Pros and cons of a pushover analysis of seismic performance evaluation

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The static pushover analysis is becoming a popular tool for seismic performance evaluation of existing and new structures. The expectation is that the pushover analysis will provide adequate information on seismic demands imposed by the design ground motion on the structural system and its components. The purpose of the paper is to summarize basic concepts on which the pushover analysis can be based, assess the accuracy of pushover predictions, identify conditions under which the pushover will provide adequate information and, perhaps more importantly, identify cases in which the pushover predictions will be inadequate or even misleading.

Keywords: pushover analysis, performance evaluation, inelastic behavior

1. Introduction

In the simplest case, seismic design can be viewed as a two step process. The first, and usually most important one, is the conception of an effective structural system that needs to be configured with due regard to all important seismic performance objectives, ranging from serviceability considerations to life safety and collapse prevention. This step comprises the art of seismic engineering, since no rigid rules can, or should, be imposed on the engineer's creativity to devise a system that not only fulfills seismic performance objectives, but also pays tribute to functional and economic constraints imposed by the owner, the architect, and other professionals involved in the design and construction of a building. By default, this process of creation is based on judgment, experience, and understanding of seismic behavior, rather than rigorous mathematical formulations. Rules of thumb for strength and stiffness targets, based on the fundamental knowledge of ground motion and elastic and inelastic dynamic response characteristics, should suffice to configure and rough-size an effective structural system.

Elaborate mathematical/physical models can only be built once a structural system has been created. Such models are needed to evaluate seismic performance of an existing system and to modify component behavior characteristics (strength, stiffness, deformation capacity) to better suit the specified performance criteria. This second step of the design process should involve a demand/capacity evaluation at all important performance levels, which requires identification of important capacity parameters and prescription of acceptable values of these parameters, as well as the prediction of the demands imposed by ground motions. Suitable capacity parameters and their acceptable values, as well as suitable methods for demand prediction will depend on the performance level to be evaluated. This paper is concerned only with demand prediction at low performance levels, such as life safety and collapse prevention, at which it is expected that the structure will have to undergo significant inelastic deformations.

In an ideal world there would be no debate about the proper method of demand prediction and performance evaluation at low performance levels. Clearly, inelastic time history analysis that predicts with sufficient reliability the forces and cumulative deformation (damage) demands in every element of the structural system is the final solution. The implementation of this solution requires the availability of a set of ground motion records (each with three components) that account for the uncertainties and differences in severity, frequency characteristics, and duration due to rupture characteristics and distances of the various faults that may cause motions at the site. It requires further
the capability to model adequately the cyclic load-deformation characteristics of all important elements of the three-dimensional soil-foundation structure system, and the availability of efficient tools to implement the solution process within the time and financial constraints imposed on an engineering office. Moreover, it requires the adequate knowledge of element deformation capacities with due regard to deterioration characteristics that define the limit state of acceptable performance.

We need to work towards this final solution, but we also need to recognize the limitations of today’s states of knowledge and practice. It is fair to say that at this time none of the afore-mentioned capabilities have been adequately developed and that efficient tools for implementation do not exist. Recognizing these limitations, the task is to perform an evaluation process that is relatively simple, but captures the essential features that significantly affect the performance goal. In this context, the accuracy of demand prediction is desirable, but it may not be essential, since neither seismic input nor capacities are known with accuracy. The inelastic pushover analysis, which is the subject of this paper, serves this purpose provided its limitations and pitfalls are fully recognized.

2. Purpose of pushover analysis

The purpose of the pushover analysis is to evaluate the expected performance of a structural system by estimating its strength and deformation demands in design earthquakes by means of a static inelastic analysis, and comparing these demands to available capacities at the performance levels of interest. The evaluation is based on an assessment of important performance parameters, including global drift, interstory drift, inelastic element deformations (either absolute or normalized with respect to a yield value), deformations between elements, and element and connection forces (for elements and connections that cannot sustain inelastic deformations). The inelastic static pushover analysis can be viewed as a method for predicting seismic force and deformation demands, which accounts in an approximate manner for the redistribution of internal forces occurring when the structure is subjected to inertia forces that no longer can be resisted within the elastic range of structural behavior.

The pushover is expected to provide information on many response characteristics that cannot be obtained from an elastic static or dynamic analysis. The following are examples of such response characteristics:

- The realistic force demands on potentially brittle elements, such as axial force demands on columns, force demands on brace connections, moment demands on beam-to-column connections, shear force demands in deep reinforced concrete spandrel beams, shear force demands in unreinforced masonry wall piers, etc.
- Estimates of the deformation demands for elements that have to deform inelastically in order to dissipate the energy imparted to the structure by ground motions.
- Consequences of the strength deterioration of individual elements on the behavior of the structural system.
- Identification of the critical regions in which the deformation demands are expected to be high and that have to become the focus of thorough detailing.
- Identification of the strength discontinuities in plan or elevation that will lead to changes in the dynamic characteristics in the inelastic range.
- Estimates of the interstory drifts that account for strength or stiffness discontinuities and that may be used to control damage and to evaluate P-delta effects.
- Verification of the completeness and adequacy of load path, considering all the elements of the structural system, all the connections, the stiff nonstructural elements of significant strength, and the foundation system.

The last item is perhaps the most relevant one, provided the analytical model incorporates all elements, whether structural or nonstructural, that contribute significantly to lateral load distribution. For instance, load transfer across connections between ductile elements can be checked with realistic forces; the effects of stiff partial-height infill walls on shear forces in columns (short columns) can be evaluated; and the maximum overturning moment in walls, which is often limited by the uplift capacity of foundation elements, can be estimated.

Clearly, these benefits come at the cost of additional analysis effort, associated with incorporating all important elements, modeling their inelastic load-deformation characteristics, and executing incremental inelastic analysis, preferably with a three-dimensional analytical model.

At this time, with few exceptions, adequate analytical tools for this purpose are either very cumbersome or not available, but several good tools are under development, since the demand for the pushover analysis has been established, primarily through the recent pre-publication of the FEMA 273 document. This two-volume document (guidelines and commentary), which is the result of a 5-year development effort, also includes extensive recommendations for the load-deformation modeling of individual elements and for acceptable values of force and deformation parameters for performance evaluation.

3. Background to pushover analysis

The static pushover analysis has no rigorous theoretical foundation. It is based on the assumption that the response of the structure can be related to the response of an equivalent single degree-of-freedom (SDF) system. This implies that the response is controlled by a single mode, and that the shape of this mode remains constant throughout the time history response. Clearly, both assumptions are incorrect, but pilot studies carried out by several investigators have indicated that these assumptions lead to rather good predictions of the maximum seismic response of multi degree-of-freedom (MDOF) structures, provided their response is dominated by a single mode.

The formulation of the equivalent SDOF system is not unique, but the basic underlying assumption common to all approaches is that the deflected shape of the MDOF system can be represented by a shape vector ($\Phi$) that remains constant throughout the time history, regardless of the level of deformation. Accepting this assumption and defining the relative displacement vector $X$ of an MDOF system as $X = [\Phi]_x$ ($x_t =$ roof displacement), the governing differential equation of an MDOF system can be written as

$$M[\phi]_t + C[\phi]_t + Q = -M[\phi]_g$$

where $M$ and $C$ are the mass and damping matrices, $Q$
denotes the story force vector, and \( \ddot{x}_y \) is the ground acceleration.

If we define the reference SDOF displacement \( x^* \) as

\[
x^* = [\Phi]^T \mathbf{M} [\Phi] x_t,
\]

and pre-multiply equation (1) by \([\Phi]^T\), and substitute for \( x_t \) using equation (2), we obtain the following differential equation for the response of the equivalent SDOF system:

\[
M^* \ddot{x}^* + C^* \dot{x}^* + Q^* = -M^* \ddot{x}_y.
\]

\( M^*, C^* \) and \( Q^* \) denote the properties of the equivalent SDOF system and are given by

\[
M^* = [\Phi]^T \mathbf{M} [\Phi] \quad (4)
\]

\[
Q^* = [\Phi]^T \mathbf{Q} \quad (5)
\]

\[
C^* = [\Phi]^T \mathbf{C} [\Phi] \quad (6)
\]

Presuming that the shape vector \([\Phi]\) is known, the force-deformation characteristics of the equivalent SDOF system \((Q^* - \ddot{x}^* \text{ relationship})\) can be determined from the results of a nonlinear incremental static analysis of the MDOF structure, which usually produces a base shear \((V) - \text{roof displacement} (x_t, \delta_{t,y})\) diagram of the type shown in Figure 2. In order to identify nominal global strength and displacement quantities, the multilinear \(V - \delta\) diagram needs to be represented by a bilinear relationship that defines a 'yield' strength, \( V_y \), an effective 'elastic' stiffness, \( K_e = V_y/\delta_{t,y} \), and a hardening (or softening) stiffness,

\[
K_s = \alpha K_e
\]

for the structure. Some judgment may be needed to define these properties. The simplified bilinear base shear–roof displacement response curve, which is shown also in Figure 1a, is needed to define the properties of the equivalent SDOF system.

The yield value of the base shear \((V_y)\) and the corresponding roof displacement \((x_{t,y})\) from Figure 1a are used together with equations (2) and (5) to compute the force–displacement relationship for the equivalent SDOF system as follows:

\[
x^*_t = [\Phi]^T \mathbf{M} [\Phi] x_t, \quad Q^*_t = [\Phi]^T \mathbf{Q}_t
\]

where \( \mathbf{Q}_t \) is the story force vector at yield, i.e. \( V_y = [\mathbf{1}]^T \mathbf{Q}_y \).

The initial period of the equivalent SDOF system \( T_{eq} \) can be computed as

\[
T_{eq} = 2\pi \left[ \frac{x^*_t M^*}{Q^*_t} \right]^{1/2}
\]

The strain hardening ratio \((\alpha)\) of the \(V-x_t\) relationship of the MDOF structure defines the strain hardening ratio of the equivalent SDOF system.

The basic properties of the equivalent SDOF system are now known. The fundamental question in the execution of the pushover analysis is the magnitude of the target displacement at which seismic performance evaluation of the structure is to be performed. The target displacement serves as an estimate of the global displacement the structure is expected to experience in a design earthquake. A convenient definition of target displacement is the roof displacement at the center of mass of the structure. The properties of the equivalent SDOF system, together with spectral information for inelastic SDOF system which have been developed over the last few years, provide the information necessary to estimate the target displacement.

The roof displacement of the structure, \( x_n \), is related to the equivalent SDOF displacement, \( x^*_t \), by means of equation (2). Thus, the target displacement can be found if the displacement demand of the equivalent SDOF system can be estimated for the design earthquake. For an elastic SDOF system the displacement demand is given by the spectral displacement. For inelastic systems the SDOF displacement demand needs to be obtained from inelastic spectra as is discussed later. The utilization of inelastic spectral demand information requires the estimation of the ratio of elastic strength demand to yield strength of the equivalent SDOF system, usually referred to as the R-factor. Since inelastic spectra are usually obtained for unit
mass systems, it is convenient to divide equation (3) by $M^*$ to obtain the differential equation of the unit mass equivalent SDOF system:

$$\ddot{x} + \frac{C^*}{M^*} \dot{x} + \frac{Q^*}{M^*} x = -\ddot{x}_t.$$  \hspace{1cm} (9)

Equation (9) describes the response of a unit mass SDOF system with period $T_{eq}$ and yield strength $F_{y,eq}$ given as

$$F_{y,eq} = \frac{Q^*}{M^*}.$$ \hspace{1cm} (10)

If the elastic response spectrum is known, the elastic strength demand of the unit mass equivalent SDOF system can be computed as

$$F_{c,eq} = S_c(T_{eq})$$ \hspace{1cm} (11)

where $S_c(T_{eq})$ is the spectral ordinate of the elastic acceleration spectrum. The strength reduction factor $R$ can then be obtained from the relationship

$$R = \frac{F_{c,eq}}{F_{y,eq}} = \frac{S_c(T_{eq})M^*}{Q^*}.$$ \hspace{1cm} (12)

The utilization of the R-factor for estimating inelastic displacement demands is discussed later.

Both the R-factor and the target displacement depend on the choice of the shape vector $\{\Phi\}$. Most of the investigators who have utilized the pushover in pilot studies have recommended the use of the normalized displacement profile at the target displacement level as shape vector. Since this displacement is not known a priori, an interaction process will have to be performed if this shape vector is selected.

The use of $T_{eq}$ and of the afore-mentioned shape vector for estimating the properties of the equivalent SDOF system and the target displacement requires elaborate computations and time consuming iterations. Recognizing all the assumptions and approximations inherent in the pushover procedure, there is no good justification to be rigorous in the computations leading to the estimate of the target displacement, and accuracy can often be sacrificed for the sake of simplicity.

Sensitivity studies have shown that the difference between $T_1$ (first mode structure period) and $T_{eq}$ is usually small and its effect on the target displacement can be neglected unless the design spectrum is very sensitive to small variations in period. Simplifications in the shape vector $\{\Phi\}$ should also be acceptable. The use of a shape vector corresponding to the deflected shape at the target displacement is only a recommendation and has no theoretical foundation. In all cases studied by the writers the use of a simple predetermined shape vector (such as the elastic first mode shape vector) resulted in good predictions of the target displacement. In fact, in extensive studies of soft story structures, it was found that the use of a straight line shape vector gave better predictions of the roof displacement than the use of the shape vector corresponding to the deflected shape at the maximum displacement. Employment of the afore-mentioned simplifications facilitates the computation of the equivalent SDOF properties and eliminates the need for iterations.

There are many additional considerations that will affect the accuracy of seismic demand predictions by means of a pushover analysis. These considerations have to do primarily with the estimate of the target displacement and the selection of load patterns that are supposed to deform the structure in a manner similar to that experienced in a design earthquake. Some of the important issues are discussed in the following section.

4. Target displacement

In the pushover analysis it is assumed that the target displacement for the MDOF structure can be estimated as the displacement demand for the corresponding equivalent SDOF system transformed to the SDOF domain through the use of a shape vector and equation (2). This assumption, which is always an approximation, can only be accepted within limitations and only if great care is taken in incorporating in the predicted SDOF displacement demand all the important ground motion and structural response characteristics that significantly affect the maximum displacement of the MDOF structure.

Inherent in this approach is the assumption that the maximum MDOF displacement is controlled by a single shape vector without regards to higher mode effects. Parameter studies have shown that for frame and wall structures with a first mode period of less than 2 s this assumption is rather accurate for elastic systems and conservative (overestimates the MDOF displacement) for inelastic systems. The assumption was not checked for systems with longer periods, since in the writers’ opinion the pushover should not be employed in its present form for long period structures.

Incorporation of all important structural response characteristics in the prediction of the SDOF displacement demand implies the ability to represent the global load–deformation response of the structure by an equivalent SDOF system with appropriate hysteretic characteristics. For this purpose the simplified bilinear base shear–roof displacement diagram shown in Figure 2 may serve as a skeleton defining a yield level and an effective elastic and post-elastic stiffness. The skeleton alone does not necessarily define the hysteretic characteristics of the SDOF system. Depending on the structural system and material, the restoring force characteristics may be bilinear, or may exhibit stiffness degradation or pinching, or may even exhibit strength deterioration. If the displacement demand depends strongly on these characteristics, their incorporation in the equivalent SDOF model will be necessary.

In general, there are several steps involved in deriving the inelastic displacement demand for the equivalent SDOF model. The conversion of the structure’s base shear–roof displacement response to the equivalent SDOF skeleton curve (Figure 1) and the estimation of the associated R-factor (equation (12)) have been discussed previously. Once the R-factor is known, the SDOF displacement demand can be computed, either directly or indirectly, with due regard given to the hysteretic characteristics of the equivalent SDOF system. If the seismic input is represented by a time history record, the inelastic displacement demand can be computed directly through a single time history analysis using the equivalent SDOF system with properly modeled hysteretic characteristics.

The purpose of a pushover analysis is rarely the prediction of demands for a specific ground motion. It is employed mostly as a design evaluation tool. For this purpose, the seismic input is usually represented by a
smoothed elastic response spectrum rather than individual ground motion records. Thus, the inelastic displacement demand cannot be computed directly, but needs to be deduced from spectral data and auxiliary information that accounts for differences between the elastic and inelastic displacement demand. There is no unique way to achieve this objective. The following approach is only one of several feasible ones, but it has the advantages of being based on transparent physical concepts and being able to take advantage of seismic demand information for inelastic SDOF systems generated by the writers and many others.

Given the spectral acceleration $S_0$, the elastic SDOF displacement demand can be computed as $(T^2/4\pi^2)S_0$. This displacement becomes the base line for predicting the inelastic displacement demand, which needs to be accomplished with due consideration given to the yield strength and hysteretic characteristics of the equivalent SDOF system. Both the effects of yield strength and hysteretic characteristics can be accounted for through cumulative modification factors applied to the elastic displacement demand. The following discussion addresses some of these modifications.

4.1. Modification for yield strength

Much information has been generated during the last 30 years on the effect of yield strength on SDOF seismic demands$^{6-13}$. The yield strength is related to the elastic strength demand by the previously discussed R-factor [see equation (12)]. Many studies have been performed that relate this R-factor to the ductility demand $\mu$, resulting in $R-\mu-T$ relationships for different hysteresis systems and site soil conditions (see Ref. 8 for a summary of various results). As a representative example, Figure 3 shows $R-\mu-T$ relationships obtained from a statistical study of bilinear SDOF systems subjected to a set of 15 rock and firm soil (soil type $S_1$) ground motion records$^9$. The results of most studies reported in the literature are based on bilinear nondegrading SDOF systems with positive or zero strain hardening. In general, using zero strain hardening ($\alpha = 0$) will result in the smallest R-factor (or largest displacement demand).

Since $R$ is, in general, not equal to $\mu$, the inelastic displacement demand, $\delta_{\mu}$, will differ from the elastic demand, $\delta_0$ (since $\delta_0/\delta_0 = \mu/R$). Thus, a basic modification to the elastic spectral displacement is the ratio $\delta_0/\delta_\mu$, assuming that the SDOF hysteretic system is bilinear with zero strain hardening. Typical results for this modification factor are shown in Figures 4 and 5. Figure 4 shows the mean of the ratio $\delta_0/\delta_\mu$ for the 15 soil type $S_1$ records on which Figure 3 is based. Figure 5 shows the mean ratio for 10 generated records in a soft soil with a predominant period $T_s = 1.25$ s$^{11}$. Clearly, the ratio $\delta_0/\delta_\mu$ for soft soil records follows a different pattern than for rock and firm soil records. It was found that this ratio is very sensitive to the soil period $T_s$, but once normalized to this period, the period dependence of $\delta_0/\delta_\mu$ closely follows the pattern shown in Figure 5 regardless of the value of $T_s$.

4.2. Modification for stiffness degradation or pinching

Often the effect of a degrading reloading stiffness on the inelastic displacement demand is of much concern. In an SDOF system this effect can be represented by a peak-oriented hysteresis model (reloading is directed towards the peak of the previous loading history) or by a pinching model of the type shown in Figure 6. The displacement amplification, compared to a bilinear nondegrading model, has been studied with the same 15 $S_1$ records referenced previously. It was found that stiffness degradation or pinching does not have a significant effect on the inelastic displacement demand except for very short period systems. This is illustrated in Figure 7, which shows the displacement amplification for a severely pinched hysteresis model (reloading target strength $F_p$ is 25% of yield strength $F_y$). This result indicates that stiffness degradation is not an important consideration except for very short period structures. This conclusion has to be viewed with caution, since

Figure 3  Strength reduction factors for the $S_1$ set of records

Figure 4 Normalized inelastic displacement demands for bilinear systems, $S_1$ set of records

Figure 5 Normalized inelastic displacement demands for soft soil records
it is based only on SDOF studies and needs to be verified on MDOF structural systems.

4.3. Modification for strength deterioration
Strength deterioration may have a significant effect on the inelastic displacement demand. Unfortunately, there is no simple answer to the magnitude of its effect, which will depend strongly on the assumed rate of deterioration. A typical example of the displacement amplification due to strength deterioration, which is assumed to be proportional to cumulative energy dissipation, is illustrated in Figure 8\(^2\). If a structure has the potential for significant strength deterioration, its effect on displacement demand may overpower most of the other modifications discussed here. More research on this subject is urgently needed.

4.4. Modification for P–delta effect
The structure P–delta effect (caused by gravity loads acting on the deformed configuration of the structure) will always lead to an increase in lateral displacements. If the P–delta effect causes a negative post-yield stiffness in any one story, it may affect significantly the interstory drift and the target displacement. Such a negative stiffness will lead to the drifting of the displacement response (increase in displacements in one direction). The extent of drifting will depend on the ratio of the negative post-yield stiffness to the effective elastic stiffness, the fundamental period of the structure, the strength reduction factor R, the hysteretic load–deformation characteristics of each story, the frequency characteristics of the ground motion, and the duration of the strong motion portion of the ground motion.

Because of the many parameters involved, the effect of P–delta on the drifting of the seismic response (increase in target displacement) is difficult to describe with a single modification factor, but since the P–delta effect may lead to a significant amplification of displacements and may even lead to incremental collapse, this effect cannot be ignored. The results of mean displacement amplifications for SDOF systems with 5% negative stiffness (\(\alpha = -0.05\)) obtained from the previously mentioned 15 S\(_1\) records, are presented in Figure 9. It can be seen that the amplification is large for bilinear systems with short periods and low strength (large R-factor). The amplification is much smaller for peak-orientated stiffness degrading systems.

At this time it is not clear whether these SDOF displacement amplifications should be applied directly to the prediction of the MDOF target displacement. In most structures the negative stiffness is not mobilized before significant inelastic deformations have occurred. This decreases the displacement drifting compared to systems with bilinear response characteristics. On the other hand, it must be recognized that the negative stiffness represented in the global base shear vs roof displacement response may not be representative of the negative stiffness existing in the critical story, which is likely to be at the bottom of the structure. P–delta is a story drift amplification problem and not a global drift amplification problem, and the P–delta
effects are usually highest in the lowest stories, in which the gravity loads are largest. Again, more research is needed to develop a reliable procedure that permits the incorporation of the MDOF story $P$−$\delta$ effect in the SDOF target displacement estimate.

4.5. Modification for effective viscous damping

The previous modifications are presented for systems with 5% viscous damping. The inelastic displacement demands need to be modified if the effective viscous damping is judged to be significantly different from 5%. Figure 10 presents mean data (using the same 15 $S_1$ records) on the dependence of inelastic displacement demands on the percentage of critical damping, $\xi$, for bilinear systems with an R-factor of 3.0. The results are normalized with respect to systems with 5% damping. Results for other R-factors lead to the conclusion that the displacement ratios are rather insensitive to the R-factor and that the presented results can be used with reasonable accuracy for all R-factors.

4.6. Other modifications

There are other issues that may affect the expected target roof displacement, which cannot be incorporated in the SDOF displacement demand and are not represented adequately in the shape vector $\{\Phi\}$ that converts the SDOF displacement demand into an MDOF target displacement. For instance, nonlinear time history analysis of MDOF frame structures has shown that the sum of maximum interstory drifts is about 10−30% larger than the maximum roof displacement\(^\text{14}\). Since the purpose of a pushover analysis is a performance evaluation at the element level, it would be appropriate to increase the target roof displacement accordingly. Also, foundation uplift, torsional effects and semi-rigid floor diaphragms are expected to affect the target displacement. At this time inadequate information exists to incorporate these effects in the prediction.

The following summary observations can be made with respect to the prediction of the target displacement for inelastic MDOF structures. The equivalent SDOF displacement demand can be estimated with good accuracy for standard conditions of ground motion spectra and restoring force characteristics. Greater approximations are involved if the design spectrum represents soft soil conditions or if the structural system experiences strength deterioration or significant $P$−$\delta$ effect. A good choice for the shape vector $\{\Phi\}$, which is used to convert the SDOF displacement demand into an MDOF target displacement, is the elastic first mode shape.

The prediction of the target displacement will usually be rather accurate for structures without significant strength or stiffness irregularities and without the formation of weak story mechanisms, but it should not be expected that the target displacement can be predicted with great accuracy for all possible cases. The writers must confess that they do not consider this a major drawback of the pushover analysis. A reasonable prediction of the displacement, which can be achieved in most practical cases, is believed to be adequate in the context of all the other approximations involved in this evaluation procedure. What is believed to be more important is the explicit consideration of the different phenomena that may affect the displacement response and an approximate evaluation of their importance. The data presented in Figures 3−10 serves this purpose.

5. Lateral load patterns

For a performance evaluation the load pattern selection is likely to be more critical than the accurate determination of the target displacement. The load patterns are intended to represent and bound the distribution of inertia forces in a design earthquake. It is clear that the distribution of inertia forces will vary with the severity of the earthquake (extent of inelastic deformations) and with time within an earthquake. If an invariant load pattern is used, the basic assumptions are that the distribution of inertia forces will be reasonably constant throughout the earthquake and that the maximum deformations obtained from this invariant load pattern will be comparable to those expected in the design earthquake. These assumptions may be close to the truth in some cases, but not in others. They are likely to be reasonable if: (a) the structure response is not severely affected by higher mode effects; or (b) the structure has
only a single load yielding mechanism that can be detected by an invariant load pattern.

In such cases carefully selected invariant load patterns may provide adequate predictions of element deformation demands. Since no single load pattern can capture the variations in the local demands expected in a design earthquake, the use of at least two load patterns that are expected to bound inertia force distributions is recommended. One should be a 'uniform' load pattern (story forces proportional to story masses), which emphasizes the demands in lower stories compared to the demands in upper stories and magnifies the relative importance of story shear forces compared to overturning moments. The other could be the design load pattern used in present codes or, preferably, a load pattern that accounts for elastic higher mode effects, such as a load pattern derived from SRSS story shears.

Clearly, none of these invariant load patterns can account for a redistribution of inertia forces, which may occur when a local mechanism forms and the dynamic properties of the structure change accordingly. Thus, it is attractive to utilize adaptive load patterns that follow more closely the time variant distribution of inertia forces. Different suggestions have been made in this regard, including the use of story loads that are proportional to the deflected shape of the structure, the use of SRSS load patterns based on mode shapes derived from secant sti""nesses at each load step, and the use of patterns in which the applied story loads are proportional to story shear resistances at the previous step. At this time there is no consensus on the advantages of these adaptive load patterns, but there is no doubt that improved load patterns need to be developed in order to make a demand prediction by means of a static pushover analysis a more reliable process.

The writers believe that the load pattern issue is at this time the weak point of the pushover analysis procedure. The use of invariant patterns may lead to misleading predictions, particularly for long period structures with localized yielding mechanisms. These problems will be discussed later in the section on limitations of the pushover analysis. The suggested adaptive patterns may improve the prediction in some cases, but none have proven to be universally applicable. Further improvements of the pushover procedure need to focus on the load pattern issue.

6. Implementation of pushover analysis

The process is to represent the structure in a two- or three-dimensional analytical model that accounts for all important linear and nonlinear response characteristics, apply gravity loads followed by lateral loads in predetermined or adaptive patterns that represent approximately the relative inertia forces generated at locations of substantial masses, and push the structure under these load patterns to target displacements that are associated with specific performance levels. The internal forces and deformations computed at these target displacements are used as estimates of the strength and deformation demands, which need to be compared to available capacities.

A simple example of a pushover analysis is illustrated in Figure 11. The shown two-dimensional frame could represent the lateral load resisting system for a steel perimeter frame structure. In simple cases the analysis can be performed as a series of elastic analyses in which, for instance, points where the bending strength has been reached are treated as hinges in the application of additional lateral loads. Bilinear or multilinear load–deformation diagrams need to be incorporated if strain hardening or softening are important characteristics of element response. The analysis continues until the target displacement, \( \delta_t \), is attained, resulting in a base shear vs roof displacement response of the type shown in Figure 11. In most cases it will be necessary to perform the analysis with displacement rather than force control, since the target displacement may be associated with a very small positive or even a negative lateral stiffness because of the development of mechanisms and \( P-\delta \) effects. At the target displacement level the member forces and deformations (e.g. the plastic hinge rotation at point 1) are compared to the available capacities for performance evaluation.

The basic concepts for the determination of target displacements and the formulation of representative lateral load patterns have been discussed in the previous section. There are many issues associated with the implementation of these concepts, ranging from specific structural modeling issues to quantifying the discussed modifications to the inelastic SDOF displacement demand. The reader is referred to Ref. 1 for the implementation of a 'nonlinear static procedure' that utilizes many of the discussed concepts, provides detailed modeling procedures for a pushover analysis, and presents guidelines for a performance evaluation based on comparing computed element force and deformation demands to corresponding capacities. The document provides extensive quantitative recommendations for acceptable force and deformation values (capacities) associated with different performance levels. These recommendations are applicable for elements made of structural steel, reinforced concrete, wood and masonry.

In the context of the expressed concerns about the accuracy of demand predictions by means of a pushover analysis, it must be said that equal concerns exist in regard to capacity predictions. At this time much judgment needs to be employed in establishing acceptability criteria, particularly if they are associated with deformation quantities. This may not be a major drawback since accurate predictions are desirable, but not critical, particularly for elements that deteriorate in a gradual manner. What is more important is the realization that collapse or life safety hazards are caused primarily by brittle failure modes in components and connections that are important parts of the gravity and lateral load paths. Thus, the emphasis in performance evaluation needs to be on:

- verification that an adequate load path exists;
- verification that the load path remains sound at the deformations associated with the target displacement level;
- verification that critical connections remain capable of transferring loads between the elements that form part of the load path;
- verification that individual elements that may fail in a brittle mode and are important parts of the load path are not overloaded;
- verification that localized failures (should they occur) do not pose a collapse or life safety hazard, i.e. that the loads tributary to the failed element(s) can be transferred safely to other elements and that the failed element itself does not pose a falling hazard.

A thoughtful application of the pushover analysis will provide adequate answers in many cases. The exceptions will be discussed later. However, there are many unre-
solved issues that need to be addressed through more research and development. Examples of the important issues that need to be investigated are:

- Incorporation of torsional effects (due to mass, stiffness and strength irregularities).
- 3-D problems (orthogonality effects, direction of loading, semi-rigid diaphragms, etc.).
- Use of site specific spectra.
- Cumulative damage issues.
- Most importantly, the consideration of higher mode effects once a local mechanism has formed.

The simple conclusion is that much more work needs to be done to make the static pushover analysis a general tool applicable to all structures, which can be employed with confidence by engineers who are not necessarily experts on the evaluation of demands and capacities in structures that respond inelastically to earthquake ground motion.

7. A successful pushover example

There are many examples in the literature that have demonstrated that the pushover analysis can predict local and global demands with good accuracy. Most of the success stories are limited to structures with relatively few stories (about 6 or fewer), in which inelasticity is either distributed over the height of the structure or is concentrated in one weak story. Summarized here is an example that may be unique insofar as it represents a case study in which pushover demand predictions and corresponding dynamic analysis predictions were performed for nine different ground motions, permitting a statistical evaluation of the accuracy of demand predictions.

The case study building is a four-story steel perimeter frame structure that was severely damaged in the 1994 Northridge earthquake. In the NS direction the structure, whose fundamental period is 0.91 s, can be represented by the moment resisting frame shown in Figure 12. The base shear–roof displacement diagram for this frame, obtained from a pushover analysis using a triangular load pattern, is shown in Figure 13. The frame was subjected to nine ground motion records whose mean and mean ± σ response spectra are presented in Figure 14. The target displacement for each record was computed from inelastic response spectra and using the procedure outlined earlier (for bilinear nondegrading hysteretic systems). Local and global seismic demands were evaluated from the pushover analysis results at the target displacements associated with the individual records. These are the demands to be compared with the demands obtained from the nine time history analyses.

Comparisons of representative demand predictions
Pushover analysis for seismic performance evaluation: H. Krawinkler and G. D. P. K. Seneviratna

Table 1 Interstory drifts – pushover predictions vs time history results

<table>
<thead>
<tr>
<th>Story</th>
<th>Gravity + Station 1</th>
<th>Gravity + Station 2</th>
<th>Gravity + Station 3</th>
<th>Gravity + Station 4</th>
<th>Gravity + Station 5</th>
<th>Gravity + Station 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Time history abs. max 0.009</td>
<td>0.010</td>
<td>0.009</td>
<td>0.018</td>
<td>0.017</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>Pushover 0.008</td>
<td>0.010</td>
<td>0.008</td>
<td>0.013</td>
<td>0.011</td>
<td>0.005</td>
</tr>
<tr>
<td>2</td>
<td>Time history abs. max 0.014</td>
<td>0.017</td>
<td>0.014</td>
<td>0.023</td>
<td>0.021</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>Pushover 0.014</td>
<td>0.017</td>
<td>0.014</td>
<td>0.021</td>
<td>0.019</td>
<td>0.008</td>
</tr>
<tr>
<td>3</td>
<td>Time history abs. max 0.013</td>
<td>0.017</td>
<td>0.013</td>
<td>0.022</td>
<td>0.020</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>Pushover 0.013</td>
<td>0.017</td>
<td>0.013</td>
<td>0.022</td>
<td>0.020</td>
<td>0.007</td>
</tr>
<tr>
<td>4</td>
<td>Time history abs. max 0.012</td>
<td>0.016</td>
<td>0.008</td>
<td>0.022</td>
<td>0.016</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>Pushover 0.009</td>
<td>0.013</td>
<td>0.009</td>
<td>0.019</td>
<td>0.016</td>
<td>0.005</td>
</tr>
</tbody>
</table>

obtained from the pushover and time history analyses are presented in Tables 1 and 2. The tables show values for each of the nine records as well as mean values and coefficients of variation. Table 1 presents maximum interstory drifts (\(\delta/s\)), and Table 2 presents plastic hinge rotations at a second floor exterior beam end \([B-123 \text{ end } (i)]\) and panel zone plastic shear distortions at a second floor interior joint \((J-23)\). In general, the variation in demands between records is much larger than the differences between pushover predictions and dynamic analysis results. The only significant difference exists in the first story drift, where the dynamic characteristics of records four, five and seven cause significantly larger drifts than predicted from the pushover analysis. As the mean story drifts show, inelastic deformations are distributed rather uniformly over the height of this frame.

The conclusion to be drawn is that for regular low-rise structures in which higher mode effects are not very important and in which inelasticity is distributed rather uniformly over the height, the pushover analysis provides very good predictions of seismic demands.

8. Limitations of pushover analysis

There are good reasons for advocating the use of the inelastic pushover analysis for demand prediction, since in many cases it will provide much more relevant information than an elastic static or even dynamic analysis, but it would be counterproductive to advocate this method as a general solution technique for all cases. The pushover analysis is a useful, but not infallible, tool for assessing inelastic strength and deformation demands and for exposing design weaknesses. Its foremost advantage is that it encourages the design engineer to recognize important seismic response quantities and to use sound judgment concerning the force and deformation demands and capacities that control the seismic response close to failure, but it needs to be recognized that in some cases it may provide a false feeling of security if its shortcomings and pitfalls are not recognized.

A carefully performed pushover analysis will provide insight into structural aspects that control performance during severe earthquakes. For structures that vibrate primarily in the fundamental mode, such an analysis will very likely provide good estimates of global as well as local inelastic deformation demands. It will also expose design weaknesses that may remain hidden in an elastic analysis. Such weaknesses include story mechanisms, excessive deformation demands, strength irregularities, and overloads on potentially brittle elements, such as columns and connections.

It must be emphasized that the pushover analysis is approximate in nature and is based on static loading. As

Table 2 Member plastic deformation demands – pushover predictions vs time history results

<table>
<thead>
<tr>
<th>Element</th>
<th>Gravity + Station 1</th>
<th>Gravity + Station 2</th>
<th>Gravity + Station 3</th>
<th>Gravity + Station 4</th>
<th>Gravity + Station 5</th>
<th>Gravity + Station 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-123 end (i)</td>
<td>Time history abs. max 0.008</td>
<td>0.010</td>
<td>0.007</td>
<td>0.016</td>
<td>0.014</td>
<td>0.001</td>
</tr>
<tr>
<td>Pushover</td>
<td>0.006</td>
<td>0.010</td>
<td>0.006</td>
<td>0.015</td>
<td>0.013</td>
<td>0.000</td>
</tr>
<tr>
<td>J-23</td>
<td>Time history abs. max 0.008</td>
<td>0.012</td>
<td>0.008</td>
<td>0.017</td>
<td>0.015</td>
<td>0.002</td>
</tr>
<tr>
<td>Pushover</td>
<td>0.008</td>
<td>0.012</td>
<td>0.008</td>
<td>0.017</td>
<td>0.015</td>
<td>0.002</td>
</tr>
<tr>
<td>Element</td>
<td>Gravity + Station 7</td>
<td>Gravity + Station 8</td>
<td>Gravity + Station 9</td>
<td>Mean</td>
<td>C.O.V.</td>
<td></td>
</tr>
</tbody>
</table>

| B-123 end (i) | Time history abs. max 0.016 | 0.006 | 0.002 | 0.009 | 0.009 | 0.630 |
| Pushover | 0.015 | 0.005 | 0.002 | 0.008 | 0.008 | 0.690 |
| J-23 | Time history abs. max 0.016 | 0.007 | 0.005 | 0.010 | 0.010 | 0.507 |
| Pushover | 0.017 | 0.007 | 0.004 | 0.010 | 0.010 | 0.533 |
such it cannot represent dynamic phenomena with a large degree of accuracy. It may not detect some important deformation modes that may occur in a structure subjected to severe earthquakes, and it may exaggerate others. Inelastic dynamic response may differ significantly from predictions based on invariant or adaptive static load patterns, particularly if higher mode effects become important.

Limitations are imposed also by the load pattern choices. Whatever load pattern is chosen, it is likely to favor certain deformation modes that are triggered by the load pattern and miss others that are initiated and propagated by the ground motion and inelastic dynamic response characteristics of the structure. The simplest example is a structure with a weak top story. Any invariant load pattern will lead to a concentration of inelastic deformations in the top story and may never initiate inelastic deformations in any of the other stories. Thus, good judgment needs to be employed in selecting load patterns and in interpreting the results obtained from selected load patterns.

A comprehensive assessment of the accuracy of pushover demand predictions will require the execution of a great number of case studies for many different configurations. Until this is done, a preliminary assessment needs to be based on past studies whose primary objective may not have been an evaluation of the pushover procedure. One such study, which was concerned with a general evaluation of inelastic MDOF effects in frame and wall structures\cite{4}, provides extensive data that can be used to draw conclusions on specific aspects of pushover demand predictions. Some of the conclusions drawn from this study are discussed next.

In order to put the presented results in perspective, the analyses performed in this study and the relationship between the structure strength and corresponding SDOF spectral information utilized in this study need to be explained. The previously mentioned 15 S1 records formed the basis for analysis and design. Elastic and inelastic spectra (for ductility ratios \( \mu(SDOF) \) from 2 to 8) were generated for these ground motion records, and the frame and wall structures were designed for a base shear strength equal to the inelastic strength demand of the first mode period SDOF system for a specific ductility ratio. In other words, a structure associated with \( \mu(SDOF) = 4 \) has a base shear strength equal to the inelastic strength demand that will limit the ductility of the first mode period SDOF system to a ductility of four. Triangular as well as SRSS load patterns were used to define the shear strength of the individual stories of the MDOF systems. Structures from 2 to 40 stories were investigated, with the first mode period given by the code equation \( T = 0.02h_0 \). Using a story height of 12 ft (3.6 m), the periods of 2, 5, 10, 20, 30 and 40-story structures are computed as 0.22, 0.43, 0.73, 1.22, 1.65 and 2.05 s, respectively. These periods may be unrealistically short for frame structures, but are quite realistic for wall structures. They were chosen to be equal for wall and frame structures in order to permit a direct comparison between the two types of systems.

An evaluation of the analysis results from this study leads to the conclusion that the prediction of the target roof displacement by means of the previously discussed procedure is very good in all cases, whether the structures are frames with beam hinge mechanisms, column hinge mechanisms, or weak story mechanisms, or walls with flexural hinging at the base, but there are many other aspects which show clear limitations of the pushover analysis predictions.

In general, these limitations do not apply to low-rise structures (2- and 5-stories), but show up in increasing magnitude as the structures become taller, i.e. as the higher mode effects become more important.

An example of the expected accuracy of story drift predictions for a relatively tall frame structure (20-stories, \( T = 1.22 \) s) is illustrated in Figure 15. This structure is designed to have a straight line deflected shape and to yield simultaneously in every story under a static code load pattern. If a pushover analysis with the code load pattern is applied, the deflected shape will remain linear and the same story drift will be predicted in each story. Thus, the predicted story drift index \( \delta_i/h_i \) will be the same in each story and will be equal to the global drift index \( \delta_i/h_i \). The latter value, obtained from dynamic analysis with a particular record, is indicated with a solid vertical line in Figure 15. The individual story drift indices obtained from dynamic analysis with this record are shown with a stepped line. They vary significantly over the height and have a maximum value in the first story of more than twice the value predicted by the pushover analysis. This large discrepancy in prediction is caused by higher mode effects. It clearly demonstrates a basic limitation of the pushover analysis for taller structures.

The discrepancy in maximum story drift predictions between pushover analysis and dynamic analysis is a function of the period and yield strength of the structure (the latter is characterized in this study by \( \mu(SDOF) \)). For the type of frame structures, discussed in the previous paragraph, this discrepancy is illustrated in Figure 16, which shows mean values of the ratio of maximum story drift obtained from dynamic analysis to story drift predicted
from a pushover analysis. For low-rise structures this ratio is close to 1.0 regardless of the strength of the structure, but for tall structures it increases up to a value of 3. This result reinforces the previously made statement that the pushover analysis will not provide good predictions for tall structures, in which higher mode effects are important. This statement needs to be qualified, since the pushover prediction accuracy depends on the selected load pattern. If an SRSS load pattern is used, the quality of prediction improves somewhat, but not by a drastic amount. Adaptive load patterns have not been investigated in this study. On the positive side, it can be said that the pushover prediction of maximum interstory drift was found to be reasonably accurate for structures of all heights if the deformation mode was a weak story mechanism in the first story.

An example that demonstrates other potential problems with the pushover analysis is that of multi-story wall structures modeled by a single shear wall. In these wall structures it is assumed that the bending strength of the wall is constant over the height, and that the shear strength and stiffness are large so that the behavior of the wall is controlled by bending. It is also assumed that no strain hardening exists once a plastic hinge has formed in the wall. A pushover analysis will predict hinging at the base of the wall for all rational load patterns. A mechanism exists once this single plastic hinge has formed, the wall will rotate around its base, and the lateral loads can no longer be increased. Thus, a pushover analysis will not permit propagation of plastic hinging to other stories and will predict a base shear demand that corresponds to the sum of lateral loads needed to create the plastic hinge at the base.

Nonlinear dynamic time history analysis gives very different results. For taller wall structures higher mode effects significantly amplify the story shear forces that can be generated in the wall once a plastic hinge has formed at the base. This is illustrated in Figure 17, which shows mean values of base shear amplification, defined as the maximum base shear from dynamic analysis over pushover base shear causing plastic hinging at the base. The amplification depends on the period (number of stories) of the wall structure and on the wall bending strength (represented by $\mu$ (SDOF)). The diagram shows that the amplification of base shear demands may be as high as five for tall wall structures with reasonable bending strength ($\mu$ (SDOF) ≤ 4). This amplification implies that the base shear demand may be much higher than the base shear obtained from the lateral loads that cause flexural hinging at the base of the structure. Thus, wall shear failure may occur even though the pushover analysis indicates flexural hinging at the base.

The nonlinear dynamic time history analysis also shows that flexural hinging in tall structures is not necessarily limited to the first story. It may propagate into other stories to an extent that depends on the period and flexural strength of the structure. This is illustrated in the story overturning moment envelopes presented in Figure 18 for a wall structure with a period of 2.05 s (40-stories). The moment envelope obtained from dynamic analyses is very different from that obtained from a code type load pattern (solid line). Thus, if such a code load pattern is used in the pushover analysis, a very misleading picture of the story overturning moment demand is obtained. The use of an SRSS load pattern does not change the behavior by much.

No static pushover analysis could have predicted this behavior. This example shows that additional measures need to be taken in some cases to allow a realistic performance assessment. Such measures need to be derived from nonlinear dynamic analyses and need to be formalized to the extent that they can be incorporated systematically in a pushover analysis procedure.

9. Conclusions

The engineering profession in the US is being faced with a new challenge, namely the implementation in the engineering practice of an approximate performance evaluation procedure, commonly referred to as the pushover analysis. If implemented with caution and good judgment, and with due consideration given to its many limitations, the pushover analysis will be a great improvement over presently employed elastic evaluation procedures. This applies particularly to the seismic evaluation of existing structures whose element behavior cannot be evaluated in the context of presently employed global system quality factors such as the $R$ and $R_w$ factors used in present US seismic codes.

On the positive side, a carefully performed pushover analysis will provide insight into structural aspects that control performance during severe earthquakes. For structures that vibrate primarily in the fundamental mode, the pushover analysis will very likely provide good estimates of global, as well as local inelastic, deformation demands. This analysis will also expose design weaknesses that may remain hidden in an elastic analysis. Such weaknesses include story mechanisms, excessive deformation demands, strength irregularities and loads on potentially brittle elements such as columns and connections.
On the negative side, deformation estimates obtained from a pushover analysis may be very inaccurate (on the high or low side) for structures in which higher mode effects are significant and in which the story shear force vs story drift relationships are sensitive to the applied load pattern. This problem can be mitigated, but usually not eliminated, by applying more than one load pattern, including load patterns that account for elastic higher mode effects (e.g. SRSS load patterns). Perhaps most critical is the concern that the pushover analysis may detect only the first local mechanism that will form in an earthquake and may not expose other weaknesses that will be generated when the structure’s dynamic characteristics change after formation of the first local mechanism.

In the writers’ opinion the pushover analysis can be implemented for all structures, but it should be complemented with other evaluation procedures if higher mode effects are judged to be important. No single criterion can be established for this condition, since the importance of higher mode effects depends on the number of stories as well as on the relative position of the modal periods with respect to the peak(s) and plateau(s) of the design spectrum. Candidates for additional evaluation procedures are, in order of preference, inelastic dynamic analysis with a representative suite of ground motions (probably unfeasible in most practical cases), and elastic dynamic (modal) analysis using the unreduced design spectrum and a suitable modal combination procedure (SRSS, CQC). The latter procedure will provide estimates of elastic demand/capacity ratios which need to be compared to acceptable values. Again, the FEMA 273 document provides many recommendations for this procedure.

Acknowledgements

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