modifications to solve problems in the problem-solving section of this chapter.

The procedure to determine the factor of safety of slopes using the method of slices, with reference to Fig. 13.10, is as follows:

1. Draw the slope to scale and note the positions and magnitudes of any external loads.
2. Draw a trial slip surface and identify its point of rotation.
3. Draw the phreatic surface, if necessary (Chapter 11).
4. If the soil is fine-grained, calculate the depth of the tension crack and sketch in a possible location of the tension crack. You can start at the crest and locate a point on your slip surface where the depth matches the depth of the tension crack.
5. Divide the soil mass above the slip surface into a convenient number of slices. More than five slices are needed for most problems.
6. For each slice:
 - (a) Measure the width, b_j .
 - (b) Determine W_j —the total weight of a slice including any external load. For example, for the two-layer soil profile shown in Fig. 13.10, the weight of slice ② ($j = 2$) is $W_2 = b_2(q_s + z_a(\gamma)_{s2} + z_b(\gamma_{\text{sat}})_{s2} + z_c(\gamma_{\text{sat}})_{s1})$, where s_1 and s_2 denote soil layers 1 and 2, q_s is the surface load per unit area, and z_a , z_b , z_c are the mean heights as shown in Fig. 13.10.
 - (c) Measure the angle θ_j for each slice or you can calculate it if you measure the length, $l_j[\theta_j = \cos^{-1}(b_j/l_j)]$. The angle θ_j can be negative. Angles left of the center of rotation are negative. For example, the value of θ for slice ⑦ is negative but for slice ②, θ is positive. Alternatively, negative values of the slope of the slip surface in a slice give negative values of θ_j .
 - (d) Sketch an equipotential line starting from the intersection of the vertical center line and the slip surface to intersect the phreatic surface at $\sim 90^\circ$. The vertical projection of the equipotential line is the porewater pressure head, $(z_w)_j$.
 - (e) Calculate $r_u = \gamma_w(z_w)_j / \sum_{j=1}^n \gamma_j z_j$.
7. You now have values for all the required parameters to calculate the factor of safety. Prepare a table or use a spreadsheet program to carry out the calculations. To facilitate calculations, using a nonprogrammable calculator, a chart for m_j is shown in Fig. 13.11. You have to guess a value of FS and then iterate until the guessed value of FS and the calculated value of FS are the same or within a small tolerance (≈ 0.01). If a tension crack is present, set the term $W_j(1 - r_u)(\tan \phi'_j)m_j$

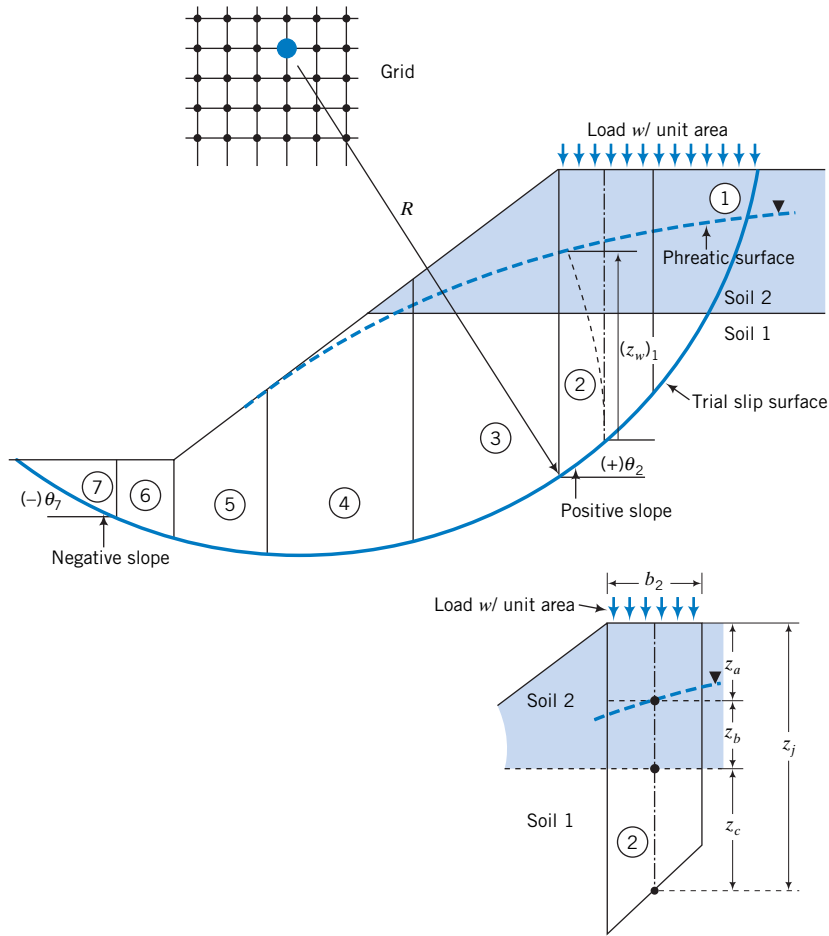


FIGURE 13.10 Method of slices.

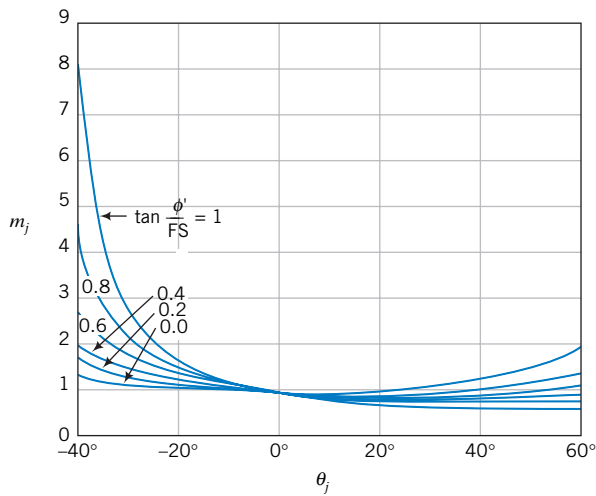


FIGURE 13.11 The m_j to be used in Bishop's method.

to zero but keep the term $W_j \sin \theta_j$ for the slices above the tension crack when you are considering an ESA; for a TSA, set $s_u = 0$ for the slices above the tension crack but keep the term $W_j \sin \theta_j$.

8. Repeat the procedure from item 2 to item 7 until the smallest factor of safety is found. There are several techniques that are used to reduce the number of trial slip surfaces. One simple technique is to draw a grid and selectively use the nodal points as centers of rotation. Commercially available programs use different methods to optimize the search for the slip plane with the least factor of safety.

The essential points are:

1. The appropriate value of ϕ' to use is ϕ'_{cs} , except for fissured overconsolidated fine-grained soils, where you should use $\phi' = \phi'_r$.
2. Tension cracks in fine-grained soils reduce the factor of safety of a slope. Tension cracks may also provide channels for water to introduce seepage forces and weaken underlying soil layers.
3. For slopes adjacent to bodies of water, you should consider the effects of operating and environmental conditions on their stability.

EXAMPLE 13.3 Slope Stability Using Bishop's Method

Computer Program Utility

Access Chapter 13 on the CD and click Spreadsheet on the sidebar to view a spreadsheet program for this and other examples.

Use Bishop's simplified method for the factors of safety of the slope shown in Fig. E13.3a. Assume the soil above the phreatic surface to be saturated. Consider three cases: Case 1—no tension crack; Case 2—tension crack; and Case 3—the tension crack in Case 2 is filled with water.

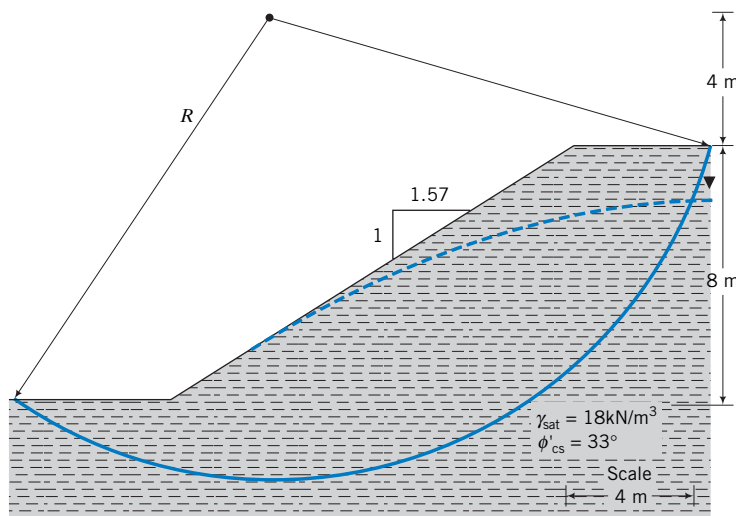


FIGURE E13.3a

Strategy Set up a table to carry out the calculations following the procedure in Section 13.10. If you have access to a spreadsheet program, use it. For the solution of this example, the author used Excel. Before dividing the sliding mass into slices, determine the depth of the tension crack and locate it on the crest of the slope as a side of a slice.

Solution 13.3

Step 1: Redraw the figure to scale.

See Fig. E13.3b.

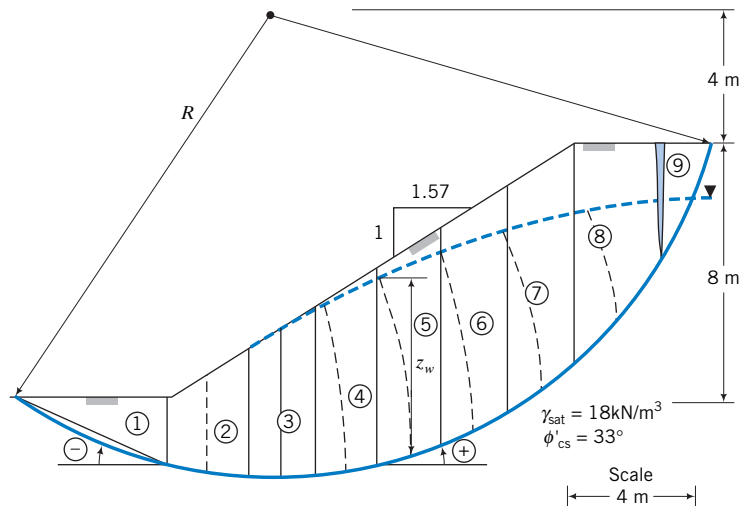


FIGURE E13.3b

Step 2: Find the depth of the tension crack.

$$z_{cr} = \frac{2s_u}{\gamma} = \frac{2 \times 30}{18} = 3.33 \text{ m}$$

Step 3: Divide the sliding mass into slices.

Find a height from the crest to the failure surface that equals z_{cr} and sketch in the tension crack. Use this location of the tension crack as a side of a slice. In Fig. E13.3b, the sliding mass is divided into 9 slices.

Step 4: Set up a spreadsheet.

See Table E13.3.

Step 5: Extract the required values.

Follow the procedure in Section 13.10. The weight $W = \gamma_{\text{sat}} b z$. When the tension crack is considered, the shear resistance of slice ⑨ is neglected. When water fills the tension crack, the moment of the hydrostatic pressure in the tension crack is

$$\text{TCM} = \frac{1}{2} \gamma_w z_{\text{cr}}^2 (z_s + \frac{2}{3} z_{\text{cr}})$$

For an ESA, assume a value of FS and then change this value until it becomes equal to the calculated value.

TABLE E13.3 Bishop's Simplified Method

Homogeneous soil

 s_u 30 kPa ϕ' 33° γ_w 9.8 kN/m³ γ_{sat} 18 kN/m³ z_{cr} 3.33 m

FS 1.05 assumed

No tension crack

Slice	b (m)	z (m)	$W = \gamma b z$ (kN)	z_w (m)	r_u	θ (deg)	m_j	$W \sin \theta$	ESA	TSA
									$W(1 - r_u)(\tan \phi')m_j$	$s_u b / \cos \theta$
1	4.9	1	88.2	1	0.54	-23	1.47	-34.5	38.3	159.7
2	2.5	3.6	162.0	3.6	0.54	-10	1.14	-28.1	54.6	76.2
3	2	4.6	165.6	4.6	0.54	0	1.00	0.0	49.0	60.0
4	2	5.6	201.6	5	0.49	9	0.92	31.5	62.1	60.7
5	2	6.5	234.0	5.5	0.46	17	0.88	68.4	72.2	62.7
6	2	6.9	248.4	5.3	0.42	29	0.85	120.4	80.1	68.6
7	2	6.8	244.8	4.5	0.36	39.5	0.86	155.7	87.6	77.8
8	2.5	5.3	238.5	2.9	0.30	49.5	0.90	181.4	97.5	115.5
9	1.6	1.6	46.1	0.1	0.03	65	1.02	41.8	29.6	113.6
Sum									570.9	794.8
FS									1.06	1.48

Tension crack

FS = 1 assumed

Slice	b (m)	z (m)	$W = \gamma b z$ (kN)	z_w (m)	r_u	θ (deg)	m_j	$W \sin \theta$	ESA	TSA
									$W(1 - r_u)(\tan \phi')m_j$	$s_u b / \cos \theta$
1	4.9	1	88.2	1	0.54	-23	1.50	-34.5	39.1	159.7
2	2.5	3.6	162.0	3.6	0.54	-10	1.15	-28.1	55.0	76.2
3	2	4.6	165.6	4.6	0.54	0	1.00	0.0	49.0	60.0
4	2	5.6	201.6	5	0.49	9	0.92	31.5	61.8	60.7
5	2	6.5	234.0	5.5	0.46	17	0.87	68.4	71.5	62.7
6	2	6.9	248.4	5.3	0.42	29	0.84	120.4	78.9	68.6
7	2	6.8	244.8	4.5	0.36	39.5	0.84	155.7	85.8	77.8
8	2.5	5.3	238.5	2.9	0.30	49.5	0.87	181.4	95.1	115.5
9	1.6	1.6	46.1	0.1	0.03	65	0.99	41.8	0.0	0.0
Sum									536.2	681.2
FS									1.00	1.27

(continued)

TABLE E13.3 (Continued)

Tension crack filled with water										
$R = 14.3 \text{ m}$ $TCM/R = 23.7 \text{ kN}$ $FS = 0.95 \text{ assumed}$										
Slice	b (m)	z (m)	$W = \gamma bz$ (kN)	z_w (m)	r_u	θ (deg)	m_j	$W \sin \theta$	ESA	TSA
									$W(1 - r_u)(\tan \phi')m_j$	$s_u b / \cos \theta$
1	4.9	1	88.2	1	0.54	-23	1.53	-34.5	39.9	159.7
2	2.5	3.6	162.0	3.6	0.54	-10	1.15	-28.1	55.3	76.2
3	2	4.6	165.6	4.6	0.54	0	1.00	0.0	49.0	60.0
4	2	5.6	201.6	5	0.49	9	0.91	31.5	61.5	60.7
5	2	6.5	234.0	5.5	0.46	17	0.86	68.4	70.9	62.7
6	2	6.9	248.4	5.3	0.42	29	0.83	120.4	77.8	68.6
7	2	6.8	244.8	4.5	0.36	39.5	0.83	155.7	84.3	77.8
8	2.5	5.3	238.5	2.9	0.30	49.5	0.86	181.4	93.0	115.5
9	1.6	1.6	46.1	0.1	0.03	65	0.96	41.8	0.0	0.0
Sum								536.6	531.7	681.2
FS									0.95	1.22

Step 6: Compare the factors of safety.

Condition	FS	
	ESA	TSA
Without tension crack	1.06	1.48
With tension crack	1.00	1.27
Tension crack filled with water	0.95	1.22

The smallest factor of safety occurs using an ESA with the tension crack filled with water. The slope, of course, fails because $FS < 1$. ■

EXAMPLE 13.4 Slope Stability Using Bishop's Method for Two-Layer Soils

Determine the factor of safety of the slope shown in Fig. E13.4a. Assume no tension crack.

Strategy Follow the same strategy as described in Example 13.3.

Solution 13.4

Step 1: Redraw the figure to scale.
See Fig. E13.4b.

Step 2: Divide the sliding mass into slices as shown in Fig. E13.4b.

Step 3: Set up a spreadsheet.
See Table E13.4.

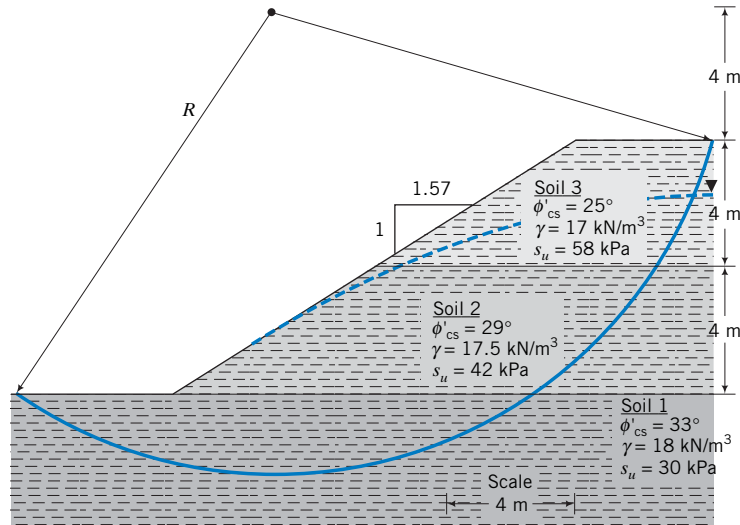


FIGURE E13.4a

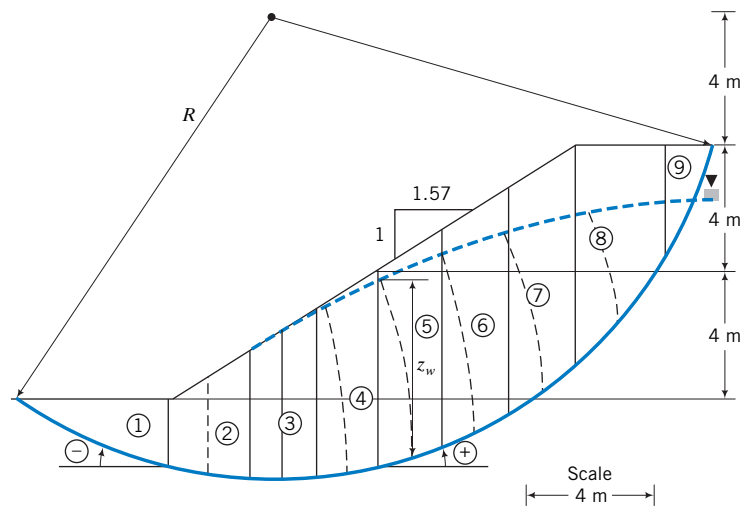


FIGURE E13.4b

Step 4: Extract the required values and perform the calculations. Let z_1 , z_2 , and z_3 be the heights of soil 1, 2, and 3, respectively, in each slice. Slice ①, for example, only contains soil 1 while slice ⑥ contains each soil. Use the appropriate value of ϕ'_{cs} and s_u for each of the slices.

TABLE 13.4 Three Soil Layers

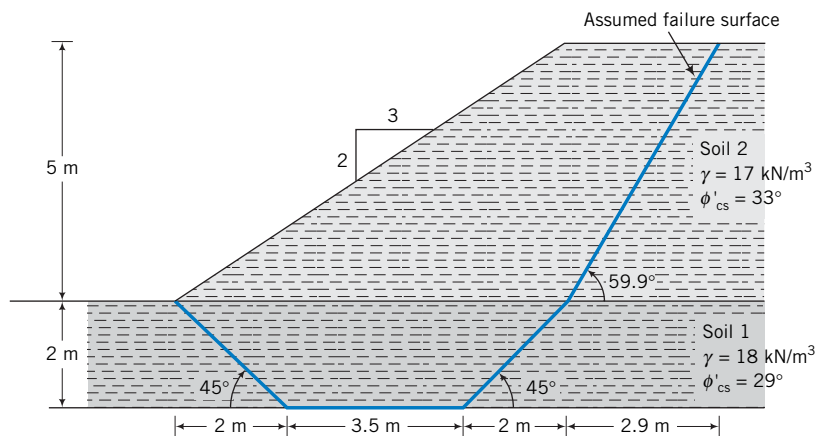
	Soil 1	Soil 2	Soil 3	
s_u	30	42	58	kPa
ϕ'	33°	29°	25°	
γ_w	9.8			kN/m ³
γ_{sat}	18	17.5	17	kN/m ³
FS	1.01	assumed		

											ESA	TSA
Slice	b (m)	z_1 (m)	z_2 (m)	z_3 (m)	$W = \gamma b z$ (kN)	z_w (m)	r_u	θ (deg)	m_j	$W \sin \theta$	$W(1 - r_u)(\tan \phi') m_j$	$s_u b / \cos \theta$
1	4.9	1	0	0	88.2	1	0.54	-23	1.49	-34.5	39.0	159.7
2	2.5	2.3	1.3	0	160.4	3.6	0.55	-10	1.15	-27.8	53.7	76.2
3	2	2.4	2.2	0	163.4	4.6	0.55	0	1.00	0.0	47.6	60.0
4	2	2	3.6	0	198.0	5	0.49	9	0.92	31.0	59.7	60.7
5	2	0.9	4.1	1.5	226.9	5.5	0.48	17	0.87	66.3	67.6	62.7
6	2	0.8	4.1	2	240.3	5.3	0.43	29	0.84	116.5	74.7	68.6
7	2	0	3.7	3.1	234.9	4.5	0.38	39.5	0.89	149.4	72.6	108.9
8	2.5	0	1.5	3.8	227.1	2.9	0.31	49.5	0.94	172.7	81.1	161.7
9	1.6	0	0	1.6	43.5	0.1	0.04	65	1.19	39.4	23.3	219.6
Sum											519.1	978.1
FS											1.01	1.91

For example, $\phi'_{cs} = 33^\circ$ and $s_u = 30$ kPa for soil 1 are applicable to slices ① through ⑥, while $\phi'_{cs} = 24^\circ$ and $s_u = 58$ kPa are applicable to slice ⑨.

EXAMPLE 13.5 Slope Stability Using Janbu's Method

A coarse-grained fill was placed on saturated clay. A noncircular slip surface was assumed, as shown in Fig. E13.5a. Determine the factor of safety of the slope using an ESA. The groundwater level is below the assumed slip surface.

**FIGURE E13.5a**

Strategy Since a noncircular slip surface is assumed, you should use Janbu's method. Groundwater is below the slip surface; that is, $r_u = 0$.

Solution 13.5

Step 1: Redraw the figure to scale.
See Fig. E13.5b.

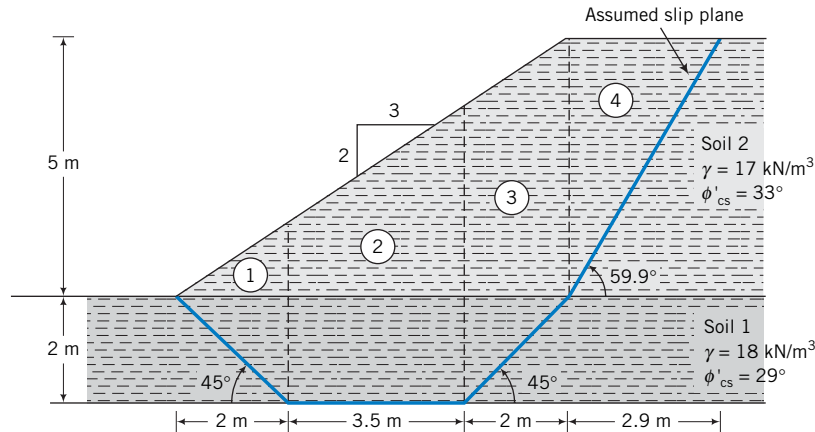


FIGURE E13.5b

Step 2: Divide the sliding mass into a number of slices.
In this case, four slices are sufficient.

Step 3: Extract the required parameters.

Step 4: Carry out the calculations.
Use a spreadsheet as shown in Table E13.5.

TABLE E13.5 Janbu's Method

	Soil 1	Soil 2	
ϕ'	29°	33.5°	
γ_w	9.8		kN/m ³
γ_{sat}	18	17	kN/m ³
d	4.5		m
l	11.5		
d/l	0.39	f_o	1.06
FS	1.04	assumed	

							ESA	
Slice	b (m)	z_1 (m)	z_2 (m)	$W = \gamma bz$ (kN)	θ (deg)	m_j	$W \tan \theta$	$W \tan \phi'(\cos \theta) m_j$
1	2	1	0.7	59.8	-45	3.03	-59.8	71.0
2	3.5	2	2.5	274.8	0	1.00	0.0	152.3
3	2	1	4.3	182.2	45	0.92	182.2	65.9
4	2.9	0	2.5	123.3	59.9	0.95	212.6	38.9
Sum							335.0	328.0
							FS	1.04

What's next . . . Charts can be prepared to allow you to quickly estimate the stability of slopes with simple geometry in homogeneous soils. In the next section, we present some of the popular charts.

13.11 STABILITY OF SLOPES WITH SIMPLE GEOMETRY

13.11.1 Taylor's Method

Let us reconsider the stability of a slope using a TSA as expressed by Eq. (13.34). We can rewrite Eq. (13.34) as

$$FS = N_o \frac{\sum (s_u)_j}{\sum (\gamma z)_j} \quad (13.43)$$

where N_o is called stability number and depends mainly on the geometry of the slope. Taylor (1948) used Eq. (13.43) to prepare a chart to determine the stability of slopes in a homogeneous deposit of soil underlain by a much stiffer soil or rock. He assumed no tension crack, failure occurring by rotation, no surcharge or external loading, and no open water outside the slope.

The procedure to use Taylor's chart to determine the safe slope in a homogeneous deposit of soil using a TSA, with reference to Fig. 13.12, is as follows:

1. Calculate $n_d = D_o/H_o$, where D_o is the depth from the toe to the top of the stiff layer and H_o is the height of the slope.
2. Calculate $N_o = FS(\gamma H_o/s_u)$.
3. Read the value of α_s at the intersection of n_d and N_o .

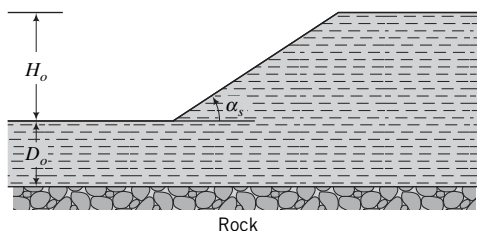
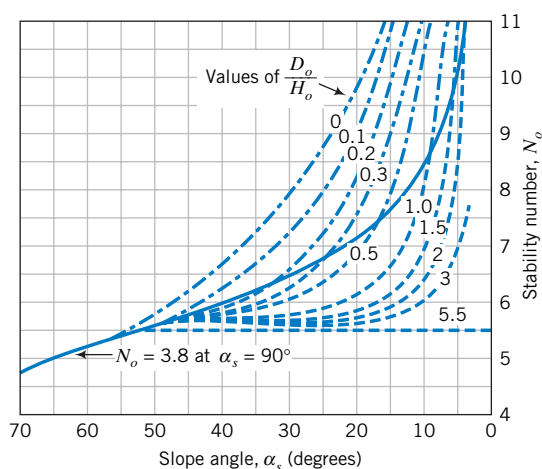


FIGURE 13.12 Taylor's curves for determining the stability of simple slopes.

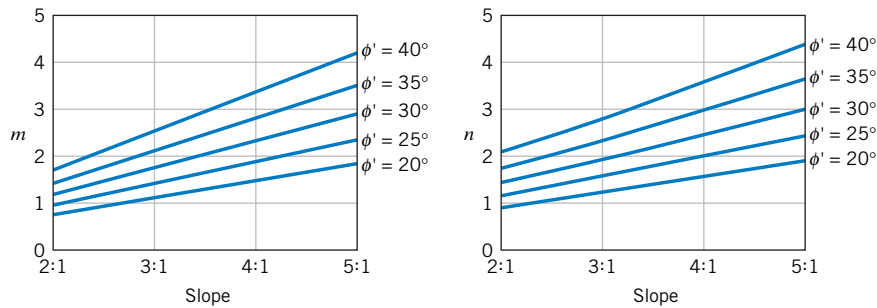


FIGURE 13.13 Values for m and n for the Bishop–Morgenstern method.

If you wish to check the factor of safety of an existing slope or a desired slope, the procedure is as follows:

1. Calculate $n_d = D_o/H_o$.
2. Read the value of N_o at the intersection of α_s and n_d .
3. Calculate $FS = N_o s_u / \gamma H_o$.

13.11.2 Bishop–Morgenstern Method

Bishop and Morgenstern (1960) prepared a number of charts for homogeneous soil slopes with simple geometry using Bishop's simplified method. Equation (13.30) was written as

$$FS = m - nr_u \quad (13.44)$$

where m and n are stability coefficients (Fig. 13.13) that depend on the friction angle and the geometry of the slope.

The procedure to use the Bishop–Morgenstern method is as follows:

1. Assume a circular slip surface.
2. Draw the phreatic surface (Chapter 11).
3. Calculate $r_u = \gamma_w(z_w)_j / \gamma_j z_j$ (see Fig. 13.10). Use a weighted average value of r_u within the sliding mass. A practical range of values of r_u is $\frac{1}{3}$ to $\frac{1}{2}$.
4. With $\phi' = \phi'_{cs}$ and the assumed slope angle, determine the values of m and n from Fig. 13.13.
5. Calculate FS using Eq. (13.44).

EXAMPLE 13.6 Slope Stability Using Taylor's Method

Determine the factor of safety of the slope shown in Fig. E13.6.

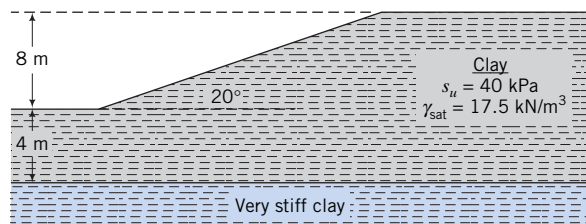


FIGURE E13.6

Strategy Follow the procedures in Section 13.11.

Solution 13.6

Step 1: Calculate n_d .

$$H_o = 8 \text{ m}, D_o = 4 \text{ m}.$$

$$n_d = \frac{D_o}{H_o} = \frac{4}{8} = 0.5$$

Step 2: Determine N_o .

From Fig. 13.12, for $\alpha_s = 20^\circ$ and $n_d = 0.5$, we get $N_o = 6.8$.

Step 3: Calculate FS.

$$\text{FS} = \frac{N_o s_u}{\gamma H_o} = \frac{6.8 \times 40}{17.5 \times 8} = 1.94$$

EXAMPLE 13.7 Slope Stability Using Bishop and Morgenstern Method

Determine the factor of safety of the slope shown in Fig. E13.7.

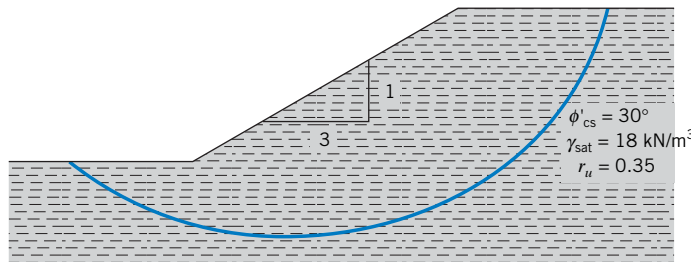


FIGURE E13.7

Strategy Use the Bishop and Morgenstern (1960) charts and equations. Follow the procedures in Section 13.11. Since you are given r_u , you only need to do Steps 4 and 5.

Solution 13.7

Step 1: Determine m and n .

From Fig. 13.13, $m = 1.73$ and $n = 1.92$ for a slope of 3:1 and $\phi'_{cs} = 30^\circ$.

Step 2: Calculate FS.

$$\text{FS} = m - nr_u = 1.73 - 1.92 \times 0.35 = 1.06$$

13.12 FACTOR OF SAFETY (FS)

It is difficult to specify a particular FS for slopes because it is dependent on many factors, including the geological conditions, the population density, types and density of existing and anticipated structures, reliability of soil parameters, groundwater and environmental conditions, and natural hazards. Consequently, the decision on what factor of safety to use is subjective. The usual range of factor of safety is 1.15 to 1.5. In the mining industry, tailing dams are designed with $\text{FS} \approx 1.1$ to 1.2. As a general guide, $\text{FS} \approx 1.3$ is suitable for common slopes such as a cut for a highway. For a dam, $\text{FS} \approx 1.4$ is common.

13.13 SUMMARY

In this chapter, we examined the stability of simple slopes. Slope failures are often catastrophic and may incur extensive destruction and deaths. Slopes usually fail from natural causes (erosion, seepage, and earthquakes) and by construction activities (excavation, change of land surface, etc.). The analyses we considered were based on limit equilibrium, which requires simplifying assumptions. Careful judgment and experience are needed to evaluate slope stability. The geology of a site is of particular importance in determining slope stability. You should consider both an effective stress analysis and a total stress analysis for slopes in fine-grained soils and an effective stress analysis for slopes in coarse-grained soils. The main sources of errors in slope stability analysis are the shear strength parameters, especially s_u , and the determination of the porewater pressures.

Practical Example

EXAMPLE 13.8 *Design of a Slope in a Canal*

A canal is required around a housing project for drainage and recreation. The soil profile is shown in Fig. E13.8a. The maximum level of water expected in the canal is 1 m below the existing surface.

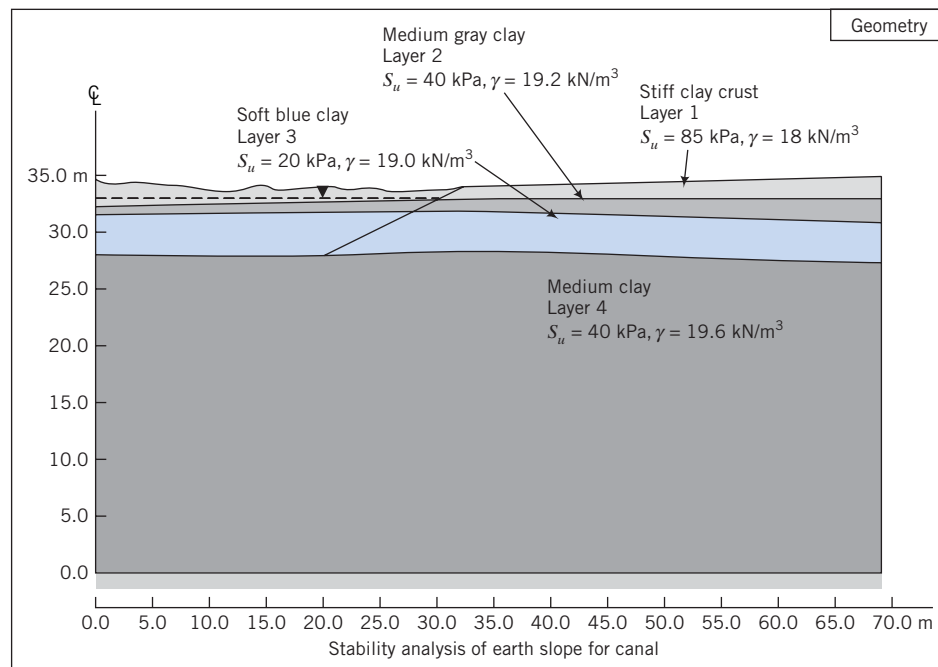


FIGURE E13.8a

It is possible that rapid drawdown could occur. The surcharge is 10 kPa. The proposed slope of the canal is 2(H): 1(V). Determine the factor of safety of the canal slope.

Strategy It is best to use a slope stability program. We will use a freely available student version of the program, Xslope, available from <http://www.civil.usyd.edu.au/cgr/software.php#StudentVersions>.

This program limits the number of layers to 4 and also limits the values of undrained shear strength and friction angles. The soil is assumed to be a cohesive-frictional material. In this book, we assume soil to be a dilatant-frictional material. To use this program for undrained conditions, substitute the undrained shear strength for cohesion and input a friction angle of zero. The worse-case condition is rapid drawdown. However, we will investigate both the normal condition and rapid drawdown condition. This example is intended to show you some of the issues you need to consider when analyzing slope failures using computer programs.

Solution 13.8

Step 1: Input data.

Define nodal points to establish the top of each soil layer as shown in Fig. E13.8b. The input data for this problem for normal condition are shown below.

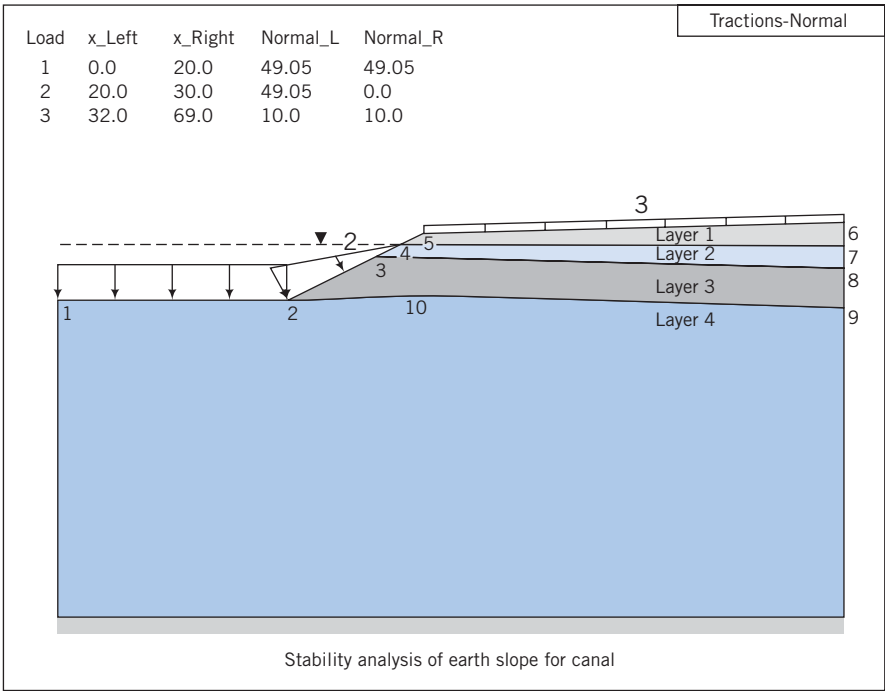


FIGURE E13.8b

[MAXIMUM VALUES]

[Slices Layers Ndes/1 Ndes/t Loads Piezo PP_XG PP_YG Circs]

50 15 40 75 15 15 21 21 10000

[TITLE]

Stability analysis of earth slope for canal

[WATER] Unit weight of water

9.810

[GEOMETRY] No. of layers

4

[GEOMETRY] No. of nodes defining each layer surface

6 5 4 4

[GEOMETRY] Node points defining each layer surface (New line - each layer)

1	2	3	4	5	6
1	2	3	4	7	
1	2	3	8		
1	2	10	9		

[GEOMETRY] Node point, (x)-coordinate, (y)-coordinate

1	0.000	28.000
2	20.000	28.000
3	28.000	32.000
4	30.000	33.000
5	32.000	34.000
6	69.000	35.000
7	69.000	33.000
8	69.000	31.000
9	69.000	27.500
10	31.000	28.500

[PORE PRESSURE FLAG] IPIEZ = (1,2,3, or 4)

1

[MATERIAL PROPERTIES] Layer, Cohesion, Friction angle, Bulk unit weight, {R(u)}

1	85.000	0.000	18.000
2	40.000	0.000	19.200
3	20.000	0.000	19.000
4	40.000	0.000	19.600

[NON-UNIFORM STRENGTH PROPERTIES] Layer, Coh:(x)(y)(m), Phi:(x)(y)(m), y(top), Uniform?

1	0.000	0.000	0.000	0.000	0.000	0.000	0.000 0
2	0.000	0.000	0.000	0.000	0.000	0.000	0.000 0
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000 0
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000 0

[PIEZOMETRIC SURFACE] {IPIEZ=1,2 or 4?} NPP - No. of points along surface

2

[PIEZOMETRIC SURFACE] {IPIEZ=1,2 or 4?} (x,y) coords. of NPP points

0.000	33.000
69.000	33.000

[LOADS] LOADS - No. of external tractions

3

[LOADS] {LOADS>0} (x)-Left,(x)-Right,Normal-L,Shear-L,Normal-R,Shear-R
 0.000 20.000 49.050 0.000 49.050 0.000
 20.000 30.000 49.050 0.000 0.000 0.000
 32.000 69.000 10.000 0.000 10.000 0.000

[CIRCLE ROTATION] IROTAT =0 Clockwise =1 Anti-clockwise
 0

[CIRCLE CODE] NCODE >0 Specify =0 Grid -n Automate n "g" Graphics -1000

[CIRCLE GRID] {NCODE=0} Xbot, Ybot, Xtop, Ytop, NOX, NOY, NOC

[CIRCLES] {NCODE > 0} (x)-Centre, (y)-Centre, Radius

[OUTPUT CODE] IOUT = (1,2,3 or 4)
 1

[SEISMIC COEFFICIENTS-FUNCTIONS] Number of points defining functions
 0

[SEISMIC COEFFICIENTS FUNCTION] Point No., y_coord, Horiz_k, Vertical_k

Step 2: Evaluate results.

The minimum factor of safety is 2.57 and the critical circle is shown in Fig. E13.8c. The circle intersects both vertical boundaries of the soil. Consequently, you need to extend the vertical boundaries and reanalyze the slope. Unfortunately, the student version of Xslope limits the lateral dimension.

Step 3: Analyze rapid drawdown condition.

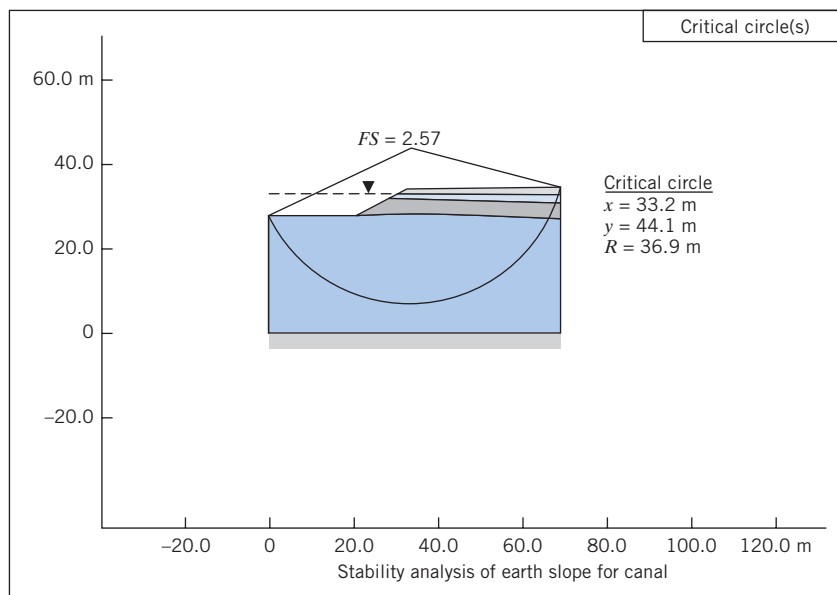


FIGURE E13.8c

For this condition, the groundwater level within the slope remains unchanged but the water in the canal is removed. This is accomplished by setting the loading due to the water on the face of the slope to zero. That is, the surface traction for rapid drawdown changes from

[LOADS]	[LOADS > 0]	(x)-Left,(x)-Right,Normal-L,Shear-L,Normal-R, Shear-R
0.000	20.000	49.050 0.000 49.050 0.000
20.000	30.000	49.050 0.000 0.000 0.000
32.000	69.000	10.000 0.000 10.000 0.000

to

[LOADS]	[LOADS > 0]	(x)-Left,(x)-Right,Normal-L,Shear-L,Normal-R,Shear-R
32.000	69.000	10.000 0.000 10.000 0.000

Step 4: Evaluate rapid drawdown results.

The results from the program (Fig. E13.8d) indicate that piping will occur at the toe of the slope. Piping is a serious matter and must be prevented from occurring in geotechnical systems. However, you need to evaluate the results carefully to check whether piping will indeed occur or it is a result of a numerical problem. The failure circle shown in Fig. E13.8d is very shallow and occurs near the toe of the slope indicating a localized failure. This could occur if the critical hydraulic gradient is achieved or if the porewater pressure is greater than the normal stress near the toe. We are analyzing rapid drawdown (seepage does not occur). Consequently, it appears that the indication of piping from the program is a numerical error possibly caused by the size of the slice (too large). In a situation like this you should use the program to investigate specific circles with low FS and increase the number of slices. In addition, a hand calculation should be carried out as a further check.

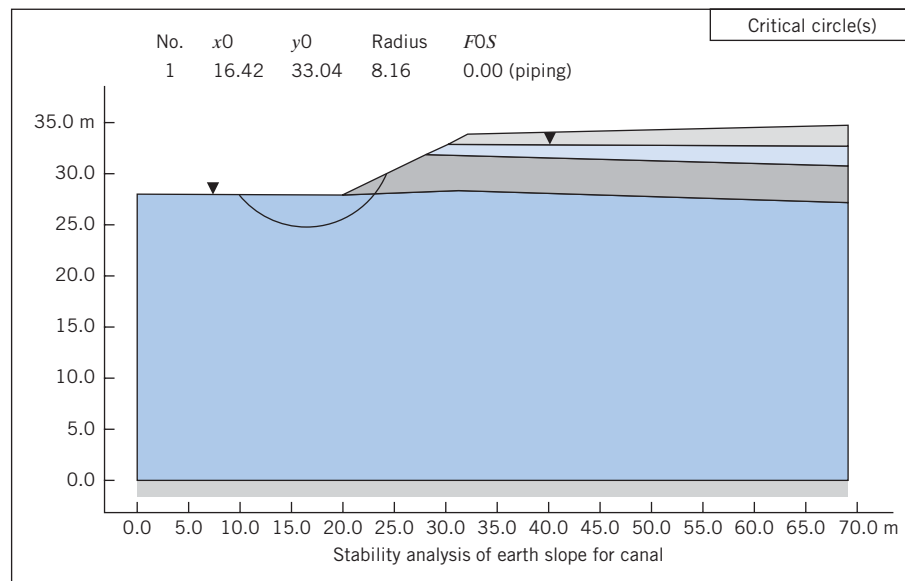


FIGURE E13.8d

EXERCISES

Theory

- 13.1** A clay slope fails as shown in Fig. P13.1. Derive an equation for the undrained shear strength of the clay.

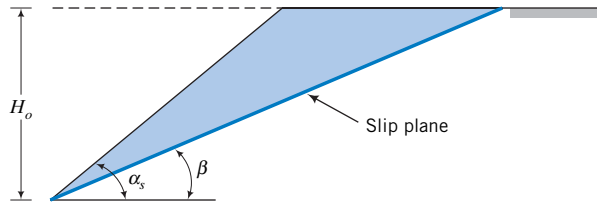


FIGURE P13.1

- 13.2** Derive an equation for the factor of safety of the slope in Fig. P13.2 using the mechanism shown.

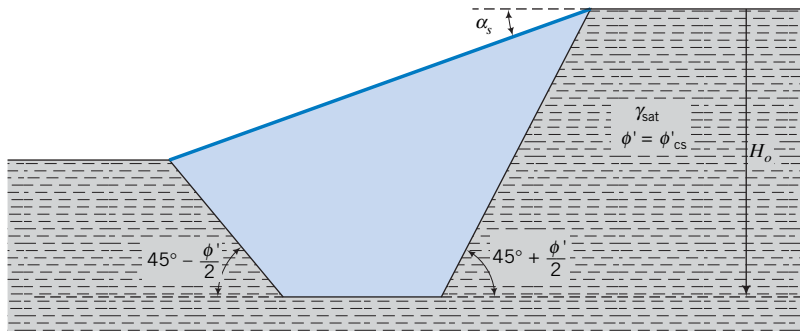


FIGURE P13.2

- 13.3** Figure P13.3 shows the profile of a beach on a lake. It is proposed to drawdown the lake by 2 m. Determine the slope angle of the beach below the high water level after the drawdown. You may assume an infinite slope failure mechanism.

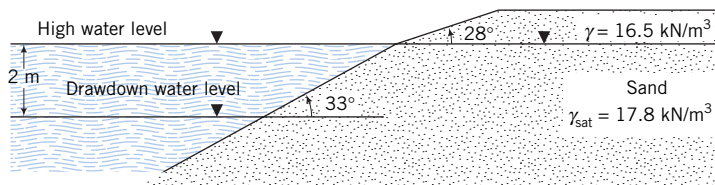


FIGURE P13.3

Problem Solving

- 13.4** A cut for a highway is shown in Fig. P13.4. Determine the factor of safety of the slope using an ESA and a TSA. Assume a center of rotation, O , such that the slip surface passes through the toe of the slope.

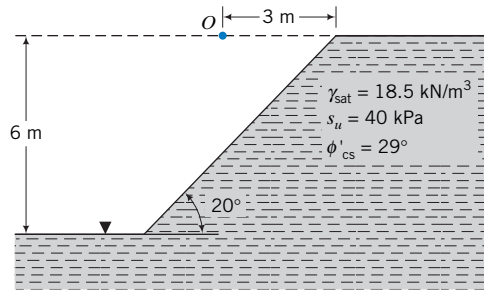


FIGURE P13.4

- 13.5** Determine the factor of safety of the slope shown in Fig. P13.5 using an ESA and a TSA. The point of rotation is indicated by O and the line representing the top of the stiff soil is a tangent to the slip plane.
- 13.6** A compacted earth fill is constructed on a soft, saturated clay (Fig. P13.6). The fill was compacted to an average dry unit of 19 kN/m^3 and water content of 15%. The shearing strength of the fill was determined by CU tests on samples compacted to representative field conditions. The shear strength parameters are $s_u = 35 \text{ kPa}$, $\phi'_p = 34^\circ$, and $\phi'_{cs} = 28^\circ$. The variation of undrained shear strength of the soft clay with depth as determined by simple shear tests is shown in Fig. P13.6 and the friction angle at the critical state is

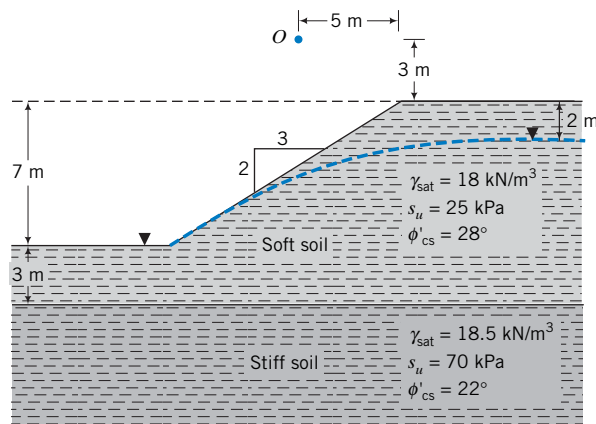


FIGURE P13.5

$\phi'_{cs} = 30^\circ$. The average water content of the soft clay is 55%. Compute the factor of safety using Bishop's simplified method. Assume that a tension crack will develop in the fill.

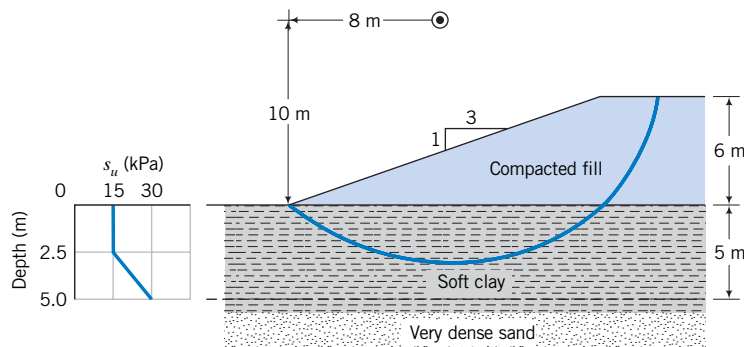


FIGURE P13.6

13.7 A cross section of a canal is shown in Fig. P13.7. Determine the factor of safety for (a) the existing condition and (b) a rapid drawdown of the water level in the canal. Use Bishop's method. The center of rotation of the sliding mass is at coordinates $x = 113$ m and $y = 133$ m. The rock surface is tangent to the slip plane. The properties of the soil are as follows:

Soil	Description	γ_{sat} (kN/m ³)	s_u (kPa)	ϕ'_{cs}
A	Clay	17.8	34	30
B	Clay	18.0	21	28

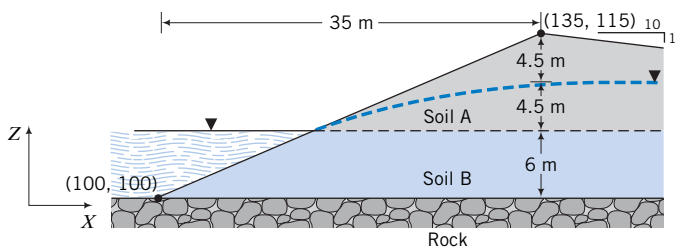


FIGURE P13.7

13.8 Use Janbu's method to determine the factor of safety of the slope shown in Fig. P13.8.

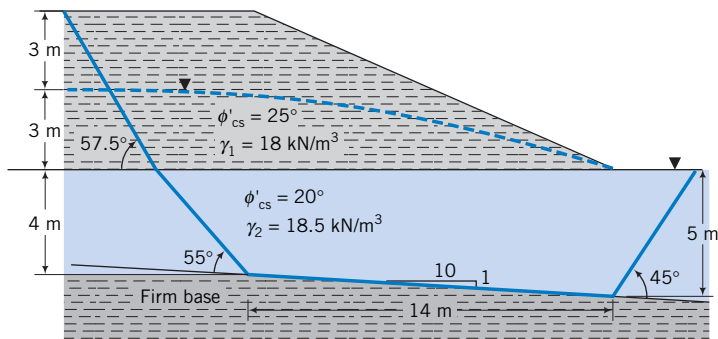


FIGURE P13.8

13.9 Use Taylor's method to determine the factor of safety of the slope shown in Fig. P13.9.

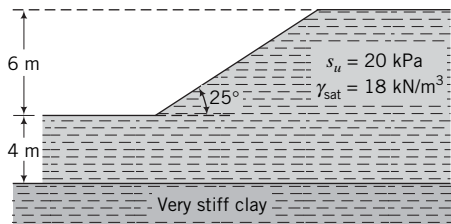


FIGURE P13.9

- 13.10** Use Taylor's method to determine the slope in Fig. P13.10 for $FS = 1.25$.

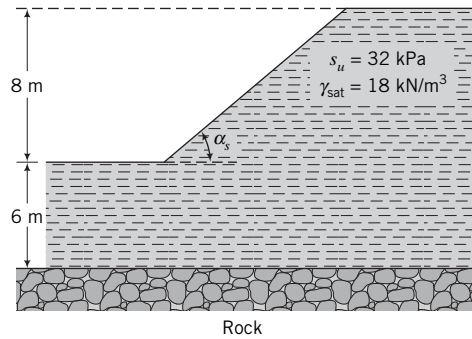


FIGURE P13.10

- 13.11** Determine the factor of safety of a 2:1 slope with $r_u = 0.25$, $\phi'_{cs} = 27^\circ$ using the Bishop–Morgenstern method.

Practical

- 13.12** The soil at a site is shown in Fig. P13.12. A cut was made as shown, and one possible slip plane is shown. Determine the factor of safety.

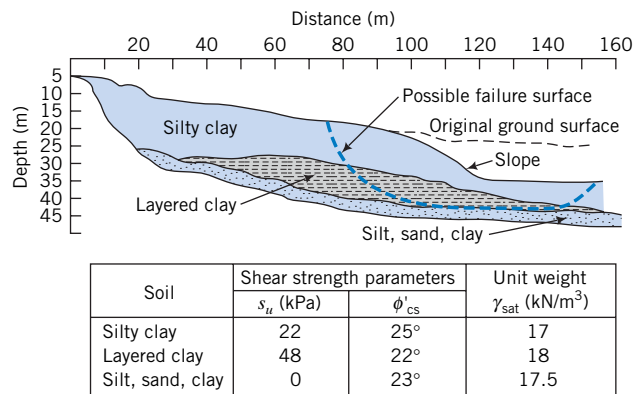


FIGURE P13.12

APPENDIX

A

A COLLECTION OF FREQUENTLY USED SOIL PARAMETERS AND CORRELATIONS

TABLE A.1 Typical Values of Unit Weight for Soils

Soil type	γ_{sat} (kN/m ³)	γ_d (kN/m ³)
Gravel	20–22	15–17
Sand	18–20	13–16
Silt	18–20	14–18
Clay	16–22	14–21

TABLE A.2 Description Based on Relative Density

D_r (%)	Description
0–20	Very loose
20–40	Loose
40–70	Medium dense or firm
70–85	Dense
86–100	Very dense

TABLE A.3 Soil Types, Description, and Average Grain Size According to USCS

Soil type	Description	Average grain size
Gravel	Rounded and/or angular bulky hard rock	Coarse: 75 mm to 19 mm Fine: 19 mm to 4.75 mm
Sand	Rounded and/or angular bulky hard rock	Coarse: 4.75 mm to 2.0 mm Medium: 2.0 mm to 0.425 mm Fine: 0.425 mm to 0.075 mm 0.075 mm to 0.002 mm
Silt	Particles smaller than 0.075 mm exhibit little or no strength when dried	
Clay	Particles smaller than 0.002 mm exhibit significant strength when dried; water reduces strength	<0.002 mm

TABLE A.4 Description of Soil Strength Based on Liquidity Index

Values of LI	Description of soil strength
LI < 0	Semisolid state—high strength, brittle (sudden) fracture expected
0 < LI < 1	Plastic state—intermediate strength, soil deforms like a plastic material
LI > 1	Liquid state—low strength, soil deforms like a viscous fluid

TABLE A.5 Atterberg Limits for Typical Soils

Soil type	LL (%)	PL (%)	PI (%)
Sand		Nonplastic	
Silt	30–40	20–25	10–15
Clay	40–150	25–50	15–100

TABLE A.6 Hydraulic conductivity for Common Soil Types

Soil type	k_z (cm/s)
Clean gravel	>1.0
Clean sands, clean sand and gravel mixtures	1.0 to 10^{-3}
Fine sands, silts, mixtures comprising sands, silts, and clays	10^{-3} to 10^{-7}
Homogeneous clays	$<10^{-7}$

TABLE A.7 Typical Values of Poisson's Ratio

Soil type	Description	ν'
Clay	Soft	0.35–0.4
	Medium	0.3–0.35
	Stiff	0.2–0.3
Sand	Loose	0.15–0.25
	Medium	0.25–0.3
	Dense	0.25–0.35

TABLE A.8 Typical Values of E and G

Soil type	Description	E^a (MPa)	G (MPa)
Clay	Soft	1–15	0.4–5
	Medium	15–30	5–11
	Stiff	30–100	11–38
Sand	Loose	10–20	4–8
	Medium	20–40	8–16
	Dense	40–80	16–32

^aThese are secant values at peak deviatoric stress for dense and stiff soils, and when the maximum deviatoric stress is attained for loose, medium, and soft soils.

TABLE A.9 Ranges of Friction Angles (degrees) for Soils

Soil type	ϕ'_{cs}	ϕ'_p	ϕ'_r
Gravel	30–35	30–50	
Mixture of gravel and sand with fine-grained soils	28–33	30–40	
Sand	27–37 ^a	32–50	
Silt or silty sand	24–32	27–35	
Clays	15–30	20–30	5–15

^aHigher values (32° to 37°) in the range are for sands with significant amount of feldspar (Bolton, 1986). Lower values (27° to 32°) in the range are for quartz sands. The peak dilation angle, α_p , ranges from 0 to 15°.

TABLE A.10 A_f Values

Type of Clay	A_f
Highly sensitive	$\frac{3}{4}$ to 1.0
Normally consolidated	$\frac{1}{2}$ to 1
Compacted sandy clay	$\frac{1}{4}$ to $\frac{3}{4}$
Lightly overconsolidated clays	0 to $\frac{1}{2}$
Compacted clay-gravel	$-\frac{1}{4}$ to $\frac{1}{4}$
Heavily overconsolidated clays	$-\frac{1}{2}$ to 0

SOURCE: After Skempton (1954).

TABLE A.11 Correlation of N , N_{60} , γ , D_r , and ϕ' for Coarse-Grained Soils

N	N_{60}	Compactness	γ (kN/m ³)	D_r (%)	ϕ' (degrees)
0–4	0–3	Very loose	11–13	0–20	26–28
4–10	3–9	Loose	14–16	20–40	29–34
10–30	9–25	Medium	17–19	40–70	35–40 ^a
30–50	25–45	Dense	20–21	70–85	38–45 ^a
>50	>45	Very dense	>21	>85	>45 ^a

^aThese values correspond to ϕ'_p .**TABLE A.12** Correlation of N_{60} and s_u for Saturated Fine-Grained Soils

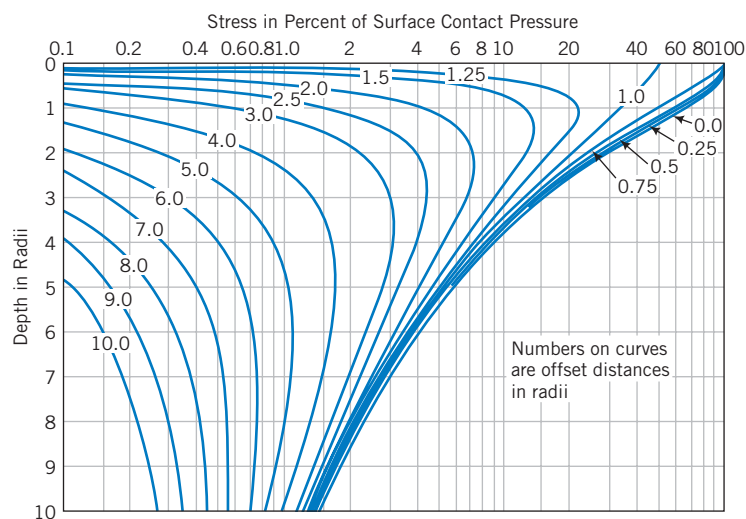
N_{60}	Description	s_u (kPa)
0–2	Very soft	<10
3–5	Soft	10–25
6–9	Medium	25–50
10–15	Stiff	50–100
15–30	Very stiff	100–200
>30	Extremely stiff	>200

TABLE A.13 Empirical Soil Strength Relationships

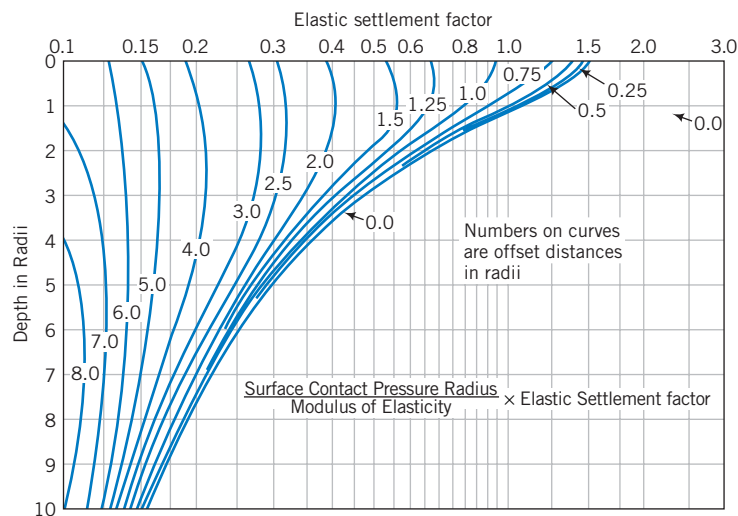
Soil type	Equation	Reference
Normally consolidated clays	$\left(\frac{s_u}{\sigma'_z}\right)_{nc} = 0.11 + 0.0037 \text{ PI}$	Skempton (1957)
Overconsolidated clays	$\frac{(s_u/\sigma'_z)_{oc}}{(s_u/\sigma'_z)_{nc}} = (\text{OCR})^{0.8}$	Ladd et al. (1977)
	$\frac{s_u}{\sigma'_z} = (0.23 \pm 0.04) \text{OCR}^{0.8}$	Jamiolkowski et al. (1985)
All clays	$\frac{s_u}{\sigma'_{zc}} = 0.22$	Mesri (1975)
Clean quartz sand	$\phi'_p = \phi'_{cs} + 3D_r(10 - \ln p'_f) - 3$, where p'_f is the mean effective stress at failure (in kPa) and D_r is relative density. This equation should only be used if $12 > (\phi'_p - \phi'_{cs}) > 0$.	Bolton (1986)

APPENDIX **B**

DISTRIBUTION OF VERTICAL STRESS AND ELASTIC DISPLACEMENT UNDER A UNIFORM CIRCULAR LOAD



(a) Vertical stress increase



(b) Elastic settlement

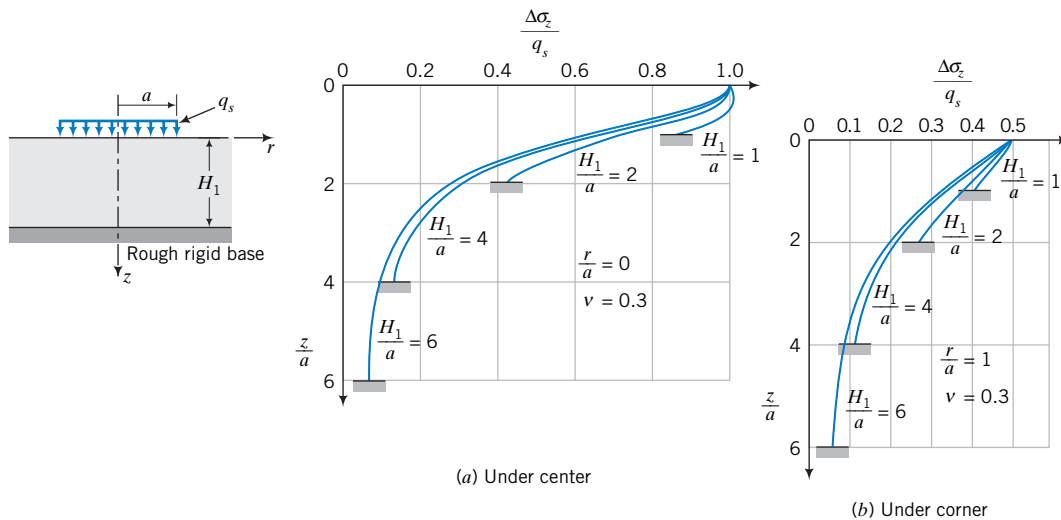
Sources: Foster and Alvin (1954); Poulos and Davis (1974).

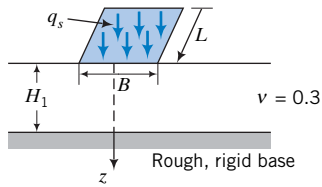
APPENDIX C

DISTRIBUTION OF SURFACE STRESSES WITHIN FINITE SOIL LAYERS

C1 VERTICAL STRESSES IN A FINITE SOIL LAYER DUE TO A UNIFORM SURFACE LOAD ON A CIRCULAR AREA AND A RECTANGULAR AREA

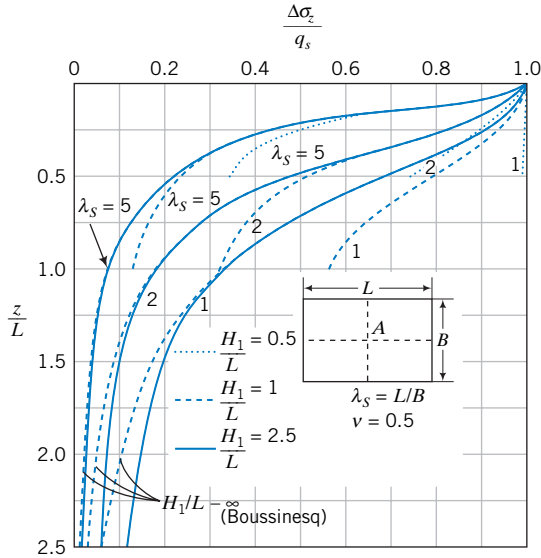
C1.1 Circular Area (Milovic, 1970)



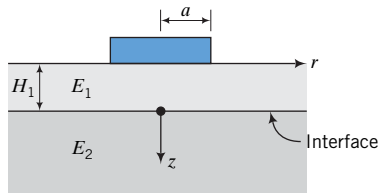
C1.2 Rectangular Area with Rough, Rigid Base (Milovic and Tournier, 1971)

		L/B			L/B		
		1	2	5	1	2	5
		Center			Corner		
H_1/B	z/B	$\Delta\sigma_z/q_s$			$\Delta\sigma_z/q_s$		
1	0.000	1.000	1.000	1.000	0.250	0.250	0.250
	0.100	0.974	0.992	0.996	0.250	0.250	0.250
	0.200	0.943	0.977	0.981	0.250	0.250	0.250
	0.400	0.842	0.924	0.922	0.250	0.250	0.250
	0.600	0.690	0.827	0.832	0.250	0.250	0.250
	0.800	0.570	0.734	0.751	0.238	0.244	0.244
	1.000	0.468	0.638	0.672	0.220	0.232	0.233
2	0.000	1.000	1.000	1.000	0.250	0.250	0.250
	0.100	0.970	0.985	0.990	0.250	0.250	0.250
	0.200	0.930	0.963	0.971	0.250	0.250	0.250
	0.400	0.802	0.878	0.890	0.244	0.249	0.247
	0.800	0.464	0.619	0.670	0.211	0.231	0.230
	1.200	0.286	0.441	0.528	0.172	0.207	0.208
	1.600	0.204	0.340	0.443	0.142	0.183	0.188
3	2.000	0.155	0.269	0.377	0.117	0.160	0.168
	0.000	1.000	1.000	1.000	0.250	0.250	0.250
	0.100	0.970	0.982	0.990	0.250	0.250	0.250
	0.200	0.930	0.962	0.969	0.249	0.250	0.249
	0.400	0.799	0.872	0.884	0.241	0.246	0.246
	0.800	0.453	0.599	0.650	0.203	0.222	0.225
	1.200	0.264	0.405	0.492	0.158	0.191	0.197
5	1.600	0.172	0.289	0.395	0.122	0.163	0.174
	2.000	0.124	0.220	0.333	0.098	0.141	0.156
	2.500	0.093	0.171	0.281	0.078	0.120	0.138
	3.000	0.073	0.137	0.238	0.064	0.102	0.123
	0.000	1.000	1.000	1.000	0.250	0.250	0.250
	0.100	0.970	0.981	0.990	0.250	0.250	0.250
	0.200	0.930	0.961	0.969	0.249	0.250	0.249
5	0.400	0.798	0.870	0.881	0.241	0.245	0.245
	0.800	0.450	0.594	0.641	0.200	0.219	0.221
	1.200	0.258	0.394	0.475	0.153	0.184	0.191
	1.600	0.162	0.271	0.368	0.114	0.151	0.164
	2.000	0.111	0.195	0.296	0.087	0.125	0.143
	2.500	0.075	0.139	0.235	0.064	0.100	0.123
	3.000	0.056	0.105	0.193	0.050	0.082	0.108
5	3.500	0.044	0.085	0.165	0.040	0.069	0.097
	4.000	0.037	0.071	0.144	0.034	0.060	0.089
	4.500	0.032	0.062	0.128	0.030	0.053	0.082
	5.000	0.027	0.053	0.113	0.026	0.047	0.075

C1.3 Rectangular Area with Smooth, Rigid Base (Sovinc, 1961)



C2 VERTICAL STRESSES IN A TWO-LAYER SOIL UNDER THE CENTER OF A UNIFORMLY LOADED CIRCULAR AREA (Fox, 1948)



		E_1/E_2				E_1/E_2			
		1	10	100	1000	1	10	100	1000
		Rough interface				Smooth			
a/H_1	z	$\Delta\sigma_z/q_s$				$\Delta\sigma_z/q_s$			
1/2	0	0.284	0.101	0.0238	0.0051	0.31	0.105	0.0241	0.0051
	H_1	0.087	0.047	0.0158	0.0042	0.141	0.063	0.0183	0.0045
	$2H_1$	0.0403	0.0278	0.0117	0.0035	0.064	0.0367	0.0136	0.0038
	$3H_1$	0.023	0.0184	0.0091	0.0031	0.0346	0.0235	0.0105	0.0033
	$4H_1$	0.0148	0.0129	0.0074	0.0028	0.0212	0.0161	0.0083	0.0029
1	0	0.646	0.292	0.081	0.0185	0.722	0.305	0.082	0.019
	H_1	0.284	0.168	0.06	0.0162	0.437	0.217	0.068	0.0172
	$2H_1$	0.145	0.105	0.046	0.0143	0.225	0.136	0.0525	0.0151
	$3H_1$	0.087	0.07	0.036	0.0124	0.128	0.089	0.0409	0.0133
	$4H_1$	0.057	0.05	0.029	0.011	0.081	0.062	0.0326	0.0117
2	0	0.911	0.644	0.246	0.071	1.025	0.677	0.249	0.067
	H_1	0.646	0.48	0.205	0.0606	0.869	0.576	0.225	0.063
	$2H_1$	0.424	0.34	0.165	0.0542	0.596	0.421	0.186	0.057
	$3H_1$	0.284	0.244	0.133	0.048	0.396	0.302	0.15	0.051
	$4H_1$	0.2	0.181	0.108	0.0428	0.271	0.22	0.122	0.0454

APPENDIX D

LATERAL EARTH PRESSURE COEFFICIENTS (KERISEL AND ABSI, 1970)

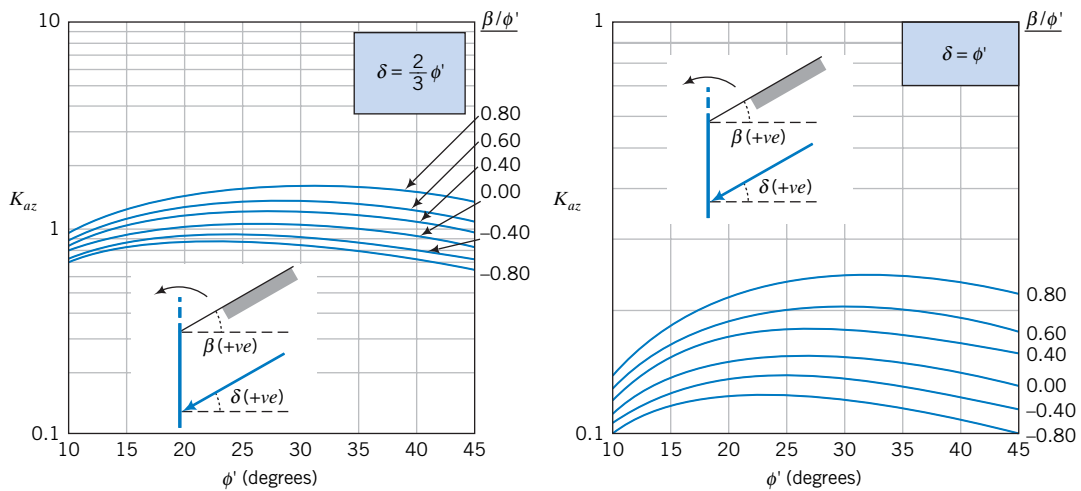


FIGURE D.1 Vertical component of the active lateral pressure coefficient. (Plotted from data published by Kerisel and Absi, 1990.)

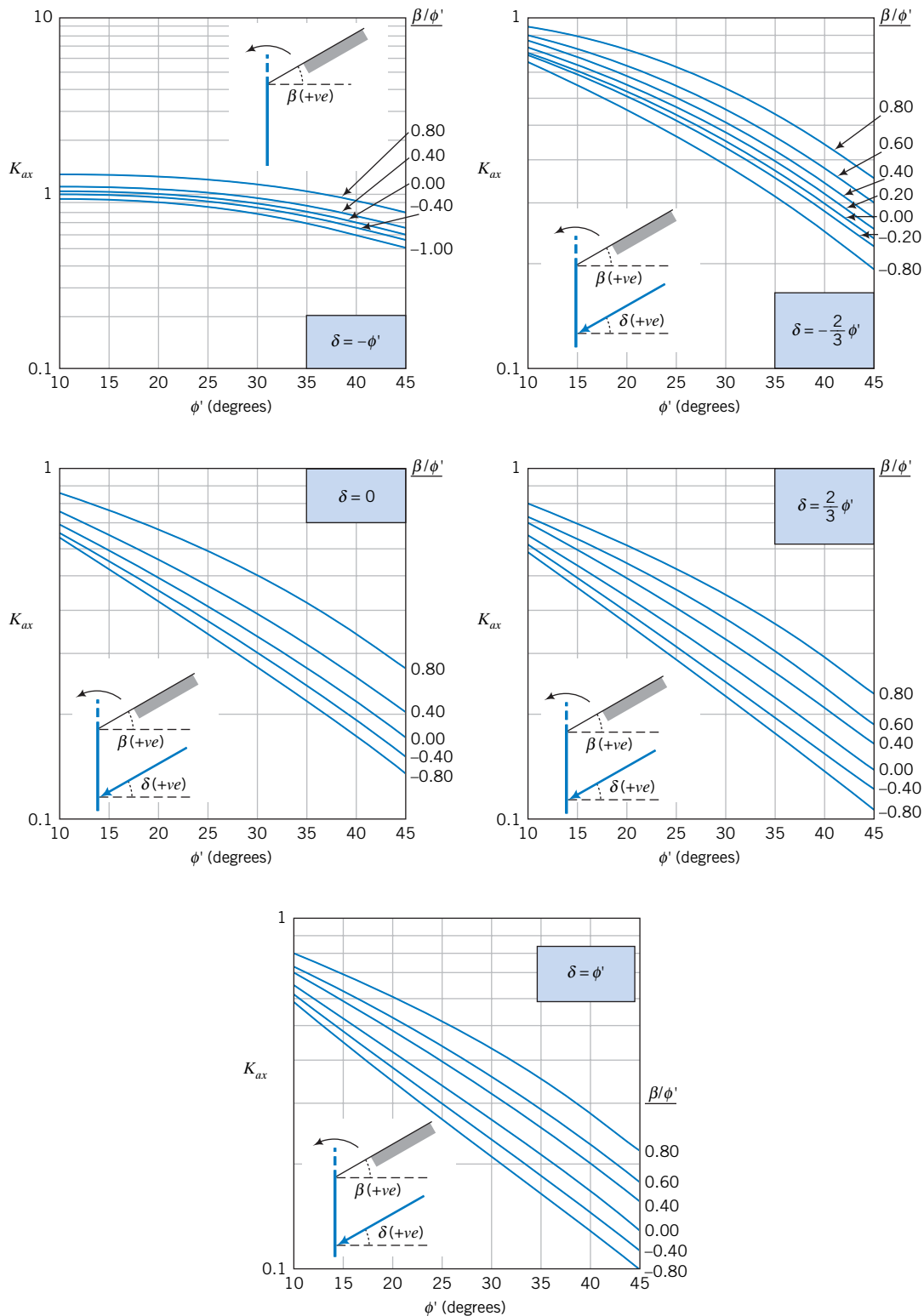


FIGURE D.2 Horizontal component of the active lateral pressure coefficient. (Plotted from data published by Kerisel and Absi, 1990.)

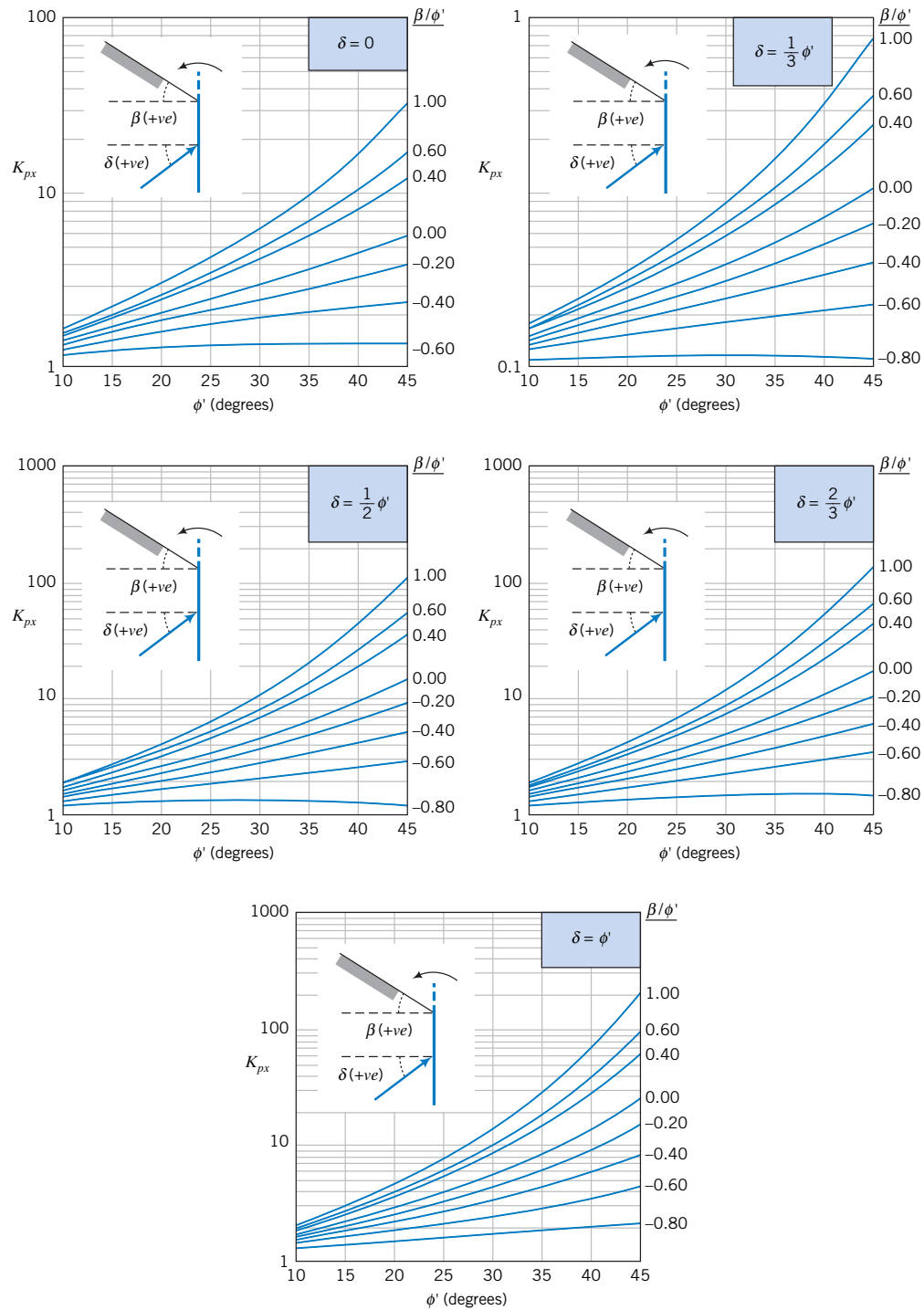


FIGURE D.3 Horizontal component of the passive lateral pressure coefficient. (Plotted from data published by Kerisel and Absi, 1990.)

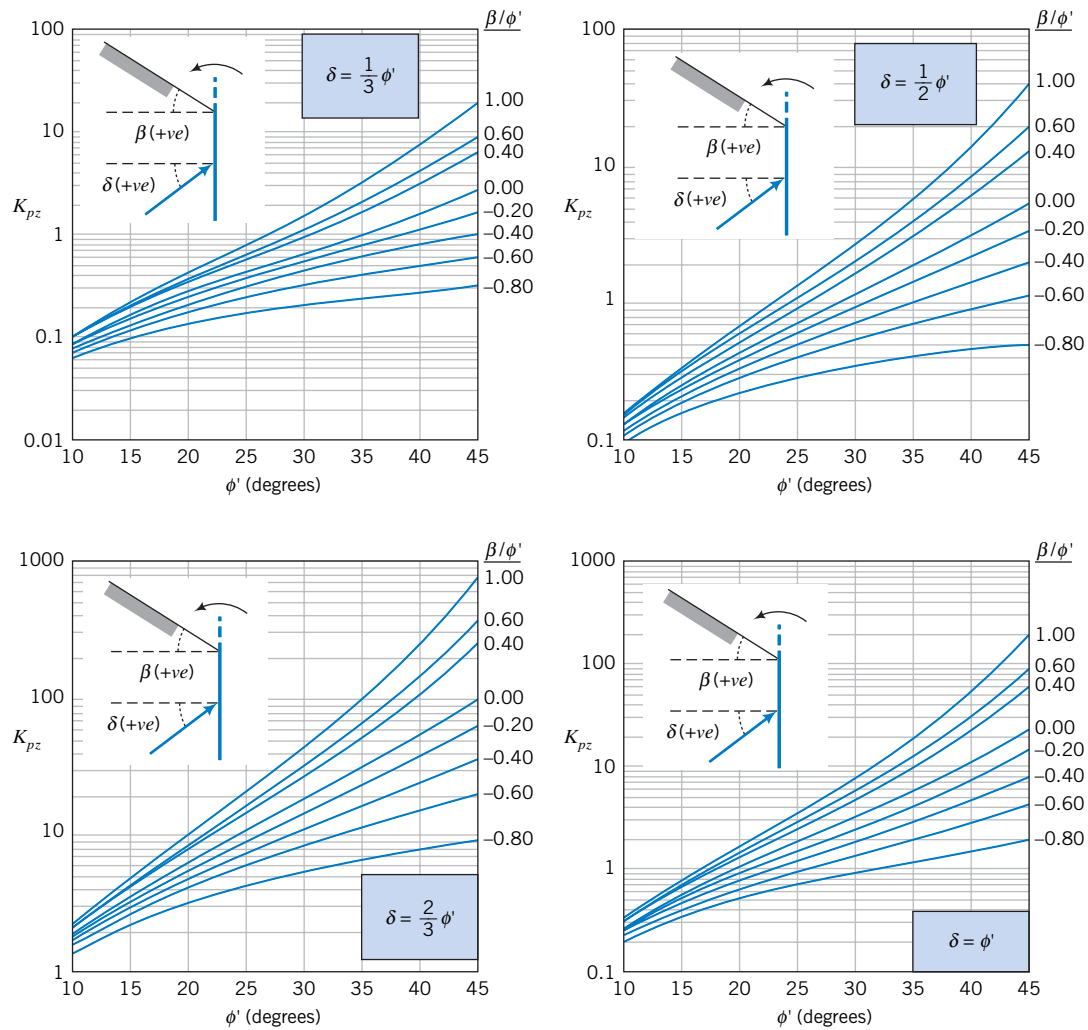


FIGURE D.4 Vertical component of the passive lateral pressure coefficient. (Plotted from data published by Kerisel and Absi, 1990.)

ANSWERS to Selected Problems

Answers to Even-Numbered Problem-Solving Questions

Chapter 3

- 2) $D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}} = \frac{\gamma_d - \gamma_{d_{\min}}}{\gamma_{d_{\max}} - \gamma_{d_{\min}}} \left(\frac{\gamma_{d_{\max}}}{\gamma_d} \right)$
- 4) $e_{\min} = 0.35$, $e_{\max} = 0.91$, $n_{\min} = 0.2595$, $n_{\max} = 0.4764$
- 6) 2.67
- 8) $e = 0.49$, $S = 0.55$ or 55%, % of air voids = 45%
- 10) $w = 6.7\%$, $e = 0.64$, $S = 28\%$
- 12) (a) $\gamma = 16 \text{ kN/m}^3$, $\gamma' = 7.6 \text{ kN/m}^3$ (b) $\gamma = 17.1 \text{ kN/m}^3$, $\gamma' = 7.3 \text{ kN/m}^3$
(c) $\gamma_{\text{sat}} = 17.4 \text{ kN/m}^3$, % error = 1.8%
- 14) Soil A: **SC**, Soil B: **SW**, Soil C: **SP**
- 16) $LL = 40\%$, $LI = 0.88$, brittle failure is unlikely.
- 18) $w_{\text{opt}} = 15.5\%$, $(\gamma_d)_{\max} = 17.7 \text{ kN/m}^3$, S at $(\gamma_d)_{\max} = 84\%$
- 20) (b) The water content at depth = 3 m is very low in comparison with the other values and is rather suspicious, (c) The soil at depth = 3 m is likely a thin sand layer.
- 22) Pit 1: cost = \$51,374,752, Pit 2: cost = \$46,334,156, Pit 1 contains a better soil for the embankment than Pit 2.

Chapter 4

2)

Point	Elev. Head (m)	Pressure Head (m)	Total Head (m)
A	0.25	0.71	0.96
B	0.90	1.02	1.92
C (datum)	0.0	0.0	0.0

- 4) $k = 0.028 \text{ cm/s}$; $n = 55\%$, $v = 0.045 \text{ cm/s}$, $v_s = 0.081 \text{ cm/s}$; $n = 35\%$, $k = 0.0054 \text{ cm/sec}$
- 6) $(k_x)_{\text{eq}} = 0.051 \text{ cm/sec}$, $(k_z)_{\text{eq}} = 2.0 \times 10^{-6} \text{ cm/sec}$, $K_{\text{eq}} = 3.2 \times 10^{-4} \text{ cm/sec}$.
- 8) $45.5 \times 10^{-7} \text{ m}^3/\text{sec} = 0.455 \text{ m}^3/\text{day}$
- 10) $1.2 \times 10^{-6} \text{ m}^3/\text{sec}$

Chapter 5

- 2) $\Delta u = \Delta \sigma_3 + \frac{1}{3}(\Delta \sigma_1 - \Delta \sigma_3)$; $A = \frac{1}{3}$
 4) (a) plot with slope $\frac{\Delta q}{\Delta p} = \frac{3}{4}$, (b) plot with slope $\frac{\Delta q}{\Delta p} = \infty$
 6) $\varepsilon_1 = 2.24 \times 10^{-3}$; $\varepsilon_3 = -3.25 \times 10^{-5}$
 8) (a) $\sigma_1 = 882.8 \text{ kPa}$, $\sigma_3 = 317.2 \text{ kPa}$ (b) $\psi = 22.5^\circ$ (c) $\tau_{\max} = 282.8 \text{ kPa}$ (d) $\sigma_\theta = 624.7 \text{ kPa}$, $\tau_\theta = 281.8 \text{ kPa}$

10)

Depth	σ (kPa)	u (kPa)	σ' (kPa)
0	0	0	0
4.5	85.6	0	85.6
9.5	181.8	49.0	132.8

12) $z = 10.4 \text{ m}$

- 14) (a) $z_c \approx 2 \text{ m}$
 (b)

Depth (m)	σ (kPa)	u (kPa)	σ' (kPa)
0	0	0	0
1	19.9	-19.6	39.5
3	60.3	0	60.3
10	201.7	68.6	133.1

- 16) $\Delta \sigma_z = 1 \text{ kPa}$, $\Delta z = 0.026 \text{ mm}$
 18) $\Delta \sigma_z = 47 \text{ kPa}$
 20) $\frac{\Delta q}{\Delta p} = \frac{-200}{133.3} = -\frac{3}{2}$, $\Delta u = \Delta p = 133.3 \text{ kPa}$
 22) 100.6 kPa
 24) (b) outside edge = 56.3 mm , center = 91.2 mm , inside edge = 72 mm (c) Tilt angle of the silo = 0.11 degrees with respect to vertical (d) No, the silos will not touch.

Chapter 6

- 6) $w = 0.311 + 0.0185 \log\left(\frac{12}{z}\right)$
 8) $\rho_{pc} = 87 \text{ mm}$, difference in settlement = 57 mm
 10) Differential settlement = 135 mm
 12) (b) $C_c = 0.206$, $C_r = 0.018$ (c) $m_v = 1.55 \times 10^{-4} \text{ m}^2/\text{kN}$

- 14) (a) $e_f = 0.610$

(b)

Load	0	50	100	200	400	800	0
Void ratio	0.665	0.638	0.628	0.611	0.596	0.575	0.61

- (c) $H_o = 31.02 \text{ mm}$ (d) $m_v = 1.2 \times 10^{-4} \text{ m}^2/\text{kN}$

16)

U (%)	T	t_2 (yr)	S_t (mm)
0	0	0	0
10	0.008	0.09	6
25	0.049	0.55	15
30	0.071	0.8	18
40	0.126	1.41	24
50	0.197	2.21	30
60	0.287	3.22	36
70	0.403	4.52	42
80	0.567	6.36	48
90	0.848	9.518	54

- 18) Total Settlement = 403.7 mm, $t_{50} = 17.75$ years

- 20) Settlement due to subsidence = 23 mm, the ground surface does not return to its original elevation. Recovered settlement after recharging the aquifer = 2.9 mm.

Chapter 7

2)

$$\tau_{cs} = \frac{\sigma'_1(1-K)}{2} \cos \phi'_{cs}$$

$$\tau_{zx} = \frac{\sigma'_1(1-K_o)}{2}$$

- 4) Soil will not fail, since $\tau_f > 30 \text{ kPa}$ and $FS = \frac{\tau_f}{\tau} = \frac{46.6}{30} = 1.55$
- 6) (a) $\phi'_{cs} = 22^\circ$, (b) $\phi_p = 45.6^\circ$
- 8) $\phi'_{cs} = 38.4^\circ$, $\phi_p = 43.8^\circ$, $\alpha_p = 5.4^\circ$
- 10) $s_u = 58.6 \text{ kPa}$
- 12) (b) The consolidation stress is different (c) test 1: $\phi' = 25.3^\circ$, test 2: $\phi' = 20.4^\circ$
 (d) $s_u = 182.5 \text{ kPa}$ (e) test 1: $\tau_f = 144.7 \text{ kPa}$, test 2: $\tau_f = 171 \text{ kPa}$
- 14) (b) test 1: $\phi' = 22.7^\circ$, test 2: $\phi' = 22.9^\circ$, test 3: $\phi' = 22.5^\circ$ (c) $\sigma'_3 = 132.9 \text{ kPa}$
 (d) (1) $\theta = 56.4^\circ$, (2) $\theta = 90^\circ$ (e) test 1: $(\sigma'_3)_f = 98 \text{ kPa}$, $(\sigma'_1)_f = 221 \text{ kPa}$, $\tau_f = 56.7 \text{ kPa}$, $\sigma'_{nf} = 134.2 \text{ kPa}$
 (f) No, the failure stress occurs on the plane of maximum stress obliquity.

- 16) (a) $\phi'_{(1)} = 22.6^\circ$, $\phi'_{(2)} = 23.2^\circ$, $\phi'_{(3)} = 22.9^\circ$ (b) $s_u = 103.8 \text{ kPa}$ (c) $\tau_{\max}$: (1) 43.8 kPa, (2) 103.8 kPa, and (3) 172.5 kPa $\tau_f = 40.4 \text{ kPa}$, 95.4 kPa, 159.9 kPa.
- 18) $\phi' = 31.1^\circ$
- 20) (a) $\sigma'_1 = 311.8 \text{ kPa}$, $\sigma'_3 = 88.2 \text{ kPa}$ (b) $\phi' = 34^\circ$ (c) $s_u = 111.8 \text{ kPa}$
- 22) $\phi' = 27.8^\circ$, $S_u = 165.5 \text{ kPa}$
- 24) FS (short term) = 2.9 (b) FS (long term) = 1.28

Chapter 8

- 6) $e = 1.485 - 0.15 \ln \left(1 + \frac{\Delta p'}{p'} \right)$
- 8) (a) yield surface : $F = (p'_o)^2 - (p'_o)(p'_c) + \frac{q_o^2}{M^2} = 0$
(b) plot $q = 0.92p' \ln(q, p')$ space
plot $e = e_o - \lambda \ln \frac{p'}{p'_o} = 1.22 - 0.25 \ln \frac{p'}{15}$ in (e, p') space
- 10) Failure line: $q = Mp' = 0.98p'$, $e_\Gamma = 2.911$, CSL: $e = 2.911 - 0.357 \ln p'$,
NCL: $e = 2.3 - 0.35 \ln \frac{p'}{10}$, URL: $e = 1.35 - 0.07 \ln \frac{p'}{150}$
- 12) $(s_u)_A = 151 \text{ kPa}$
- 14) Use the Spreadsheet in the CD.
- 16) Use the Spreadsheet in the CD.
- 18)

Depth	Δu
m	kPa
2	35.4
5	21.7
10	27.1
20	74.9

Chapter 9

- 2) $P_u = \frac{585.7}{\left(\frac{5}{2} \pm 0.75\right)} = 180.2 \text{ kN}$ or 334.7 kN
- 4) Strip = 116 kPa, square = 104 kPa
- 6) Position (1): $D_f = 1.5 \text{ m}$, $q_{a1} = 236 \text{ kPa}$, Position (2): $d_f = 0$, $q_{a2} = 206 \text{ kPa}$
- 8) TSA: $B = 3.1 \text{ m}$
- 10) FS = 23.8, Total settlement = 82 mm
- 12) ESA: FS = 0.73 when groundwater rises to surface.
- 14) 1.25 m

Chapter 10

- 2) ESA governs: $Q_a = 683 \text{ kN}$
- 4) $Q_a = 246 \text{ kN}$
- 6) TSA governs design: $Q_{\text{ult}} = 5223 \text{ kN}$
- 8) TSA-Single file failure mode governs: $Q_a = 1824 \text{ kN}$, Total settlement = 33 mm
- 10) Single file failure mode governs: $Q_{\text{ult}} = 56.5 \text{ MN}$, FS = 5.65

Chapter 11

- 2) (b) $q = 0.53 \text{ cm}^3/\text{sec}$.
- (c)

point	$h_p \gamma_w$ (kPa)
A	58.8
B	109.8
C	105.8
D	0

- (d) ($i = 0.8 < i_{cr} = 1.1$) no piping.
- 4) (b) $q = 0.51 \text{ cm}^3/\text{sec}$.
- (c)

point	$h_p \gamma_w$ (kPa)
A	58.8
B	109.8
C	105.8
D	0

- (d) ($i = 0.85 < i_{cr} = 1.1$) no piping
- 6) $q = 6.0 \times 10^{-4} \text{ cm}^3/\text{sec}$
- 8) $q = 12.8 \times 10^{-3} \text{ cm}^3/\text{sec}$.

Chapter 12

- 2) If no tension crack develops, $z_{cr} = 0$, then $q_s \geq 2s_u$
- 4) $(\sigma_a)_x$ at 12 m = 61.6 kPa, $(\sigma_p)_x$ at 12 m = 106.4 kPa
- 6) $P_{aC} = 168.7 \text{ kN}$ acting at $(\delta + \eta) = 30^\circ$ from the horizontal plane.
- 8) Left wall: $K_{aC} \cos(15) = 0.29$, right wall: $K_{aC} \cos(45) = 0.485$, therefore right wall gives the larger horizontal force

- 10) Rotation: $\frac{B}{6} = 0.75 > e$; satisfactory, Translation: $(FS)_T = 1.3 < 1.5$; unsatisfactory, Bearing: short term: $(FS)_B = \frac{q_u}{(\sigma_z)_{\max}} = 3.9 > 3$; satisfactory for short term loading, long term: $(FS)_B = \frac{q_u}{(\sigma_z)_{\max}} = 3.8 > 3$; satisfactory for long term loading.
- 12) Design depth = 5.68 m, $M_{\max} = 35.3 \text{ kN} \cdot \text{m}$
- 14) FSM ($F_\phi = 1.25$): $d = 12 \text{ m}$, Moment = $996.3 \text{ kN} \cdot \text{m}$, NPPM ($FS_r = 1.5$): $d = 11.2 \text{ m}$, Moment = $98.5 \text{ kN} \cdot \text{m}$.
- 16) Strut Loads: $A = 29.3 \text{ kN/m}$, $B = 31.2 \text{ kN/m}$, $C = 28.3 \text{ kN/m}$, $D = 38.8 \text{ kN/m}$.
- 18) Use $S_z = 150 \text{ mm}$ for bottom half and 300 mm for top half, Use $L_b = 6 \text{ m}$, $L = 3.55 \text{ m}$
Short term: $(FS)_B = \frac{q_{\text{ult}}}{(\sigma_x)_{\max}} = 3.4 > 3$: okay, Long term: $(FS)_B = \frac{521.7}{105} \approx 5 > 3$: okay.
- 20) Use length = 8 m , use $t = 3 \text{ mm}$, Bearing capacity: $(FS)b = \frac{q_{\text{ult}}}{(\sigma_x)_{\max}} = 17 > 3$; therefore bearing capacity is satisfactory.
- 22) Rotation: $\frac{B}{6} = 0.75 > e$; satisfactory, Translation: $(FS)_T = 1.2 < 1.5$; unsatisfactory, Bearing: short term: $(FS)_B = \frac{q_{\text{ult}}}{(\sigma_z)_{\max}} = 2 < 3$; unsatisfactory
- 24) Rotation: $\frac{B}{6} = 0.71 < e$; unsatisfactory, Translation: $(FS)_T = 1.0 < 1.5$; unsatisfactory, Bearing: short term: $(FS)_B = \frac{q_{\text{ult}}}{(\sigma_z)_{\max}} = 0.7 < 3$; unsatisfactory

Chapter 13

- 2) $FS = \frac{\tan \phi'}{R_3 \cos \beta - R_1 \cos \alpha} (W_2 + R_3 \sin \alpha + R_1 \sin \beta)$
- 4) ESA: $FS = 2.65$, TSA: $FS = 1.58$
- 6) TSA: $FS = 2.38$, ESA: $FS = 0.61$
- 8) $FS = 0.59$
- 10) From Taylor's chart: $\alpha_s = 42^\circ$

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